GRG Vol.6, No.5 (1975), pp. 439 - 447.

RESPONSE OF DOPPLER SPACECRAFT TRACKING TO GRAVITATIONAL RADIATION†

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Revised version received 16 *January* 1975

ABSTRACT

A calculation is made of the effect of gravity waves on the observed Doppler shift of a sinusoidal electromagnetic signal transmitted to, and transponded from, a distant spacecraft. We find that the effect of plane gravity waves on such observations is not intuitively immediate and in fact can have surprisingly different spectral signatures for different spacecraft directions and distances. We suggest the possibility of detecting such plane waves by simultaneous coherent Doppler tracking of several spacecraft.

§(1):

A standard technique for tracking distant spacecraft is precise monitoring of the Doppler shift of a sinusoidal electromagnetic signal, continuously transmitted to the spacecraft and coherently re-transmitted back to earth. We have calculated the contribution to this Doppler shift expected from a passing train of plane gravity waves. The result is surprisingly different from one's first expectation, based on a picture of the passing waves of strain simply changing the separation of the freely falling earth and spacecraft. For gravity wave frequencies ω rad sec^{-1} which are high multiples of $2\pi/T$ (where T is the round-trip travel time of the electromagnetic signals) the effects of spatial strains, delay time effects, and propagation effects owing to the time varying metric along the signal path all seem to resonate. For plane wave trains incident from various directions the resulting 'Doppler' frequency shifts are so characteristic as to give hope that this

† This paper presents the results of one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, under Contract No. NAS7-100, sponsored by the National Aeronautics and Space Administration.

effect might eventually be experimentally distinguishable from the very different effects of intervening plasma clouds (cf, e.g., [*1*]), and non-geodesic motions of the spacecraft. With sufficiently high precision time-keeping, this would make possible gravitational wave observations in the VLF frequency region (periods from 10 sec to 10^4 sec) [*2*].

The present theoretical calculation was motivated by some preliminary spectral analyses of the Doppler residuals in high precision tracking data from the Pioneer 10 spacecraft. These analyses were performed at the Jet Propulsion Laboratory by Allen Joel Anderson†, who originally suggested that a distribution of isotropically incident gravity waves might be detected in this manner using the NASA-JPL Deep Space Net tracking system. Anderson's initial results show definite structure which is generally similar to some of the theoretical curves presented here, but it is certainly not yet possible to attribute them to gravitational waves. Not only should all competing physical effects be further analyzed theoretically, but in particular the stability of even the best available clocks—hydrogen maser frequency standards with accuracies approaching one part in 10^{15}—seems clearly insufficient presently to allow detection of *reasonable* intensities of gravitational radiation. Our results below do, however, show the effects of clock instability to be clearly separable from comparable Doppler shifts resulting from beamed gravity waves.

V. Braginsky [*3*] has recently discussed electron beam stabilization of very high Q superconducting microwave cavity oscillators, a technique which promises an advance in time-keeping to the level of one in 10^{18} or beyond. This would correspond to measurement sensitivity of passing strains of 10^{-18} or less, a level at which gravitational waves from various sources begin to be suspected. The referee has commented on the unlikelihood of there being any monochromatic source approaching this amplitude, as a nearby pair of condensed solar-mass stars, in an orbit of a few thousand kilometers, would be required. Nevertheless, the recent discovery of a binary pulsar may give some support to such a possibility. Braginsky, and Kovács and Thorne [*3*] have discussed 'flyby' radiation from relativistically moving stars in the galactic nucleus; their first calculations suggest VLF bursts of broad-band plane waves incident on the solar system with amplitudes as high as 10^{-17}. A giant black hole in the galactic nucleus, capturing a solar-mass star, is estimated to produce VLF burst of amplitude $\sim 10^{-19}$ [*2*].

Experimental techniques using dual frequencies, and two or more antennas, can discriminate against effects of intervening plasmas; in any event these effects (as well as those from non-geodesic spacecraft motions) are random, depending on the immediate spacecraft environment, and have spectral characteristics very different from those we calculate for gravitational waves. As will be

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seen in the following section, we find that the effect of *plane* gravity waves on Doppler tracking observations has surprisingly distinct and characteristic signatures depending sensitively on spacecraft direction and distances. This strongly indicates that waves from a discrete source could be detected in the presence of large amplitude environmental interference by cross-correlating simultaneous Doppler tracking of several spacecraft. Thus it will certainly be worthwhile to investigate further Anderson's suggested use of Doppler detection schemes for gravity waves, which may well become feasible as time standards are developed.

