Tsinghua University

Discrete Optimistic

Fall 2024

**Homework 1**

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- **Collaborators:** I finish this homework by myself.

**Problem 1.** (a) When OPT $\geq c$, assume with $\frac{1}{T}$ algorithm $A$ outputs a solution of value at least $s$. $T \in O(poly(n))$ Run algorithm $A$ for $T \cdot n$ iterations. Then with $(1-\frac{1}{T})^{Tn} < e^{-n}$ probability, the algorithm $A$ outputs a solution of value less than $s$.

So with at least $1 - e^{-n}$ probability, the algorithm $A$ outputs a solution of value at least $s$.

(b)

$$s = \mathbb{E}[outputs] \leq \Pr[outputs \geq s - \frac{1}{n^a}] \cdot poly(n) + (1 - \Pr[outputs \geq s - \frac{1}{n^a}]) \cdot (s - \frac{1}{n^a})$$

Then

$$\Pr[outputs \geq s - \frac{1}{n^a}] \geq \frac{\frac{1}{n^a}}{poly(n) - s + \frac{1}{n^a}} = \frac{1}{n^a(poly(n) - s) + 1}$$

Here we end the proof.