

## Original research article

# Integrative optimization of the practical wavefront coding systems for depth-of-field extension



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## ABSTRACT

Optimization of the phase mask is one of the key tasks in the design of wavefront coding systems for depth of field extension. Instead of optimization of the phase mask for ideal optical systems, which was the focus of the previous research, this paper proposes an integrative optimization method for practical optical systems considering the various optical aberrations. A practical wavefront coding system based on a doublet is designed using the integrative optimization method, while an ideal system with an optimized phase mask is also used for comparison. The simulation results show that the practical wavefront coding system designed by the integrative optimization method works well on both the on-axis and off-axis fields, while the ideal system with the optimized phase mask had a poor performance on the off-axis fields. Off-axis fields should also be considered as well as on-axis field in the design of practical optical systems, therefore the integrative optimization method will be significant for the design of practical wavefront coding systems.

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## 1. Introduction

Wavefront coding imaging technology is commonly used to extend the depth of field in incoherent optical systems [1]. It consists of optical coding and digital decoding. By inserting a phase mask in the pupil plane, the optical transfer function (OTF) or the point spread function (PSF) is modulated to be insensitive to the misfocus. Immediate images captured in the image plane are blurred, but they can be restored to be sharp ones by digital decoding.

It is well-known that the design of the phase mask is one of the key tasks in wavefront coding technology. Therefore, various phase masks [2–5] were proposed to extend the depth of field such as cubic phase mask, sinusoidal phase mask, exponential phase mask and logarithmic phase mask. Cubic phase mask and logarithmic phase mask are chosen as examples of the phase masks in this paper. The reason is that the cubic phase mask is one of the classic masks and has been widely studied [6,7]; logarithmic phase mask can further extend the depth of field with lower surface relief phase structures fabricated in photoresist [8,9].

In the previous research, different optimization algorithms [10–13] were applied to acquire the optimal phase masks of the wavefront coding system. However, these algorithms were studied in ideal optical imaging systems in order to

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greatly reduce the complexity of the design. In other words, various aberrations except misfocus are ignored in ideal optical systems including spherical aberration, chromatism, coma aberration, curvature of field, astigmatism and distortion. These aberrations take an important role in practical optical systems. Therefore, the optimal phase mask designed in the ideal optical system may not applicable to the practical wavefront coding system. Consequently, an integrative optimization method for the design of practical wavefront coding systems is proposed in this paper. Firstly, the parameters of both the phase mask and other optical elements are considered as variables in the integrative optimization; Secondly, various optical aberrations are taken into account. To ensure the reliability, the whole system is optimized and analyzed in the commercial optical softwares. Zemax is used as an example of the softwares in this paper.

The remainder of this paper is organized as follows: Section 2 describes the theoretical analysis of the wavefront coding imaging technology. The Wavefront coding imaging system is designed and optimized based on the ideal and the practical optical system respectively in Section 3. Then the optimization results of ideal optical system and practical one are also compared in this section. The conclusion is given in Section 4.

## 2. Theoretical analysis

### 2.1. Wavefront coding

The pupil function  $P(x, y)$  of a wavefront coding system can be expressed as:

$$P(x, y) = p(x, y) \exp \{ j [\phi(x, y) + \psi(x^2 + y^2) + Z] \} \quad (1)$$

where  $x$  and  $y$  denote the normalized coordinates in the pupil plane;  $j = \sqrt{-1}$ ;  $p(x, y)$  denotes the amplitude pupil function, where  $p(x, y) = 1$  when  $x^2 + y^2 \leq D^2$ , if not,  $p(x, y) = 0$ ;  $Z$  denotes the optical aberrations of the system except misfocus; and  $\phi(x, y)$  describes the phase mask placed in the pupil plane;  $\psi$  is the misfocus parameter, which is defined as:

$$\psi = \frac{\pi D^2}{4\lambda} \left( \frac{1}{f} - \frac{1}{d_o} - \frac{1}{d_i + \Delta d} \right) \quad (2)$$

