MPC with Improved Gradient Based Parameter Estimator for Lane Change Maneuver in Road Section with Uncertain Road Grade

li zhaolun

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In practical applications, uncertain environmental parameters can lead to inaccuracies in predictions and sub-optimal control performance of the MPC. Therefore, it is crucial to incorporate a parameter estimator in the MPC framework to overcome this challenge. This estimator can provide online estimation of the uncertain environmental parameters, ensuring accurate predictions and optimal control performance. This need is particularly crucial in safety-critical applications such as autonomous driving, where precise control is essential to avoid accidents and ensure passengers' safety.

To address this issue, an improved gradient-based approach is proposed for adapting to the uncertain environmental parameters, inspired by the gradient-based parameter adaptation method traditionally used for linear systems (as discussed in Chapter 3 of [1]). Specifically, we assume that the coefficient of adhesion μ and the rolling resistance γ are constants, while the ego vehicle navigates through a road section with varying road grade ϕ. In this case, an improved gradient adaptation law can be utilized to estimate the uncertain variable $a(\phi)$, which is a function of the uncertain road grade ϕ.

1 MPC Design for Double Integrator

The dynamic bicycle model is a widely used mathematical model to represent the motion of a vehicle. The model assumes that the vehicle can be approximated as a two-wheeled bicycle, where the vehicle's motion is described by its position, velocity, and orientation. There are several benefits of using the dynamic bicycle model to represent an autonomous vehicle. Firstly, the dynamic bicycle model provides a realistic representation of the motion of a vehicle, which is important for accurately predicting the vehicle's behavior in different situations. The model takes into account the vehicle's dynamic properties, such as its mass, moment of inertia, and tire characteristics, which can affect the vehicle's motion. Additionally, the dynamic bicycle model is a relatively simple model that can be easily implemented in a simulation environment, making it computationally efficient and allowing for faster simulations.

Thus, we will use the dynamic bicycle model to represent both the prediction model and the actual model of the vehicle in the following programs. The dynamic bicycle model in a road section with uncertain road grade $\phi(k)$ is represented by (see Figure 1),

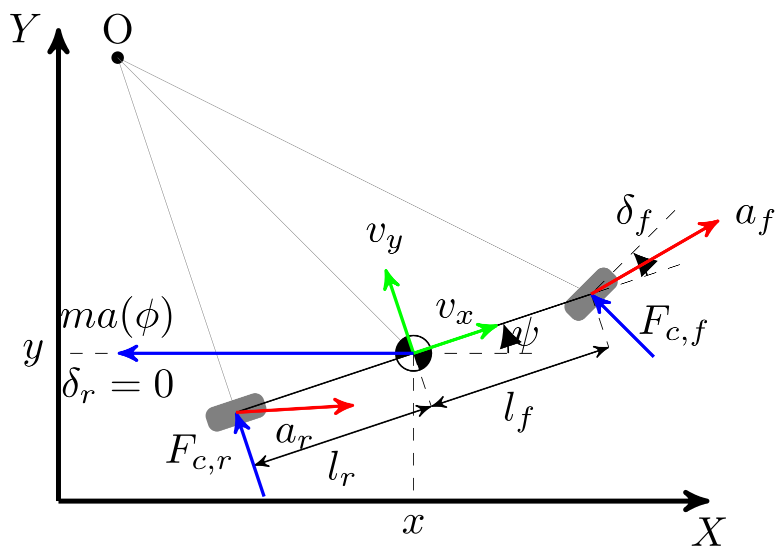

Figure 1: Dynamic Bicycle Model on a slope

$$
\begin{aligned}
v_x(k+1) &= v_x(k) + [\dot{\psi}(k)v_y(k) + a_x(k) - a(\phi(k))\cos(\psi(k))]T_s, \\
v_y(k+1) &= v_y(k) + [-\dot{\psi}(k)v_x(k) + \frac{1}{m}(F_{c,f}(k) + F_{c,r}(k)) + a(\phi(k))\sin(\psi(k))]T_s, \\
\dot{\psi}(k+1) &= \dot{\psi}(k) + [\frac{1}{I_Z}(l_f F_{c,f}(k) - l_r F_{c,r}(k))]T_s, \\
\psi(k+1) &= \psi(k) + \dot{\psi}(k)T_s, \\
X(k+1) &= X(k) + [v_x(k)\cos(\psi(k)) - v_y(k)\sin(\psi(k))]T_s, \\
Y(k+1) &= Y(k) + [v_x(k)\sin(\psi(k)) + v_y(k)\cos(\psi(k))]T_s,
\end{aligned} \tag{1}
$$

