Homework 2
AOE6744/ME6544/ECE6744: Linear Control Theory
Spring 2018

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The HW is due on February 15th in class.

1 Assignments

1.1 Problem 1

Consider the following LTI system with a free parameter k,

$$\dot{x} = \begin{bmatrix} -1 & 0 \\ k & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u,$$
$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} x.$$

- Draw a block diagram.
- Find the transition matrix $\Phi(t)$.

□

1.2 Problem 2

Consider the following system,

$$\dot{x} = \begin{bmatrix} 1 & -1 \\ -9 & -5 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} u,$$
$$y = \begin{bmatrix} 2 & 2 \\ -4 & -1 \end{bmatrix} x.$$

- Draw a block diagram.
- Find the transfer function.
- Derive a second order ode for the system.
- Find poles and zeros.
- Is the system minimal?
- Find minimal realization (A, B, C, D).

□

1.3 Problem 3

Consider the following system,

$$\begin{aligned}\dot{x} &= \begin{bmatrix} 1 & 1 \\ 3 & 5 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u, \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} x.\end{aligned}$$

- Find poles and zeros.
- Find transfer function.
- Find $\Phi(s)$ and $\phi(t)$.
- Find impulse response and use matlab to plot it.
- Find step response and use matlab to plot it.
- Given that $u = 0,\ \forall t$ and that the initial condition $x(0) = \begin{bmatrix} 3 & -1 \end{bmatrix}$ then determine the value of the output $y(t)$ at $t = 2$.

□

1.4 Problem 4

Consider the following system,

$$\dot{x} = \begin{bmatrix} -\frac{1}{t+1} & 0 \\ -\frac{1}{t+1} & 0 \end{bmatrix} x.$$

- Find $x(t)$.
- Does the system have an asymptotically stable equilibrium point?

□

1.5 Problem 5

Consider the following system,

$$\dot{x} = \begin{bmatrix} -1 & e^{2t} \\ 0 & -1 \end{bmatrix} x.$$

- Find $x(t)$. Please substitute $x(0) = \begin{bmatrix} 0 & 1 \end{bmatrix}$.
- What are the eigenvalues of $A(t),\ \forall t \geqslant 0$? Are the trajectories bounded? What can you say about the stability of the equilibrium point?

$\square$