Homework 2

1. Determine if the equilibrium $x_1 = x_2 = 0$ is unstable, stable, asymptotically stable, exponentially stable for:

$$\dot{x}_1 = x_2 - x_1^3 + x_1^5$$
$$\dot{x}_2 = -x_1$$
, $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2$.

State the strongest claim you can make and provide support for the conclusion. Consider only local properties (do not worry about global properties).
Hint: The function $V(x) = \frac{1}{2}(x_1^2 + x_2^2)$ may help.

2. For:

$$\dot{x}_1 = x_1 - x_1 x_2$$
$$\dot{x}_2 = (x_1 - x_2)(2 - x_2)$$
, $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2$

a) Find <u>all</u> equilibria
b) Analyze stability of each equilibrium, i.e. if it is unstable, stable, locally asymptotically or exponentially stable.
c) For exponentially stable equilibria outline an approach to estimate the region of attraction.
Hint: Using the Lyapunov equation may help.

3. For:

$$\dot{x}_1 = -x_1^3 - x_2$$
$$\dot{x}_2 = x_1$$

, $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2$

show that the equilibrium $x_1 = x_2 = 0$ is G.A.S. (globally asymptotically stable).

4. For:

$$\dot{x}_1 = x_1 + x_2 - x_1(x_1^2 + x_2^2)$$
$$\dot{x}_2 = -x_1 + x_2 - x_2(x_1^2 + x_2^2)$$

, $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2$

show that the equilibrium $x_1 = x_2 = 0$ is unstable. using Chetaev's theorem.