

Grasping [[Prattichizzo, 2016](#)]

George Kontoudis

ME5984 Motion Planning Analysis
Spring 2017

Mechanical Engineering Department, Virginia Tech

October 2, 2018

Outline

Motivation

Grasping

Grasping Notation

Object Modeling

Hand Modeling

Contact Modeling

Conclusions

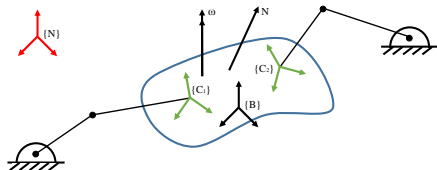
References

Motivation

- ▶ Mathematical models for robotic grasping
- ▶ Object and robot hand behaviour
- ▶ Grasping simulations
- ▶ Robot hands applications

Grasping Notation

- ▶ $\{N\}$: inertial frame
- ▶ $\{C_i\}$: frame at contact i
- ▶ $\{B\}$: fixed frame in object (CoM)
- ▶ N : object's translational velocity
- ▶ ω : object's angular velocity



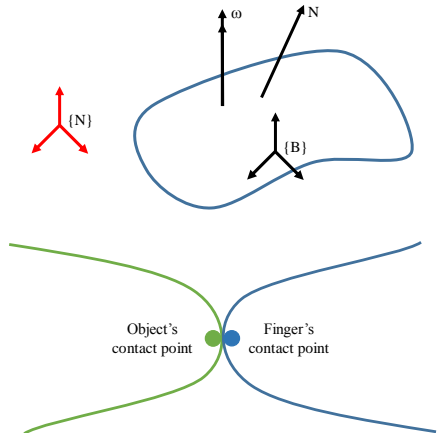
Twist and Contacts

In spatial systems (3D), $n_\nu=6$, $n_b=1$

$$\nu = \nu^N = \begin{bmatrix} N_x \\ N_y \\ N_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}, \quad \nu : \text{twist}$$

Two i-th contact points

- ▶ object
- ▶ hand



Twist of Contact on Object

Goal: Given object's twist ν , compute twist of i -th contact on body

$$\nu_{i,obj}^C = \tilde{G}_i^T \nu^N,$$

- ▶ $\nu_{i,obj}^C$: twist expressed in frame C_i
- ▶ ν^N : twist expressed in inertial frame N
- ▶ \tilde{G}_i^T : partial grasp matrix

Translational and Angular Velocity

Object's points have different translational velocity N

$$N_i^N = N_{\text{cm}}^N + \omega^N \times (r_i^N - r_{\text{cm}}^N) = N_{\text{cm}}^N - (r_i^N - r_{\text{cm}}^N) \times \omega^N = N_{\text{cm}}^N - S(r_i^N - r_{\text{cm}}^N) \omega^N,$$

S : Skew-symmetric matrix

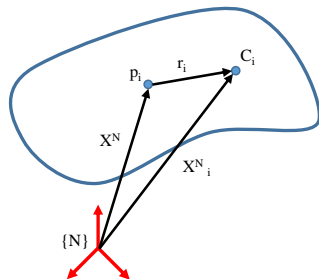
$$S(a) = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} = -S(a)^T$$

Angular velocity ω of all points on an object is the same

$$\omega^{C_i} = R_N^{C_i} \omega^N$$

Rigid Body Equation

- ▶ Position of contact point i $X_i^N = X^N + r_i$
- ▶ Euler theorem: $r_i = Rr_i^N$, with $R^T R = I$
- ▶ $X_i^N = X^N + Rr_i^N$
- ▶ $N_i^N = N^N + \frac{dR}{dt} r_i^N = N^N + \frac{dR}{dt} R^T R r_i^N = N^N + \frac{dR}{dt} R^T r_i = N^N + \Omega r_i$
- ▶ $\Omega r = \omega \times r$, Ω : angular velocity tensor
- ▶ $N_i^N = N_{cm}^N + \omega^N \times (r_i^N - r_{cm}^N)$



Twist Transformation to Contact Frame

The twist results

$$\begin{bmatrix} N_i^N \\ \omega_i^N \end{bmatrix} = \begin{bmatrix} I_{3 \times 3} & S(r_i^N - r_{cm}^N)^\top \\ 0_{3 \times 3} & I_{3 \times 3} \end{bmatrix} \begin{bmatrix} N^N \\ \omega^N \end{bmatrix}$$

Vector expression from contact frame C_i to inertial frame N ,
 $d^N = R_{C_i}^N d^{C_i} \Rightarrow (R_{C_i}^N)^\top d^N = (R_{C_i}^N)^\top R_{C_i}^N d^{C_i} \Rightarrow (R_{C_i}^N)^\top d^N = Id^{C_i}$

Thus the twist yields

$$\nu_{i,obj} = \begin{bmatrix} N_{i,obj}^{C_i} \\ \omega_{i,obj}^{C_i} \end{bmatrix} = \begin{bmatrix} (R_{C_i}^N)^\top & 0 \\ 0 & (R_{C_i}^N)^\top \end{bmatrix} \begin{bmatrix} I & S(r_i^N - r_{cm}^N)^\top \\ 0 & I \end{bmatrix} \begin{bmatrix} N^N \\ \omega^N \end{bmatrix} = \tilde{G}_i^\top \nu^N$$

Twist of Contact in Hand

Goal: Given joint angles \dot{q} , compute twist of i -th contact in hand

$$\nu_{i,hand}^C = \tilde{J}_i \dot{q},$$

- ▶ $\nu_{i,hand}^C$: twist expressed in frame C_i
- ▶ \dot{q} : joint angles
- ▶ \tilde{J}_i : partial hand Jacobian matrix

