

**Adaptive Control Systems**  
**Linear Adaptive Control Project**  
**ME6574/ECE6774/AOE6774**

The purpose of this project is to design a “direct” model reference adaptive controller for a DC motor drive, so that velocity tracking may be achieved without knowledge of motor or load parameters.

The dynamics of a brush-commutated permanent-magnet DC motor turning an inertial load are described by

$$\begin{aligned} J \frac{d\omega}{dt} &= -B\omega(t) + Ki(t), \\ L \frac{di}{dt} &= -K\omega(t) - Ri(t) + v(t), \end{aligned}$$

where  $\omega(t)$  is the mechanical state variable (angular velocity),  $i(t)$  is the electrical state variable (armature current), and  $v(t)$  is the control input (armature voltage). The constant but unknown mechanical parameters are the inertia  $J$  and viscous friction  $B$ , and the constant but unknown electrical parameters are armature inductance  $L$ , armature resistance  $R$ , and motor constant  $K$ . Each of the unknown parameters, i.e.  $J$ ,  $B$ ,  $L$ ,  $R$ ,  $K$ , are known to be strictly positive.

The objective is to find a voltage control input  $v(t)$  such that the output velocity  $\omega(t)$  tracks a desired velocity signal  $\omega_d(t)$ , provided as the output of a reference model. The only measurement of the plant responses available to the control system is the output  $\omega(t)$  (i.e.  $i(t)$  is not available). The desired output is specified to be

$$\omega_d(t) = 10 \sin(2t),$$

but the choice of the reference model (and corresponding reference signal  $r(t)$ ) is not specified. For purpose of plant simulation, the true values of the unknown parameters are

$$J = 0.0026, \quad B = 0.00057, \quad L = 0.0045, \quad R = 0.5, \quad K = 0.56.$$

All initial conditions may be set to zero.

Write software that implements the plant dynamics, the reference model dynamics, the filter dynamics (if necessary), and the adaptation dynamics, for the three cases described below. Demonstrate the operation of each case; at minimum, plot (1) the actual plant output  $\omega$  along with the reference model output  $\omega_d$ , and (2) the adjustable controller parameters along with their unknown perfectly tuned constant values, as functions of time, to determine how well, if at all, the output tracking objective is met (and to what extent parameter convergence is achieved).

- i)* It is common to neglect electrical transients when controlling mechanical motions. For example, if the armature inductance is negligibly small, i.e. if  $L = 0$ , then the second-order electromechanical model reduces to a first-order mechanical model. For the first simulation, make this approximation and design a “direct” MRAC for the first-order mechanical plant model, and also use the first-order mechanical plant model in the simulation as if it were perfectly accurate. Does the output  $\omega(t)$  converge to  $\omega_d(t)$  as  $t \rightarrow \infty$ ? Do the adjustable controller parameters converge to their unknown perfectly tuned values as  $t \rightarrow \infty$ ?
- ii)* Whether or not the above plant modeling approximation is well justified depends on several factors, including the relative sizes of the electrical and mechanical time constants, as well as the motor velocity magnitude. For the second simulation, test how well the actual

second-order plant is controlled using the reduced-order “direct” MRAC design approach, by leaving the controller unchanged but by including the true second-order electromechanical plant dynamics in the simulation. Does the output  $\omega(t)$  converge to  $\omega_d(t)$  as  $t \rightarrow \infty$ ? Do there exist constant, unknown perfectly tuned values, for the adjustable controller parameters in this case? What is the behavior of the adjustable controller parameters as  $t \rightarrow \infty$ ?

- iii) It is not necessary to make any modeling approximation prior to the design of an adaptive controller for the second-order electromechanical model of the plant. For the third simulation, use the relative-degree-two approach to “direct” MRAC design, without introducing any modeling approximations for design or for simulation. The controller will, of course, have more dynamics in this case than in the prior case. Does the output  $\omega(t)$  converge to  $\omega_d(t)$  as  $t \rightarrow \infty$ ? Do the adjustable controller parameters converge to their unknown perfectly tuned values as  $t \rightarrow \infty$ ?
- iv) Finally, change the state representation of the system dynamics so that the new state variables are  $\omega(t)$  and  $\dot{\omega}(t)$ . Then derive a “direct” MRAC control using the full state representation, hence using both  $\omega(t)$  and  $\dot{\omega}(t)$  for feedback purposes. Finally, since  $\dot{\omega}(t)$  is not an available measurement, replace it with a filtered version of such derivative, i.e.  $\dot{\omega}(t) \simeq \mathcal{L}^{-1} \left( \frac{s}{1+\tau s} \mathcal{L}(\omega(t)) \right)$  with  $\tau \ll 1$ . Compare the results you obtained in this way with those obtained using the actual  $\dot{\omega}(t)$  as well those obtained at the previous points.