Recursion

Chapter 11

5TH EDITION

Lewis & Loftus

java

Software Solutions

Foundations of Program Design

Recursion

- **Recursion is a fundamental programming technique that can provide an elegant solution certain kinds of problems**

- **Chapter 11 focuses on:**
  - **thinking in a recursive manner**
  - **programming in a recursive manner**
  - **the correct use of recursion**
  - **recursion examples**

Outline

- Recursive Thinking
- Recursive Programming
- Using Recursion
- Recursion in Graphics

Recursive Thinking

- **A *recursive definition* is one which uses the word or concept being defined in the definition itself**
- **When defining an English word, a recursive definition is often not helpful**
- **But in other situations, a recursive definition can be an appropriate way to express a concept**
- **Before applying recursion to programming, it is best to practice thinking recursively**

Recursive Definitions

- **Consider the following list of numbers:**

```
24, 88, 40, 37
```

- **Such a list can be defined as follows:**

```
A LIST is a:  number
        or a:  number  comma  LIST
```

- **That is, a LIST is defined to be a single number, or a number followed by a comma followed by a LIST**

- **The concept of a LIST is used to define itself**

Recursive Definitions

- **The recursive part of the LIST definition is used several times, terminating with the non-recursive part:**

```
number comma LIST
  24     ,   88, 40, 37

             number comma LIST
               88     ,   40, 37

                          number comma LIST
                            40     ,   37

                                       number
                                         37
```

Infinite Recursion

- **All recursive definitions have to have a non-recursive part**
- **If they didn't, there would be no way to terminate the recursive path**
- **Such a definition would cause *infinite recursion***
- **This problem is similar to an infinite loop, but the non-terminating "loop" is part of the definition itself**
- **The non-recursive part is often called the *base case***

Recursive Definitions

- **N!, for any positive integer N, is defined to be the product of all integers between 1 and N inclusive**
- **This definition can be expressed recursively as:**

```
1!  =  1
N!  =  N * (N-1)!
```

- **A factorial is defined in terms of another factorial**
- **Eventually, the base case of 1! is reached**

Recursive Definitions

5! $\boxed{120}$

5 * 4! $\boxed{24}$

4 * 3! $\boxed{6}$

3 * 2! $\boxed{2}$

2 * 1!

1

Outline

Recursive Thinking

Recursive Programming

Using Recursion

Recursion in Graphics

Recursive Programming

- **A method in Java can invoke itself; if set up that way, it is called a *recursive method***
- **The code of a recursive method must be structured to handle both the base case and the recursive case**
- **Each call to the method sets up a new execution environment, with new parameters and local variables**
- **As with any method call, when the method completes, control returns to the method that invoked it (which may be an earlier invocation of itself)**

Recursive Programming

- **Consider the problem of computing the sum of all the numbers between 1 and any positive integer N**
- **This problem can be recursively defined as:**

