# Learning sensory representations for flexible computation with recurrent circuits

A common approach to modeling time-dependent tasks is to optimize the connectivity of a recurrent neural network (RNN) so that it reproduces a desired behavior. But neural circuits are presumably optimized over a lifetime of experience to solve a wide range of ethologically relevant tasks. One might thus expect that, when learning a new task, this prior structure would be exploited for computation. This seems to be the case in motor cortical circuits, which are able to learn a new motor task by recycling the same activity patterns used during a previously learned task [Golub et al '18, *Nat Neuro*]. Here we propose a model in which a general-purpose RNN is exploited for solving new tasks, without needing to continuously re-optimize the recurrent connectivity. Instead, learning a new task boils down to learning the appropriate representation of the task-relevant stimuli that will drive the RNN to produce the correct responses to task cues. Moreover, incorporating low-dimensional structure in the stimulus can drastically reduce the number of parameters that need to be learned, leading to much faster learning. In particular, rather than having to learn all N^2 weights of the recurrent connectivity, only the weights specifying how the stimulus is represented and how the RNN's activity is read out need to be learned. This setting in principle lends itself well to biologically plausible learning algorithms. We empirically show that such an architecture can be utilized to accurately model behavioral performance on the ready-set-go task [Jazayeri & Shadlen '15, *Cur Bio*], as well as a custom delay-copy task, with performance comparable to that of fully-trained RNNs.
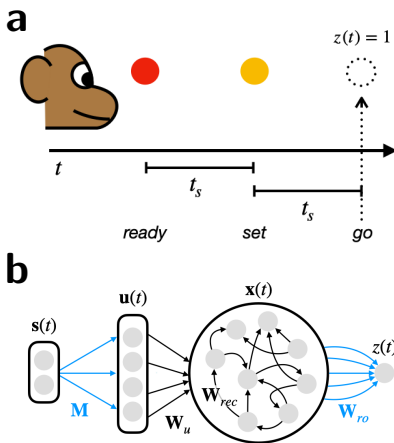
Figure 1: a) ready-set-go task. **b)** RNN model architecture.

Additional information. In each trial of the *ready-set-go* (RSG) task, a monkey sees two flashes of light separated by a target time interval of t_s milliseconds; those flashes indicate the *ready* and *set* cues. The subject's goal is to generate a saccade exactly t_s milliseconds after the set cue is shown, termed the *go* time (Figure 1a). RNNs trained to complete a variant of RSG [Remington et al '18, *Neuron*] exhibited dynamics that were qualitatively similar to those in dorsomedial frontal cortex (DMFC) recordings from monkeys performing the task. We consider roughly the same model, a simple rate-based recurrent network governed by the following dynamics:

$$\tau \frac{d\mathbf{x}}{dt} = -\mathbf{x} + \tanh(\mathbf{W}_{rec}\mathbf{x} + \mathbf{W}_u\mathbf{u}), \quad z = \exp(\mathbf{W}_{ro}\mathbf{x}) \qquad (1)$$

where $\mathbf{x} \in \mathbb{R}^N$ are the firing rates of the N-neuron network, $\mathbf{u} \in \mathbb{R}^D$ is the sensory input to the network, and $z \in \mathbb{R}$ is the readout. In the RSG task, $z(t)$ represents the decision variable controlling saccade generation, so that a saccade is generated once $z(t) \geq 1$. Therefore, the goal is for $z(t)$ to be less than 1 before the trial *go* time and at least 1 after that. We included additional bias and noise terms (omitted from (1) for simplicity).

Typically, the synaptic weights of such a network (the $\mathbf{W}$'s) are optimized to complete the task. However, learning in large recurrent networks is slow, making it an unlikely candidate for tasks like RSG. Therefore, instead of optimizing the weights of the recurrent network, we propose to keep them fixed and rather learn a good *representation* $\mathbf{u}(t)$ of the sensory *stimulus* $\mathbf{s}(t) \in \mathbb{R}^2$ (whose components are $s_1(t) = \delta(t - t_0)$, $s_2(t) = \delta(t - t_0 - t_s)$ where $\delta(\cdot)$ is the Dirac delta function, t_0 is the time of the ready cue, and $t_0 + t_s$ is the time of the set cue) in addition to the readout weights. We parameterize the sensory representation as a simple linear map from $\mathbf{s}(t)$ to $\mathbf{u}(t)$ via

$$\mathbf{u}(t) = \mathbf{M}\mathbf{s}(t) \qquad (2)$$

so that $\mathbf{M}$ is a $D \times 2$ matrix of $2D$ encoding weights. A more complex functional mapping could be used, but we show that the linear map is sufficient for the tasks we consider. By fixing D to a small number, the dimensionality of the learning problem can be reduced massively. For comparison, previous work assumed $\mathbf{u}(t) = \mathbf{s}(t)$, and instead optimized the $N^2 + 3N$ weights in $\mathbf{W}_{rec}$, $\mathbf{W}_u$ and $\mathbf{W}_{ro}$.

