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Physics-0 Lecture Notes

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1.3 Forces in Nature, Statics

1.3.1 The four fundamental forces in nature

Our universe is governed by the **four fundamental forces** that play a crucial role in how everything interacts with one another. These four forces are

- gravitational force
- electromagnetic force
- weak nuclear force
- strong nuclear force.

Each of these forces plays a unique role in the workings of the universe, from the behavior of objects on a planetary scale to the interactions of subatomic particles.

In quantum theory, the concept of forces as we know it in classical physics becomes somewhat ambiguous. Instead, what we call forces are usually interpreted as the exchange of particles between objects, which is more accurately described as an interaction. Therefore, it is more appropriate to refer to the "**four fundamental interactions**" rather than the "four fundamental forces".

Here is a brief summary of these four fundamental forces:

1. **Gravitational Force.** Gravity is the force that governs the behavior of objects on a large or macroscopic scale. It is responsible for keeping planets in orbit around stars, and causing apples and other objects to fall towards the ground. This force is described by Newton's law of universal gravitation, which states that every particle in the universe is attracted to every other particle with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them:

$$\vec{F} = -\frac{Gm_1m_2}{r^2}\hat{r}. \tag{1.3.1}$$

   Comparing to other forces, the Newtonian constant of gravitation $G \approx 6.67 \times 10^{-11}\text{m}^3/\text{kg}\cdot\text{s}^2$ is relatively small. We will discuss Newton's theory of gravity in detail in section 1.6 of this lecture.

   The best theory for gravity to date is Einstein's theory of General Relativity. It describes gravity as the curvature of spacetime manifold caused by the presence of massive objects. And masses move according to the curvature of the manifold. The basic equation of general relativity is Einstein's equation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}, \tag{1.3.2}$$

where the left-hand side describes the geometry of spacetime and the right-hand side describes the distribution of matter and energy. J. A. Wheeler's famous statement summarizes the core idea of general relativity as *"Spacetime tells matter how to move; matter tells spacetime how to curve."*

2. **Electromagnetic Force.** Electromagnetic force unifies the electric force, which describes how electric charges attract or repel each other and interact with electric fields, and the magnetic force, which describes the interaction of moving charged objects and magnetic fields. For example, the Coulomb's law states that the force between two stationary, electrically charged particles is

$$\vec{F} = \frac{kq_1q_2}{r^2}\hat{r}, \tag{1.3.3}$$

which is very similar to Newton's law of universal gravitation Eq. (1.3.1). Because almost all everyday objects have electric charges, the electromagnetic force plays a significant role in many everyday phenomena, including the operation of electronic devices, the behavior of a compass, the colors we see, and the heat and light we feel from the sun. Our lecture will delve deeper into the topic of electromagnetism in the second half, offering a more detailed and comprehensive discussion on the subject.

3. **Weak Nuclear Force.** The weak interaction plays a critical role in governing the decay of unstable subatomic particles, initiating nuclear fusion reactions in stars like the Sun, and underlying some forms of radioactivity. It involves the exchange of force-carrier particles called W and Z particles, which are relatively heavy with masses around 100 times that of a proton. Interestingly, the weak interaction is the only fundamental interaction known to break parity symmetry. The idea of parity violation was proposed in the mid-1950s by Chen-Ning Yang and Tsung-Dao Lee and later confirmed experimentally by Chien Shiung Wu. The weak force violates parity symmetry, which means that it treats left-handed and right-handed particles differently. As a result, the concept of left-handedness and right-handedness has physical significance in the weak interaction, making them fundamentally different: The nature is left/right handed.

4. **Strong Nuclear Force.** The strong interaction is the force that binds protons and neutrons together in the nucleus of an atom. The strong interaction is responsible for the stability of atomic nuclei, and without it, the nucleus would quickly fall apart. The strong force is mediated by particles known as gluons, which are exchanged between quarks to hold them together. The strong force is very strong at very short distances, but it quickly decreases in strength at longer distances.