$$\sum_{i=1}^{N} i = N + \sum_{i=1}^{N-1} i = N + N-1 + \sum_{i=1}^{N-2} i$$

$$= N + N-1 + N-2 + \sum_{i=1}^{N-3} i$$

$$\vdots$$

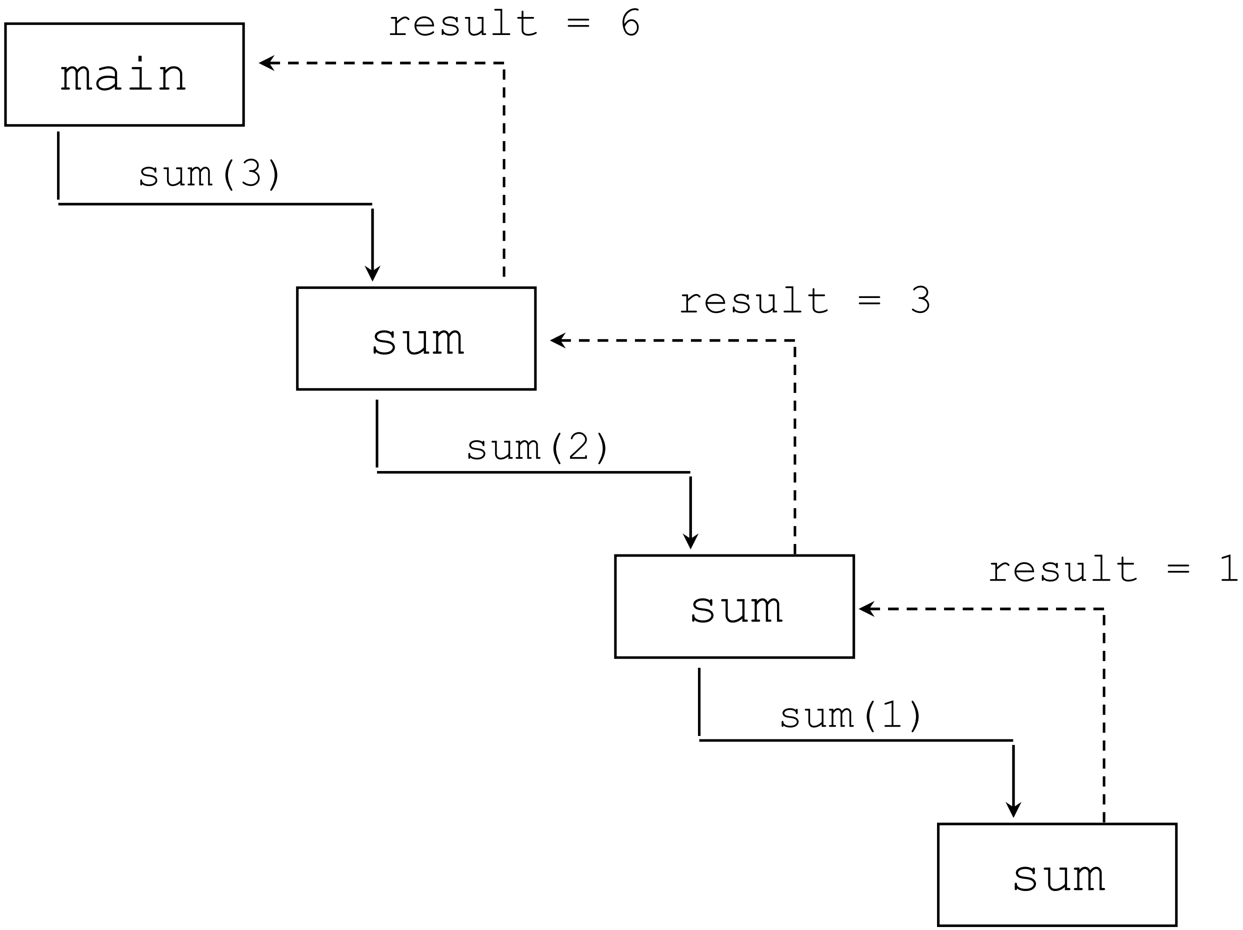
Recursive Programming

```
// This method returns the sum of 1 to num
public int sum (int num)
{
    int result;

    if (num == 1)
        result = 1;
    else
        result = num + sum (n-1);

    return result;
}
```

