Banker's Problem

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1 Overview

The following summarizes the optimization problem faced by a banker in a multiple bank-household pair GK environment with cross-pair capital sales and capital adjustment costs on the part of banks, paid for with output in the period the investment takes place. The new investment does not generate output until the following period. Risk neutral bankers maximize the present discounted value of dividends subject to the law of motion for capital, a balance sheet constraint, and an incentive compatibility constraint. Dividends are taken to be a constant fraction of net worth after adjustment costs are paid.

2 Banker's Problem

In extensive form, the banker's problem is to maximize the objective:

$$V(k_{t-1}^{b,i}, d_{t-1}^{i}) = \max_{k_{t+s},\, s\geq 0} E_t \left[\sum_{s=0}^{\infty} \beta^s (1-\sigma)((Z_t^i + Q_t)k_{t-1}^{b,i} - R_t^i d_{t-1}^i - h(k_t^i, k_{t-1}^{b,i}) + W) \right],$$

where

- β is the rate of time preference
- $(1-\sigma)$ is the exogenous constant dividend rate
- Z_t^i is the productivity shock of the banker for production in date t with capital stock carried over from date t-1, taken as exogenous and revealed at date t. It may be IID or persistent.
- Q_t is the price of capital in period t, taken as exogenous
- $k_t^{b,i}$ is the quantity of capital held from period t which produces output in period t+*1*
- R_t^i is the interest rate paid on deposits maturing in date t, which is taken as exogenous
- d_t^i are the deposits issued in date t, maturing at date t+*1*
- $h(k_t^i, k_{t-1}^{b,i})$ is the adjustment cost paid in date t, a function of that period's investment and capital held coming into that period. It is assumed to be everywhere at least weakly convex in investment, with h=0 when i=0 and with a derivative with respect to i of 0 when i=0, where i is the difference between capital starting and ending the period.
- W is the

The objective can be formulated in the Bellman equation form (z denotes all factors other than the particular banker's capital held and deposits owed):

$$V(k_{t-1}^{b,i}, d_{t-1}^i, z_t) = \max_{i_t} E_t \begin{bmatrix} (1-\sigma)((Z_t^i + Q_t)k_{t-1}^{b,i} - R_t^i d_{t-1}^i - h(i_t^i, k_{t-1}^{b,i}) + W) \\ +\beta V(k_t^{b,i}, d_t^i, z_{t+1}) \end{bmatrix} \tag{1}$$

The constraints the banker faces are as follows:

1. The balance sheet constraint:

$$Q_t k_t^{b,i} = d_t^i + \sigma((Z_t^i + Q_t)k_{t-1}^{b,i} - R_t^i d_{t-1}^i - h(k_t^i, k_{t-1}^{b,i}) + W)$$

   which can be rewritten as a law of motion for deposits:

$$d_t^i = Q_t k_t^{b,i} - \sigma((Z_t^i + Q_t)k_{t-1}^{b,i} - R_t^i d_{t-1}^i - h(k_t^i, k_{t-1}^{b,i}) + W) \tag{2}$$

   which states that deposits are equal to the total price of capital held less the ex-dividend value of existing capital, its output, and adjustment costs, and adding back the ex-dividend proportion of maturing deposits.

2. The incentive compatibility constraint:

$$\theta Q_t^i K_t^{b,i} \leq \beta E_t[V(k_t^{b,i}, d_t^i, z_{t+1})] \tag{3}$$

   which holds that the liquidation value of an absconding banker can be no greater than the going-concern value of the bank next period given capital held, deposits owed, and all other factors not controlled by the banker. Note that if the incentive compatibility constraint binds with equality both this period and next period, then

$$\theta Q_t^i K_t^{b,i} = \beta E_t \left[(1-\sigma)((Z_{t+1}^i + Q_{t+1})k_t^{b,i} - R_{t+1}^i d_t^i - h(k_{t+1}^i, k_t^{b,i}) + W) + \theta Q_{t+1}^i K_{t+1}^{b,i}\right]$$

3 Outcome

The above problem yields the following first order condition in investment:

$$0 \leq (1-\sigma)(-\frac{\partial h}{\partial k'}(k_t^i, k_{t-1}^{b,i})) + \beta E_t[V_k(k_t^{b,i}, d_t^i, z_{t+1}) + \sigma(Q_t + \frac{\partial h}{\partial k'}(k_t^i, k_{t-1}^{b,i}))V_d(k_t^{b,i}, d_t^i, z_{t+1})] \tag{4}$$

The envelope condition for deposits is:

$$V_d(k_{t-1}^{b,i}, d_{t-1}^i, z_t) = E_t \left[(1-\sigma)(-R_t^i) + \beta\sigma R_t^i V_d(k_t^{b,i}, d_t^i, z_{t+1})\right] \tag{5}$$

The envelope condition for capital is:

$$V_k(k_{t-1}^{b,i}, d_{t-1}^i, z_t) = E_t \begin{bmatrix} (1-\sigma)((Z_t^i + Q_t) - \frac{\partial h}{\partial k}(k_t^i, k_{t-1}^{b,i})) \\ +\beta \left(-\sigma \left(Z_t^i + Q_t + \frac{\partial h}{\partial k}(k_t^i, k_{t-1}^{b,i})\right) V_d(k_t^{b,i}, d_t^i, z_{t+1})\right) \end{bmatrix} \tag{6}$$