1 The Economy and Financial System, No Anticipation of Runs

The model is defined by the following system of equations. Terms with br are for managing the mechanics of bank equity and entrance during and following a bank run, household utility is log, and management and transactions costs are quadratic (capital management cost $\frac{\alpha}{2}\left(K^{h,i}/w^i\right)^2 w^i$ and risk free asset transaction cost $scale\frac{\alpha}{2}\left(\frac{\max\left(0,g^i_{t+1}\right)}{Gw^i}-1\right)^2 Gw^i$.

An adding up requirement that the total stock G of risk free assets be held by households:

$$\sum_i \max\left(0, g^i_t\right) = G \tag{1}$$

A complementary slackness condition that households can't short the risk free asset:

$$\lambda^i_t g^i_t = 0 \tag{2}$$

In practice, parameters (principally the *scale* parameter) were chosen so that the condition would not become binding in the exercises performed.

Uses of funds (household consumption, bank consumption/dividends, capital management and transactions costs) equal sources (output from capital and labor, retained/purchsed risk-free household income stream, and entering bankers' endowments).

$$\begin{aligned} C^{h,i}_t + \frac{1-\sigma}{\sigma}\left(N^i_t - w^i W^b\left(1-br^i_t\right)\right) + \frac{\alpha}{2}\left(K^{h,i}/w^i\right)^2 w^i + scale\frac{\alpha}{2}\left(\frac{\max\left(0,g^i_{t+1}\right)}{Gw^i}-1\right)^2 Gw^i \\ = Z^i_t(1+W^h)w^i + \max\left(0,g^i_t\right) + w^i W^b\left(1-br^i_t\right) + \left(1-br^i_{t-1}\right)\xi w^i W^b \end{aligned} \tag{3}$$

Each household must be indifferent between consuming now and saving using the capital asset at the margin, where the :

$$1 = E(\beta\frac{C^{h,i}_t}{C^{h,i}_{t+1}}\frac{Z^i_t + Q_t}{Q^i_t + \alpha K^{h,i}}) \tag{4}$$

Each household must be indifferent between consuming now and saving in deposits at their paired bank:

$$1 = E(\beta\frac{C^{h,i}_t}{C^{h,i}_{t+1}}R^i_t) \tag{5}$$

Each household must be indifferent between consuming now and saving through risk free asset purchases:

$$p_t = -scale * \alpha(\frac{\max\left(0, g^i_{t+1}\right)}{Gw^i} - 1) + E(C^{h,i}_t \left(\beta \frac{1}{C^{h,i}_{t+1}} + \max\left(0, \lambda^i_{t+1}\right)\right)) \quad (6)$$

A bank's assets are equal in value to its debt issued plus its net worth:

$$Q_t K^{b,i}_t = D^i_t + N^i_t \quad (7)$$

The leverage ratio is the ratio of bank assets to net worth:

$$\phi^i_t N^i_t = Q^i_t K^{b,i}_t \quad (8)$$

Aggregate bank net worth evolves according to net income from holding capital less deposit redemptions. A fraction $(1-\sigma)$ of bankers exit and consume their net worth, leaving a fraction σ of the net worth remaining, to which is added endowments of entering bankers:

$$\begin{aligned} N^i_t = &\left(((Z^i_t + Q_t)K^{b,i}_{t-1} - R^i_t D^i_{t-1})\sigma + w^i W^b\right)\left(1 - br^i_t\right) \\ &+ \left(1 + \xi\sigma\right) w^i W^b \left(br^i_{t-1}\right)\left(1 - br^i_t\right) \end{aligned} \quad (9)$$

The bank's incentive compatibility constraint, which is that the expected value of operating honestly (the expected dividend next period and the value of the bank as a going concern next period, rhs) is at least as great as that of absconding with the bank's assets (of which the banker is able to convert a fraction θ to the consumption good, lhs).

