

机器学习入门

既然咱们学 Python，也肯定要讲讲机器学习。因为它的很多库都是用 Python 写的。开始先装上它们玩一下。

Tensorflow

安装一下。

```
$ pip install tensorflow
ERROR: Could not find a version that satisfies the requirement tensorflow
ERROR: No matching distribution found for tensorflow
```

```
$ type python
python is aliased to `/usr/local/Cellar/python@3.9/3.9.1_6/bin/python3'
```

然而 Tensorflow 2 只支持 Python 3.5-3.8。我们用的是 3.9。

```
% type python3
python3 is /usr/bin/python3
% python3 -V
Python 3.8.2
```

注意到我系统里的 python3 是 3.8.2 版本。这个 Python 版本对应的 pip 安装到哪儿呢。

```
% python3 -m pip -V
pip 21.0.1 from /Users/lzw/Library/Python/3.8/lib/python/site-packages/pip (python 3.8)
```

对应的 pip 在这里。那我更改一下 .zprofile 文件里。最近我更改了我的 shell。 .zprofile 就相当于之前的 .bash_profile。加入一行。

```
alias pip3=/Users/lzw/Library/Python/3.8/bin/pip3
```

这样，我们用 python3 和 pip3 来玩 Tensorflow。

```
% pip3 install tensorflow
```

```
...
```

```
Successfully installed absl-py-0.12.0 astunparse-1.6.3 cachetools-4.2.1 certifi-2020.12.5 chardet-4.0.0
```

安装了很多库。用上官网的一个例子。

```
import tensorflow as tf
```

```
mnist = tf.keras.datasets.mnist
```

```
(x_train, y_train), (x_test, y_test) = mnist.load_data()
x_train, x_test = x_train / 255.0, x_test / 255.0
```

```
model = tf.keras.models.Sequential([
    tf.keras.layers.Flatten(input_shape=(28, 28)),
    tf.keras.layers.Dense(128, activation='relu'),
    tf.keras.layers.Dropout(0.2),
    tf.keras.layers.Dense(10)
])
```

```
predictions = model(x_train[:1]).numpy()
print(predictions)
```

运行一下。

```
$ /usr/bin/python3 tf.py
```

```
Downloading data from https://storage.googleapis.com/tensorflow/tf-keras-datasets/mnist.npz
11493376/11490434 [=====] - 10s 1us/step
[[ 0.15477428 -0.3877643  0.0994779  0.07474922 -0.26219758 -0.03550266
   0.32226565 -0.37141111  0.10925996 -0.0115255 ]]
```

可见下载了数据集，接着输出了结果。

接下来，看看图片分类的例子。

```
# TensorFlow and tf.keras
import tensorflow as tf

# Helper libraries
import numpy as np
import matplotlib.pyplot as plt

print(tf.__version__)
```

报错。

```
ModuleNotFoundError: No module named 'matplotlib'
```

安装一下。

```
% pip3 install matplotlib
```

正确了。

```
$ /usr/bin/python3 image.py
```

2.4.1

进行复制粘贴例子代码。

```
# TensorFlow and tf.keras
```

```
import tensorflow as tf
```

```
# Helper libraries
```

```
import numpy as np
```

```
import matplotlib.pyplot as plt
```

```
fashion_mnist = tf.keras.datasets.fashion_mnist
```

```
(train_images, train_labels), (test_images, test_labels) = fashion_mnist.load_data()
```

```
class_names = ['T-shirt/top', 'Trouser', 'Pullover', 'Dress', 'Coat',  
               'Sandal', 'Shirt', 'Sneaker', 'Bag', 'Ankle boot']
```

```
print(train_images.shape)
```

```
print(len(train_labels))
```

输出了结果。注意到这里有 train_images、train_labels、test_images、test_labels。就是分为训练数据集和测试数据集。

```
(60000, 28, 28)
```

```
60000
```