Recursive Programming



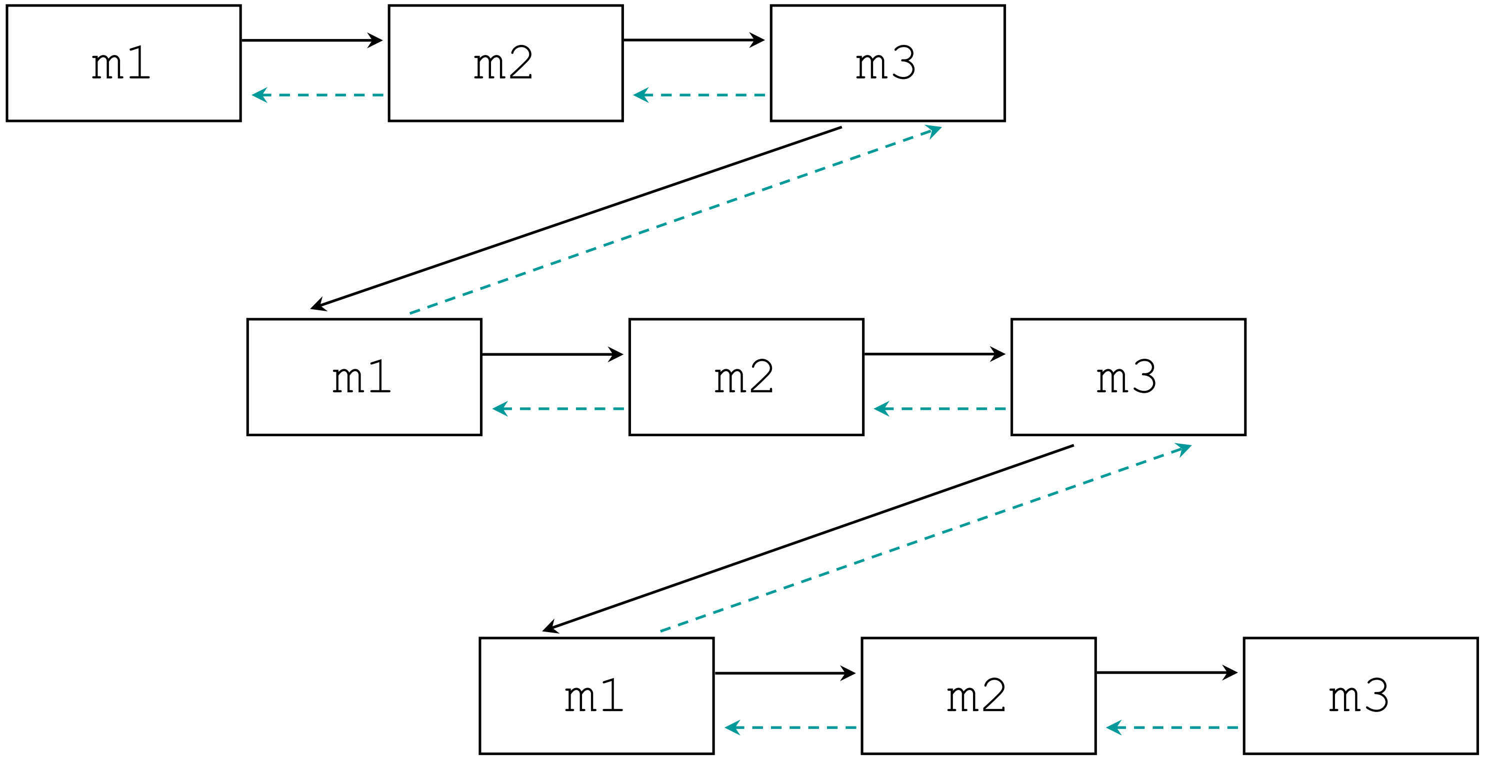
Recursive Programming

- **Note that just because we can use recursion to solve a problem, doesn't mean we should**
- **For instance, we usually would not use recursion to solve the sum of 1 to N problem, because the iterative version is easier to understand**
- **However, for some problems, recursion provides an elegant solution, often cleaner than an iterative version**
- **You must carefully decide whether recursion is the correct technique for any problem**

Indirect Recursion

- **A method invoking itself is considered to be *direct recursion***
- **A method could invoke another method, which invokes another, etc., until eventually the original method is invoked again**
- **For example, method `m1` could invoke `m2`, which invokes `m3`, which in turn invokes `m1` again**
- **This is called *indirect recursion*, and requires all the same care as direct recursion**
- **It is often more difficult to trace and debug**

Indirect Recursion



Outline

Recursive Thinking

Recursive Programming

Using Recursion

Recursion in Graphics

Maze Traversal

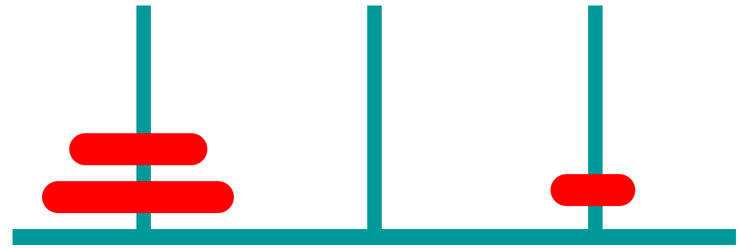
- **We can use recursion to find a path through a maze**
- **From each location, we can search in each direction**
- **Recursion keeps track of the path through the maze**
- **The base case is an invalid move or reaching the final destination**
- **See `MazeSearch.java` (page 587)**
- **See `Maze.java` (page 588)**

