### 1. Models Equations

**(a) Fixed effects ANOVA**

HM:

Level 1:  where  and independent.

Level2:  where  and  are fixed population parameters, and j=1, 2, 3…146.

LMM:



MM:



**(b) ANCOVA with OCmath as a covariate**

HM:

Level 1:  where  and independent.

Level 2:  

LMM:



MM:



**(c) Radom effects ANOVA**

HM:

Level 1:  where  and independent.

Level 2:  where  and independent over j and from.

LMM:



MM:



**(d) Null model (same as (c))**

HM:

Level 1:  where  and independent.

Level 2:  where  and independent over j and from.

LMM:



MM:



**(e) Random intercept with math**

HM:

Level 1:  where  and independent.

Level 2:   where  and independent over j and from.

LMM:



MM:



**(f) Random intercept with OCmath**

HM:

Level 1:  where  and independent.

Level 2:   where  and independent over j and from.

LMM:



MM:



**(g) Random intercept with grpCmah**

HM:

Level 1:  where  and independent.

Level 2:   where  and independent over j and from.

LMM:



MM:



**(h) Random intercept with grpMmath**

HM:

Level 1:  where  and independent.

Level 2:  where  and independent over j and from.

LMM:



MM:



**(i) Random intercept with grpCmath and grpMmath**

HM:

Level 1:  where  and independent.

Level 2:   where  and independent over j and from.

LMM:



MM:



(j) Random intercept with grpCmath, grpMmath, gender, and grade

HM:

Level 1:  where  and independent.

Level 2:     where  and independent over j and from.

LMM:



MM:



|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | Fixed Effects | | | Random Effects | | | | Fit Statistics | |
|  |  |  |  |  | Between | | Within | |  | |
| M | No.of est pars | Names | ’s | SE |  | SE |  | SE | -2 loglike | *AIC* |
| a | 147 |  |  |  |  |  | 76.9537 | 1.2918 | 50964.1 | 51258.1 |
| b | 148 |  |  |  |  |  | 51.1241 | .8582 | 48061.8 | 48357.8 |
| c | 3 | i | 150.06 | .3955 | 20.9437 | 2.6867 | 78.5815 | 1.3331 | 51489.0 | 51495.0 |
| d | 3 | i | 150.06 | .3955 | 20.9437 | 2.6867 | 78.5815 | 1.3331 | 51489.0 | 51495.0 |
| e | 4 | i  math | 62.5902  0.5800 | 1.4336  0.0094 | 6.0149 | 0.8675 | 52.2296 | 0.8864 | 48482.2 | 48490.4 |
| f | 4 | i  OC | 150.22  .5800 | .2227  .0094 | 6.0149 | 0.8675 | 52.2296 | 0.8864 | 48482.2 | 48490.4 |
| g | 4 | i  grpC | 150.04  .5671 | .3964  .0096 | 21.6691 | 2.6952 | 52.2039 | .8856 | 48646.8 | 48654.8 |
| h | 4 | i  grpM | 14.0879  .9016 | 5.9217  .0392 | 3.0484 | .5719 | 78.6205 | 1.3340 | 51267.2 | 51275.2 |
| i | 5 | i  grpM  grpC | 14.2280  .9006  .5671 | 5.9517  .0394  .0096 | 3.7105 | .5851 | 52.2304 | .8863 | 48426.4 | 484436.4 |
| j | 7 | i  grpMmath  grpCmath  3rd  4th  boy  girl | 14.0460  .9002  .5528  -.9253  0  1.1154  0 | 5.9717  .03965  .0100  .1953  .  .1720  . | 3.7542 | .5884 | 51.7541 | .8782 | 48363.7 | 48377.7 |
| k | 10 | i  grpC  grpM  b  g  3rd  4th  none  little  some  lot | 15.4795  .5528  .8926  1.1193  0  -.9233  0  -.04519  -.9318  0  .03570 | 6.0041  0.01001  0.04014  .1720  .  .1953  .  .5859  .6303  .  1.0732 | 3.6278 | .5711 | 51.7470 | .8780 | 48359.0 | 48408.8 |
| l | 8 | i  grpC  grpM  b  g  grade shortage | 11.0335  .5528  .8961  1.1156  0  .9261  -.1035 | 6.1988  .01001  .04056  .1720  .  .1953  .2254 | 3.7509 | .5878 | 51.7533 | .8782 | 48363.5 | 48403.4 |

**3.** There are 146 schools and a total 7079 observations.

**4.** Yes, it appears that a random coefficient model is needed, because in model (c), the value of, which is 20.9437, is pretty large considering its s.e., which is 2.6867.



**5.** Similarities:

(1) Both models are linear models;

(2) Both models assume that ~N (0, ), and are independent.