These four fundamental forces behave very differently. For example, the interaction range of them is different. The strong nuclear force has a very short range, only acting over distances of the order of a few femtometers (10^{-15} meters). The weak nuclear force has a range of about 10^{-18} meters, while the electromagnetic force and gravity have infinite range. These differences originate from the masses of the mediating particles. As a result, in our everyday life, we can only observe and experience the electromagnetic force and the gravitational force easily.

Besides the interaction ranges, the strengths of these four fundamental forces are also different:

$$\text{strong nuclear force} > \text{electromagnetic force} > \text{weak nuclear force} > \text{gravitational force}.$$

A notable example that illustrates the relative strength of electromagnetic force compared to gravitational force is that we are able to stand on the ground, rather than being pulled directly to the center of the Earth by gravity. This is because the electromagnetic force between the atoms in our feet and the atoms in the ground (only atoms near the feet!) is much stronger than the gravitational attraction between us and the Earth (the whole Earth!).

The history of physics is a history of unification, where theories are developed to explain multiple phenomena with a single framework. For example, Newton's theory of gravity unified the falling of an apple and the motion of the moon around the Earth. Maxwell's theory unified electricity and magnetism. While three of the four fundamental forces have been successfully unified by gauge theory, gravity remains incompatible with quantum theory. Despite decades of effort, it is still unknown how to reconcile Einstein's theory of gravity with quantum mechanics.

1.3.2 Some particular forces

There are several types of forces that we encounter in our daily life quite frequently.

Gravitational force and weight

The gravitational force on Earth is a fundamental force that is exerted by the Earth on all objects with mass, including human beings, animals, and objects. The weight of an object near the surface of the Earth is defined by Newton's law of universal gravitation (1.3.1) as:

$$W = mg = \frac{GMm}{R^2}, \tag{1.3.4}$$

where M is the mass of the Earth, m is the mass of the object, R is the radius of the Earth, and g is the acceleration due to gravity. The direction of the gravitational force is always towards the center of the Earth. From the expression of the free-fall acceleration

$$g = \frac{GM}{R^2}, \tag{1.3.5}$$

we find notably that this acceleration is independent of the mass of the object, meaning that all objects will experience the same acceleration under the influence of gravity near the Earth's surface. Numerically, the free-fall acceleration is approximately $g \approx 9.8\ \mathrm{m/s^2}$.

Normal force

When an object is placed on a table, it experiences a normal force with direction perpendicular to the table. Essentially, this normal force comes from the electromagnetic repulsion between the closely spaced molecules of the object and the molecules of the table. The magnitude of the force is not fixed.

Friction

Friction is a force that opposes motion between two surfaces in contact. When two objects are in contact, the roughness of their surfaces causes them to "stick" together slightly. This makes it harder to move one object relative to the other. Friction can be thought of as a microscopic "drag" force that acts opposite to the direction of motion or the direction of an applied force.

There are two types of friction: *static friction* and *kinetic (or sliding) friction.* Static friction occurs when an object is stationary and is about to be moved. The force of static friction prevents the object from being moved until a sufficient external force is applied. Once the object starts to move, kinetic friction takes over, which is generally less than static friction.

The magnitude of static friction has a maximum value, which is proportional to the normal force. So we have

$$f_s \leq \mu_s F_N, \tag{1.3.6}$$

where μ_s is a dimensionless coefficient of static friction and F_N is the magnitude of the normal force. On the other hand, if the body begins to slide along the surface, the magnitude of the kinetic frictional force is

$$f_k = \mu_k F_N, \tag{1.3.7}$$

where μ_k is the coefficient of kinetic friction. Usually, the maximum static friction is bigger than the kinetic friction: $\mu_s > \mu_k$.

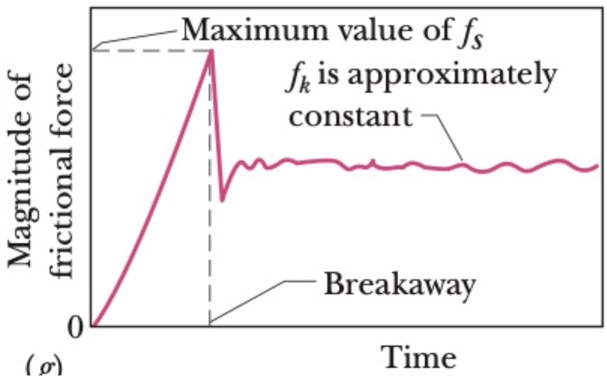

Figure 1.6: Friction.

