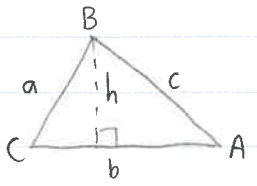
Problem Set 0

1. $\because h = a\sin C \quad \therefore K = \frac{1}{2}\left(\frac{\sin B}{\sin A}\right) a \cdot h$

$\because b\sin A = a\sin B$

$a = \frac{b\sin A}{\sin B}$

$\therefore K = \frac{1}{2}\left(\frac{\cancel{\sin B}\cdot b\cdot \cancel{\sin A}}{\cancel{\sin A}\cdot \cancel{\sin B}}\right)\cdot h = \frac{1}{2}bh$



2. $\because$ Law of Sines

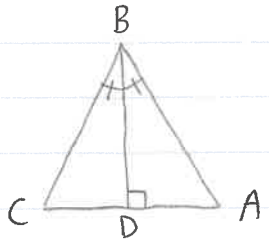
$\therefore \frac{\sin\angle BDA}{BA} = \frac{\sin\angle DBA}{DA} \quad \therefore \frac{DA}{BA} = \frac{\sin\angle DBA}{\sin\angle BDA}$

$\because \frac{\sin\angle BDC}{BC} = \frac{\sin\angle CBD}{CD} \quad \therefore \frac{CD}{BC} = \frac{\sin\angle CBD}{\sin\angle BDC}$

$\because \angle BDC + \angle BDA = 180^0 \quad \therefore \sin\angle BDC = \sin\angle BDA$

$\because \angle DBA = \angle CBD \quad \therefore \sin\angle DBA = \sin\angle CBD$

$\therefore \frac{DA}{BA} = \frac{CD}{BC} \quad \therefore \frac{BC}{BA} = \frac{CD}{DA}$



$a^2 = b^2 + c^2 - 2bc\cos A$

$-2bc$

3. $\because x^2 = \left(\frac{c}{2}\right)^2 + b^2 - 2\left(\frac{c}{2}\right)\cdot b\cdot\cos A$

$\because \cos A = \frac{a^2 - b^2 - c^2}{-2cb} = \frac{b^2+c^2-a^2}{2cb}$

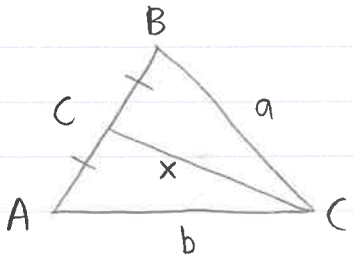
$\therefore x^2 = \frac{1}{4}c^2 + b^2 - \cancel{cb}\cdot\frac{b^2+c^2-a^2}{2\cancel{cb}}$

$x^2 = \frac{1}{4}c^2 + b^2 + \frac{1}{2}a^2 - \frac{1}{2}b^2 - \frac{1}{2}c^2$

$x^2 = \frac{1}{2}a^2 + \frac{1}{2}b^2 - \frac{1}{4}c^2$

$x = \sqrt{\frac{1}{4}(2a^2 + 2b^2 - c^2)}$

$x = \frac{1}{2}\sqrt{2a^2 + 2b^2 - c^2}$

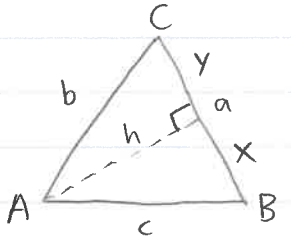


4. $\because h = \frac{a}{\cot B + \cot C}$

$\therefore h = \frac{a}{\frac{\cos B}{\sin B} + \frac{\cos C}{\sin C}} = \frac{a}{\frac{x}{\cancel{h}}\cdot\frac{\cancel{h}}{h} + \frac{y}{\cancel{b}}\cdot\frac{\cancel{b}}{h}} = \frac{a}{\frac{x+y}{h}}$

$\because a = x + y$

$\therefore h = \cancel{a}\cdot\frac{h}{\cancel{a}} = h$



5. $\because \cos X = \frac{RX}{z}, \quad \cos X = \frac{SX}{y}$

$RS^2 = RX^2 + SX^2 - 2(RX\cdot SX)\cos x$

$\therefore RS^2 = (z\cos x)^2 + (y\cos x)^2 - 2z\cos x\cdot y\cos x\cdot\cos x$

$RS^2 = z^2\cos^2 x + y^2\cos^2 x - 2zy\cos^3 x$

$RS^2 = \cos^2 x\,(z^2 + y^2 - 2zy\cos x)$

$RS^2 = x^2\cos^2 x \rightarrow RS = x\cdot\cos X$