

TABLE 2.1. Simple exponential families with dispersion parameter

$$f(y|\theta, \phi, \omega) = \exp \left\{ \frac{y\theta - b(\theta)}{\phi} \omega + c(y, \phi, \omega) \right\}$$

(a) Components of the exponential family

Distribution		$\theta(\mu)$	$b(\theta)$	ϕ
Normal	$N(\mu, \sigma^2)$	μ	$\theta^2/2$	σ^2
Bernoulli	$B(1, \pi)$	$\log(\pi/(1 - \pi))$	$\log(1 + \exp(\theta))$	1
Poisson	$P(\lambda)$	$\log \lambda$	$\exp(\theta)$	1
Gamma	$G(\mu, \nu)$	$-1/\mu$	$-\log(-\theta)$	ν^{-1}
Inverse Gaussian	$IG(\mu, \sigma^2)$	$1/\mu^2$	$-(-2\theta)^{1/2}$	σ^2

(b) Expectation and variance

Distribution	$E(y) = b'(\theta)$	var. fct. $b''(\theta)$	$\text{var}(y) = b''(\theta)\phi/\omega$
Normal	$\mu = \theta$	1	σ^2/ω
Bernoulli	$\pi = \frac{\exp(\theta)}{1 + \exp(\theta)}$	$\pi(1 - \pi)$	$\pi(1 - \pi)/\omega$
Poisson	$\lambda = \exp(\theta)$	λ	λ/ω
Gamma	$\mu = -1/\theta$	μ^2	$\mu^2 \nu^{-1}/\omega$
Inverse Gaussian	$\mu = (-2\theta)^{-1/2}$	μ^3	$\mu^3 \sigma^2/\omega$