Translational Velocity

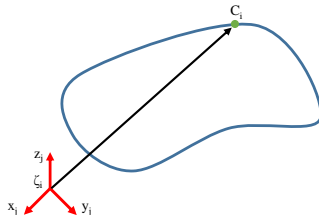
Hand's translational N, similar w/ Jacobian in velocity kinematics

$$N_{ij} = d_{ij}^N \dot{q} = \begin{bmatrix} 0 \\ \hat{z}_j^N \dot{q}_j \\ \hat{z}_j^N \dot{q}_j \times (C_i^N - \zeta_j^N) \end{bmatrix} \begin{array}{l} \rightarrow \text{no contact} \\ \rightarrow \text{prismatic joint} \\ \rightarrow \text{revolute joint} \end{array}$$

Revolute joint can be written

$$\hat{z}_j^N \dot{q}_j \times (C_i^N - \zeta_j^N) = -(C_i^N - \zeta_j^N) \times \hat{z}_j^N \dot{q}_j = S^T(C_i^N - \zeta_j^N) \hat{z}_j^N \dot{q}_j$$

- ▶ ζ_j : origin of joint frame
- ▶ j : joint



Angular Velocity and Finger Contacts

Angular velocity $\omega_{ij}^N = I_{ij}^N \dot{q}_{ij}$,

$$I_{ij} = \begin{bmatrix} 0 \\ 0 \\ \hat{z}_j^N \end{bmatrix} \begin{array}{l} \rightarrow \text{no contact} \\ \rightarrow \text{prismatic joint} \\ \rightarrow \text{revolute joint} \end{array}$$

which yields

$$\nu_{ij}^N = \begin{bmatrix} d_{ij}^N \\ I_{ij}^N \end{bmatrix} \dot{q}_i$$

For all finger contacts

$$\nu_{i,hand}^N = \begin{bmatrix} d_{i,1}^N \cdots d_{i,n_q}^N \\ I_{i,1}^N \cdots I_{i,n_q}^N \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \vdots \\ \dot{q}_{n_q} \end{bmatrix} = Z_i \dot{q}$$

Partial Hand Jacobian

- Transform twist to contact frame

$$\nu_{i,hand}^C = \overline{R}_i^T Z_i \dot{q} = \tilde{J}_i \dot{q}, \quad \tilde{J}_i \in \mathbb{R}^{6n_c \times n_q}$$

- Partial hand Jacobian \tilde{J}_i , maps joint velocities with the contact twists of the hand

Complete Hand Jacobian and Grasp Matrix

For all contacts we get

$$\nu_{C,hand} = \begin{bmatrix} \nu_{1,hand} \\ \vdots \\ \nu_{n_c,hand} \end{bmatrix}, \quad \nu_{C,obj} = \begin{bmatrix} \nu_{1,obj} \\ \vdots \\ \nu_{n_c,obj} \end{bmatrix}$$

Complete hand Jacobian and grasp matrix

$$\tilde{J} = \begin{bmatrix} \tilde{J}_1 \\ \vdots \\ \tilde{J}_{n_c} \end{bmatrix}, \quad \tilde{G}^T = \begin{bmatrix} \tilde{G}_1^T \\ \vdots \\ \tilde{G}_{n_c}^T \end{bmatrix}$$

Models of interest

Three models of interest

- ▶ Point contact w/o friction (PwoF)
- ▶ Hard finger (HF)
- ▶ Soft finger (SF)

Relative twist at i -th contact

$$(\tilde{J}_i - \tilde{G}_i^T) \begin{pmatrix} \dot{q} \\ \nu \end{pmatrix} = (\nu_{i,hand} - \nu_{i,obj})$$

Particular contact model through homogeneous $H_i \in \mathbb{R}^{l_i \times 6}$

$$H_i(\nu_{i,hand} - \nu_{i,obj}) = 0$$

Homogeneous matrix

- Homogeneous matrix defined as

$$H_i = \begin{bmatrix} H_{iF} & 0 \\ 0 & H_{iM} \end{bmatrix},$$

H_{iF} : Translational components

H_{iM} : Rotational components

- Contact models matrices selection

Model	l_i	H_{iF}	H_{iM}
PwoF	1	(1 0 0)	0
HF	3	$I_{3 \times 3}$	0
SF	4	$I_{3 \times 3}$	(1 0 0)

l_i : transmitted twist components

Hand Jacobian and Grasp Matrix

- ▶ From the complete hand Jacobian and grasp matrix we get

$$H(\nu_{C,hand} - \nu_{C,obj}) = H(\tilde{J}\dot{q} - \tilde{G}^T\nu) = (H\tilde{J} - H\tilde{G}^T) \begin{pmatrix} \dot{q} \\ \nu \end{pmatrix} = 0$$

- ▶ Hand Jacobian and grasp matrix are

$$J\dot{q} = \nu_{cc,obj} = \nu_{cc,hand} = G^T\nu$$

$J = H\tilde{J}$, hand Jacobian matrix

$G^T = H\tilde{G}^T$, grasp matrix

Conclusions

Robot grasping procedure

- ▶ Given: Contact points C_i , hand kinematic structure, object velocity twists ν , and joint velocities \dot{q}
- ▶ Compute: Velocities of contact points on objects $\nu_{C,hand}$, and velocities of contact points in hand $\nu_{C,obj}$
- ▶ Given: Contact model
- ▶ Compute: Hand Jacobian matrix, Grasp matrix

References



D.Prattichizzo and J.Trinkle (2016)

Grasping

Springer handbook of robotics, 955–988, 2016.

Thank You!