Towers of Hanoi

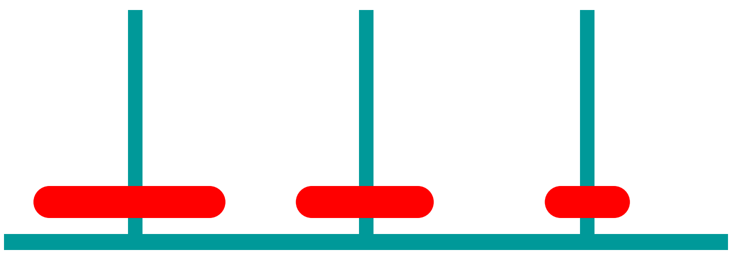
- **The *Towers of Hanoi* is a puzzle made up of three vertical pegs and several disks that slide on the pegs**
- **The disks are of varying size, initially placed on one peg with the largest disk on the bottom with increasingly smaller ones on top**
- **The goal is to move all of the disks from one peg to another under the following rules:**
  - **We can move only one disk at a time**
  - **We cannot move a larger disk on top of a smaller one**

Towers of Hanoi

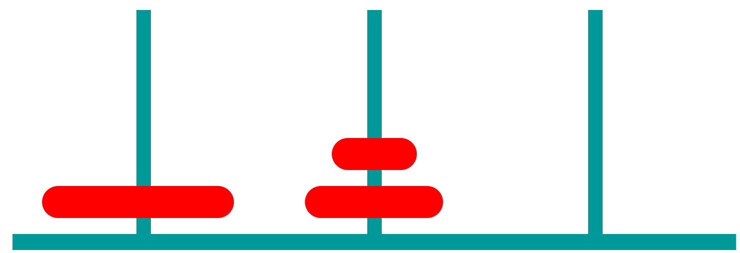
Original Configuration



Move 1

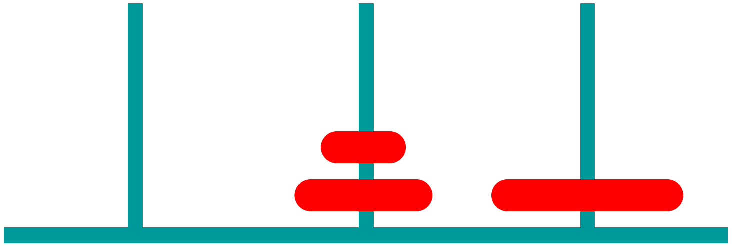


Move 2

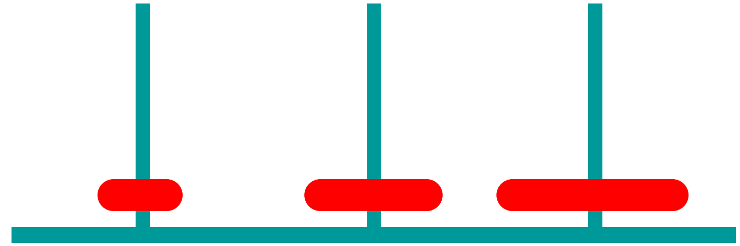


Move 3

Towers of Hanoi



Move 4



Move 5

Move 6

Move 7 (done)

Towers of Hanoi

- **An iterative solution to the Towers of Hanoi is quite complex**
- **A recursive solution is much shorter and more elegant**
- **See `SolveTowers.java` (page 594)**
- **See `TowersOfHanoi.java` (page 595)**

