Homework 4, NSL

P1. Consider the system:

$$\dot{x}_1 = \tanh x_1 (a x_1 + x_2)$$
$$\dot{x}_2 = b x_1 x_2 + \frac{1}{1+x_2^2} u$$

Suppose a = 2 and b = 3. Use backstepping to design a feedback control law $u(x_1, x_2)$ which asymptotically stabilizes the equilibrium at the origin. Simulate the closed loop system response for initial conditions $x_1(0) = x_2(0) = 1$. Show plots of $x_1(t)$ and $x_2(t)$ versus time and a plot of $x_1(t)$ versus $x_2(t)$. Prove that indeed you have asymptotic stability for the first closed loop used in backstepping. There is no need to do that for the second closed loop system.
Hint: Cast the equations in the format used in class and use $V = \eta^2/2$, and $\phi(\eta) = -a\eta - \eta^2$. For the constant k use $k = 1$ in your simulations (and you are free to explore other values to see how the system behaves).

P2. Consider the same system like in P1 but with a and b uncertain (unknown). The only knowledge you have is that $a \in [1,3], b \in [2,4]$. Use sliding mode control to design a feedback control law $u(x_1, x_2)$ which asymptotically stabilizes the equilibrium at the origin. Simulate the closed loop system response for initial conditions $x_1(0) = x_2(0) = 1$. Show plots of $x_1(t)$ and $x_2(t)$ versus time and a plot of $x_1(t)$ versus $x_2(t)$. In your sliding mode control law implementation use the saturation function instead of the signum function. Use your simulations to explore the effect of different values of the small parameter ε.
Hint: will be provided in class.