Exercise 3.4 $\varphi(y+1-(x-1)^3)=0$. Let $l := y+1-(x-1)^3$

Then for $f \in \mathbb{C}[x,y]$, let $f = ml+n$, where $m \in \mathbb{C}[x,y], n \in \mathbb{C}[x]$. $(*)$

Then $\varphi(f) = \varphi(n)$. $\Rightarrow$

$$\varphi(f) = 0 \Leftrightarrow \varphi(n) = 0 \Leftrightarrow n(t+1) = 0 \Leftrightarrow n = 0$$

Hence $K = l\mathbb{C}[x,y] = (l)$.

By Correspondence theorem, the ideal I of $\mathbb{C}[x,y]$ that contains K gets an ideal I/K of $\mathbb{C}[x,y]/K$. And by $(*)$, $\mathbb{C}[x,y]/K = \mathbb{C}[x]$.

By Prop 11.3.22, $I/K = (\overline{x})$ for some $\overline{x} \in \mathbb{C}[x,y]/K$. Then $I = (x,l)$ since $\forall i \in I, \overline{i} = \overline{x} \cdot \overline{f}, \overline{f} \in \mathbb{C}[x,y]/K \Rightarrow i = (x+k_1)(f+k_2)+k_3 \in (x,l)$ for some $k_1, k_2, k_3 \in K = (l)$.