Numerical Analysis

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目录

拉格朗日和 Hermite 的插值估计

$$s_n f(x) = \frac{a_0}{2} + \sum_{k=1}^{n} a_k \cos kx + b_k \sin kx$$

为 Fourier 级数投影，则

Theorem 0.1. *$T_\lambda f = f(x+\lambda)$.*

$$s_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} T_{-\lambda} L T_\lambda f(x) \mathrm{d}\lambda$$

where L is the projection on triangular space.

$$\|s_n\| = \lambda_n = \frac{1}{\pi}\int_{-\pi}^{\pi} |D_n(t)|\mathrm{d}t > \frac{4}{\pi^2}\log n$$

where $D_n(t) = \dfrac{\sin(n+\frac{1}{2})t}{2\sin\frac{1}{2}t}$

Theorem 0.2 (单调算子定理)**.** 对于 $C[a,b]$ 上的单调线性算子序列 L_n，以下条件等价

1. *$\forall f \in C[a,b]$, $\|L_n f - f\| \to 0$.*

2. *for $f = 1, x, x^2$, $\|L_n f - f\| \to 0$*

3. *$\|L_n 1 - 1\|_\infty \to 0$ and*

$$(L_n \phi_t)(t) \rightrightarrows 0 \textit{ uniformly for } t,\ \phi_t(x) = (t-x)^2$$

证明. Key idea: Find φ_t such that $\varphi_t(t) = 0$,

$$|t-x| \leqslant \alpha\varphi_t(x)$$

when $|t-x|$ large.

try treating as $f_0 = 1, f_1 = x, f_2 = x^2$ functions.

f_0 is a functions $\Rightarrow$ a constant of x, like $f(t)$ can be treated as functions. $\square$

Theorem 0.3. 一型边界条件:

$$\|f - s\|_\infty \leqslant \frac{5}{384} h^4 \|f^{(4)}\|_\infty$$

$$m_j - f_j'| \geqslant \frac{1}{24} h^3 \|f^{(4)}\|_\infty$$

正交多项式内积可以通过 $(xf_n, f_{n-2}) = (f_n, xf_{n-2})$ 转化!

我们有结论

Theorem 0.4. φ_n 关于 ρ 的正交多项式序列。

$$\varphi_n = (\alpha_n x + \beta_n)\varphi_{n-1} + \gamma_n \varphi_{n-2}$$

在考虑正交性的证明的时候，往往要换取别的基证明，或者考虑更一般情况

Legendre 多项式和 Chebyshev 多项式的正交性

Lemma 0.5 (Darboux).

$$\sum_{i=0}^{n} \bar{Q}_i(t)\bar{Q}_i(x) = \lambda_n^{-2} \frac{\bar{Q}_{n+1}(x)\bar{Q}_n(t) - \bar{Q}_n(x)\bar{Q}_{n+1}(t)}{x - t}$$

where λ_n^{-1} *is the coefficient of* x^n *term in* $\bar{Q}_n$

List of Theorems