Tsinghua University

Numerical Analysis

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Homework 1

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First, V has dimension $2N+3$.

Actualy, for $u \in V$, it should satisfy

$$u(x_i) = u(x_{i+1}), u'(x_i) = u'(x_{i+1}, u''(x_i) = u''(x_{i+1}))$$

There are already $3N-3$ linear equations.

u is composed by N polynoimal, so it is in the polynoimal space with free elements $5N$.

Hence the dimension of V is $5N - 3N + 3 = 2N + 3$.

Now for $f_i \in V$, denote $f_{i,j}$ to be $f_i|_{[x_j, x_{j+1}]}$.

We construct basis V such that the length of supp$(f_{i,j}) \le \frac{3}{N}$.

First, take 5 funtion $f_i(x)$ with support in $[x_1, x_4]$ such that

$$f_{i,0}(x_1) = f_{i,1}(x_1), f'_{i,0}(x_1) = f'_{i,1}(x_1), f''_{i,0}(x_1) = f''_{i,1}(x_1)$$

$$f_{i,1}(x_2) = f_{i,2}(x_2), f'_{i,1}(x_2) = f'_{i,2}(x_2), f''_{i,1}(x_2) = f''_{i,2}(x_2)$$

$$f_{i,2}(x_3) = f_{i,3}(x_3), f'_{i,2}(x_3) = f'_{i,3}(x_3), f''_{i,2}(x_3) = f''_{i,3}(x_3)$$

$$f_{i,3}(x_4) = 0, f'_{i,3}(x_4) = 0, f''_{i,3}(x_4) = 0$$

Noticed that There are 12 linear equations but the free coeffients have number of 25, so it can be founded with f_i linearly independent.

Keep this process, similarly we can find functions with support in $[x_j, x_{j+3}]$ linearly independt.

There are actually $3N-3$ funcions. So we have found those $3N-3$ linearly independent functions.