Memorandum: Alterations to Gertler and Kiyotaki 2015

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1 Overview

The following memo includes updates to the GK model with heterogeneous agents. Additional sections include 2) Responses to the 7/5 memo from Professor King, 9) A brief discussion of numerical solution methods and the challenges heterogeneity add, and 10) Proposals for how to simplify and/or narrow the project sufficiently to allow for results in the next month. Throughout, more significant changes are in bold. Section 8 has been substantially retooled as well. Selected derivations are included in the file SLZ20170717addendum.pdf attached to the email this memo was sent in.

2 Responses to RK Memo 7/5/17

2.1 Starting Point

The understanding of the GK model as including

1. Financial accelerater channels in banking
2. Endogenous determination of collateral value in markets
3. Bank runs characterized by asset fire sales as assets are transferred to households, who have a comarpative disadvantage in holding capital
4. Constant returns to scale in banking

is correct, as is the lack of a safe asset held by the households.

2.2 Two basic comments

Following further work on the basic model without equity issuance, I am inclined to agree that a model with heterogeneous productivity and no equity issuance is appropriate for the next month, and not restrict functional forms of utility and cost functions at this time. I am likewise proceeding with the addition of short term safe assets.

- Point 4: In the homogenous productivity case, if there are wealthy bankers, my intuition is that the banking market equilibrium likely involves risk neutral bankers bearing the productivity uncertainty (absent bank runs) while household receives stable consumption. Is this correct in GK? I note that GK assume that there is a household endowment which is proportional to Z, presumably for balanced growth reasons and that this would appear to make household consumption risk even if saving is not.

With sufficiently wealthy bankers, the entire capital stock will be intermediated, even if the banks are not fully capitalized by equity and thus still issue deposits. In that case, households will not be exposed to income risk through wealth, but you are correct that they are still exposed through the endowment proportional to Z.

- Point 5: Is the GK assumption that the productivity shocks are IID?

No, they are an AR(1) process, which in simulation is 1) assumed after a fixed period of time to return precisely to the long-run value and 2) has subsequent shocks turned off in order to calculate impulse responses. I expect that it will be necessary to retain both features to retain a tractable problem.

- Point 6: In thinking about the GK reference model, it became clear to me that I did not really understand the supply of deposits by a household to its paired bank. Would you please clarify this? How does it depend on the nature of productivity shocks? On the net worth of the bank?

There are two factors which affect the supply of deposits to the bank. The expected future price of the asset and productivity shock, together with the current price, define the rate of return on capital before management costs; this marginal expected return adjusted for management costs (which increase the effective price of capital bought in the current period) and stochastic discount factors must equal the expected return on deposits, which is the promised rate of return in no-run states and the recovery ratio in run states, scaled by their respective probabilities and SDFs.

This is the most direct effect on the supply of deposits via productivity shocks: they are a component of the return on holding marginal capital:

$$R_t^{h'} = \frac{(Z_{t+1} + Q_{t+1})}{Q_t + f'(K_t^h)}$$

IID shocks would tend to result in more stable capital prices compared to consumption (driven entirely by the difference in SDF for the current period compared to later), persistence would bake in an expected path with in high states lower marginal utility now than in the future and higher expected production now than in the future implying a declining path of prices.

The second factor is the bank's incentive compatibility constraint. Here the productivity is factored in indirectly through the price of capital and the continuation value of the banking firm. The directly relevant factor is the bank's asset holdings compared to its net worth. CRS implies a constant franchise value per unit of net worth conditioned on the decision rule. In order for the incentive compatibility constraint to hold, the franchise value per unit must be greater than the current asset value per unit of net worth which could be absconded with: thus with a given decision rule and holding everything else constant, an increase of X% net worth implies an increase in deposits supplied by X%. This would not precisely hold absent perfect competition as it would change the marginal cost of holding capital for the household, and thus returns and prices.