where  $\lambda$  is the wavelength of the light; visible light is used in the following examples of this paper and  $0.587 \mu\text{m}$  is used as the reference wavelength;  $f$  is the focal length;  $d_o$  is the object distance;  $d_i$  is the image distance of the in-focus image plane which satisfying Gaussian lens formula  $1/d_o + 1/d_i = 1/f$ ;  $\Delta d$  is the distance between the in-focus image plane and the practical image plane. Obviously,  $\Delta d = 0$  and  $\psi = 0$  when the image plane is in focus according to Eq. (2). Therefore, there are two methods to describe the misfocus aberration. One is the distance between the in-focus image plane and the practical image plane  $\Delta d$  and it is usually used in practical optical imaging systems. The other is the phase variation  $\psi$  caused by misfocus and it is usually used in ideal systems. They can be converted to each other via Eq. (2).

### 2.2. Merit function for optimization

To make the system be insensitive to the misfocus is one of the key tasks in wavefront coding technology. In other words, the consistency of MTF with different misfocus can be one of the merit functions in the design of a wavefront coding system. Fisher information [14], described in Eq. (3), can evaluate the consistency of the object function. Thus Fisher Information is used to merit the sensitivity of the MTF to the misfocus.

$$J(\varphi) = \int_{-\psi_{\max}}^{\psi_{\max}} \int_0^{U_{\max}} \int_0^{U_{\max}} |H(u, v, \varphi, \psi)| \left[ \frac{\partial}{\partial \psi} \ln |H(u, v, \varphi, \psi)| \right] \left[ \frac{\partial}{\partial \psi} \ln |H(u, v, \varphi, \psi)| \right]^T du dv d\psi \quad (3)$$

Where  $|H(u, v, \varphi, \psi)|$  denotes the MTF of the imaging system with different misfocus parameter  $\psi$  and phase mask  $\varphi$ ;  $\psi_{\max}$  is the maximum of the misfocus aberration;  $u$  and  $v$  denote the tangential and sagittal spatial frequency of the optical system respectively. The smaller the Fisher information is, the more consistent the MTFs with different misfocus is, and the system is the more insensitive to the misfocus. On other hand, the MTF with the spatial frequency above the Nyquist frequency [15]  $u_{\max}$  is insignificant to optical imaging systems and will be ignored in this paper. Therefore, only the MTF with the frequency less than  $u_{\max}$  will be taken into consideration. Notice that the noise will be amplified in the digital decoding, thus the signal to noise ratio in optical coding should be great enough to get sharp images after digital decoding. Since the noise is mainly determined by the digital sensor, the signal cannot be too small. The integral of MTF over the frequency is widely used to evaluate the signal, and 0.32–0.35 is considered as an acceptable minimum value [2,5,8,9]. The value of MTF at the Nyquist frequency is about 0.1 if the integral of MTF was 0.32 when the cubic phase mask is used. For the simplicity, 0.1 is used as the threshold of the MTF with the frequency above the Nyquist frequency.

The whole merit function  $M$  evaluating the consistency of the object function for the optimization is described as Eq. (4).

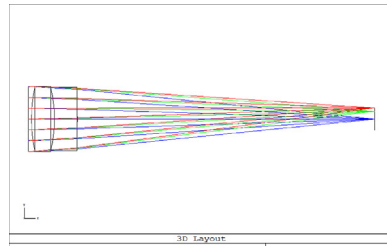
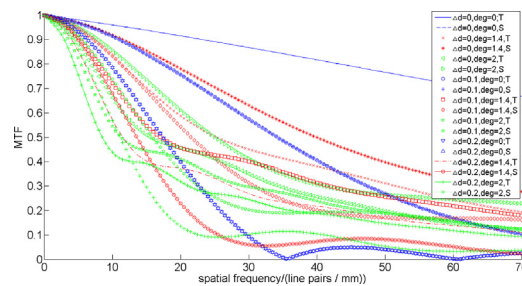
$$\begin{aligned} M &= \min J(\varphi) \\ \text{subject to } |H(u, v, \alpha, \psi)|_{u \leq U_{\max}, v \leq U_{\max}} &\geq th \end{aligned} \quad (4)$$

where  $th$  is the threshold.