接着试试打印出图片来。

```
print(train_images[0])
```

看下结果。

```
[[ 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  
   0  0  0  0  0  0  0  0  0  0  0]  
[ 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  
   0  0  0  0  0  0  0  0  0  0  0]  
[ 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  
   0  0  0  0  0  0  0  0  0  0  0]  
[ 0  0  0  0  0  0  0  0  0  0  0  0  1  0  0 13 73  0  
   0  1  4  0  0  0  0  1  1  0]
```

```
[ 0  0  0  0  0  0  0  0  0  0  0  0  0  3  0 36 136 127 62
 54  0  0  0  1  3  4  0  0  3]
[ 0  0  0  0  0  0  0  0  0  0  0  0  0  6  0 102 204 176 134
144 123 23  0  0  0  0 12 10  0]
[ 0  0  0  0  0  0  0  0  0  0  0  0  0  0  0 155 236 207 178
107 156 161 109 64 23 77 130 72 15]
[ 0  0  0  0  0  0  0  0  0  0  0  0  1  0 69 207 223 218 216
216 163 127 121 122 146 141 88 172 66]]
....
```

这里节选了部分结果。

```
print(len(train_images[0][0]))
```

输出 28。所以很清楚，这是一个横宽为 28 的矩阵。继续打印。

```
print(len(train_images[0][0][0]))
```

```
TypeError: object of type 'numpy.uint8' has no len()
```

所以很明白。每张图片都是 28*28*3 的数组。最后一维数组保存的是 rgb 值。然而发现我们的想法可能是错的。

```
print(train_images[0][1][20])
```

```
0
```

```
print(train_images[0][1])
```

```
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
```

说明每张图片是 28*28 的数组。捣鼓了一阵。我们终于知道了秘密。

先来看看输出的图。

```
plt.figure()
```

```
plt.imshow(train_images[0])
```

```
plt.colorbar()
```

```
plt.grid(False)
```

```
plt.show()
```

看到右边的颜色条吗。0 到 250。原来这是在两种颜色里的渐变。可是它怎么知道是哪两种颜色。我们哪里告诉它了。

接着我们把第二张图也打印出来。

```
plt.imshow(train_images[1])
```

很有意思。这难道是 pyplot 依赖库默认的吗。继续运行官网给的代码。

```
plt.figure(figsize=(10,10))
for i in range(25):
    plt.subplot(5,5,i+1)
    plt.xticks([])
    plt.yticks([])
    plt.grid(False)
    plt.imshow(train_images[i], cmap=plt.cm.binary)
    plt.xlabel(class_names[train_labels[i]])
plt.show()
```

注意到这里显示了图片以及它们的分类。终于我们知道了 `cmap` 参数。如果 `cmap` 什么都不写，一定会是刚刚我们那种色彩的。果然。

```
plt.imshow(train_images[i])
```

这会我们搜索 `pyplot cmap`。找到一些资料。

```
plt.imshow(train_images[i], cmap=plt.cm.PiYG)
```

改一下代码。

```
plt.figure(figsize=(10,10))
for i in range(25):
    plt.subplot(2,5,i+1)  ## 改这行
    plt.xticks([])
    plt.yticks([])
    plt.grid(False)
    plt.imshow(train_images[i], cmap=plt.cm.Blues)
    plt.xlabel(class_names[train_labels[i]])
plt.show()
```

然而报错了。

```
ValueError: num must be 1 <= num <= 10, not 11
```

这意味着什么。之前的 `5,5,i+1` 到底什么意思。为什么改成 `2` 就不行了。尽管我们直观地知道大概是 `5` 行 `5` 列的意思。但为什么会报这个错误。`11` 是怎么计算出来的。`num` 又是什么意思。`10` 是什么意思。注意到 `2*5=10`。所以也许当 `i=11` 的时候出错了。当改成 `for i in range(10):` 时，得到了以下结果。

这会稍微看一下文档，得知 `subplot(nrows, ncols, index, **kwargs)`。嗯，到此我们很明白了。

plot_scale

Figure 1: plot_scale

```
plt.figure(figsize=(10,10))
for i in range(25):
    plt.subplot(5,5,i+1)
    # plt.xticks([])
    plt.yticks([])
    plt.grid(False)
    plt.imshow(train_images[i], cmap=plt.cm.Blues)
    plt.xlabel(class_names[train_labels[i]])
plt.show()
```