2.3 The Central Question in the Near Term

- Point 8: As I understand the direction outlined in the memo, the idea is that there are no traded claims on the output of capital, but that there can be trade in capital across households and banks. Is my understanding of this correct? If so, then there are some important analogies that can be made to other economies with restricted contingent claims (as in consumption theory and international business cycles).

This is correct. I have been thinking in terms of this looking something like international business cycles, which can also provide a basis for thinking about fixed banking relationships between household-banker pairs.

- Point 9: I am not clear about how the capital market works with idiosyncratic shocks, but I do understand that you will need to have some reallocation costs to avoid all capital flowing to highest return activity if there is serial correlation in the idiosyncratic shocks. ... More substantitively, are you envisioning that the main mechanism is that "resource flush" banks (in terms of position taking into account a good productivity shock and the past net wealth) use retained earnings to acquire capital from other banks, while "resource poor" banks undertake sales of capital? It would seem important to think about how this is affected by the persistence of the idiosyncratic productivity shock

This is correct. Resource rich banks will tend to have relatively resource and productivity rich households in their pair, enabling an expansion of capital holdings financed by both equity and debt. Resource poor will need to contract due to the incentive compatibility constraint, even as their cost of (even risk free) debt financing is pushed up due to lower management costs from their paired household. With persistent idiosyncratic shocks, lower expected output yields would also push in this direction.

- Point 10: I am not clear about whether there is depositor reallocation across banks. Tentatively, I am assuming that there is not. Then, it seems that a key issue is the "supply of deposits" by the household in the presence of idiosyncratic shocks to the returns on its paired bank. This would seem to depend importantly on the extent to which bank deposits are risky due to these shock and, as a related matter, about the extent to which deposit rates are heterogeneous due to individual bank risk. Above, I asked for a clarification of deposit supply in the basic GK model. Essentially, this is the natural extension of that question.

There is not depositor reallocation across banks. The deposit supply functions in substantially the same fashion as in the baseline GK model, with some important changes. Marginal rates of transformation will vary across households, as will their SDFs. This will feed through to different deposit rates, as will dispersion in default probability.

3 Key Features

Equations (1) through (4) detail the fundamental assets and production technologies of the GK model: a fixed quantity of a durable asset "capital" set to unity which may be operated by either bankers or households, but more efficiently by bankers.

$$K_t^b + K_t^h = 1 \tag{1}$$

Capital operated by bankers K_t^b requires no other input and generates output of a final good as well as returning the undepreciated capital :

$$\begin{gathered} date\, t \rightarrow date\, t+1 \\ K_t^b\ capital \rightarrow \begin{cases} Z_{t+1}K_t^b & output\ , \\ K_t^b & capital \end{cases} \end{gathered} \tag{2}$$

with Z_{t+1} an aggregate productivity shock.

Households operated with an additional management cost $f(K_t^h)$, paid for with period t final goods, to produce the same output the following period:

$$\textit{date}\ t \rightarrow \textit{date}\ t+1$$
$$\begin{cases} K_t^h & \textit{capital} \\ f(K_t^h) & \textit{goods} \end{cases} \rightarrow \begin{cases} Z_{t+1}K_t^h & \textit{output}\ , \\ K_t^h & \textit{capital} \end{cases} \tag{3}$$

The management was taken to be increasing, convex, and costless when no household operation took place:

$$f(K_t^h) = \frac{\alpha}{2}(K_t^h)^2 \tag{4}$$

In the planned extension, the key features remain the same with the following exceptions:

1. $K_t^b = \int_i K_t^{b,i} = \sum_i K_t^{b,i}$ (likewise K_t^h), the latter in the case of discrete household-bank pairs. This is made explicit because the main purpose of the extension is to explore bank heterogeneity.

2. The production functions (2), (3), and (4) operate at the level of the individual household or bank, rather than the aggregate.

3. The productivity shock Z_{t+1} is a combined firm-specific and aggregate shock:

   (a.1): $Z_{t+1}^i = Z_{t+1}^{agg} S_{t+1}^i$.

