

$$3.1 \quad \tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$1 - \tan \alpha \tan \beta = \frac{\tan \alpha + \tan \beta}{\tan(\alpha + \beta)}$$

$$\tan \alpha \tan \beta = 1 - \frac{\tan \alpha + \tan \beta}{\tan(\alpha + \beta)}$$

$$3.2 \quad \text{Calculate } \tan\left(\frac{\pi}{12}\right) \tan\left(\frac{2\pi}{12}\right) \tan\left(\frac{3\pi}{12}\right) \tan\left(\frac{4\pi}{12}\right) \tan\left(\frac{5\pi}{12}\right)$$

Take $\tan\left(\frac{\pi}{12}\right) \tan\left(\frac{5\pi}{12}\right)$ and use formula in 3.1

$$\tan\left(\frac{\pi}{12}\right) \tan\left(\frac{5\pi}{12}\right) = 1 - \frac{\tan \frac{\pi}{12} + \tan \frac{5\pi}{12}}{\tan\left(\frac{6\pi}{12}\right)}$$

$$\therefore \tan \frac{6\pi}{12} = \tan \frac{\pi}{2} = \text{undefined} \rightarrow \text{infinite}$$

$$\therefore \tan \frac{\pi}{12} \text{ is finite, } \tan \frac{5\pi}{12} \text{ is finite, so } \tan \frac{\pi}{12} + \tan \frac{5\pi}{12} \text{ is finite}$$

$$\therefore \text{Finite} \div \text{infinite} = 0$$

$$\therefore 1 - 0 = 1$$

$$\therefore \tan\left(\frac{\pi}{12}\right) \tan\left(\frac{5\pi}{12}\right) = 1$$

$$\tan\left(\frac{2\pi}{12}\right) \tan\left(\frac{4\pi}{12}\right) \text{ is also } 1 \text{ because } \frac{2\pi}{12} + \frac{4\pi}{12} = \frac{\pi}{2} \rightarrow \text{same case}$$

$$\text{Now we have } 1 \times 1 \times \tan\left(\frac{3\pi}{12}\right)$$

$$\tan\left(\frac{3\pi}{12}\right) = \tan\left(\frac{\pi}{4}\right)$$

$$= 1$$

$$4.1 \quad g(x) = 8x^3 + 4\sqrt{3}x^2 - 2x - \sqrt{3}$$

$$= (4x^2 - 1)(2x + \sqrt{3})$$

$$= (2x + 1)(2x - 1)(2x + \sqrt{3})$$

$$x = -\frac{1}{2}, \frac{1}{2}, -\frac{\sqrt{3}}{2}$$

$$4.2 \quad f(x) = 8\cos^3 x + 4\sqrt{3}\cos^2(x) - 2\cos x - \sqrt{3}, \quad -\pi \leq x \leq \pi$$

$$\cos x = -\frac{1}{2}$$

$$x = \left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$$

$$x = \frac{2\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{6}, -\frac{2\pi}{3}, -\frac{\pi}{3}, -\frac{5\pi}{6}$$

$$\cos x = \frac{1}{2}$$

$$x = \left(\frac{\pi}{3}, \frac{5\pi}{3}\right)$$

$$\cos x = -\frac{\sqrt{3}}{2}$$

$$x = \left(\frac{5\pi}{6}, \frac{7\pi}{6}\right)$$

$$\text{b/c } \cos x = \cos(2\pi n - x)$$

$$4.2 \quad f(x) \text{ for } -2\pi \leq x \leq 2\pi \rightarrow \text{domain}$$

$$\text{period: } 2\pi$$

$$\text{range: } 11.196 \geq y \geq -1.858$$

$$\text{frequency: } 2$$