where $a_x(k)$ represents the longitudinal acceleration of the vehicle at time step k, while $\psi(k)$ denotes the yaw angle at that time step. The longitudinal and lateral speeds in the vehicle body frame are denoted as $v_x(k)$ and $v_y(k)$, respectively. The vehicle's position on the ground frame is represented by $X(k)$ for longitudinal and $Y(k)$ for lateral position.

The vehicle's mass and yaw inertia are denoted by m and I_Z, respectively. The distances from the vehicle's center of mass to the front and rear axles are denoted by l_f and l_r, respectively. The lateral forces generated by the front and rear wheels can be denoted as $F_{c,f}(k)$ and $F_{c,r}(k)$, respectively. The sampling time is denoted by T_s.

Apart from that, $a(\phi(k))$ is a function of the uncertain road grade $\phi(k)$ and is presented as follows,

$$a(\phi(k)) = g\left(\sin(\phi(k)) + \gamma\cos(\phi(k))\right), \tag{2}$$

where $g = 9.8$ is the gravitational acceleration and $\gamma = 0.006$ is the rolling resistance coefficient.

In addition, the magic formula from [2] is used to calculate the lateral forces $F_{c,f}(k)$ and $F_{c,r}(k)$ on the front and rear wheels as

$$F_{c,i}(k) = D\sin(C\arctan(Ba_i(k) - E(Ba_i - \arctan(Ba_i(k))))) \tag{3}$$

where $i \in \{f, r\}$ and B, C, D, E are coefficients depending on constants b_n , $n \in [1, 2, \ldots, 6]$ which are defined as,

$$\begin{aligned} D &= 0.8\mu F_{z,i}(k), \\ C &= 1.3, \\ BCD &= b_1\sin(b_2\arctan(b_3 * F_{z,i}(k)))/10, \\ B &= \frac{BCD}{CD}, \\ E &= b_4F_{z,i}^2(k) + b_5F_{z,i}(k) + b_6, \end{aligned} \tag{4}$$

where μ is the road coefficient of adhesion and $F_{z,i}(k) \in \{F_{z,f}(k), F_{z,r}(k)\}$ and $a_i(k) \in [a_f(k), a_r(k)]$. The slipping angles $a_i(k) \in [a_f(k), a_r(k)]$ for the front and rear wheels are defined as,

$$\begin{aligned} a_f(k) &= \delta_f(k) - \arctan\frac{v_y(k) + l_f\dot{\psi}(k)}{v_x(k)}, \\ a_r(k) &= -\arctan\frac{v_y(k) - l_r\dot{\psi}(k)}{v_x(k)}, \end{aligned} \tag{5}$$

and $F_{z,i}(k) \in [F_{z,f}(k), F_{z,r}(k)]$ are defined as,

$$\begin{aligned} F_{z,f}(k) &= \frac{mgl_r}{l_f + l_r}\cos(\phi(k)), \\ F_{z,r}(k) &= \frac{mgl_f}{l_f + l_r}\cos(\phi(k)). \end{aligned} \tag{6}$$

It should be noted that the term $\cos(\phi(k))$ is included to account for the normal forces applied on the slope and constants b_n, $n \in \{1, \ldots, 6\}$ are presented in Table 1.

The downloaded file (*e.g.* Lane Change in Road Section with Uncertain Road Grade (IG)) contains a function named 'vehicle_dynamic_model_6_states_theta_magic_formula', which serves as the prediction model used in the MPC. This function is based

Table 1: Values of Constant b_n

$b_1 = 300$	$b_2 = 1.82$	$b_3 = 0.208$	$b_4 = 0$	$b_5 = -0.354$	$b_6 = 0.707$

on the dynamic bicycle model and its specific parameters are elaborated upon in the comments.

It is important to acknowledge that the accuracy of the normal forces ($F_{z,f}$ and $F_{z,r}$) utilized in the prediction model is not 100% accurate. This is due to the improved gradient adaptation law's inability to estimate the uncertain uncertain road grade $\phi(k)$ directly. To address this, we have extracted the term $\cos(\phi(k))$ from the normal force calculation.