   This is added because even with added frictions, if all agents are subject to the same shocks motivating dispersion in outcomes across agents is infeasible. **The aggregate shock and the idiosyncratic shock need not follow the same processes.**

4. Banks continuing in operation must pay an adjustment cost when changing the quantity of capital operated, $h(k_t^{h,i}, k_{t-1}^{h,i})$, **equivalently** $h(i_t^{h,i}, k_{t-1}^{h,i})$. On the part of buyers this reflects higher oversight costs when purchasing loans or investment projects from other banks or households, and initiation costs such as closing costs on mortgages which are in excess of costs of ongoing servicing of existing loans and projects. Costs to sellers reflect due diligence costs to assure the buyer of the quality of the assets, collections or litigation costs and haircuts on loans not rolled over, and shutdown costs of non-continuing investment projects. **The transaction costs are not assumed to take any particular form at this time. However, there are some characteristics I expect I will want the function to follow. It should be at least weakly increasing as the magnitude of the change increases, as should its first derivative. It should be differentiable. It would be undesirable for there to be a situation in which a bank needs to liquidate assets and the marginal cost of asset disposal is greater than the price of the asset.**

5. **The household's untility function is left undefined at this time.**

4 Households

Households are permanently paired with banks where they deposit savings, receiving a promised rate of return if no bank run returns, and the promised rate of return times a recovery rate if one does.

$$R^i_{t+1} = \begin{cases} \bar{R}^i{}_{t+1} & no-run \\ x^i_{t+1}\bar{R}^i{}_{t+1} & run \end{cases} \tag{5}$$

Households face a period by period budget constraint

$$C^{h,i}_t + D^i_t + Q_t K^{h,i}_t + f(K^{h,i}_t) + p_t G^i_{t+1} = Z^i_t W^h + R^i_t D^i_{t-1} + (Z^i_t + Q^i_t)K^{h,i}_{t-1} + G^i_t \tag{6}$$

The left side of the equation is uses of funds (consumption, new deposits, capital held for next period, risk free asset holdings, and the management cost), the right is sources (an endowment proportional to the aggregate shock, deposit and risk free asset returns, gross return from capital held last period). The households' optimal holdings follow these conditions:

$$E(\Lambda^i_{t,t+1} R^i_{t+1}) = 1 \tag{7}$$

$$\Lambda^i_{t,t+s} = \beta^s \frac{u(C^{h,i}_{t+s})}{u(C^{h,i}_t)} \tag{8}$$

$$E(\Lambda^i_{t,t+1} R^{h,i}_{t+1}) = 1 \tag{9}$$

$$R^{h,i}_t = \frac{(Z^i_{t+1} + Q_{t+1})}{Q^i_t + f'(K^{h,i}_t)} \tag{10}$$

The presence of a tradable risk free asset guarantees a uniform SDF and risk free rate across household-bank pairs, *provided an interior optimum.*

5 Banks

In the GK model, risk neutral bankers earn income from operations and exit with constant exogenous probability $(1-\sigma)$ to motivate dividend payments and avoid saving indefinitely to avoid financial constraints (exiting bankers consume their entire net worth); bankers also stopped operations if their net worth fell to 0. I have modified this assumption to operate within each bank: there are a large numer of bankers at each bank, with equal ownership shares, and each period a fraction $(1-\sigma)$ exit and take with them their portion of the bank's net worth after costs in cash. **I have eliminated entering bankers of existing types, so that bankers enter only following a bank run. They could be reintroduced, but require a means of handling their transaction costs, which could be a problem for various forms.** The remaining bankers continue with the reduced fraction of the bank's net worth. Bankers have the value function

$$V_t = E_t\left[\sum_{s=0}^{\infty} \beta^s (1-\sigma) n^{b,i}_{t+s}\right],$$

the discounted value of expected terminal consumption.