注意到 0 25 这种就叫 `xticks`。当我们放大缩小这个框的时候，会有不同的展示。

注意到放大缩小框，`xticks` 和 `xlabels` 会有不同的显示。

```
model = tf.keras.Sequential([
    tf.keras.layers.Flatten(input_shape=(28, 28)),
    tf.keras.layers.Dense(128, activation='relu'),
    tf.keras.layers.Dense(10)
])

model.compile(optimizer='adam',
              loss=tf.keras.losses.SparseCategoricalCrossentropy(from_logits=True),
              metrics=['accuracy'])

model.fit(train_images, train_labels, epochs=10)

test_loss, test_acc = model.evaluate(test_images, test_labels, verbose=2)

print('\nTest accuracy:', test_acc)
```

注意到了这里定义 `model` 的方式，用到了类 `Sequential`。注意这些参数，28,28、128、relu、10。

注意到需要 `compile` 和 `fit`。`fit` 是拟合的意思。注意到 28,28 就是图形大小。

Epoch 1/10

1875/1875 [=====] - 2s 928us/step - loss: 0.6331 - accuracy: 0.7769

Epoch 2/10

1875/1875 [=====] - 2s 961us/step - loss: 0.3860 - accuracy: 0.8615

```

Epoch 3/10
1875/1875 [=====] - 2s 930us/step - loss: 0.3395 - accuracy: 0.8755
Epoch 4/10
1875/1875 [=====] - 2s 1ms/step - loss: 0.3071 - accuracy: 0.8890
Epoch 5/10
1875/1875 [=====] - 2s 1ms/step - loss: 0.2964 - accuracy: 0.8927
Epoch 6/10
1875/1875 [=====] - 2s 985us/step - loss: 0.2764 - accuracy: 0.8955
Epoch 7/10
1875/1875 [=====] - 2s 961us/step - loss: 0.2653 - accuracy: 0.8996
Epoch 8/10
1875/1875 [=====] - 2s 1ms/step - loss: 0.2549 - accuracy: 0.9052
Epoch 9/10
1875/1875 [=====] - 2s 1ms/step - loss: 0.2416 - accuracy: 0.9090
Epoch 10/10
1875/1875 [=====] - 2s 1ms/step - loss: 0.2372 - accuracy: 0.9086
313/313 - 0s - loss: 0.3422 - accuracy: 0.8798

```

Test accuracy: 0.879800021648407

模型已经训练出来了。来改下参数。

```

model = tf.keras.Sequential([
    tf.keras.layers.Flatten(input_shape=(28, 28)),
    tf.keras.layers.Dense(28, activation='relu'),    # 128 -> 28
    tf.keras.layers.Dense(10)
])

```

修改一下 Dense 的第一个参数。

```

Epoch 1/10
1875/1875 [=====] - 2s 714us/step - loss: 6.9774 - accuracy: 0.3294
Epoch 2/10
1875/1875 [=====] - 1s 715us/step - loss: 1.3038 - accuracy: 0.4831
Epoch 3/10
1875/1875 [=====] - 1s 747us/step - loss: 1.0160 - accuracy: 0.6197
Epoch 4/10
1875/1875 [=====] - 1s 800us/step - loss: 0.7963 - accuracy: 0.6939
Epoch 5/10

```

```

1875/1875 [=====] - 2s 893us/step - loss: 0.7006 - accuracy: 0.7183
Epoch 6/10
1875/1875 [=====] - 1s 747us/step - loss: 0.6675 - accuracy: 0.7299
Epoch 7/10
1875/1875 [=====] - 1s 694us/step - loss: 0.6681 - accuracy: 0.7330
Epoch 8/10
1875/1875 [=====] - 1s 702us/step - loss: 0.6675 - accuracy: 0.7356
Epoch 9/10
1875/1875 [=====] - 1s 778us/step - loss: 0.6508 - accuracy: 0.7363
Epoch 10/10
1875/1875 [=====] - 1s 732us/step - loss: 0.6532 - accuracy: 0.7350
313/313 - 0s - loss: 0.6816 - accuracy: 0.7230