Differences:

(1) Model (a) has only the within school variance, while model (c) has both within school variance  as well as between school variance.

(2) Models (a) indicates that between-school differences are fixed, while model (c) indicates that the between-school differences are random.

(3) Model (a) is only interested in the schools that are selected, while model (c) considers the selected school as a sample from the population, which is what model (c) is really interested in.

(4) In model (a), Var (Y) =, while in model (c), Var (Y) =+.

**6.** Similarities:

(1) Both models are linear models;

(2) Both models assume that ~N (0, ), and are independent.

(3) Both models have OCmath as the predictor, and the effect of OCmath is fixed over all schools and individuals.

Differences:

(1) Model (b) has only the within school variance, while model (f) has both within school variance  as well as between school variance.

(2) Models (b) indicates that between-school differences are fixed, while model (f) indicates that the between-school differences are random.

(3) Model (b) is only interested in the schools that are selected, while model (f) considers the selected school as a sample from the population, which is what model (f) is really interested in.

(4) In model (b), Var (Y) =, while in model (f), Var (Y) =+.

**7.** Model (c) and model (d) are the same.

**8. Substantive considerations:**

I think we should consider within and between groups differences separately, because how a students is doing compared with other people in the same school is rather separate from how a school as whole is doing compared with other schools. Therefore, I think model (g) is more appropriate.

**Statistical considerations:**

(1) Models (e) and (f) have the same goodness of fit to the data, in that  in both models have the same estimates and standard error. This means that the two models are statistically equivalent.

(2) In model (g),  has slightly smaller estimate and slightly larger standard error, but the goodness of fit is not that much worse than models (e) and (f). However, based on statistical considerations, models (e), raw math scores model, and (f), overall mean centered math scores model are better.

**Interpretational considerations:**

(1) Model (f) is a better choice than model (e) because model (f) uses overall mean centering, which means that the intercept in model (f) can be interpreted as the science score of the students’ mean math score.

(2) Model (g), by including both raw math scores and grpMmath, takes into consideration both individual and school levels, and therefore Model (g) is superior to models (e) and (f) in terms of interpretation.

To conclude, I think we should use model (g).

**9.**

|  |  |  |  |
| --- | --- | --- | --- |
| Model | “Effects” |  |  |
| (d) | Null model | 20.9437 | 78.5815 |
| (f) | Micro (OCmath)ij | 6.0149 | 52.2296 |
| (g) | Micro (grpCmath)ij | 21.6691 | 52.2039 |
| (h) | Macro (grpMmath)j | 3.0484 | 78.6205 |
| (i) | Macro and micro | 3.7105 | 52.2304 |

(1) For model (f), because the micro variable (OCmath)ij  varies systematically from school to school, we can observe in model (f) decreases in both  and  compared with model (d), which is the null model.

(2) For model (g), because the micro variable (grpCmath)ij  does not vary systematically from school to school, we observe only a decrease in the within school variance estimate  but not in the between school variance estimate , when comparing it to the null model.

(3) Because model (h) has only a macro variable and no micro variable, we observe only the decrease in the between schools variance estimate, but not in the within schools variance estimate.

(4) When a macro variable and a micro variable that does not vary systematically over schools are added in the model, we observe decreases in both unexplained between and unexplained within schools variances ( and  respectively). This showcases that the micro variable (grpCmath)ij contributes to explaining within school variance and the macro variable (grpMmath)j contributes to explaining between school variance.

**10.**

|  |  |  |  |
| --- | --- | --- | --- |
| Model | Fixed effects |  | S.E. |
| (k) | intercept  grpCmath  grpMmath  boy  girl  3rd grade  4th grade  none  little  some  lot | 15.4795  .5528  .8926  1.1193  0  -.9233  0  -.04519  -.9318  0  .03570 | 6.0041  0.01001  0.04014  .1720  .  .1953  .  .5859  .6303  .  1.0732 |

As we can see from the table above, for gender and grade, ’s are pretty large relative to S.E., while the ’s of shortages look rather small relative to the S.E.. Therefore, gender and grade are probably more useful than shortages.

**11.** My favorite model is model (j). It has both satisfactory goodness-of-fit to the data and concise structure, which means the No. of estimated parameters is rather small.

Based on the summary table, model (j) is like:

**= 14.0460+0.9002+ 0.5528- 0.9253+1.1154 **

**Interpretation:**

(1) Students whose math scores are 1 point higher than other students tend to have science scores that are 0.9002 higher.

(2) Students whose school has average math score that is 1 point higher than other schools tend to have science scores that are 0.5528 higher.

(3) Students who are in 3rd grade tend to have science scores that are 0.9253 lower.

(4) Compared with girls, science scores of boys are 1.1154 points higher.