Banks now follow a net worth process determined by current conditions and previously made portfolio decisions, scaled by the remainder after dividends are issued:

$$n_t^i = \sigma((Z_t^i + Q_t)k_{t-1}^{b,i} - R_t^i d_{t-1}^i - h(i_t^i, k_{t-1}^{b,i}));$$

net worth equal to (current shock and capital price) times capital held coming into period, less deposits repaid with interest, or 0 if they were the subject of a bank run.

Assets held in the current period are:

$$k_t^{b,i} = i_t^i + k_{t-1}^{b,i}$$

and follow the balance sheet constraint

$$Q_t k_t^{b,i} = d_t^i + n_t^i$$

and incentive compatibility constraint

$$\theta Q_t^i K_t^{b,i} \leq \beta E_t[V_{t+1}^i]$$

The bankers' problem in Bellman equation form is thus as follows:

$$V^i(k_{t-1}^{b,i}, d_{t-1}^i, z_t) = \max_{i_{t+s}} E_t\left[\sum_{s=0}^{\infty} \beta^s (1-\sigma) n_{t+s}^{b,i}\right] = \max_{i_{t+s}} E_t\left[(1-\sigma)n_t^{b,i} + \sum_{s=1}^{\infty} \beta^s (1-\sigma)(n_{t+s}^{b,i})\right]$$

$$= \max_{i_t} E_t\left[(1-\sigma)((Z_t^i + Q_t)k_{t-1}^{b,i} - R_t^i d_{t-1}^i - h(i_t^i, k_{t-1}^{b,i})) + \beta V^i(k_t^{b,i}, d_t^i, z_{t+1})\right]$$

This has, for a given bank, state variables $k_t^{b,i}, d_t^{b,i}$, with z_t reflecting all other factors including the expected sequence of shocks as well as endogenous factors including current and future prices of capital.

This gives envelope conditions that can be solved forward (**maintaining for now assumptions of perfect competition**) to give values that can in principle be computed recursively backward in time taking the path of prices as exogenous **(see attached scan page 7/15 3 for derivation, and 7/15 1 and 2 for related with dividends paid on basis of pre-investment cost net worth):**

$$V_d^i(k_{t-1}^{b,i}, d_{t-1}^i, z_t) = -E_t[\sum_{s=o}^{\infty} (\beta\sigma)^s (\prod_{q=0}^{s} R_{t+q}^i)(1-\sigma)R_{t+s}^i] = -A_t$$

$$V_k^i(k_{t-1}^{b,i}, d_{t-1}^i, z_t) =$$

$$E_t\left\{\sum_{s=0}^{\infty} \beta^s \left[\begin{array}{c} (1-\sigma)(R_{t+s}^b Q_{t+s-1} - \dfrac{\partial h}{\partial k}(i_{t+s}, k_{t+s-1})) - \\ \beta\left(Q_{t+s} - \sigma R_{t+s}^b Q_{t+s-1} + \dfrac{\partial h}{\partial k}(i_{t+s}, k_{t+s-1})\right) A_{t+s+1} \end{array}\right]\right\}$$

The expected excess return on the marginal unit of banker's capital is, after accounting for the balance sheet constraint and compressing notation:

$$r' = E(\frac{Q_{t+1} + Z_{t+1} - h'(i', k')}{Q_t + h'(i,k)} - R)$$

However, because it is no longer the case that the value function is CRS (and even if the transaction cost function were also CRS in capital, it would after dividing by capital yield an envelope condition in terms of the sequence of future investments anyway, see attached page 7/16 1) and it now depends on future choice variables that are themselves products of the optimization, this cannot be scaled down to give a simple rule about deposit demand such as if the excess return is positive, purchase as much capital as possible, if it is negative, reduce capital until no deposits remain outstanding, and accept any amount of deposits where the excess expected return is 0, which GK used to demonstrate that the bank would (under their parameterizations and shocks) always operate with a binding incentive contraint, now:

$$\theta Q_t k_t^{b,i} \leq \beta E_t[V_{t+1}^i]$$

6 Aggregation and Equilibrium without Bank Runs

The value of assets held by the banking system is equal to the sum over all banks, or equivalently the sum of their continuing net worths times their leverage $\phi_t^i = \frac{Q_t^i k_t^{b,i}}{n_t^i}$:

$$Q_t K_t^b = \sum Q_t k_t^{b,i} = \sum \phi_t^i n_t^i$$

The same holds after an aggregation in the expanded model, integrating/summing each side reflecting the individual banks.

