Tsinghua University

NUMERICAL ANALYSIS

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**Homework 1**

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- **Collaborators:** I finish this homework by myself.

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**Problem 1.** By Theorem 3.3.6, since Chebyshev polynomial is normal w.r.t. the kernal $\omega = \frac{1}{\square 1-x^2}$

$$\lim_{n\to\infty} \|L_n f - f\|_\omega = 0$$

Noticed that

$$\|L_n f - f\|_\omega = \int_{-1}^{1} \omega(x) |L_n f(x) - f(x)|^2 \, \mathrm{d}x$$

**Problem 2.**

**Problem 3.** That's because, $\bar{Q}_i(x)$, $0 \le i \le n$ is an orthogonormal basis of polynomial space of degree $n$, $\mathbb{P}_n$. So

$$p(x) = \sum_{i=0}^{n} \left\langle p, \bar{Q}_i \right\rangle \bar{Q}_i(x) = \int_a^b p(t)\bar{Q}_i(t)\omega(t)\,\mathrm{d}t\bar{Q}_i(x) = \int_a^b p(t)K_n(t,x)\omega(t)\,\mathrm{d}t$$

**Problem 4.** If $\lim\limits_{n\to\infty} \|L_n f - f\|_\infty = 0$, obviously it holds for $f = 1, \sin x, \cos x$.
Conversely, if it holds for $f = 1, \sin x, \cos x$, then for any $f \in \mathrm{Span}\{1, \sin x, \cos x\}$, $\lim\limits_{n\to\infty} \|L_n f - f\|_\infty = 0$