Report of Project4-Car

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1. Problem 1: Bayesian Network Basics

a) In subproblem (a), we want to calculate $P(C_2 = 1|D_2 = 0)$. Since the posterior is proportional to the product of prior and likelihood, we can get the reasoning process below:

$$P(C_2|D_2 = 0) = \alpha P(C_2, D_2 = 0)$$

$$P(C_2, D_2 = 0) = \sum_{c_1}\sum_{d_1}\sum_{c_3}\sum_{d_3} P(c_1, d_1, C_2, D_2 = 0, c_3, d_3)$$

$$= P(D_2 = 0|C_2)\sum_{c_1} P(c_1)P(C_2|c_1)$$

From the equation above, we can easily know:

$$P(C_2 = 1, D_2 = 0) = \frac{\eta}{2} \quad , P(C_2 = 0, D_2 = 0) = \frac{1-\eta}{2}$$

Thus, we can conduct normalization and get our target value:

$$P(C_2 = 1|D_2 = 0) = \frac{P(C_2 = 1, D_2 = 0)}{P(C_2 = 1, D_2 = 0) + P(C_2 = 0, D_2 = 0)} = \eta$$

b) In subproblem (b), we want to calculate $P(C_2 = 1|D_2 = 0, D_3 = 1)$. The process to calculate it is very similar to the last one. Here is it:

$$P(C_2|D_2 = 0, D_3 = 1) = \alpha P(C_2, D_2 = 0, D_3 = 1)$$

$$P(C_2, D_2 = 0, D_3 = 1) = \sum_{c_1}\sum_{d_1}\sum_{c_3} P(c_1)P(d_1|c_1)P(C_2|c_1)P(D_2 = 0|C_2)P(c_3|C_2)P(D_3 = 1|c_3)$$

$$= P(D_2 = 0|C_2)(\sum_{c_1} P(c_1)P(C_2|c_1))(\sum_{c_3} P(c_3|C_2)P(D_3 = 1|c_3))$$

According to the equation above, we can derive that:

$$P(C_2 = 1, D_2 = 0, D_3 = 1) = \frac{2\epsilon\eta^2 + \eta - \eta^2 - \epsilon\eta}{2}$$

$$P(C_2 = 0, D_2 = 0, D_3 = 1) = \frac{2\epsilon\eta^2 + \eta - \eta^2 - 3\epsilon\eta + \epsilon}{2}$$

In the same way, we have:

$$P(C_2=1|D_2=0,D_3=1)=\frac{P(C_2=1|D_2=0,D_3=1)}{P(C_2=1|D_2=0,D_3=1)+P(C_2=0|D_2=0,D_3=1)}$$

$$=\frac{2\epsilon\eta^2+\eta-\eta^2-\epsilon\eta}{4\epsilon\eta^2+2\eta-2\eta^2-4\epsilon\eta+\epsilon}$$

c) This part will answer subproblem (c). $\boldsymbol{\epsilon}=\mathbf{0.1}$ and $\boldsymbol{\eta}=\mathbf{0.2}$.

i. From (a), we know:

$$P(C_2=1|D_2=0)=\eta=0.2000$$

From (b), we know:

$$P(C_2=1|D_2=0,D_3=1)=\frac{2\epsilon\eta^2+\eta-\eta^2-\epsilon\eta}{4\epsilon\eta^2+2\eta-2\eta^2-4\epsilon\eta+\epsilon}\approx 0.4157$$

We can see that $P(C_2=1|D_2=0,D_3=1)$ is greater than $P(C_2=1|D_2=0)$.

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ii. These two results imply that after we get new evidence that $D_3=1$, the probability of $C_2=1$ will have a dramatic change and will be higher than before.

From the transition probability formula, the parameters ϵ and η represents the probability of the car's move and the probability of the sensor's wrong response. First, $\epsilon=0.1$ implies that there is a high probability that the car will not move at the next timestamp. That is, $P(C_2=C_3)$ is high. Similarly, $\eta=0.2$ implies that there is a high probability that the sensor's report is correct, which is equivalent to say is $P(D_3=C_3)$ high. Thus, intuitively, $P(C_2=D_3)$ is more plausible than $P(C_2\neq D_3)$. So, if the sensor reports that $D_3=1$, it is more rational to think that $C_2=1$ as well and this is shown by the increasement from $P(C_2=1|D_2=0)$ to $P(C_2=1|D_2=0,D_3=1)$.

iii. Let $P(C_2=1|D_2=0)=P(C_2=1|D_2=0,D_3=1)$, which is equivalent to:

$$\eta=\frac{2\epsilon\eta^2+\eta-\eta^2-\epsilon\eta}{4\epsilon\eta^2+2\eta-2\eta^2-4\epsilon\eta+\epsilon}$$

$$\leftrightarrow\ \epsilon(4\eta^2-6\eta+2)=2\eta^2-3\eta+1$$

$$\leftrightarrow\ \epsilon=0.5$$

From the reasoning process above, we know that if we want to make the equation hold, we should set $\boldsymbol{\epsilon}$ be $\mathbf{0.5}$ while keeping $\eta=0.2$.

From ii, setting ϵ be 0.5 is to say there is no preference between move or stay for the specific car. It is like telling us that there is no connection between C_2 and C_3. Thus, when we observe that $D_3=1$, it will only help us judge whether C_3 is equal to 1. For the judgement about C_2, it's useless. So, intuitively, $P(C_2=1|D_2=0)=P(C_2=1|D_2=0,D_3=1)$.