Tension

Tension is a pulling force that is transmitted through a flexible material, such as a rope, cable, or string, when it is pulled tight by opposing forces. It is a type of force that acts along the length of the material, and its magnitude is proportional to the amount of force being applied. Tension is used in a wide range of applications, such as in bridges, cables, pulleys, and other structures that rely on the strength of flexible materials.

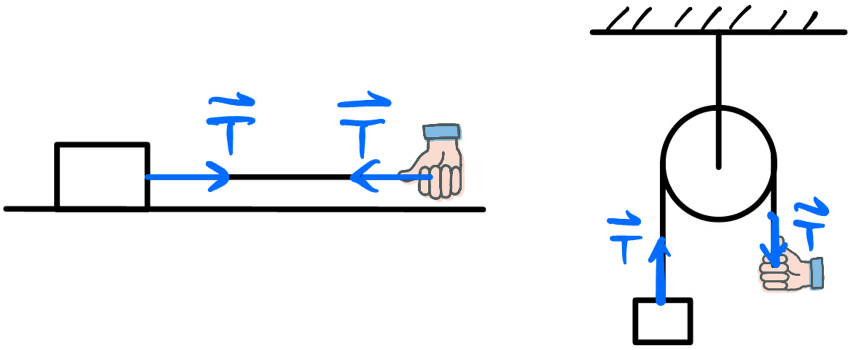

Figure 1.7: Tensions of ropes. (a) Pulling an object on a table. (b) The rope runs around a pulley.

Spring force

Spring force is a force exerted by a compressed or stretched spring, which tends to restore the spring to its equilibrium length. The magnitude of the spring force is proportional to the displacement of the spring from its equilibrium position. It can be expressed mathematically as

$$\vec{F} = -k\vec{x}, \tag{1.3.8}$$

where F is the spring force, k is the spring constant, and x is the displacement from the equilibrium position.

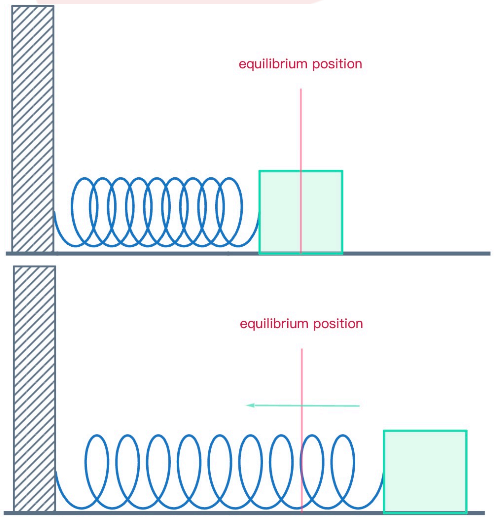

Figure 1.8: Spring force.

1.3.3 Statics I: forces

When an object or system remains unchanged over time in a particular reference frame, it is said to be in a state of equilibrium or static equilibrium. The branch of classical mechanics that studies systems in this state is known as statics.

Definition 8 (Body)**.** *A body refers to a physical object, which can consist of one or multiple parts, that can be treated as a single entity for the purpose of analysis.*

The concept of body is useful for simplifying the description and calculation of the motion, forces, and other properties of objects, especially when the individual components are too numerous or complicated to handle separately. By treating a complex system as a single body, we can apply the principles of mechanics, such as Newton's laws, to describe its behavior.

In the field of statics, we analyze an object by considering the forces acting on it as a whole and by examining each of its individual parts, which can be thought of as distinct bodies. The selection of a particular body to analyze is an engineering decision that is made based on the specific goals of the analysis. For instance, when designing the foundation of a high-rise building, we may consider the entire building as a single body. However, when evaluating the strength of the building's individual components, such as columns and beams, we would examine them separately to ensure that they can effectively perform their intended functions.