The function that represents the controlled vehicle's actual model has been named 'vehicle_dynamic_model_6_states_phi_magic_formula', and further elaboration can be found within the comments inside the function. Another function, namely 'get_Fy0', has been designed to calculate the lateral force acting upon the front and rear wheels, utilizing the magic formula tire model. Detailed information about this function can also be found within its respective comments.

1.1 Improved Gradient Based Parameter Estimator

The adaptation law is expressed as follows:

$$a(\hat{\phi}(k+1)) = a(\hat{\phi}(k)) + \alpha_{gain} \frac{[\hat{v}_x(k+1) - v_x(k+1)] \cos(\psi(k)) T_s}{1 + \alpha_{gain} \cos^2(\psi(k)) T_s^2} \tag{7}$$

where $a_{gain} > 0$ is the adaptation gain, and $\hat{v}_x(k+1)$ is the predicted longitudinal speed which is defined as,

$$\hat{v}_x(k+1) = v_x(k) + [\dot{\psi}(k) v_y(k) + a_x(k)] T_s - a(\hat{\phi}(k)) \cos(\psi(k)) T_s. \tag{8}$$

Theorem 1. *If the road grade is assumed to be a constant such that $\phi(k+1) = \phi(k)$, then the adaptation law from (7) guarantees convergence of the estimated road grade to its true value.*

Proof. The rearranged equation for longitudinal speed $v_x(k+1)$ from (1) is presented as,

$$v_x(k+1) = v_x(k) + [\dot{\psi}(k) v_y(k) + a_x(k)] T_s - a(\phi(k)) \cos(\psi(k)) T_s. \tag{9}$$

Then it is easy to get the following equation from (8)-(9) as,

$$\hat{v}_x(k+1) - v_x(k+1) = [a(\phi(k)) - a(\hat{\phi}(k))] \cos(\psi(k)) T_s \tag{10}$$

Since the unknown road grade is assumed to be a constant (*i.e.*, $\phi(k+1) = \phi(k)$) and two new variables are introduced as $\theta(k) = a(\phi(k))$, $\hat{\theta}(k) = a(\hat{\phi}(k))$, the uncertain variable $a(\phi)$ would follow the equation, $a(\phi(k+1)) = a(\phi(k)) = \theta$. Then based on (10), the adaptation law from (7) can be rewritten as,

$$\hat{\theta}(k+1) = \hat{\theta}(k) + \alpha_{gain} \frac{[\theta - \hat{\theta}(k)] \cos^2(\psi(k)) T_s^2}{1 + \alpha_{gain} \cos^2(\psi(k)) T_s^2} \tag{11}$$

The estimation error is defined as $\tilde{\theta}(k) = \hat{\theta}(k) - \theta$ (*i.e.*, $\tilde{\theta}(k) = \alpha(\hat{\phi}(k)) - \alpha(\phi(k))$). Based on this definition, if θ is deducted from both sides of (11), the new equation is presented as,

$$\begin{aligned}\tilde{\theta}(k+1) &= \tilde{\theta}(k) - \frac{\alpha_{gain}\tilde{\theta}(k)\cos^2(\psi(k))T_s^2}{1+\alpha_{gain}\cos^2(\psi(k))T_s^2} \\ &= \frac{\tilde{\theta}(k)}{1+\alpha_{gain}\cos^2(\psi(k))T_s^2},\end{aligned} \tag{12}$$

From (12), it is clear that the estimation error $\tilde{\theta}(k)$ decreases with every iteration of the adaptation. Therefore, the convergence of the improved gradient adaptation law is proved. □

Remark 1. *Although the proof of convergence is given based on the assumption that the unknown road grade is a constant, the proposed improved gradient adaptation law can still adapt to time-varying road grades.*

The improved gradient based parameter estimator is included the program named 'lane_change_nominal_MPC_with_IG_adaptation_slope' from line 151 to line 155 (see the code below).

```
    zeta=cos(xlast(4))*Ts_sim;                           % use zeta to simplified the equation for the parameter adaptation law.
    vx_hat_up=get_vx_up(xlast,u,Ts_sim,theta_hat);       % vx_hat(k+1)=vx_hat_up;
    vx_up=x(1);                                          % vx(k+1) = vx_up;
    theta_hat_up=theta_hat+alpha*(vx_hat_up-vx_up)*zeta/(1+alpha*zeta^2); % Calculate a(phi(k+1))=theta_hat(k+1);
    theta_hat=theta_hat_up;                              % Update a(phi(k))
```