**Table 1**

Data of the doublet lens.

Radius/mm	Thickness/mm	Glass	Semi-Diameter/mm
47.953	7	BK7	10
−42.707	7	F2	9.651
−372.680	91.349		9.233

**Fig. 1.** Layout of the doublet lens system without phase masks.**Fig. 2.** MTF curves of the doublets without phase mask when  $\Delta d = 0$  mm, 0.1 mm, 0.2 mm,  $\text{FOV} = 0^\circ, 1.4^\circ, 2^\circ$ .

### 3. Simulation analysis

A doublet lens is regarded as an initial system within visible spectrum as Table 1 and Fig. 1 shown. The focal length is 100 mm and the F number is 5. The maximum field of view in object space is  $2^\circ$ . The object distance  $d_o$  is assumed to be infinity in this paper. According to Gaussian lens formula, the image distance  $d_i$  equals to the focal length  $f$  when the system is in focus. Based on the simulation results in Zemax as shown in Fig. 2, “T” and “S” in the legends of Fig. 2 denote the tangential and sagittal MTF respectively. It is clear that the MTF decrease as the increase of either the misfocus parameter or the field of view. The MTF curve has a zero point at 35 line pairs/mm when the departure distance  $\Delta d$  between the practical image plane and the in-focus image plane is 0.2 mm. Thus 0.2 mm is used as the maximum misfocus parameter as an example in the practical system. To keep the comparability between the ideal and the practical systems, the maximum misfocus parameter  $\psi$  in the ideal system is 10.6825 which is calculated by Eq. (2).

#### 3.1. Optimization in the ideal optical system

A cubic phase mask (CPM) and a logarithmic phase mask (LPM) have been chosen to extend the depth of field of the system, which can be expressed as:

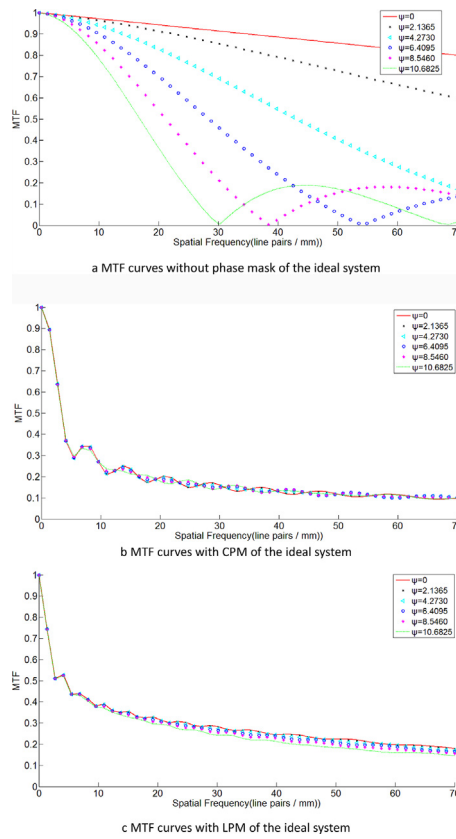
$$\varphi_{cpm}(x, y) = a_i(x^3 + y^3) \quad (5)$$

$$\varphi_{lpm}(x, y) = b_i x^4 (\log |x| + c_i) + b_i y^4 (\log |y| + c_i) \quad (6)$$

where  $a_i$  is the parameter of the cubic phase mask;  $b_i$  and  $c_i$  are the parameters of the logarithmic phase mask. The subscript  $i$  indicates the ideal optical system.

The phase masks are optimized in the ideal optical system using the methods of the previous researches [2–5,10,11]. The cut-off spatial frequency in the practical optical system is 341.52 line pairs/mm which is calculated by Zemax. The pixel width  $d$  of the digital imaging sensor is assumed as  $7 \mu\text{m}$ , then the Nyquist frequency  $u_{\max} = 1/(2d) = 70$  line pairs/mm. Considering the cut-off spatial frequency in the ideal optical system is normalized to be 2, the  $u_{\max}$  in the idea optical system is scaled to 0.4 in proportion. Here the threshold  $th = 0.1$ . The simulated annealing algorithm [16] is applied to optimize the parameters of the phase masks based on the Fisher Information merit function.