$$\theta\phi^i_t = \beta\left(1 - \sigma + \sigma\theta\phi^i_{t+1}\right)\left(\phi^i_t\left(\frac{(Z^i_{t+1} + Q^i_{t+1})}{Q^i_t} - R^i_{t+1}\right) + R^i_{t+1}\right) \quad (10)$$

Productivity of each pair follows an AR(1) process.

$$Z^i_t = \rho(Z^i_{t-1} - Z_{SS}) + \epsilon_t \quad (11)$$

2 Intuitions and Mechanisms

2.1 Inherited Model

The model inherited from GK 2015 has two key frictions. The household's capital management cost introduces a spread between the household's expected marginal return on capital (4) and the rate of return (5) they would accept on riskless deposits, provided they are managing at least some capital.

This spread provides an incentive for the banker to intermediate as much capital as possible, but the incentive compatibility constraint (10) limits the achievable leverage ratio ϕ in each state. If this limit were not in place, all capital would be intermediated and the spread would be 0.

In this context, a (for illustrative purposes, mean-reverting negative) productivity shock has two direct effects:

1. Wage income falls now and is expected to rise in the future.

2. The output of capital falls now and is expected to rise in the future.

For consumers, this has a direct effect of pushing down current consumption relative to future periods, which due to their concave utility function increases the relative price of consumption in early periods compared to later and thus the required rate of return on savings.

The price of capital falls due to a lower stream of output, as well as an increased discount rate.

For banks, concerning only backward-looking items, this has two effects which push down net worth:

1. Lower than expected current output with no reduction in current liabilities.

2. Lower price of capital owned on entering the period.

There are offsetting effects on the amount of intermediation, the capital stock operated by the bank going forward.

1. Lower bank net worth tightens the incentive compatibility constraint

2. Lower price of capital loosens the incentive compatibility constraint (in quantity terms)

3. Higher path of expected spread of capital return over deposit rate (return on capital rises more) increases the value function for continuation, loosening the IC constraint per unit of net worth, allowing higher leverage

This nets out to less intermediation and resulting amplification of the initial shock, as well as a faster return towards steady state output compared to if leverage were not able to increase in response to the rising profitability per unit of bank equity.

2.2 Additions in Presentation

My work added three new features: a parallel bank-household pair, a risk free asset traded between the households, and a friction on that trading.

In the absense of any form of friction or restriction on trading in the risk free asset, the two households would necessarily have to have the same risk free interest rates, and thus SDF ratios, for any two time periods looking forward in time. This implies (exactly in perfect foresight, in expectation for a stochastic path) the same rate of consumption growth in every period. Then temporary shocks result in permanent shifts in wealth and consumption, with no return to the preexisting steady state regardless of how long productivity has been arbitrarily close to the steady state. Additionally, extremely large balances can accrue, which might not be sustainable outside the context of perfect foresight, when further negative shocks are not possible.

The addition of a convex cost in moving away from the no net purchases state of the risk free asset gives three benefits, with the drawback of a reduced form cost with the effect that (given an external fixed price of the risk free asset and other things equal) interest rates are higher in states that are more indebted externally, and lower in states that have higher positive external balances:

1. Allowing a difference between the consumption growth rates
2. Ensuring a unique steady state (achieved in principle with even very low values of the cost)
3. Reducing the peak balances built up to levels more plausibly repayable outside of perfect foresight

The difference between consumption rates across pairs is enabled by a wedge between the global price of the risk free asset and the reciprocal of the risk free interest rate. This enables indebted households, which thus have higher deposit rates, to more quickly increase their consumption than households with positive external balances.