The net worth of continuing bankers as a whole follows:

$$N_t = \sum_i n_t^i = \sum ((Z_t^i + Q_t) k_{t-1}^{b,i} - R_t^i d_{t-1}^i - h(i_t^i, k_{t-1}^{b,i}) + W^{b,i})\sigma$$

Where in the absence of a recent bank run on i (in which case everything else would be 0), $W^{b,i} = 0$ as no new bankers are replacing failed banks.

Thus banker consumption is:

$$C_t^b = \sum_i c_t^i = (1-\sigma)\left[\sum_i (Z_t^i + Q_t) k_{t-1}^{b,i} - R_t^i d_{t-1}^i - h(k_t^{h,i}, k_{t-1}^{h,i}) + W^{b,i}\right]$$

Output is the sum of output from capital, the household's endowment (also bankers' endowment if after a run):

$$Y_t = Z_t + Z_t W^h$$

With the addition of the transaction cost, uses of output are:

$$Y_t = C_t^h + C_t^b + \sum_i f(k_t^{h,i}) + h(i_t^{b,i}, k_{t-1}^{b,i})$$

7 Unanticipated Bank Runs

The bank by bank recovery ratio with the addition of adjustment costs is:

$$x_t^i = \frac{\left(Q_t^* + Z_t^i\right) k_{t-1}^{b,i} - h(0, k_{t-1}^{b,i})}{R_t d_{t-1}} < 1$$

(Or, if the parameterization is such that *h'* can ever be above Q_t^*, the maximum net worth contingent on capital remaining above 0: this is why I want to avoid this outcome) Q_t^* is a function of the joint survival or failure of banks. Consider a case in which there are two banks, initially identical but receiving different ideosyncratic shocks. One may have a recovery ratio below 1 in the event that the price of capital is as it would be with all banks failing, and the other might retain positive net worth even in that situation. In that case, the latter could not in fact fall to a run, and the equilibrium price of capital would be higher, and Q_t^* set to that instead; this might push the weaker bank's net worth above the point where it is susceptible to a run, and it might not. In the latter case, a run equilibrium is possible for the weaker bank but not the stronger. Any fixed point where the price is the result of a set of banks failing and no others (this process will be described later), which results in that same set failing and no others, can be the outcome of a run.

With appropriately defined leverage $\phi_t^i = \frac{Q_t^i k_t^{b,i}}{n_t^i}$, the recovery ratio in terms of returns on bank assets (in event of a run), the risk free rate, and leverage is:

$$x_t^i = \frac{R_t^{b,i*}}{R_t^i} \cdot \frac{\phi_{t-1}^i}{\phi_{t-1}^i - 1} < 1$$

with return on bank capital operated

$$R_t^{b,i*} \equiv \frac{Q_t^* + Z_t^i}{Q_{t-1}^i}.$$

The liquidation price is analogous to that seen in (30) and (31) of GK:

$$1 = k_t^{h,i}$$

$$Q_t^* = E_t\left[\sum_{s=1}^{\infty} \Lambda_{t,t+s}^i (Z_{t+s}^i - \alpha k_{t+s}^{h,i})\right] - \alpha$$

Which sets the price equal to the discounted dividends of capital held by households for the indefinite future, subject to the bank net worth process:

$$n_{t+1}^i = \sigma W_b^i,$$

$$n_{t+i} = \sigma[(Z_{t+s}^i + Q_{t+s})k_{t+s-1}^{b,i} - r_{t+s}^i d_{t+s-1}^i],\ for\ all\ s \geq 2$$