Newton's second law states that the net force acting on a body is equal to its mass times its acceleration. Therefore, if a body is not accelerating, the net force acting on it must be zero. In other words, if a body is in static equilibrium, the total force acting on it should be zero:

$$\vec{F}_{\text{tot}} = m\vec{a} = \vec{0}. \tag{1.3.9}$$

So we have the following result:

The first condition of equilibrium. *In order for a body to be in static equilibrium, the net force acting on it must be zero.*

According to the principle of superposition of forces (or addition of vectors in a vector space), the total force acting on an object is the sum of all the individual forces acting on it from the environment. Mathematically, we can express this as:

$$\vec{F}_{\text{tot}} = \sum_a \vec{F}_a, \tag{1.3.10}$$

where $\vec{F}_a$ represents an individual force acting on the body from the environment. If we connect the tails of the arrows representing the force vectors to their heads, then the condition that the total force is zero is equivalent to saying that the resulting path formed by all the arrows is closed (see Figure 1.9).

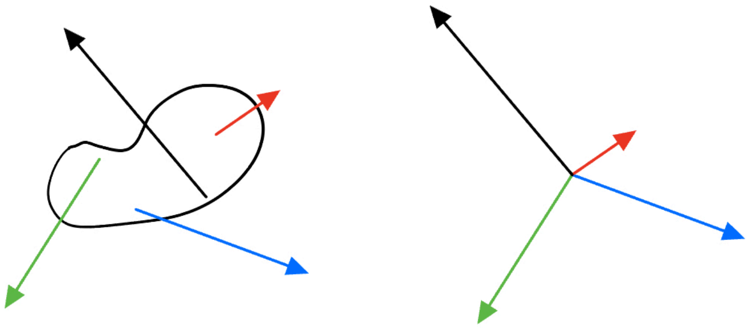

Figure 1.9: The first condition of equilibrium: Total force of a static body should be zero.

When considering a body as consisting of several parts, each part will exert internal interacting forces $\vec{F}_{ij}$ on each other. The total force acting on the body should include all external forces $\vec{F}_{aj}$ and internal forces $\vec{F}_{ij}$:

$$\vec{F}_{\text{tot}} = \sum_j \left(\sum_a \vec{F}_{aj} + \sum_i \vec{F}_{ij} \right). \tag{1.3.11}$$

Because of the Newton's third law/action-reaction law $\vec{F}_{ij} = -\vec{F}_{ji}$, the internal forces $\sum_j \sum_i \vec{F}_{ij}$ sum up to zero. Therefore, we obtain again Eq. (1.3.10) by identifying the external force as the sum of $\vec{F}_a = \sum_j \vec{F}_{aj}$.

Example. *A box is at rest on an inclined plane with an angle of inclination θ. The weight of the box is mg. The static friction coefficient for the plane and the box is μ. Determine the normal force and friction exerted by the inclined plane on the box. What is the maximum angle $\theta_{\max}$ at which the box remains in equilibrium on the plane?*

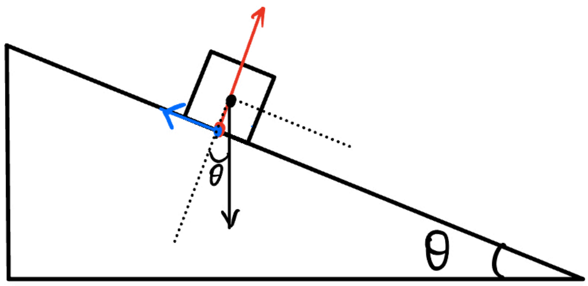

Figure 1.10: Box on an inclined plane.

Solution: We can set up a coordinate system such that the x-axis is parallel to the surface of the inclined plane and pointing down the slope, the y-axis is perpendicular to the surface and pointing upwards, and the origin is at a convenient location such as the position of the box.

The box on a slope experiences three forces: the gravitational force $m\vec{g}$, the normal force $\vec{F}_N$, and the friction $\vec{f}$. To satisfy the condition of total force, $\vec{F}_{\text{tot}} = m\vec{g} + \vec{F}_N + \vec{f}$, being zero, we need to consider two equations along the two axes:

$$f = mg\sin\theta, \tag{1.3.12}$$

$$F_N = mg\cos\theta. \tag{1.3.13}$$

These two equations completely determine the normal force and the frictional force.