§(2):

To discuss the effects of weak gravity waves on Doppler signals, we can adapt a method owing to W.L. Burke [4]. Consider a space-time with metric

$$ds^2 = -\,dt^2 + (1 + A)dx^2 + (1 - A)dy^2 + dz^2, \tag{1}$$

where

$$A = A(t - z) \tag{2}$$

is small compared with unity, and to that order describes the strain field of a train of plane gravitational waves propagating in vacuum parallel to the z axis [5]. From (1) we also have

$$g^{\mu\nu} = \begin{bmatrix} -1 & & & \\ & 1 - A & & \\ & & 1 + A & \\ & & & 1 \end{bmatrix} . \tag{3}$$

Any world line at rest in—parallel to the t axis of—these coordinates, i.e., x = constant, y = constant, z = constant, is readily shown to be a geodesic. Hence these coordinates are nicely adapted to discuss signals passing between a congruence of 'almost parallel' but inertial (i.e., freely falling) point observers. These observers are not rigid—the spatial separation between any two will depend on A; viz., there is geodesic deviation and strain as the wave passes. But if at some location A is initially zero and again finally becomes zero, then the separations of adjacent points with fixed coordinates are unchanged.

We will use three Killing vectors in this space-time, which make the calculation of geodesics immediate. They are by inspection

$$\vec{V}_1 = \frac{\partial}{\partial x}\,, \qquad \vec{V}_2 = \frac{\partial}{\partial y}\,, \qquad \vec{V}_3 = \frac{\partial}{\partial z} + \frac{\partial}{\partial t}\,. \tag{4}$$

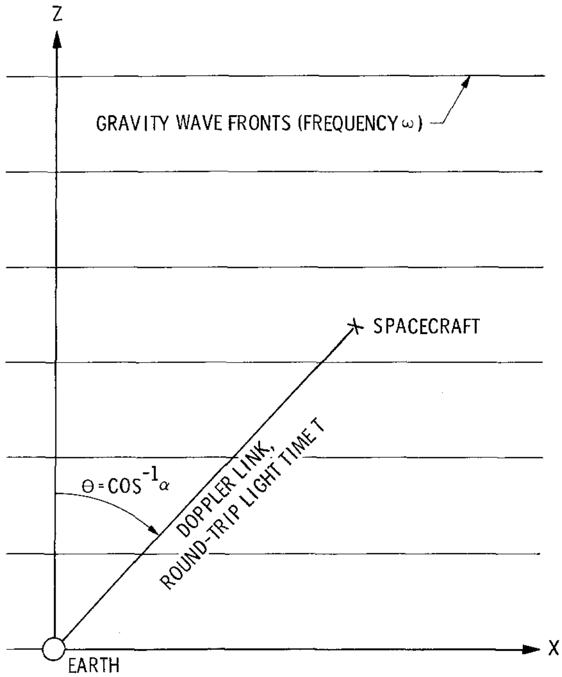

Figure 1 - Tracking Geometry.

Refer now to the diagram figure 1. A null vector at the origin $x = y = z = 0$ can be taken in the form

$$\sigma_0 = (-\nu_0)(dt - (1 + \tfrac{1}{2}A_0)\beta_0 dx - \alpha_0 dz) \tag{5}$$

where $\alpha_0{}^2 + \beta_0{}^2 = 1$ and

$$A_0 = A(t). \tag{6}$$

The null value of $\sigma_{0\mu}\sigma_{0\nu}g^{\mu\nu}$ is immediately verified. We have oriented the polarization for maximum effect by taking this vector without y component. Considering the inertial observer at the origin, whose world line has unit tangent vector $\lambda^\mu = (1,0,0,0)$, we see that $-\vec{\lambda}\cdot\sigma = \nu_0$ is the observed frequency of a photon starting out along σ. An orthonormal triad at the origin is

$${}_1\lambda^\mu = (0,1 - \tfrac{1}{2}A_0,0,0), \quad {}_2\lambda^\mu = (0,0,1 + \tfrac{1}{2}A_0,0), \quad {}_3\lambda^\mu = (0,0,0,1);$$

projecting σ_0 onto this shows that α_0 and β_0 are the locally measured direction cosines of the photon trajectory, with the z and x axes, respectively.

When the photon is received by a transponder at rest at the event labelled 1, we write σ again as

$$\sigma_1 = (-\nu_1)(dt - (1 + \tfrac{1}{2}A_1)\beta_1 dx - \alpha_1 dz), \tag{7}$$

where again $\alpha_1{}^2 + \beta_1{}^2 = 1$ and ν_1 is the frequency observed at 1.