Outline

Recursive Thinking

Recursive Programming

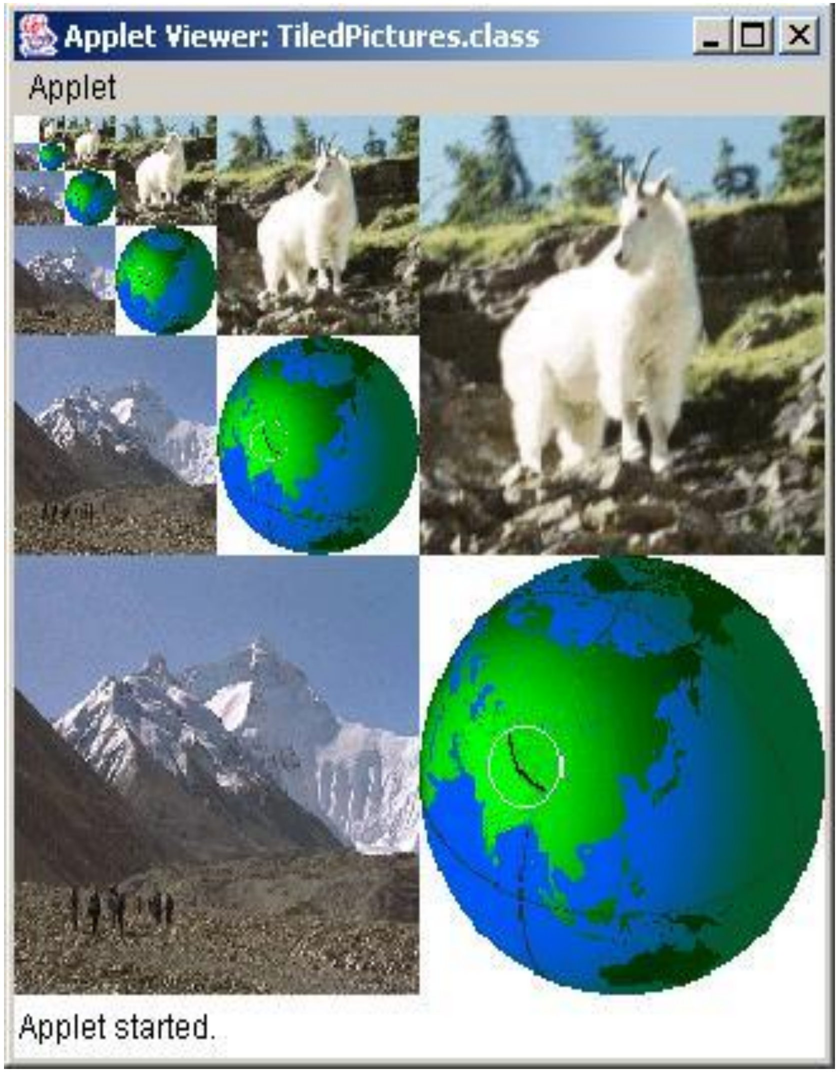
Using Recursion

Recursion in Graphics

Tiled Pictures

- **Consider the task of repeatedly displaying a set of images in a mosaic**
  - **Three quadrants contain individual images**
  - **Upper-left quadrant repeats pattern**
- **The base case is reached when the area for the images shrinks to a certain size**
- **See `TiledPictures.java` (page 598)**