8 Anticipated Bank Runs

GK continue with the case of households anticipating the possibility of systemic bank run in time *t+1* with probability p_t. This results in a different first order condition for households.

$$1 = \bar{R}_{t+1} E_t[(1 - p_t)\Lambda_{t,t+1} + p_t \Lambda^*_{t,t+1} x_{t+1}],$$

that is, that the promised rate of return times the probability of receiving that amount scaled by the applicable SDF plus the recovery amount in the event of a run times the then-applicable SDF must equal 1. The household must be indifferent to holding the deposit subject to the anticipation that the banking system might fail.

The addition of more banks that may or may not jointly fail results in a different probability distribution that is relevant. This is described as follows:

- For a given set of state variables and shocks, order the banks by $\bar{Q}^i_t$: the price they require for them to survive with positive net worth.
- Generate the set of prices $\tilde{Q}^i_t$: the price of capital in the event all banks j with $\bar{Q}^j_t \geq \bar{Q}^i_t$ fail
- Then the possible prices are those $\tilde{Q}^i_t$ such that $Q^{i-1}_t \leq \tilde{Q}^i_t < Q^i_t$, plus the price induced by all banks failing if it is below the survival price of all banks, plus the no-run equilibrium price if high enough for all banks to survive.

The recovery rate is based on the promised rather than the risk free rate, similar to GK (34),

$$x^{i,j}_{t+1} = \min\left[1, \frac{R^{b,i,j}_{t+1}}{\bar{R^i}_{t+1}} \cdot \frac{\phi^i_t}{\phi^i_t - 1}\right] = \min\left[1, \frac{\left(Q^j_{t+1} + Z^i_{t+1}\right) k^{b,i}_t}{\bar{R^i}_{t+1} d^i_t}\right]$$

Because there are a number of outcomes that can occur in addition to either the entire banking system failing or nothing failing, the promised rate of return must satisfy an equivalent to (33):

$$1 = \bar{R}^i_{t+1} E_t[(\sum_{j=0}^{B} p^{i,j}_t \Lambda^{i,j}_{t,t+1} x^{i,j}_{t+1}]$$

where

$$\Lambda^{i,j}_{t,t+1} = \beta u'(c^{h,i,j}_{t+1}) / u'(c^{h,i}_t)$$

is the SDF in period $t + 1$ when sunspot event j is realized: the j weakest (subject to realized shocks) banks face runs, with 0 being the no-run equilibrium and j=B a systemic run. Then if in a given state bank i is the lth strongest bank, we find the following **(see attached page 15 for derivation):**

$$\bar{R}_{t+1} = \frac{1 - \frac{\phi^i_t}{\phi^i_t - 1} E_t\left(\sum_{j=l}^{B} p^j_t \Lambda^{i,j}_{t,t+1} R^{b,i,j}_{t+1}\right)}{E_t\left(\sum_{j=0}^{l-1} \Lambda^{i,j}_{t,t+1}\right)}$$

(on a pair by pair basis), so that banks will take into account leverage decisions' effects on their rate of return and resources available in the event of survival. This feeds through to the appropriate maximization problem described above, with the same constraints, but updated cost of borrowed funds and thus law of motion subtracting the ex-ante risky rate from the realized rate of return rather than a risk free rate.

As GK found, this gives a way for expectations of a run to negatively affect bank equity even in the absence of one materializing.

I plan to follow GK in the assumption that if a run on a bank next period is impossible, the probability to a run on that bank is 0, and that otherwise the probability of a run is a decreasing function of the expected recovery ratio $p_t = 1 - E_t(x_{t+1})$. The more likely a bank run is to occur, and the more severe in the event one does occur, the higher the agents assign to seeing the sunspot outcome. Note that in my case, this is a pair-by-pair probability, that is, the assessed probability of that bank failing. The possibility of other banks failing while the one under consideration survives, however, will enter into the agents' consideration via price effects of capital.

