Weighted Nuclear Norm Minimization with Application to Image Denoising

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Introduction

Background

As a convex relaxation of the low rank matrix factorization problem, the nuclear norm minimization has been attracting significant research interest in recent years. The standard nuclear norm minimization regularizes each singular value equally to pursue the convexity of the objective function. However, this greatly restricts its capability and flexibility in dealing with many practical problems (e.g.,denoising), where the singular values have clear physical meanings and should be treated differently.

What we do

we study the weighted nuclear norm minimization (WNNM) problem, where the singular values are assigned different weights. The solutions of the WNNM problem are analyzed under different weighting conditions. We then apply the proposed WNNM algorithm to image denoising by exploiting the image nonlocal self-similarity.

Algorithm

Algorithm 1 Image Denoising by WNNM

Input: Noisy image $\boldsymbol{y}$

Initialize $\hat{\boldsymbol{x}}^{(0)} = \boldsymbol{y}, \boldsymbol{y}^{(0)} = \boldsymbol{y}$

for k=1:K **do**

  Iterative regularization $\boldsymbol{y}^{(k)} = \hat{\boldsymbol{x}}^{(k-1)} + \delta(\boldsymbol{y} - \hat{\boldsymbol{y}}^{(k-1)})$

  for each patch $\boldsymbol{y}_j$ in $\boldsymbol{y}^{(k)}$ **do**

    Find similar patch group $\boldsymbol{Y}_j$

    Estimate weight vector $\boldsymbol{w}$

    Singular value decomposition $[\boldsymbol{U}, \boldsymbol{\Sigma}, \boldsymbol{V}] = SVD(\boldsymbol{Y}_j)$

    Get the estimation: $\hat{\boldsymbol{X}}_j = \boldsymbol{U}\mathcal{S}_{\boldsymbol{w}}(\boldsymbol{\Sigma})\boldsymbol{V}^T$

  end for

  Aggregate $\boldsymbol{X}_j$ to form the clean image $\hat{\boldsymbol{x}}^{(k)}$

end for

Output: Clean image $\hat{\boldsymbol{x}}^{(K)}$

Algorithms

WNNM

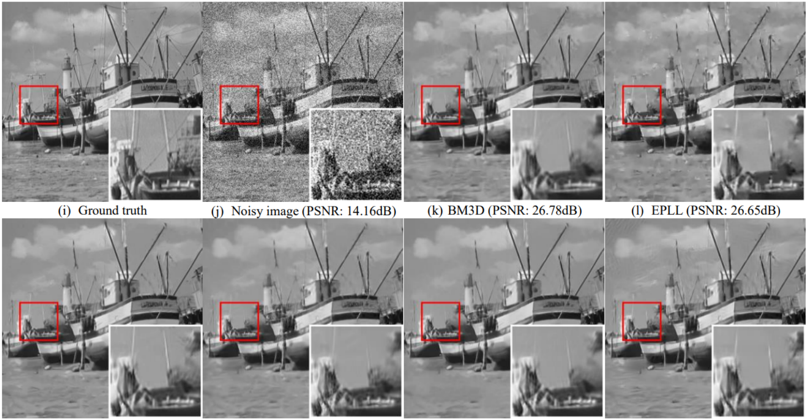
$$\hat{\boldsymbol{X}}_j = \arg\min_{\boldsymbol{X}_j} \frac{1}{\sigma_n^2} \| \boldsymbol{Y}_j - \boldsymbol{X}_j \|_F^2 + \| \boldsymbol{X}_j \|_{\boldsymbol{w},*} \tag{1}$$

$$w_i = c\sqrt{n} / \left(\sigma_i \left(\boldsymbol{X}_j \right) + \varepsilon \right) \tag{2}$$

$$\hat{\sigma}_i \left(\boldsymbol{X}_j \right) = \sqrt{\max \left(\sigma_i^2 \left(\boldsymbol{Y}_j \right) - n\sigma_n^2, 0 \right)} \tag{3}$$

Experiments

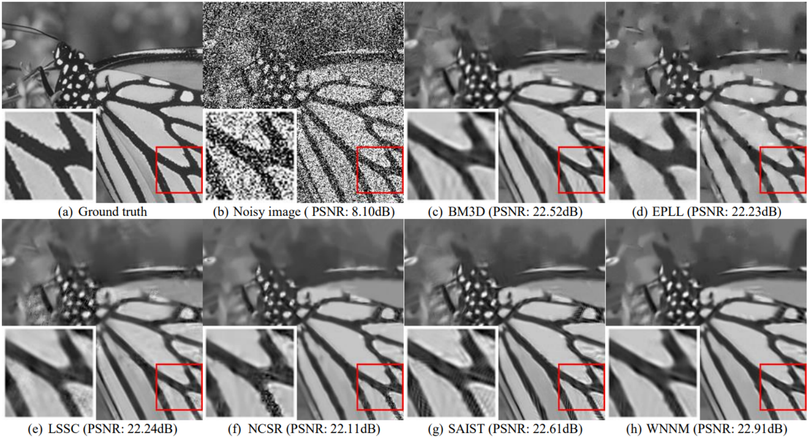
Denoising results on image Boats by different methods (noise level $\sigma_n = 50$).



(i) Ground truth
(j) Noisy image (PSNR: 14.16dB)
(k) BM3D (PSNR: 26.78dB)
(l) EPLL (PSNR: 26.65dB)
(m)LSSC (PSNR: 26.77dB)
(n) NCSR (PSNR: 26.66dB)
(o) SAIST (PSNR: 26.63dB)
(p) WNNM (PSNR: 26.98dB)

Experiments

Denoising results on image Monarch by different methods (noise level $\sigma_n = 100$).



Further

Ongoing Optimization

- how to decide thresholding?

References

- Shuhang Gu, Lei Zhang *Weighted Nuclear Norm Minimization with Application to Image Denoising*