Since the static friction should satisfy $f \le \mu F_N$, the maximum angle satisfy $\tan\theta_{\max} = \mu$, so

$$\theta_{\max} = \arctan\mu. \tag{1.3.14}$$

Example. *Calculating forces in the pulley system shown in Figure 1.11.*

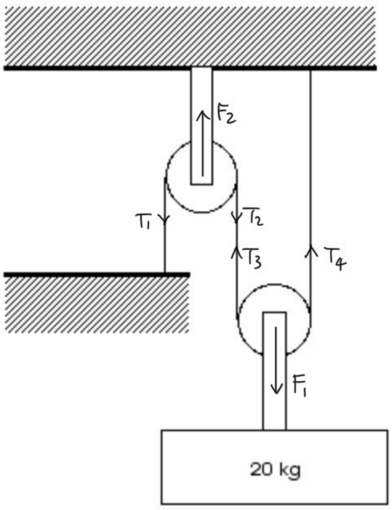

Figure 1.11: A box hanging by ropes.

Example. *If the crate has mass m in Figure 1.12, determine the forces in the boom and in the topping lift.*

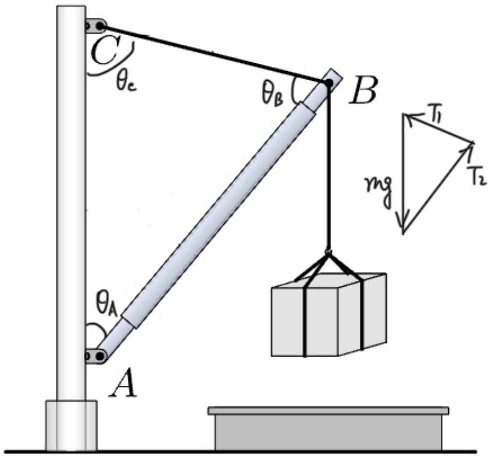

Figure 1.12: Cargo boom.

Solution: The triangle formed by the three forces acting on the static point B is similar to the triangle ABC. Therefore, we can establish the following equation:

$$\frac{T_1}{\sin\theta_A} = \frac{T_2}{\sin\theta_C} = \frac{mg}{\sin\theta_B}, \tag{1.3.15}$$

which allows us to determine all three forces.

Example. *A small ball with mass m is hanging still by ropes (see Figure 1.13). The angles between the first two ropes and the vertical direction are θ_1 and θ_2, respectively. Find the tension in each of the three ropes.*

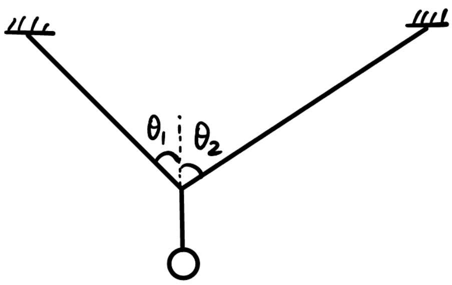

Figure 1.13: A small ball hanging by ropes.

Solution: The total force of the ball should be zero, so we have

$$T_3 = mg. \tag{1.3.16}$$

The total force acting on the connecting point of the three ropes should also be zero. The two equations in horizontal and vertical directions are

$$T_1 \sin\theta_1 = T_2 \sin\theta_2 \tag{1.3.17}$$
$$T_1 \cos\theta_1 + T_2 \cos\theta_2 = T_3. \tag{1.3.18}$$

The solutions for the three equations are

$$T_1 = mg\frac{\sin\theta_2}{\sin(\theta_1+\theta_2)}, \tag{1.3.19}$$
$$T_2 = mg\frac{\sin\theta_1}{\sin(\theta_1+\theta_2)}, \tag{1.3.20}$$
$$T_3 = mg. \tag{1.3.21}$$

Example. *Consider the cable and pulley arrangement shown in Figure 1.14. The lower block has mass M, and the upper block has mass m. The coefficient of friction between the two blocks is μ, and the coefficient of friction between the lower block and the floor is also μ. What is the maximum horizontal force F that can be exerted on the lower block before it moves? And what is the tension T in the cable?*