The parameters of the phase mask are optimized to  $a_i = 65.85$  of the CPM;  $b_i = 30.13$ ,  $c_i = 1.13$  of the LPM. MTF curves of the ideal optical systems are shown in Fig. 3. The MTF curves without phase mask shown in Fig. 3a fall rapidly with the



**Fig. 3.** MTF curves with different misfocus parameters of the ideal optical systems. a) without phase mask; b) with CPM; c) with LPM.

increase of the misfocus, and there are zero points when  $\psi \geq 6.4095$ . As shown in Fig. 3b and c, the consistency of the MTF curves of the ideal optical system with CPM or LPM is acceptable, although there are slight deviations from each other. In other words, the systems with CPM or LPM are insensitive to misfocus.

### 3.2. Verification in the practical optical systems

The phase mask optimized in the ideal optical system has a good performance as Fig. 3b and c shown. But how about the performance in the practical optical system? The optimized phase mask, which is obtained in the ideal system, is applied to the practical optical system directly. For the sake of consistency between the ideal and the practical optical system, the parameters of cubic and logarithmic phase mask in the practical system are calculated by Eq. (7) and (8) from the parameters in the ideal system respectively.

$$a_p = a_i r^3 (n_1 - n_0) \frac{2\pi}{\lambda} \quad (7)$$

$$b_p = b_i r^4 (n_1 - n_0) \frac{2\pi}{\lambda} \quad (8)$$

$$c_p = r c_i$$

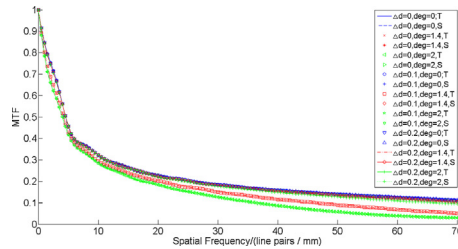
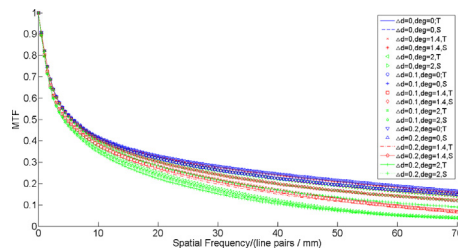
where  $n_0$  and  $n_1$  are refractive index of air and the material of the phase mask respectively. The subscript  $p$  indicates the practical optical system. Polymethyl Methacrylate (PMMA) is used as the material of the mask in this example.

The lens data of the systems with CPM and LPM are shown in Table 2. Figs. 4 and 5 illustrate the MTF curves of the practical optical systems with the cubic and the logarithmic phase mask optimized in the ideal system respectively. Although there is no zero points in the MTF curve, it is obvious that the consistency of the curves is not good, which means they are sensitive to the misfocus parameter in some extent.

**Table 2**

Lens data of the practical optical system with cubic and logarithmic phase mask.

	Radius/mm	Thickness/mm	Glass	$a_p$	$a_i$	$b_p, c_p$	$b_i, c_i$
doublet	47.953	7	BK7				
lens	–42.707	7	F2				
	–372.68	91.349					
CPM			PMMA	$1.18 \times 10^{-5}$	65.85		
LPM			PMMA			$9.17 \times 10^{-6}, 11.3$	230.13, 1.13

**Fig. 4.** MTF curves of the wavefront coding system with the CPM obtained in the ideal system when  $\Delta d = 0$  mm, 0.1 mm, 0.2 mm,  $\text{FOV} = 0^\circ, 1.4^\circ, 2^\circ$ .**Fig. 5.** MTF curves of the wavefront coding system with the LPM obtained in the ideal system when  $\Delta d = 0$  mm, 0.1 mm, 0.2 mm;  $\text{FOV} = 0^\circ, 1.4^\circ, 2^\circ$ .**Table 3**

a) Parameters of the doublets system with the CPM using the integrative optimization method. b) Parameters of the doublets system with the LPM using the integrative optimization method.