We now use Burke's method to express the fact that σ_1 is obtained by parallelly transporting σ_0 along the photon trajectory:

$$\sigma_{\mu;\nu}\sigma^\nu = 0, \tag{8}$$

from which

$$(\sigma_\mu \, {}_iV^\mu)_{;\nu}\sigma^\nu = 0, \qquad i = 1,2,3, \tag{9}$$

since the ${}_i\vec{V}$ satisfying Killing's equation,

$${}_iV_{(\mu;\nu)} = 0. \tag{10}$$

Equation (9) just requires the constancy of the components of σ on the three ${}_i\vec{V}$:

$$\nu_0(1 + \tfrac{1}{2}A_0)\beta_0 = \nu_1(1 + \tfrac{1}{2}A_1)\beta_1, \tag{11}$$

$$\nu_0(1 - \alpha_0) = \nu_1(1 - \alpha_1). \tag{12}$$

From these follows the result†

$$\frac{\nu_1 + \nu_0}{\nu_0} = \tfrac{1}{2}(1 + \alpha)(A_0 - A_1). \tag{13}$$

In this we use α for either α_0 or α_1, which differ only in first order. To the same approximation we write the coordinates of point 1 as $x = \frac{1}{2}\beta T$, $y = 0$, $z = \frac{1}{2}\alpha T$, where T is the round-trip light time, so

$$A_1 = A(t + (1 - \alpha)\tfrac{1}{2}T). \tag{14}$$

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† These expressions for the 'one-way' influence of gravity waves on red shift (or Doppler effect) have been derived by W.J. Kaufmann [6]. He also discusses possible sources in connection with earth-based observations using the Mössbauer effect.

Consider now the transponded photon, at 1, immediately re-emitted along the reverse trajectory

$$\sigma_1' = (-\nu_1)(dt + (1 + \tfrac{1}{2}A_1)\beta_1 dx + \alpha_1 dz), \tag{15}$$

and finally observed at the origin $x = y = z = 0$ at time $t = T$:

$$\sigma_2' = (-\nu_2)(dt + (1 + \tfrac{1}{2}A_2)\beta_2 dx + \alpha_2 dz). \tag{16}$$

Parallel transport gives

$$\nu_1(1 + \tfrac{1}{2}A_1)\beta_1 = \nu_2(1 + \tfrac{1}{2}A_2)\beta_2, \tag{17}$$

$$\nu_1(1 + \alpha_1) = \nu_2(1 + \alpha_2), \tag{18}$$

so again in lowest order

$$\frac{\nu_2 - \nu_1}{\nu_1} = \tfrac{1}{2}(1 - \alpha)(A_1 - A_2), \tag{19}$$

where

$$A_2 = A(t + T). \tag{20}$$

From these, the observer at 0 can calculate the Doppler shift of the returning photons:

$$\frac{\Delta\nu}{\nu} \equiv \frac{\nu_2 - \nu_0}{\nu_0} = \tfrac{1}{2}(A_1 - A_2)(1 - \alpha) + \tfrac{1}{2}(A_0 - A_1)(1 + \alpha)$$

$$= \tfrac{1}{2}(1 + \alpha)A(t) - \alpha A(t + (1 - \alpha)\tfrac{1}{2}T) - \tfrac{1}{2}(1 - \alpha)A(t + T). \tag{21}$$

If $\alpha = \cos 0° = 1$, the photons were sent out parallel to the gravity wave normals, and $\Delta\nu_{||}/\nu = 0$. If $\alpha = \cos 90°$, the photons propagate normally to the gravitons, and $\Delta\nu_{\perp}/\nu = \tfrac{1}{2}\{A(t) - A(t+T)\}$, a result obtained previously by R.W. Davies [7].

Equation (21) is a somewhat surprising linear transformation of a signal $A(t)$ to an observation $\Delta\nu(t)$, very different from other effects encountered in spacecraft tracking in its dependence on α and T. The cross-correlation of two such signals, observed in simultaneous Doppler tracking of two spacecraft, will strongly discriminate against interfering effects randomly applied to the two.

§(3):

We must also consider the effects of 'clock jitter' at the receiving station. If we consider a particular Fourier component of the gravity wave of amplitude G, setting its phase arbitrarily with t_0:

$$A(t) = G\sin\omega(t + t_0 - \tfrac{1}{2}T), \tag{22}$$

we have, first for the case of normal propagation, $\alpha = 0$,

$$\frac{\Delta\nu_\perp}{\nu} = -G\cos\omega(t + t_0)\sin\omega\tfrac{1}{2}T. \tag{23}$$

Squaring and averaging over t_0 gives the response function—the Fourier transform of the autocorrelation function of the signal—or the spectral 'energy density' that would be observed, in the Doppler measurements in the presence of incoherent gravity waves (normally incident)

$$\overline{\left|\frac{\Delta\nu_\perp}{\nu}\right|^2} = \tfrac{1}{2}G^2\sin^2\omega\tfrac{1}{2}T. \tag{24}$$

This spectrum is exactly the same as that predicted to result from random variation of the reference frequency standard (cf. [8]); it has minima when $\omega\frac{1}{2}T = n\pi$. For such observations clearly clock stability is required to be better than the desired strain sensitivity.