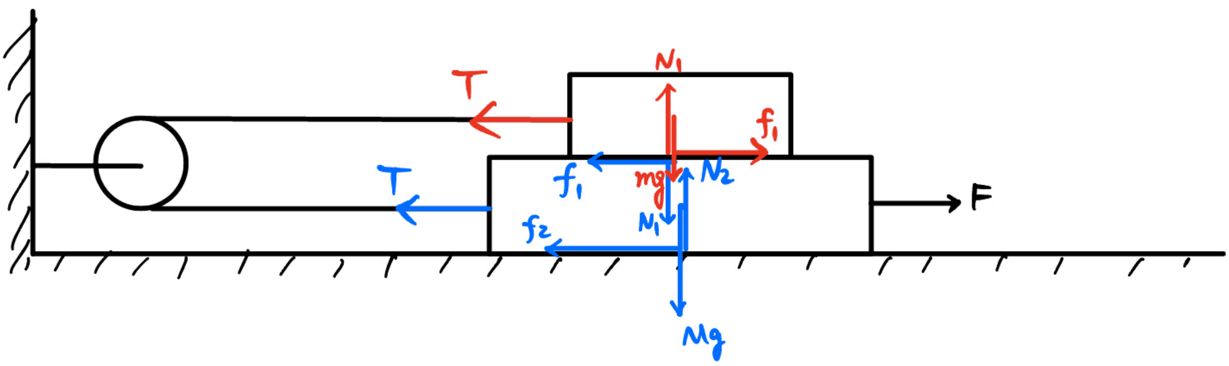

Figure 1.14

Solution: The maximum external force F that can be exerted on the lower block before it moves occurs when the friction forces between the two blocks and between the lower block and the floor are at their maximum values,

$$f_1 = \mu N_1, \tag{1.3.22}$$
$$f_2 = \mu N_2. \tag{1.3.23}$$

Applying the first condition of equilibrium on the upper block, we can set the total forces along the x and y directions to zero, giving us:

$$f_1 - T = 0, \tag{1.3.24}$$
$$N_1 - mg = 0. \tag{1.3.25}$$

For the lower block, we have

$$F - T - f_1 - f_2 = 0, \tag{1.3.26}$$

$$N_2 - N_1 - Mg = 0. \tag{1.3.27}$$

By solving these equations, we can find that the maximum external force F and the tension in the cable T are given by:

$$F = \mu(3m + M)g, \tag{1.3.28}$$

$$T = \mu m g. \tag{1.3.29}$$

1.3.4 Statics II: torques

When analyzing the motion of a body made up of several parts, it can be useful to separate their motions into the motion of the center of mass and the relative motions of the different parts. However, in order to simplify our analysis and ignore the relative motion, we introduce the following concept.

Definition 9 (Rigid body)**.** *A rigid body is a physical object that maintains its shape and size under external forces, meaning that it does not undergo deformation, bending, stretching, or twisting.*

In reality, no object is truly rigid, and all objects can deform under the application of forces. However, for many practical purposes, it is sufficient to treat an object as a rigid body. For a rigid body, we can assume that all points of it move together, so we only need to consider the motion of the object as a whole rather than the motion of each individual point on the object. This simplifies the equations of motion and makes it easier to analyze the object's behavior.

We can ask the question: what is the condition for a rigid body to be at rest or in static equilibrium? The condition that the total force acting on a rigid body is zero is not sufficient to ensure that the body is static. There is another condition related to the moment acting on the body.

Definition 10 (Cross product)**.** *Given two vectors $\vec{A}$ and $\vec{B}$ in three-dimensional Euclidean space, the cross product of them is another vector $\vec{C}$ that is perpendicular to both $\vec{A}$ and $\vec{B}$ and whose magnitude is equal to the product of the magnitudes of $\vec{A}$ and $\vec{B}$ multiplied by the sine of the angle between them. The direction of the resulting vector $\vec{C}$ is given by the right-hand rule.*

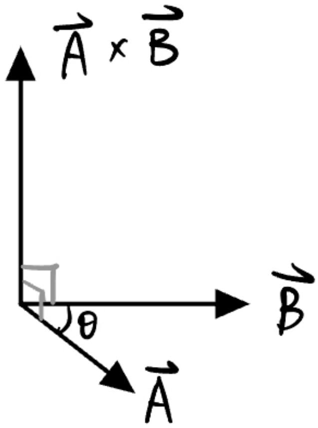

Figure 1.15: Cross product.

