Tsinghua University

DISCRETE OPTIMISTIC

Fall 2024

Homework 1

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- **Collaborators:** I finish this homework by myself.

Problem 1. Assume there exists $x_1, x_2, \cdots, x_{2n+1} \in [a, a + 2\pi) \quad s.t.$

$$\begin{pmatrix} 1 \\ \cos x_i \\ \sin x_i \\ \vdots \\ \cos nx_i \\ \sin nx_i \end{pmatrix}$$

linearly dependent.

$i.e. \quad \exists a_1, \cdots, a_{2n+1} \in \mathbb{R}$, such that

$$\sum_{i=1}^{2n+1} a_i \begin{pmatrix} 1 \\ \cos x_i \\ \sin x_i \\ \vdots \\ \cos nx_i \\ \sin nx_i \end{pmatrix} = 0$$

Since $e^{ix} = \cos x + i \sin x$, we have

$$\sum_{j=1}^{n+1} (a_{2j-1} + a_{2j}) \begin{pmatrix} 1 \\ e^{ix_1} \\ \vdots \\ e^{ix_n} \end{pmatrix}$$

which is impossible since we know that the vandermonde determinant is invertible. (In this equation, $a_{2n+2} = 0$)

Problem 2. s