Homework 3
AOE6744/ME6544/ECE6744: Linear Control Theory
Spring 2018

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The HW is due on March 1st in class.

1 Assignments

1.1 Problem 1

Consider the following 4 models,

- Inverted pendulum: $A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 10 & 0 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ 0.1 \\ 0 \\ -0.1 \end{bmatrix}$, $C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$.
- Ball balancer: $A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -9 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.1 \end{bmatrix}$, $C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$.
- Flexible-joint system: $A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & -0.2 & 1 & 0.2 \\ 0 & 0 & 0 & 1 \\ 1 & 0.2 & -10 & -2.1 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, $C_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$.
- Flexible-link system: $A_4 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & -0.1 & 1 & 0.1 \\ 0 & 0 & 0 & 1 \\ 1 & 0.1 & -23 & -0.3 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$, $C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$.

- Find poles and zeros.
- Find transfer function.
- Study the reachability and observability of the system.
- Plot step response in Matlab if the system is stable.

1.2 Problem 2

Solve the Lyapunov equation to determine stability for,

- $\dot{x} = \begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix} x.$
- $\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & -4 \end{bmatrix} x.$

1.3 Problem 3

Consider,

$$H(s) = \frac{s+3}{s^3 + 7s^2 + 14s + 8}.$$

- Find the state-space description for the reachable canonical form. Draw a block diagram.
- Find the state-space description for the observable canonical form. Draw a block diagram.
- Show that the two state-space realizations are the same if you reorder the states backwards and replace (A, B, C) by its dual.