The cross product of $\vec{A} = (A_x, A_y, A_z)$ and $\vec{B} = (B_x, B_y, B_z)$ in a coordinate system is given by:

$$\begin{aligned}\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} &= (A_yB_z - A_zB_y)\hat{x} + (A_zB_x - A_xB_z)\hat{y} + (A_xB_y - A_yB_x)\hat{z} \\ &= (A_yB_z - A_zB_y, A_zB_x - A_xB_z, A_xB_y - A_yB_x), \end{aligned} \tag{1.3.30}$$

where $\hat{x}$, $\hat{y}$, and $\hat{z}$ are the unit vectors in the x, y, and z directions, respectively. One can show directly that

$$|\vec{A} \times \vec{B}| = |\vec{A}| \cdot |\vec{B}| \cdot \sin\theta, \tag{1.3.31}$$

where θ is the angle between $\vec{A}$ and $\vec{B}$. If both $\vec{A}$ and $\vec{B}$ are coplanar and lie in the xy plane, their cross product is perpendicular to the plane:

$$\vec{A} \times \vec{B} = (A_x, A_y, 0) \times (B_x, B_y, 0) = (0, 0, A_xB_y - A_yB_x). \tag{1.3.32}$$

Therefore, the cross product of $\vec{A}$ and $\vec{B}$ in the xy plane is a vector that only has a z-component, given by $A_xB_y - A_yB_x$.

The cross product is anticommutative (that is, $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$) and is distributive over addition (that is, $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$). It is not associative, but satisfies the Jacobi identity $\vec{A}\times(\vec{B}\times\vec{C})+\vec{B}\times(\vec{C}\times\vec{A})+\vec{C}\times(\vec{A}\times\vec{B}) = 0$. It also satisfies $\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B})$ and $\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$.

The cross product allows us to define the physical concept of the *moment of a force*, also known as *torque*. The moment of a force measures the rotational effect of a force about a specific point or axis. Unlike a linear force, which causes translational motion of a body, a force that creates a moment must be applied in a way that causes the body to begin to rotate or twist. This happens when the force does not act through the centroid or center of mass of the body.

Definition 11 (Moment of a force/torque)**.** *The moment of force, or torque, is a vector defined as*

$$\vec{\tau} = \vec{r} \times \vec{F}, \tag{1.3.33}$$

where $\vec{r}$ is the position vector from the rotation center to the point where the force $\vec{F}$ is applied.

By definition, the magnitude of $\vec{\tau}$ is

$$|\vec{\tau}| = |\vec{r}| \cdot |\vec{F}| \cdot \sin\theta, \tag{1.3.34}$$

where θ is the angle between the two vectors $\vec{r}$ and $\vec{F}$. The direction of τ is given by the right-hand rule.

One can define the moment of a force with respect to a rotational axis using either $\vec{r} \times \vec{F}_{\perp}$ (see Figure 1.16b) or $(\vec{r} \times \vec{F})_{\parallel}$ (see Figure 1.16c). These two definitions coincide with each other and can be used interchangeably (show that).