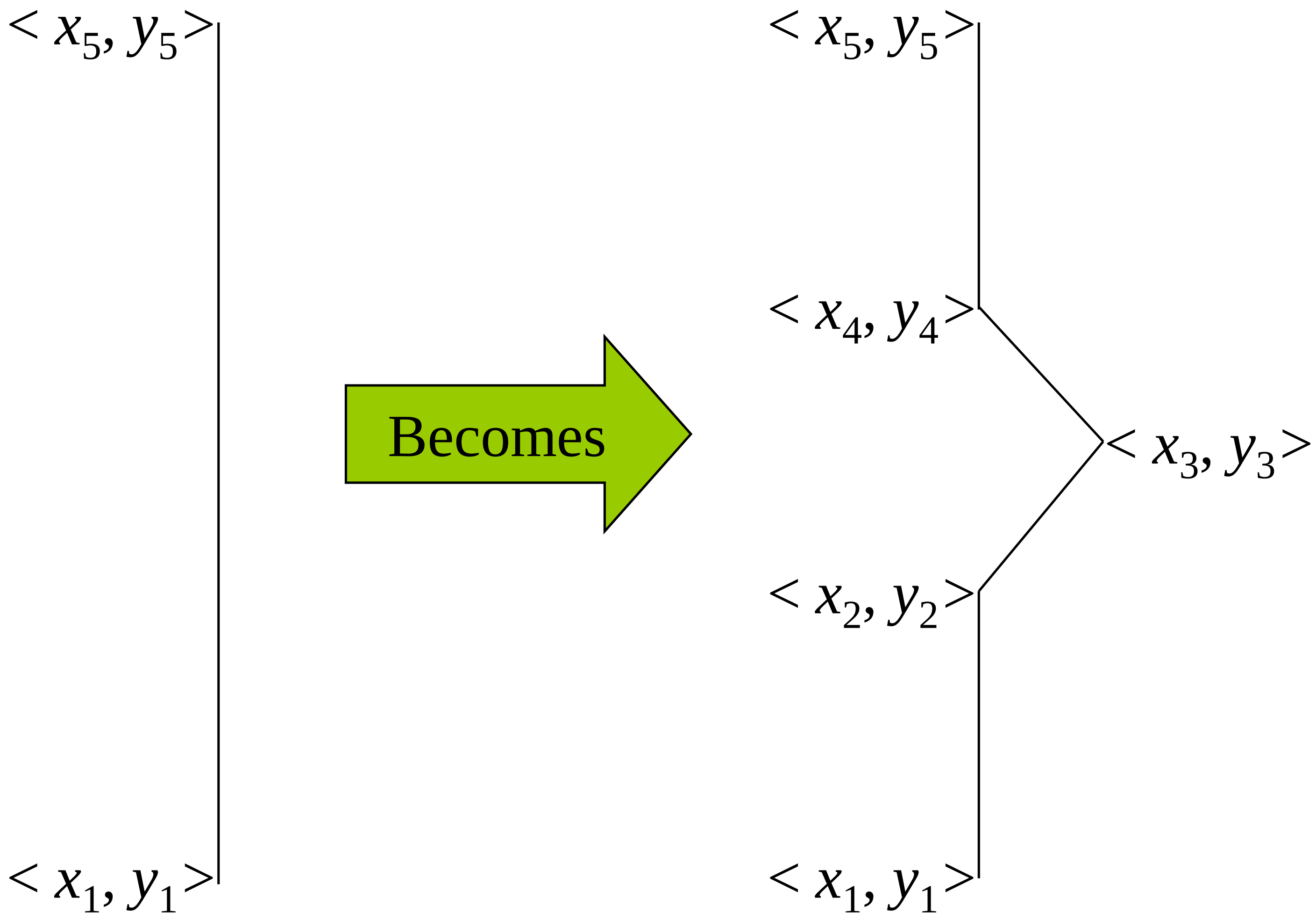
Tiled Pictures



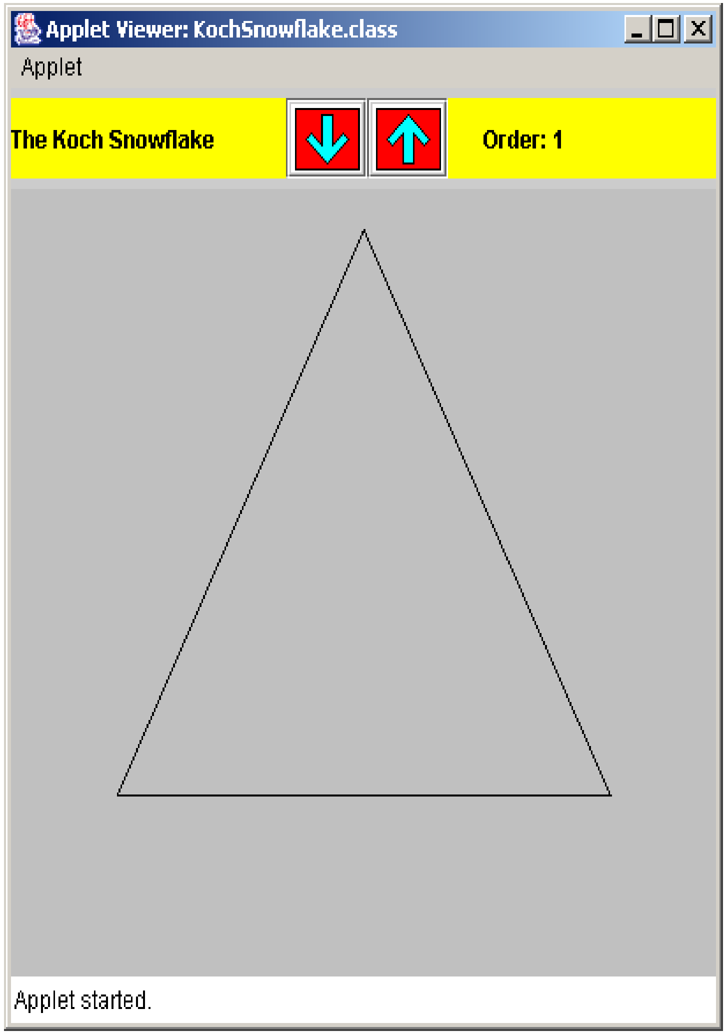
Fractals

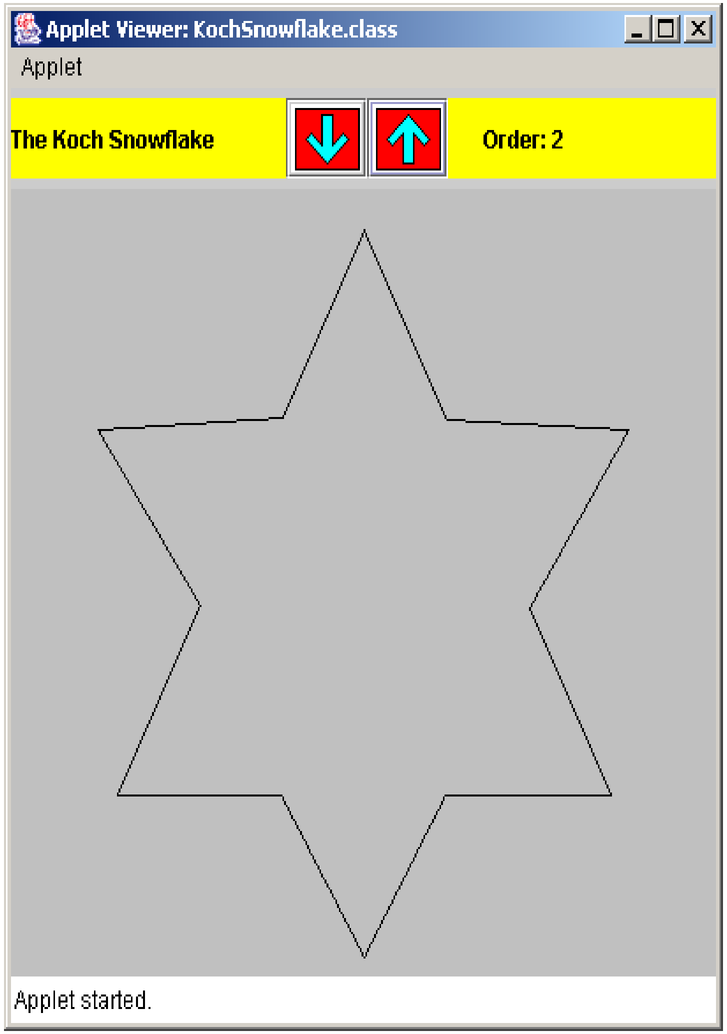
- A *fractal* is a geometric shape made up of the same pattern repeated in different sizes and orientations
- The *Koch Snowflake* is a particular fractal that begins with an equilateral triangle
- To get a higher order of the fractal, the sides of the triangle are replaced with angled line segments
- See `KochSnowflake.java` (page 601)
- See `KochPanel.java` (page 604)