This plays out in the heterogeneous case (ignoring most bank effects on the shocked side) as follows:

1. Due to effects in 2.1 the interest rate (deposit rate) in the shocked pair is above steady state, and the price of the risk free asset would be lower if it were in isolation
2. Because there is a price differential between the risk free asset in the shocked and healthy pairs, the shocked household sells to the healthy until demand is equalized with a middle global risk free rate before costs and a higher deposit rate in the shocked pair (lower deposit rate in non-shocked). Equivalently, for the shocked household, the marginal value of consumption today has risen more than that of tomorrow, while in isolation the non-shocked household has had no changed. The shocked pair trades promises of goods tomorrow for delivery of goods today, reducing their marginal value today and raising it tomorrow, while the reverse happens for the other household. The friction results in the ratios of marginal utility not being equal, and thus different consumption growth paths.
3. This leads to higher interest rates in the healthy pair, which is bad for the bank. Higher interest rates reduce the present value of capital when its productivity has remained the same, and this reduction in the value of the bank's assets reduces its net worth immediately

4. Due to leverage, this tightens the bank's IC constraint more than the lower cost of buying capital for next period loosens it, reducing the amount of capital it can intermediate

5. This is offset by an increase between the spread between the rate earned on bank capital and deposits as the wedge implied by the household's marginal management cost increases

6. Bank leverage rises

On the shocked side, this looks like the healthy pair has provided insurance as (3) through (6) effectively operate in the opposite direction.

3 Quantitative Effects of RFA Transaction Cost

I report below outcome variables for runs of the model with different values of the scale parameter which determines the price of transacting away from the steady state level of holdings of the risk free asset. The fundamental result remain the same across even extremely low parameterizations (the most extreme of which results in a difference of 0.01 basis points between the deposit rates in teh pairs on impact), which is that total bank capital operated falls more in the heterogeneous case than in the homogeneous, and that besides that bank capital operated falls considerably more in the shocked than in the non-shocked pair. Both result in larger aggregate effects on output than in the homogeneous case. However, the magnitude of the difference in effects on impact falls by about a fifth for output (from 7.58% to 6.51%) when the transaction cost parameter is reduced from 1 to 10^-6; other measures fall by more, but only in the case of deposit rates does the gap effectively disappear. (A more complete account of outcomes for various values of the transaction cost parameter is shown in the table.)

However, the transaction cost plays another important role in determining the results besides simply modulating near-term responses. For sufficiently low values of the parameter (below approximately 0.0075, resulting in a 12bp spread between deposit rates) and under the shocks in the experiments per, the restriction that household borrowing cannot exceed the household's constant endowment would become binding (though the household's debt level is still considerably below their total income). When the parameter reaches about 0.0001, results become unstable in that the system of equations does not result in a smooth return to the steady state values, instead having a relatively large jump in the last period from a trend diverging further.

A decrease in the transaction costs by a factor of 10^{-4} to 10^{-6} suffices to substantially eliminate the distinction between simulations with heterogeneous households tied to separate banks and trading a shared risk free asset, and a simulation with a representative household depositing at both banks and holding both types of capital.