a)				
	Radius/mm	Thickness/mm	Glass	$a_p$
doublet	59.140	8.210	BK7	
lens	–41.941	9.17	F2	
	–157.568	91.349		
CPM		3	PMMA	$1.319 \times 10^{-5}$
b)				
	Radius/mm	Thickness/mm	Glass	$b_p, c_p$
doublet	65.010	6.627	BK7	
lens	–38.677	8.776	F2	
	–125.448	93.417		
LPM		3	PMMA	$5.321 \times 10^{-7}, 9.585$

### 3.3. Integrative optimization in the practical optical system

To improve the insensitivity to the misfocus, an integrative optimization method is proposed here. Both the phase mask and other optical elements will be optimized as a whole. And various aberrations will be taken into consideration. Zemax is used as an example of the commercial optical software to ensure the accuracy of the data.

A cubic and a logarithmic phase mask are used as examples in the practical system as well as in the ideal system. The Fisher Information defined in Eq. (4) is applied as the user-defined merit function in Zemax. The parameters are chosen as follows,  $th = 0.1$ ,  $\Delta d = 0.2$  mm,  $u_{\max} = 70$  line pairs/mm.

Table 3 gives the data of the optimized doublet systems with the CPM and the LPM. Notice that the parameters of the doublet are changed as well as the parameters of the phase mask compared with the data shown in Table 2. The MTF curves of the practical systems with CPM and LPM after optimization are shown in Figs. 6 and 7 respectively. It can be found that there are a highly consistency in the MTF curves with different misfocus and FOV in either Fig. 6 or Fig. 7. Compared to the

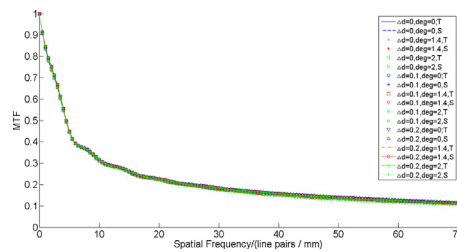


Fig. 6. MTF curves of the wavefront coding system with the CPM by integrative optimization in the practical optical system.

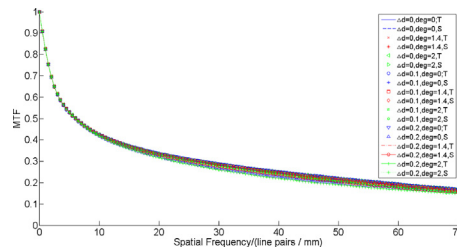


Fig. 7. MTF curves of the wavefront coding system with the LPM by integrative optimization in the practical optical system.

MTF curves of the systems using phase masks optimized in the ideal ones shown in Figs. 4 and 5, the one obtained by the integrative optimization method has a better consistency. In other words, the integrative optimized systems are insensitive to not only misfocus but also field of view. This improvement is greatly useful in digital decoding since insensitivity to misfocus is one of crucial factors in the success of wavefront coding technology.

Integrative optimization method considers various aberrations and obtains satisfying results in the practical optical systems with different phase masks. The phase mask and other optical elements are optimized integratively in the practical optical system. The consistency of the MTF curves using the integrative optimization is much better than the one optimized in the ideal system.

#### 4. Conclusion

CPM and LPM are used as examples to extend the depth of field in this paper. The phase masks obtained in the optimized ideal optical system works well on the on-axis field but poor on non-axial fields in the practical system. To solve this problem, an integrative optimization method is proposed in this paper. It not only considers all the parameters as variables including phase mask and other optical elements, but also takes various aberrations into account. The simulation results show that the wavefront coding system using the integrative optimization method is insensitive to both misfocus and FOV. And this method is effective for the system with either CPM or LPM. We believe that it will also be applicable for other phase masks. This method will be useful for the design of the practical wavefront coding systems. Experimental results will be given in future research.

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