Note that the function 'get_vx_up' is used to calculate $\hat{v}_x(k+1)$ (see the code below).

```
function vx_up=get_vx_up(xlast,u,Ts,theta_hat)
    vx_dot_last=xlast(3)*xlast(2)+u(1)-theta_hat*cos(xlast(4)); % vx_hat_dot(k)=psi_dot(k)*vy(k)+ax(k)-a(phi_hat(k))*cos(psi(k));
    vx_up= xlast(1)+vx_dot_last*Ts;                                  % vx_hat(k+1)=vx_hat(k)+vx_hat_dot(k)*Ts;
end
```

1.2 MPC Design for this scenario

Our objective is to enable the controlled vehicle to achieve and maintain a constant longitudinal speed and a constant lateral position, while utilizing the minimum possible inputs and jerks (derivation of the inputs) for this purpose. Thus the MPC can be formulated as,

$$\min_{u(k)} J = \sum_{k=1}^{N}\left[\begin{array}{c} Q\frac{(v_x^r(k+1)-v_x(k+1))^2}{25^2} + Q\frac{(Y^r(k+1)-Y(k+1))^2}{3.5^2} \\ R\frac{a_x^2(k)}{(2\sqrt{2})^2} + R\frac{a_y^2(k)}{(\frac{\pi}{6})^2} + R\frac{\Delta a_x^2(k)}{(1.5)^2} + R\frac{\Delta\delta_f^2(k)}{(\frac{\pi}{12})^2}\end{array}\right] \tag{13}$$

subject to

$$
\begin{aligned}
x(k+1) &= f(x(k), u(k), \theta(k)), \\
x(k+1) &\sim \mathcal{X}, \\
u(k) &\sim \mathcal{U}_{in}, \\
\Delta u(k) &\sim \Delta\mathcal{U}_{in},
\end{aligned}
\tag{14}
$$

where f is the dynamic bicycle model from (1) to (6). Additionally, $\mathcal{X}$, $\mathcal{U}_{in}$ and $\Delta\mathcal{U}_{in}$ represent the admissible sets of value for predicted states, system inputs and the derivative values of the system inputs, details of which are given in Table 2.

Table 2: Upper and Lower bound for the system states and inputs

$v_x \in$[0,30]m/s	$v_y \in$[-5,5]m/s	$Y \in$[-2,2]m	$a_x \in [-4, 4]$m/s^2
$\delta_f \in [-\frac{\pi}{18}, \frac{\pi}{18}]$(rad)	$\Delta a_x \in [-3, 1.5]$m/s^3	$\Delta\delta_f \in [-\frac{\pi}{36}, \frac{\pi}{36}]$(rad/s)	

It is noteworthy that the longitudinal speed reference is set to $v_x^r(k+1) = 30$ (m/s) and the lateral position reference is set to $Y^r(k+1) = 1.75$ (m). The initial states of the vehicle, denoted by $[v_x(0), v_y(0), \dot{\psi}(0), \psi(0), X(0), Y(0)]^T$, are set to $[20, 0, 0, 0, 0, -1.75]^T$. Additionally, the uncertain road grade $\phi(k)$ followed a serious of step changes detailed as follows,

$$
\phi(t) = \begin{cases} \frac{\pi}{72}\text{(rad)}, & \text{if } t < 5, \\ -\frac{\pi}{36}\text{(rad)}, & \text{if } 5 \leq t < 10, \\ \frac{\pi}{18}\text{(rad)}, & \text{if } 10 \leq t \leq 15. \end{cases}
\tag{15}
$$

Note that $\phi(k)$ represents the discrete value of $\phi(t)$ at the sampling step k.

The main program which utilizes the MPC controller to conduct simulation can be found in 'lane_change_nominal_MPC_with_IG_adaptation_slope', alongside the controller and the improved gradient based parameter estimator. For more details about this program, please refer to the comments left inside.