Of course, successful completion of the task still relies on the ability of the RNN to store and utilize the interval t_s over an extended period of time. Therefore, we initialize $\mathbf{W}_u$ and $\mathbf{W}_{rec}$ in the chaotic regime with weights drawn from $\mathcal{N}(0, \frac{1.5^2}{N})$ and $\mathcal{N}(0, \frac{1.5^2}{D})$ respectively so that they define a *reservoir* whose internal

dynamics, $\mathbf{x}$, are able to maintain inherently dynamical activity without rapidly decaying or exploding in magnitude [Sompolinsky et al. '88, *PRL*]. With the RNN weights fixed, we only need to optimize the sensory representation $\mathbf{M}$ and readout weights $\mathbf{W}_{ro}$.

To test our model, we use backpropagation through time (BPTT) to train $\mathbf{M}$ and $\mathbf{W}_{ro}$ on the RSG task. Results comparing the desired target interval with network outputs for a range of D are shown in Figure 2a; our networks are able to learn to produce very accurate behavior even for small values of D. In fact, at $D = 50$ we see that fixed-reservoir network performance can match that of trained-RNN networks.

These fixed-reservoir networks additionally reproduce Bayes-optimal behavior observed by [Sohn et al. '18, *Neuron*] in the performance of the RSG task by both non-human primates and trained RNNs in the presence of noise. We add signal-dependent noise to the sensory stimuli and dynamics and train separate fixed-reservoir networks on different sets of short and long intervals. In each case, we find that the network output is biased toward the mean of the interval range, with longer intervals exhibiting a larger bias (Figure 2b). Both of these distinctive observations are consistent with behavior observed in monkeys and fully-trained RNNs. An important future line of inquiry will be to dissect what dynamical mechanisms the learned sensory representations are exploiting that enable this bias and whether they match those hypothesized by [Sohn et al. '18] to underlie Bayesian behavior in this task.

To further illustrate the power of this approach, we tested our model on a task designed specifically to challenge RNNs with fixed recurrent weights: a *delay-copy* task, in which the stimulus is a continuous function of time, $s(t) \in \mathbb{R}$, $0 < t < T$, and the desired output is $z(t) = s(t - T)$, $T < t < 2T$. In other words, the network has to reproduce a T-length "snippet" of behavior immediately after it is shown. This task is difficult with an untrained reservoir because the dynamics in the stimuli (which need to be replicated in the output) can be made arbitrarily complex. We optimized the sensory representation and readouts weights for this task as we did with the RSG task and found that despite the difficulty, fixed-reservoir networks were able to learn the task. Output for two stimuli are shown in Figure 2c.

Our model possesses two properties that make it more plausible from a biological point of view than those that require optimizing the recurrent connectivity. First, the total number of parameters we are training is relatively low. Although we optimized $\mathbf{M}$ and $\mathbf{W}_{ro}$ with a biologically implausible implementation of BPTT, the small number of parameters makes the model more amenable to training via more plausible methods such as variants of BPTT [Murray '19, *eLife*] or node perturbation [Werfel et al. '04, *NeurIPS*]. Second, the untrained nature of the reservoir is an advantage in that it is reusable across a wide variety of tasks and contexts, especially those utilizing similar underlying dynamics. Importantly, the reservoir's weights are not specifically tailored to any particular task. We speculate that such an architecture in the mammalian motor system may underlie the flexibility with which such systems are able to learn arbitrary sensorimotor maps, like random BMI decoders [Carmena & Ganguly '09, *PLOS CB*]. This also opens up the possibility that while the RNN weights are fixed in the short term, its dynamics could be optimized over longer timescales to serve as a general-purpose reservoir of neurons for solving a large family of tasks.
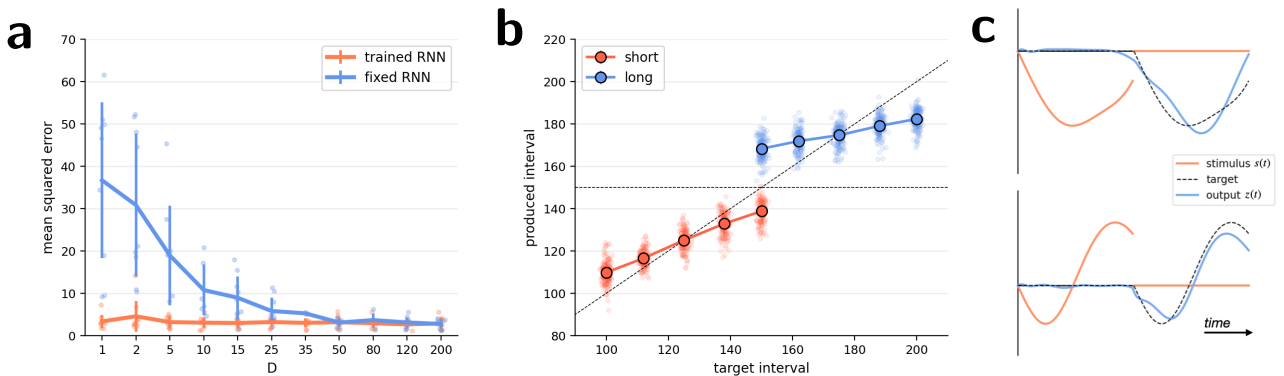
Figure 2: a) RNNs with $N = 200$ were trained to minimize an MSE loss to a target $f^*(t)$ that completes the RSG task. **b)** Fixed-reservoir networks trained on different interval ranges with noisy stimuli and dynamics show a Bayesian bias in their outputs. **c)** Outputs from a fixed-reservoir network trained on the *delay-copy* task, for two different stimuli. Although behavior is not perfect, the network is able to qualitatively complete the task.