Effect on Impact, After 10 Quarters	Baseline, No Run	Low Transaction Costs: .1x	.01x	.0075x	.002x* need to relax constraint	.001x* need to relax constraint	1e-4-- unstable	1e-6-- unstable	Representative Household
Net Output, No Shock	-0.04% -0.03%	-0.10% -0.06%	-0.19% -0.10%	-0.20% -0.11%	-0.24% -0.13%	-0.25% -0.13%	-0.27% -0.14%	-0.27% -0.14%	-0.27% -0.14%
Net Output, Shock	-7.62% -4.33%	-7.33% -4.22%	-7.07% -4.11%	-7.04% -4.10%	-6.95% -4.06%	-6.92% -4.05%	-6.89% -4.03%	-6.88% -4.03%	-6.88% -4.03%
Risk Free Asset Price	-0.11% -0.05%	-0.10% -0.04%	-0.09% -0.04%	-0.09% -0.04%	-0.08% -0.04%	-0.08% -0.04%	-0.08% -0.04%	-0.08% -0.04%	NaN NaN
Capital Price, No Shock	-0.35% 0.03%	-0.79% -0.15%	-1.36% -0.61%	-1.42% -0.67%	-1.66% -0.88%	-1.74% -0.95%	-1.84% -1.04%	-1.86% -1.06%	-1.86% -1.06%
Capital Price, Shock	-6.64% -3.86%	-5.92% -3.60%	-5.13% -3.09%	-5.04% -3.02%	-4.74% -2.80%	-4.65% -2.72%	-4.52% -2.62%	-4.50% -2.60%	-4.50% -2.60%
Deposit Rate, No Shock	0.3690 -0.0030	0.3543 0.0890	0.3479 0.1560	0.3454 0.1597	0.3372 0.1695	0.3347 0.1717	0.3316 0.1739	0.331152 0.174210	0.331148 0.174213
Deposit Rate, Shock	0.5004 0.3684	0.3835 0.2580	0.3530 0.1952	0.3495 0.1910	0.3384 0.1798	0.3353 0.1772	0.3317 0.1745	0.331153 0.174216	0.331148 0.174213
Bank Capital Operated, No Shock	-1.26% -0.93%	-2.95% -1.88%	-5.32% -3.03%	-5.60% -3.17%	-6.65% -3.68%	-7.01% -3.86%	-7.50% -4.11%	-7.57% -4.14%	-7.57% -4.14%
Bank Capital Operated, Shock	-34.97% -18.49%	-30.08% -16.00%	-25.16% -13.60%	-24.66% -13.36%	-22.92% -12.50%	-22.35% -12.22%	-21.62% -11.85%	-21.51% -11.79%	-21.51% -11.79%
Bank Net Worth, No Shock	-3.34% -1.69%	-7.51% -3.99%	-12.91% -7.15%	-13.52% -7.51%	-15.75% -8.83%	-16.51% -9.28%	-17.50% -9.87%	-17.65% -9.96%	-17.65% -9.96%
Bank Net Worth, Shock	-63.98% -38.45%	-57.15% -34.12%	-49.59% -29.37%	-48.79% -28.87%	-45.96% -27.11%	-45.03% -26.53%	-43.82% -25.78%	-43.64% -25.67%	-43.64% -25.67%
Bank Leverage Ratio, No Shock	1.79% 0.80%	4.10% 2.04%	7.24% 3.80%	7.61% 4.00%	8.97% 4.72%	9.44% 4.96%	10.07% 5.28%	10.16% 5.33%	10.16% 5.33%
Bank Leverage Ratio, Shock	68.58% 27.32%	53.49% 22.91%	40.84% 18.55%	39.70% 18.13%	35.87% 16.69%	34.70% 16.24%	33.21% 15.66%	33.00% 15.58%	32.99% 15.58%
Household Consumption, No Shock	-0.18% 0.14%	-0.42% 0.12%	-0.74% -0.13%	-0.78% -0.17%	-0.93% -0.31%	-0.98% -0.37%	-1.05% -0.44%	-1.06% -0.45%	-1.48% -0.87%
Household Consumption, Shock	-2.85% -1.73%	-2.58% -1.78%	-2.23% -1.58%	-2.19% -1.54%	-2.04% -1.42%	-1.98% -1.37%	-1.91% -1.30%	-1.90% -1.29%	NaN NaN

The aggregate effects to a heterogeneous shock against a system with two banks and a representative household continue to be more pronounced, if only slightly so, than those in a system with both representative banks and a representative household, facing an equivalent aggregate shock. Deviations from steady state values are about 1% to 5% larger on impact under the heterogeneous bank-representative household case than the representative bank-representative household case. (An exception is for household consumption, which has a smaller increase over the representative bank case and recovers somewhat more quickly due to larger bank losses.)

Period	Net Output	Capital Price	Deposit Rate	Bank-held Capital	Bank Net Worth	Household Consumption
Impact	1.14%	1.74%	5.37%	4.61%	1.71%	0.23%
I + 1	1.07%	1.60%	4.69%	4.53%	1.78%	-0.05%
I + 5	0.86%	1.15%	2.48%	4.29%	2.02%	-0.95%
I + 10	0.68%	0.75%	0.55%	4.12%	2.25%	-1.71%