It is important to note that the prediction model inputs `U` and states `X` are arranged in the following manner,

$$
\begin{aligned}
U &= \begin{bmatrix} a_x(1) & a_x(2) & \dots & a_x(N) \\ \delta_f(1) & \delta_f(2) & \dots & \delta_f(N) \end{bmatrix}, \\
X &= \begin{bmatrix} v_x(0) & v_x(1) & v_x(2) & \dots & v_x(N) \\ v_y(0) & v_y(1) & v_y(2) & \dots & v_y(N) \\ \dot{\psi}(0) & \dot{\psi}(1) & \dot{\psi}(2) & \dots & \dot{\psi}(N) \\ \psi(0) & \psi(1) & \psi(2) & \dots & \psi(N) \\ X(0) & X(1) & X(2) & \dots & X(N) \\ Y(0) & Y(1) & Y(2) & \dots & Y(N) \end{bmatrix},
\end{aligned}
\tag{16}
$$

By executing this program, we can get the data in 'nominal_mpc_IG_lane_change_slope_compare_no'.

A program that implements the same conventional MPC but lacks the improved gradient-based parameter estimator is also included in the downloadable files. It is called 'lane_change_nominal_MPC_with_slope_no_adapt'. Executing this program generates the data stored in 'nominal_mpc_lane_change_slope_no_adapt'. The file named 'Generate_plot_compare_IG_no' utilizes data from two sources: 'nominal_mpc_IG_lane_change_slope_compare_no' and 'nominal_mpc_lane_change_slope_no_adapt', to generate a performance comparison between the conventional MPC with the improved gradient based parameter estimator and the conventional MPC without a parameter estimator, in the proposed traffic scenario.

These results are presented from Figure 2 to Figure 4.

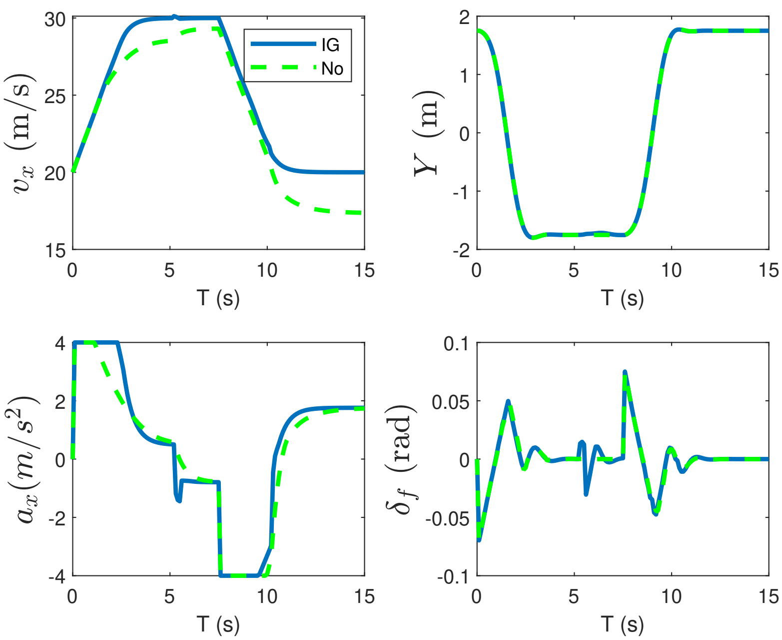

Figure 2: v_x, Y, a_x and δ_f generated by the conventional MPC with and without improved gradient method for step change road grade

It is important to note that the label 'No' and 'No adapt' in the figures correspond to the simulation results generated by the conventional MPC without a parameter estimator, while the label 'IG' and '$a(\hat{\phi})$' correspond to the results generated by the conventional MPC with the improved gradient method as its parameter estimator. Based on Figure 2-(a), it is evident that the state $v_x(k)$ generated by the improved gradient method closely follows its given reference. Conversely, the same state generated by the conventional MPC without parameter estimator deviates away from the given reference. This discrepancy can be attributed to the precise estimation of the variable $a(\hat{\phi}(k))$ from the improved gradient based parameter estimator, as demonstrated in Figure 4.

