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Contents

$\mathscr{L} = E - V$. $p_i = \dfrac{\partial \mathscr{L}}{\partial \dot{q}_i}$

$\mathscr{H} = \sum_{i=1}^{n} p_i \dot{q}_i - \mathcal{L}$.

Hamilton's equation:

$$\begin{cases} \dot{q}_i = \dfrac{\partial \mathcal{H}}{\partial p_i} = \{q_i, \mathscr{H}\} \\ \dot{p}_i = -\dfrac{\partial \mathcal{H}}{\partial q_i} = \{p_i, \mathscr{H}\} \end{cases}$$

Action

$$S = \int_{t_0}^{t_1} \left(\sum_i p_i \dot{q}_i - \mathscr{H} \right) \mathrm{d}t = \int_{t_0}^{t_1} \left(\sum_i p_i \mathrm{d}q_i - \mathscr{H} \mathrm{d}t \right)$$

Poisson bracket:

$$\{f, g\} = \sum_i \left(\frac{\partial f}{\partial q_i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q_i} \right)$$

$$\frac{\mathrm{d}}{\mathrm{d}t} f = \{f, \mathscr{H}\} + \frac{\partial f}{\partial t}$$

If $(q, p, t) \mapsto (Q, P, t)$ canonical transformation, with Hamiltonian $\mathscr{H}, \mathscr{K}$ respectively. Then

$$\sum_i p_i \mathrm{d}q_i - \mathscr{H} \mathrm{d}t - \sum_i P_i \mathrm{d}Q_i + \mathscr{K} \mathrm{d}t = \mathrm{d}F$$

Type-1 generating function $F = F_1(q, Q, t)$. Type-2 generating function $F = F_2(q, P, t)$.

$\dfrac{\partial W}{\partial q} = p$.

Theorem 0.1. *Let $W(q, t)$ be the value of the action on the extrmal curve with fixed initial point and endpoint (t, q). Then $W(q, t)$ satisfies Hamilton-Jacobi equation*

$$\frac{\partial W}{\partial t} + \mathscr{H}(q, \frac{\partial W}{\partial q}, t)$$

If $W = W_0(q_1, \cdots, q_n, \alpha_1, \cdots, \alpha_n) - Et$, α_i constant of motions, then $\beta_i = \dfrac{\partial W}{\partial \alpha_i}$. $(q, p) \mapsto (\alpha, \beta)$ canonical transformation with Hamiltonian $\mathscr{K} = 0$. And (q, p) can be represented by (α, β) Action-variation $I_i = \displaystyle\int_{\gamma_i} \sum p_i \mathrm{d}q_i$

Generating function $S = \displaystyle\int^q \sum p_k(\tilde{q}, I)\mathrm{d}\tilde{q}$.

Action-angle $\theta(I, q) = \dfrac{\partial S}{\partial I}$