Tsinghua University

Discrete Optimistic

Fall 2024

Homework 1

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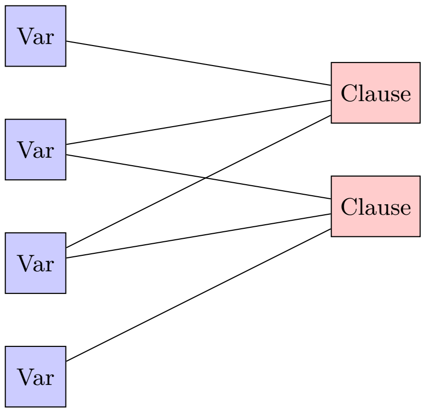
- **Collaborators:** I finish this homework by myself.

Problem 1. (a) Reduce from the instance of MAX-E3SAT-6.



Variables x_i have $\sigma(x_i) \in \{0,1\}$ and Clauses $c_i = x_{j_i}^1 \wedge x_{j_i}^2 \wedge x_{j_i}^3$ have $\sigma(c_i) \in [7]$ to represent the state of c_i.

Therefore, constraint is naturally induced.

In the instance of MAX-E3SAT, the radio of $|U|$ and $|V|$ is 2. So this is a regular Label-Cover Game for $K = 2, L = 7$ and $|V| = 2|U|$.

In the lecture we have proved that this is an instance of $\mathrm{MAX-LC}_{1,1-\epsilon}$ for some ϵ.

So $\mathrm{MAX-LC}_{1,1-\epsilon}$ is NP-Hard.

(b) We actually can construct another graph induced by (a).

We add $\bar{x}_i$ to the graph in (a) and add the induced constraints from c_i contains variable x_i to $\bar{x}_i$.

Here the Label-Cover Game is regular and symmetric.

Then for the $\mathrm{MAX-E3SAT-6}_{1,1-\epsilon}$ instance, the completeness is trivial.

Now we prove the soundness. That's because, if $\mathrm{OPT}_{\mathrm{MAX-E3SAT-6}} \leq 1-\epsilon$, consider any $\sigma : U \to \{0,1\}, V \to [7]$. At least $(1-\epsilon)|V|$ clauses are not satisfied by $\sigma|_U$. For each clause, there exists at least one variable $x_i/\bar{x}_i$ such that do not satisfy the constraint.

So Verifier rejects with probability at least $(1-\epsilon)|V|/2|V| = (1-\epsilon)/2$. So the soundness property holds if we set $\epsilon' = \frac{1+\epsilon}{2}$.

So we prove that $\mathrm{GAP-LC}(K,L)_{1,1-\epsilon}$ is NP-Hard for some ϵ and K, L even if the graph is regular and symmetric.

By Raz' Paralled Repetition Theorem, we can reduce an instance of $\mathrm{GAP}-\mathrm{LC}(K,L)_{1,\delta}$ to the instance of $\mathrm{GAP}-\mathrm{LC}_{1,\exp(-\Omega(\frac{\delta^3 t}{\log t}))}$. Therefore, we finally prove that for any $\eta > 0$, there exists K, L such that $\mathrm{GAP}-\mathrm{LP}(K,L)_{1,\eta}$ is NP-Hard.

Problem 2.

Problem 3.

$$\langle f, g\rangle = \left\langle \sum_{S\subset[n]} \hat{f}(S)\chi_S, \sum_{S\subset[n]} \hat{f}(S)\chi_S \right\rangle = \sum_{S_1,S_2\subset[n]} \hat{f}(S_1)\hat{g}(S_2)\left\langle \chi_{S_1}, \chi_{S_2}\right\rangle = \sum_{S\subset[n]} \hat{f}(S)\hat{g}(S)$$