The probability of the j weakest banks failing is as follows, given a *realized* state (this is inaccurate and should be revised to be consistent with need to add to 1, as well as the concerns in Section 9):

- If j is not an eligible outcome as described above, then $p_t^j = 0$
- For the smallest j that is consistent with j banks failing, $p_t^j = 1 - x_t^{m,j}$, where m is the largest number of banks that can consistently fail: 1 minus the recovery ratio of the worst bank when the least possible number of banks fail
- For next most comprehensive run that is consistent, $k > j$, $p_t^k = 1 - x_t^{m,k} - p_t^j = x_t^{m,j} - x_t^{m,k}$
- And so forth through the remainder of consistent failure numbers

9 Solution Methods

The online appendix to GK includes a description for a solution method for the dynamics of the system, piecewise working backwards from a return to steady state with perfect foresight of shocks but not, in general, capital, consumption, etc. Provided a solution to the optimization problems faced by the banks/working decision rule accounting for the adjustment costs and how they affect the value function, and thus leverage constraint, certain problems remain.

First, the number of endogenous state variables is multiplied by the number of banks, plus the prior period's capital holdings for each bank and holdings of the risk free asset: where GK needed essentially the leverage of the bank to solve the optimization problem, now both capital and deposits matter independently. SDFs must be calculated for B times as many households, and in principle there are up to B + 1 possible numbers of defaults in each period, given everything else, and the recovery rates in each one depend on the recovery rates in the others and the perceived probability of each sunspot occurring, in addition to those for each time step from then on.

The GK methodology depends on the fact that after a run, capital holdings are entirely determined with banks holding none and households holding everything. This enables the relatively simple calculation of a saddle path back from the steady state to the known points during and after the run, including for the case of anticipated runs. This of course does not apply to a model with the possibility of partial runs where in general multiple outcomes could have arisen, and in turn what those outcomes were depended on the history to that point.

For a given run event at time t+i after the starting point, either you need to check the entire feasible state space including jump variables for what is an equilibrium that could take a saddle path back to the steady state, then out of those determine which could be supported by optimizing agents prior to that point. And this would need to be done for every time step to calculate the probabilities.

10 Steps Forward

A full solution to this model is likely to be intractable, and I am considering ways to simplify it. The pieces I am least willing to compromise on are heterogeneity and at least some bank-household stickiness in terms of moving deposits around (given the presence of some, I suspect perfect stickiness leaves the problem the simplest).

The simplifying steps I am considering so far are:

- Assume a very small number of banks, probably 2, maybe 3
- Take as simple as possible a cost function for the household's management cost and bank's transaction costs
- Assume following the productivity shock there is a limited period of time in which agents anticipate a bank run may occur, before the probability falls to 0 exogenously: along the lines of "if we can make it through the next year, it's smooth sailing"
- Greatly simplify the assigned probability of default, making it less a function of future and more a function of current factors
- Get rid of or modify the incentive compatibility constraint (however, the restriction that there is a fixed amount of capital to be distributed could in principle bind as well), for example by using the GK policy extension of instituting a fixed maximum leverage ratio

Other concerns:

- Perfect competition: pushes in direction of always maximizing leverage subject to incentive compatibility constraint. This tends to make all banks behave more similarly (and is part of why I believe the cost function is important in maintaining differentiation), but am interested in 1) preserving differences in choices, including leverage ratios and 2) banks taking into account the decisions their choices have on other banks and the probability of a general run. However, removing perfect competition would require calculating (for example) price derivatives in terms of changes in investment, when the price is an outcome of solving for the saddle path. Unless there is a good way to insert this into the probabilities of default alone, moving away from perfect competition likely isn't an option.

11 Bibliography

Gertler, Mark and Nobuhiru Kiyotaki. "Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy." *American Economic Review*, vol. 105, no. 7, 2015, pp. 2011-2034.

Online Appendix: https://assets.aeaweb.org/assets/production/articles-attachments/aer/app/10507/20130665_app.pdf