Problem Set 3

1 Part A

$$\cancel{\sin x \sin y} \cdot (\cot x \cdot \cot y - 1) \cdot \frac{\sin x \sin y}{\cancel{\sin x \sin y}} = \frac{\cos x}{\cancel{\sin x}} \cdot \frac{\cos y}{\cancel{\sin y}} \cdot \cancel{\sin x \sin y} - 1 \cdot \sin x \sin y = \cos x \cos y - \sin x \sin y$$

$$= \cos(x+y)$$

Part B

$$\sin(\pi + x) + \sin(\pi - x) = \sin\pi \cos x \cancel{+ \cos\pi \sin x} + \sin\pi \cos x \cancel{- \cos\pi \sin x} = 2\sin\pi \cos x = 0$$

Part C

$$\frac{(1+\tan^2 x)\cdot(1-\cos 2x)}{2} = \frac{\sec^2 x \cdot (1-(2\cos^2 x - 1))}{2} = \frac{\frac{1}{\cos^2 x}\cdot(2-2\cos^2 x)}{2}$$

$$= \frac{\cancel{2}(1-\cos^2 x)}{\cos^2 x} \cdot \frac{1}{\cancel{2}} = \frac{\sin^2 x}{\cos^2 x} = \tan^2 x$$

Part D

$$\frac{\sin(3x)}{\sin(x)} - \frac{\cos(3x)}{\cos(x)} = \frac{\sin(3x)\cos(x) - \cos(3x)\sin(x)}{\sin(x)\cos(x)} = \frac{\sin(3x - x)}{\sin(x)\cos(x)} = \frac{\sin(2x)}{\sin(x)\cos(x)}$$

$$= \frac{2\sin(x)\cos(x)}{\sin(x)\cos(x)} = 2$$

2 Part A

$$\cos(3x) = 4\cdot\cos^3(x) - 3\cos(x)$$

$$\cos 2x \cos x - \sin 2x \sin x = 4\cdot\cos^3(x) - 3\cos(x)$$

$$(2\cos^2 x - 1)\cos x - 2\sin^2 x \cos x = 4\cdot\cos^3(x) - 3\cos(x)$$

$$2\cos^3 x - \cos x - 2\sin^2 x \cos x = 4\cdot\cos^3(x) - 3\cos(x)$$

$$-2\sin x^2 \cos x = 2\cos^3 x - 2\cos x$$

$$-2\sin x^2 \cos x = 2\cos x(\cos^2 x - 1)$$

$$-2\sin x^2 \cos x = 2\cos x \cdot -\sin^2 x$$

$$-2\sin x^2 \cos x = -2\cos x \sin^2 x$$

Part B

$$\tan\left(\frac{x}{2}\right) = \frac{1-\cos(x)}{\sin(x)}$$

$$\frac{\sin(\frac{x}{2})}{\cos(\frac{x}{2})} = \frac{1-\cos(x)}{\sin(x)}$$

$$\frac{\sin(\frac{x}{2})}{\cos(\frac{x}{2})}\cdot\frac{2\sin(\frac{x}{2})}{2\sin(\frac{x}{2})} = \frac{1-\cos(x)}{\sin(x)}$$

$$\frac{-2\sin^2(\frac{x}{2})}{-\sin(x)} = \frac{1-\cos(x)}{\sin(x)}$$

$$\frac{1-2\sin^2(\frac{x}{2})+1}{-\sin(x)} = \frac{1-\cos(x)}{\sin(x)}$$

$$\frac{\cos(x)+1}{-\sin(x)} = \frac{1-\cos(x)}{\sin(x)}$$

$$\frac{1-\cos(x)}{\sin(x)} = \frac{1-\cos(x)}{\sin(x)}$$