Parameter Estimation Project

The purpose of this project is to study the differences in implementation and performance when several distinct approaches are applied to the same parameter estimation problem. Submit any relevant calculations, the "source" code for your simulations, the graphical simulation plots needed to support your claims, and any explanatory text necessary, by February 21, 2018.

Consider a mass moving along a frictionless surface due to the application of the control force. The dynamic model corresponding to this situation is given by

$$m(t)\ddot{y}(t) + \dot{m}(t)\dot{y}(t) = u(t),$$

where $m(t)$ is the unknown mass, $u(t)$ is the applied control force, and $y(t)$ is the resulting displacement. The objective is to use input-output data and an on-line parameter estimation algorithm to estimate the value of $m(t)$ (or its inverse) in a recursive fashion. A typical control force which could be used to generate the input output data is given by

$$u(t) = 2\sin(t),$$

but other signals may also be considered for this purpose.

The measured signals $u_m(t)$ and $y_m(t)$ are corrupted by noise, i.e.

$$u_m(t) = u(t) + \epsilon\sin(3t),$$

$$y_m(t) = y(t) + \epsilon\sin(3t),$$

where ϵ is the noise amplitude. The true, but unknown, mass is nominally constant but drifts slowly over time, i.e.

$$m(t) = 1 + \mu\sin(0.05t),$$

where μ defines the extent of mass vibration. The best available guess for the unknown mass-related parameter is zero.

1. Write the software that implements the plant dynamics, the regressor filter dynamics, and the estimation algorithm dynamics, for three continuous time parameter estimation algorithms: gradient, least-squares, and least-squares

with exponential forgetting. Although the measurement noise and mass variation must be accounted for in the simulation of the plant and its sensor, these effects should be neglected in the design of the parameter estimator.

2. Demonstrate the operation of all three estimation algorithms for the following cases. At minimum, plot the estimated parameter (along with the true value) as a function of time to determine how well, if at all, the algorithms manage to identify the unknown mass.
   a) Noise-free constant parameter case: $\epsilon = 0$ and $\mu = 0$.
   b) Noisy constant parameter case: $\epsilon > 0$ and $\mu = 0$.
   c) Noise-free varying parameter case: $\epsilon = 0$ and $\mu > 0$.
   d) Noisy varying parameter case: $\epsilon > 0$ and $\mu > 0$.

3. Which estimation algorithm appears to perform best in each of these scenarios? What effect does the (initial) estimation gain have on the operation of these algorithm?

4. (Bonus Points). Compare the results obtained at the previous points with those obtained applying the adaptive estimation algorithm.

5. (Extra Bonus Points). When you write the dynamics in a state representation form, the A^* and B^* matrices have a particular structure, where only some elements are unknown, while other elements (0's and 1's) are known. The adaptive algorithm you implemented at the previous point completely neglects this prior knowledge and provides estimates of all the elements of such matrices. Can you think of a way to implement the adaptive algorithm so that you can use your existing knowledge and estimate only the unknown parameters?