The state $Y(k)$ obtained from the two control methods exhibits identical

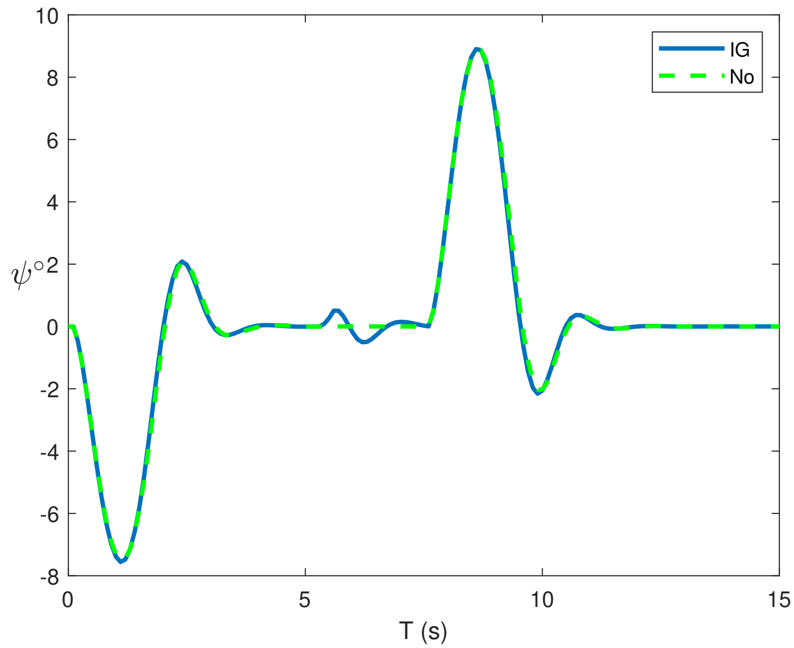

Figure 3: ψ generated by the conventional MPC with and without the improved gradient method for step change road grade

behavior, as shown in Figure 2-(b). This similarity is attributed to the fact that the yaw angle $\psi(k)$ of the ego vehicle remains small in both simulations, as evident from Figure 3. When the value of $\psi(k)$ is small, the term $a(\phi)(k)\sin(\psi(k))$, which is the only component that influences the lateral dynamics of the vehicle, becomes negligible. Therefore, the accuracy of the estimated parameter $\hat{\phi}(k)$ has no significant impact on the reference tracking of $Y(k)$.

References

[1] I. D. Landau, R. Lozano, M. M'Saad, and A. Karimi, *Adaptive control: algorithms, analysis and applications.* Springer Science & Business Media, 2011.

[2] E. Bakker, L. Nyborg, and H. B. Pacejka, "Tyre modelling for use in vehicle dynamics studies," *SAE Transactions*, pp. 190–204, 1987.

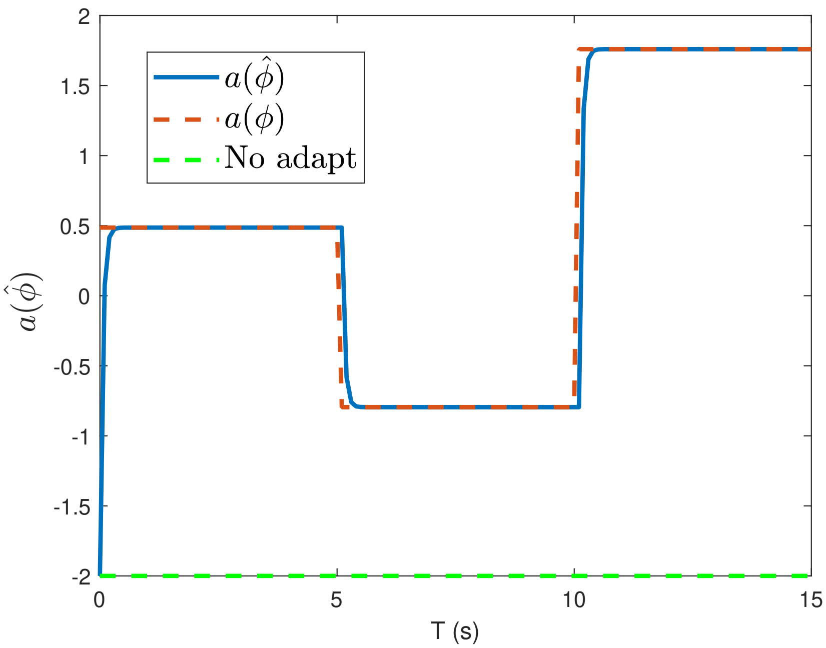

Figure 4: $a(\hat{\phi})$ generated by the conventional MPC with and without improved gradient method for step change road grade