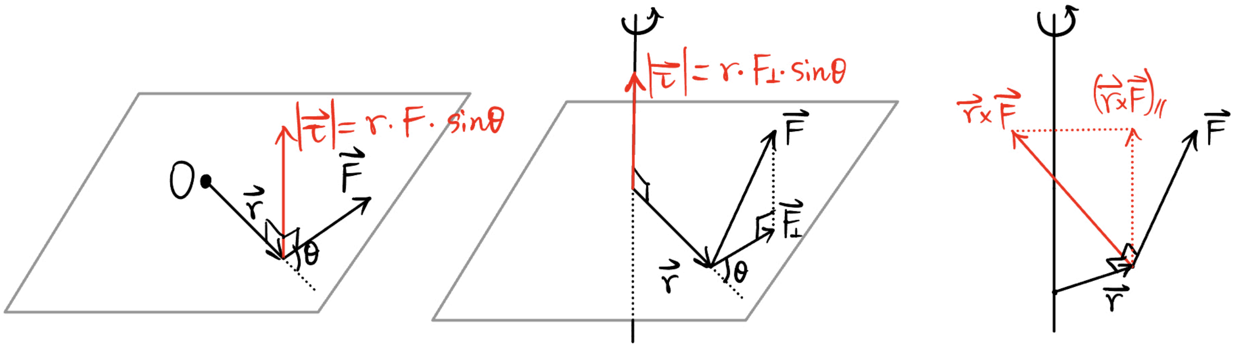

Figure 1.16: Moment of a force/torque with respect to (a) a point or (b) (c) a rotational axis.

The second condition of equilibrium. *In order for a rigid body to be in static equilibrium, the net torque acting on it with respect to any chosen axis or point must be zero.*

If we have several forces acting on different positions of a static rigid body, the total torque can be calculated as:

$$\vec{\tau}_{\text{tot}} = \sum_i \vec{r}_i \times \vec{F}_i. \tag{1.3.35}$$

The second condition of equilibrium states that the total torque $\vec{\tau}_{\text{tot}}$ acting on the rigid body as a vector is zero, which ensures that the body does not rotate.

Example. *Design a steelyard balance to find mass with torque.*

Figure 1.17: A steelyard balance.

Example. *A uniform pole of length L and weight mg is pivoted at one end to a wall. It is held at an angle of θ above the horizontal by a horizontal guy wire attached l units from the end attached to the wall. A load of Mg hangs from the upper end of the pole. Calculate the tension in the guy wire and determine the force exerted on the pole by the wall.*

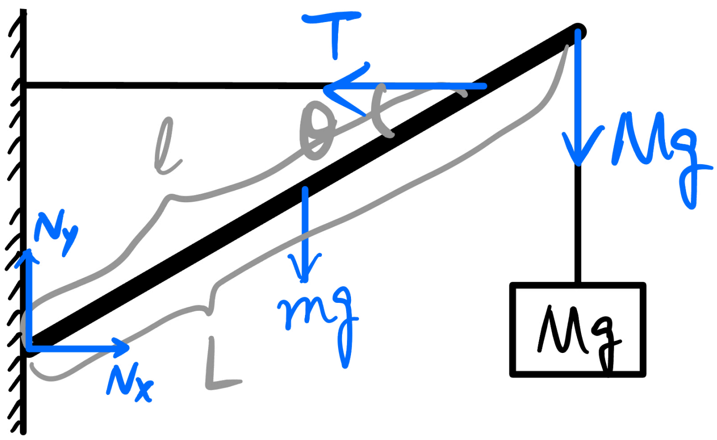

Figure 1.18: A pole at static equilibrium.

Solution: The first condition of equilibrium gives us two equations relating the forces on the pole:

$$N_x - T = 0, \tag{1.3.36}$$
$$N_y - mg - Mg = 0. \tag{1.3.37}$$

However, we have three unknown forces: N_x, N_y, and T. Therefore, we need an additional equation to solve for all three forces. This equation can be obtained from the second condition of equilibrium.

Before calculating torques, we need to choose a reference point around which we will calculate the torques. Choosing the lower left end of the pole is a good choice as it will result in two forces, N_x and N_y, having zero torque. This simplifies the equations. Using this reference point, the total torque can be written as:

$$\tau_{\text{tot}} = mg\frac{L}{2}\cos\theta + MgL\cos\theta - Tl\sin\theta = 0. \tag{1.3.38}$$

Using all the three equations from the first and second conditions of equilibrium, we can solve for the unknowns as:

$$N_y = mg + Mg, \tag{1.3.39}$$

$$N_x = T = mg\frac{L}{2l}\cos\theta + Mg\frac{L}{l}\cos\theta. \tag{1.3.40}$$