



证明: 设空间任意点 x 在超平面上投影为  $x_{\perp}$

$$(1) \quad \therefore |\omega^T \overrightarrow{xx_{\perp}}| = |\omega^T| |\overrightarrow{xx_{\perp}}| = \sqrt{(\omega^1)^2 + \dots + (\omega^N)^2} |\overrightarrow{xx_{\perp}}|$$

$$= \|\omega\| d. \quad (\text{此处 } d \text{ 即所求距离, } N \text{ 为维数})$$

$$(2) \quad \therefore |\omega^T \overrightarrow{xx_{\perp}}| = |\omega^1(x^1 - x_{\perp}^1) + \omega^2(x^2 - x_{\perp}^2) + \dots + \omega^N(x^N - x_{\perp}^N)|$$

$$= |\omega^1 x^1 + \omega^2 x^2 + \dots + \omega^N x^N - (\omega^1 x_{\perp}^1 + \dots + \omega^N x_{\perp}^N)|$$

$$= |(\omega^T x) - (\omega^T x_{\perp})|$$

$$= |\omega^T x + b| \quad \leftarrow \omega^T x_{\perp} + b = 0 \text{ (超平面)}$$

(3)  $\therefore$  结合 (1), (2)

$$\text{得 } d = \frac{|\omega^T x + b|}{\|\omega\|}$$