- "My understanding is that the Dynare PF algorithm is based on equation system of the form

$$f(y_{t+1}, y_t, y_{t-1}, u_t)$$

at each date, where y_t is a vector of endogenous variables and u_t is the vector of exogenous shock variables. Is this correct? If so, how many equations are in this system in your case? Of the full vector of variables in your model y how many are actually present in y_{t-1}?"

This is correct. There are 21 equations total, of which 2 are the complementary slackness conditions on risk free asset holdings, 2 define the evolution of the productivity shock, 1 is the requirement that all the risk free assets must be held by someone, and the rest are the endogenous processes that happen within each pair (so 8 equations, but with pair 1 and pair 2 holding). Of the endogenous variables, 4 for each pair (8 total) are predetermined at y_{t-1}: the household's RFA holdings, the capital owned by the bank, the debt owed by the bank, and the interest rate on that debt. The price of capital, bank leverage, and household consumption enter as forward-looking endogenous variables.

- My understsanding is that the Dynare PF algorithms are based on calculating

$$\frac{\partial f(y_{t+1}, y_t, y_{t-1}, u_t)}{\partial y_{t+1}}$$

$$\frac{\partial f(y_{t+1}, y_t, y_{t-1}, u_t)}{\partial y_t}$$

$$\frac{\partial f(y_{t+1}, y_t, y_{t-1}, u_t)}{\partial y_{t-1}}$$

$$\frac{\partial f(y_{t+1}, y_t, y_{t-1}, u_t)}{\partial u_t}$$

Is it possible to recover these matrices evaluated at various points on the solution path? Within the perfect foresight routines that you are using at present, which I understand generates sequences from $t = 1, \ldots T$, is there a representation of the dynamic equilibrium in the form

$$s_t = M\left(s_{t-1}, \{x_{t+k}\}_{k=0}^{T-k}\right)$$

where x_t is the future path of exogenous variables. If so, what is the dimension of s?

To the first question, which I take to mean getting the values of the Jacobian of the system of equations at different points in time for the solution arrived at (rather than at different points in the process of converging to a solution), this is not directly returned as far as I know. However, because Dynare does return vectors of all variables at all periods, this can be calculated pretty simply as long as enough is held as fixed.

For the second, I take you to ask if there is a function which returns the current state given the previous state and the shock series for various lags. Given a sufficiently large m where the last shock arrives at T-m and a stationary model, an endpoint exists. Without that, one would need to be defined separately because of the presence of forward-looking endogenous variables.

With that caveat, that is precisely what the PF solver does, taking starting and ending values of s (or more precisely, some subsets of them depending on which variables are forward or backward looking) and the shock sequence and returning s for all intermediate time periods. However, the method involved is entirely dependent on obtaining all the intermediate periods as well, essentially solving $(T+2)$*(21) equations in a sparse matrix to return all of the variables at all periods. But even that is done numerically with the Newton-Raphson method.

Roughly, the general form would be along the lines of (written for just 1 lead and one lag):

$$s_{t:t+T-1} = M\left(s_{t-1}, s_{t+T}, \{x_{t+k}\}_{k=0}^{T-k}\right)$$

Given that, in principle there should be a representation of that form of dimension 21, obtainable for example by extreme application of direct substitution, but it would be nothing resembling clear or compact.