Koch Snowflakes

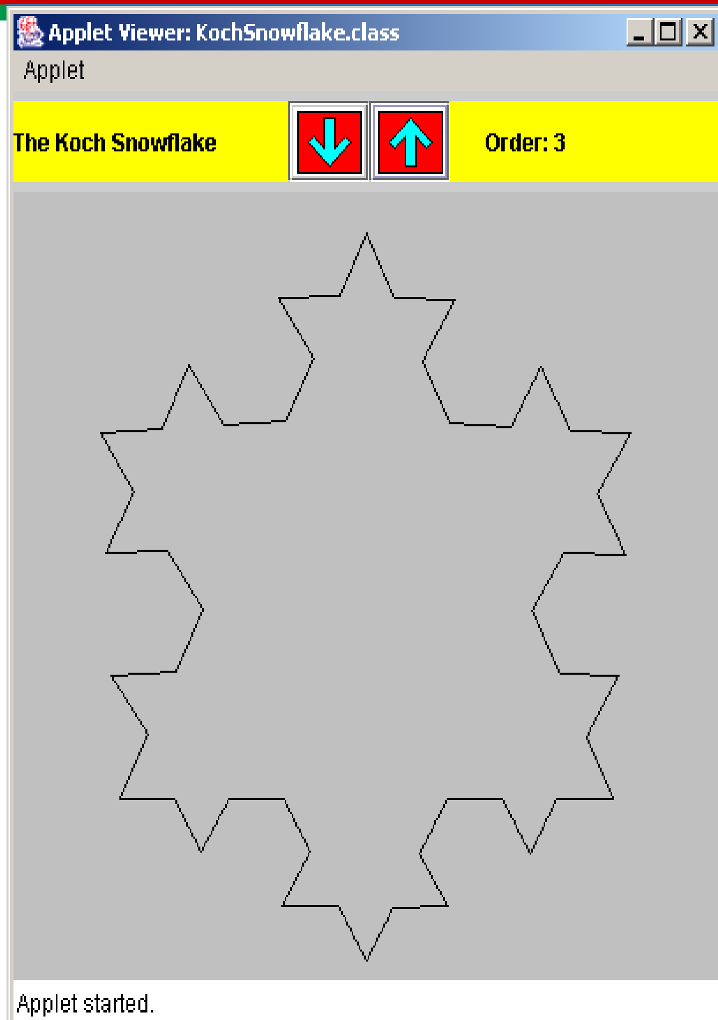


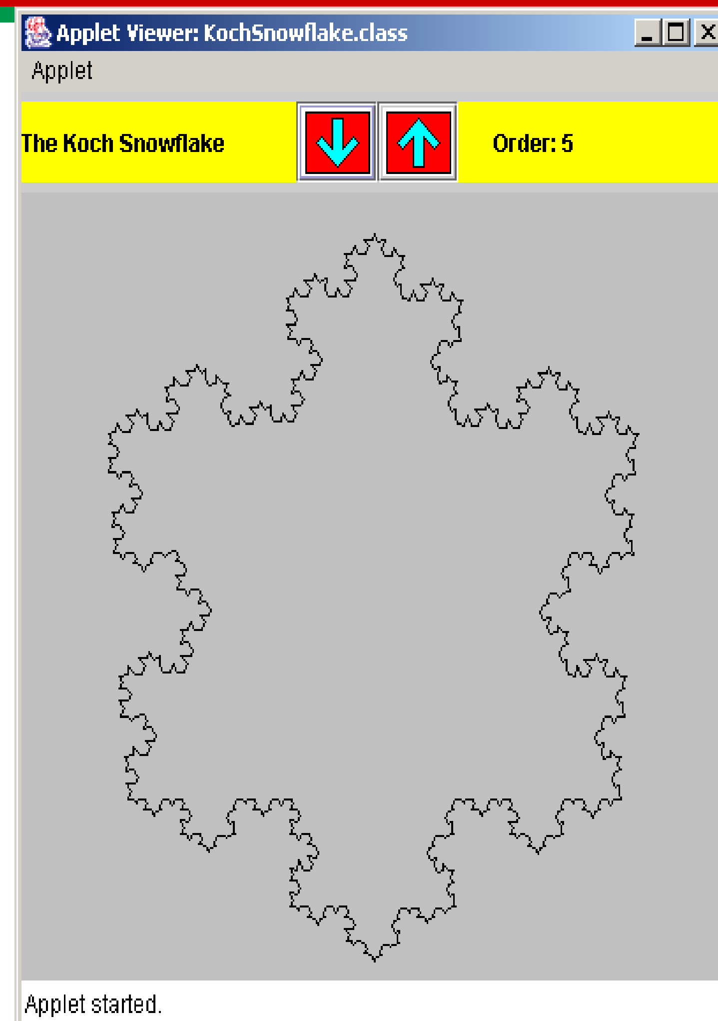
Koch Snowflakes





Koch Snowflakes





Summary

- **Chapter 11 has focused on:**
  - **thinking in a recursive manner**
  - **programming in a recursive manner**
  - **the correct use of recursion**
  - **recursion examples**