The interesting cases are thus probably the more general ones. For a Fourier component of frequency ω, probed with photons of direction cosine α, we find

$$\begin{aligned}\frac{\Delta\nu}{\nu} &= \tfrac{1}{2}(1 + \alpha)G\sin\omega(t + t_0 - \tfrac{1}{2}T) - \alpha G\sin\omega(t + t_0 - \alpha\tfrac{1}{2}T)\\ &\qquad - \tfrac{1}{2}(1 - \alpha)G\sin\omega(t + t_0 + \tfrac{1}{2}T)\\ &= G\cos\omega(t + t_0)\{\alpha\sin\alpha\omega\tfrac{1}{2}T - \sin\omega\tfrac{1}{2}T\}\\ &\qquad + \alpha G\sin\omega(t + t_0)\{\cos\omega\tfrac{1}{2}T - \cos\alpha\omega\tfrac{1}{2}T\},\end{aligned} \tag{25}$$

so

$$\begin{aligned}\overline{\left|\frac{\Delta\nu}{\nu}\right|^2} &= \tfrac{1}{2}G^2\{\alpha^2 + \sin^2\omega\tfrac{1}{2}T + \alpha^2\cos^2\omega\tfrac{1}{2}T\\ &\qquad - 2\alpha\sin\omega\tfrac{1}{2}T\sin\alpha\omega\tfrac{1}{2}T - 2\alpha^2\cos\omega\tfrac{1}{2}T\cos\alpha\omega\tfrac{1}{2}T\}.\end{aligned} \tag{26}$$

With $G^2 = 1$, the right hand sides of equation (26) are plotted in figure 2 for various α. The spectra are very different from that at

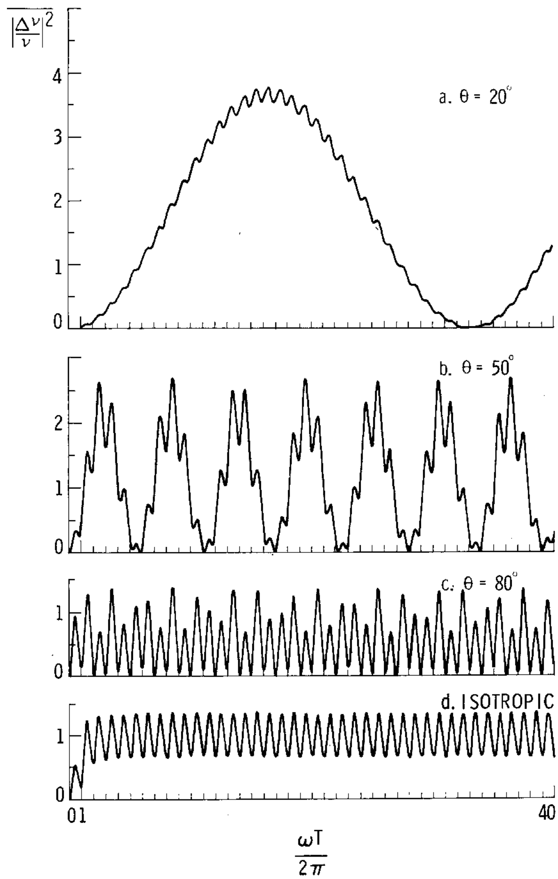

Figure 2 - Spectral response to plane waves incident at: (a) $\theta = 20°$; (b) $\theta = 50°$; (c) $\theta = 80°$; or (d) isotropically.

$\alpha = 0$. A broad band signal of known spectral content could easily be distinguished from local clock jitter at a considerably higher level. Simultaneous observation of several spacecraft will of course again greatly improve the sensitivity for observation of an unknown wave.

If, finally, we inquire about the possibility of observing the spectral content (or energy) of gravitational waves randomly incident from all directions, we average (26) over α, isotropically, to get

$$\overline{\left|\frac{\Delta\nu}{\nu}\right|^2} = \tfrac{1}{2}G^2\left\{\frac{(\omega\tfrac{1}{2}T)^2 - 3}{(\omega\tfrac{1}{2}T)^2} - \frac{(\omega\tfrac{1}{2}T)^2 + 3}{3(\omega\tfrac{1}{2}T)^2}\cos\omega T + \frac{2}{(\omega\tfrac{1}{2}T)^3}\sin\omega T\right\}. \quad (27)$$

Equation (27) is also plotted in figure 2. It again resembles the case $\alpha = 0$. We conclude that in that event again clock stability is required to be better than the desired strain sensitivity (say, $\leqslant 10^{-18}$). A source for such an isotropic signal might be found in the known galactic double star population.

ACKNOWLEDGMENTS

We thank Marcia Neugebauer for her kindness in setting up the computations required for these figures.

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