```

Test accuracy: 0.7229999899864197

注意到 Test accuracy 前后发生了变化。Epoch 这样的是 fit 函数输出的日志。注意到当是 128 时, accuracy 从 0.7769 变到 0.9086。而当是 28 时, accuracy 从 0.3294 变到 0.7350。这会注意到, 我们先用训练集, 来调优 loss 和 accuracy。接着用测试数据集来测试。先来看看 train_labels。

```

print(train_labels)
[9 0 0 ... 3 0 5]
print(len(train_labels))
60000

```

这意味着用 0 到 9 来表示这些类别。刚好 class_names 也是有 10 个。

```

class_names = ['T-shirt/top', 'Trouser', 'Pullover', 'Dress', 'Coat',
               'Sandal', 'Shirt', 'Sneaker', 'Bag', 'Ankle boot']

```

再来改一改。

```

model = tf.keras.Sequential([
    tf.keras.layers.Flatten(input_shape=(28, 28)),
    tf.keras.layers.Dense(28, activation='relu'),
    tf.keras.layers.Dense(5)    # 10 -> 5
])

model.compile(optimizer='adam',
              loss=tf.keras.losses.SparseCategoricalCrossentropy(from_logits=True),
              metrics=['accuracy'])

```



```
model.fit(train_images, train_labels, epochs=10)
```

出错了。

```
tensorflow.python.framework.errors_impl.InvalidArgumentError: Received a label value of 9 which is out of range [0-8] for label with 10 classes.
[[node sparse_categorical_crossentropy/SparseSoftmaxCrossEntropyWithLogits/SparseSoftmaxCrossEntropyWithLogits]]
```

Function call stack:

train_function

改成把 Sequential 的第三个参数 Dense 的参数改成 15 就可以了。结果区别不大。试试改改 Epoch。

```
model = tf.keras.Sequential([
    tf.keras.layers.Flatten(input_shape=(28, 28)),
    tf.keras.layers.Dense(28, activation='relu'),
    tf.keras.layers.Dense(15)
])
```

```
model.compile(optimizer='adam',
              loss=tf.keras.losses.SparseCategoricalCrossentropy(from_logits=True),
              metrics=['accuracy'])
```

```
model.fit(train_images, train_labels, epochs=15) # 10 -> 15
```

```
test_loss, test_acc = model.evaluate(test_images, test_labels, verbose=2)
```

```
print('\nTest accuracy:', test_acc)
```

Epoch 1/15

```
1875/1875 [=====] - 2s 892us/step - loss: 6.5778 - accuracy: 0.3771
```

Epoch 2/15

```
1875/1875 [=====] - 2s 872us/step - loss: 1.3121 - accuracy: 0.4910
```

Epoch 3/15

```
1875/1875 [=====] - 2s 909us/step - loss: 1.0900 - accuracy: 0.5389
```

Epoch 4/15

```
1875/1875 [=====] - 1s 730us/step - loss: 1.0422 - accuracy: 0.5577
```

Epoch 5/15

```

1875/1875 [=====] - 1s 709us/step - loss: 0.9529 - accuracy: 0.5952
Epoch 6/15
1875/1875 [=====] - 1s 714us/step - loss: 0.9888 - accuracy: 0.5950
Epoch 7/15
1875/1875 [=====] - 1s 767us/step - loss: 0.8678 - accuracy: 0.6355
Epoch 8/15
1875/1875 [=====] - 1s 715us/step - loss: 0.8247 - accuracy: 0.6611
Epoch 9/15
1875/1875 [=====] - 1s 721us/step - loss: 0.8011 - accuracy: 0.6626
Epoch 10/15
1875/1875 [=====] - 1s 711us/step - loss: 0.8024 - accuracy: 0.6622
Epoch 11/15
1875/1875 [=====] - 1s 781us/step - loss: 0.7777 - accuracy: 0.6696
Epoch 12/15
1875/1875 [=====] - 1s 724us/step - loss: 0.7764 - accuracy: 0.6728
Epoch 13/15
1875/1875 [=====] - 1s 731us/step - loss: 0.7688 - accuracy: 0.6767
Epoch 14/15
1875/1875 [=====] - 1s 715us/step - loss: 0.7592 - accuracy: 0.6793
Epoch 15/15
1875/1875 [=====] - 1s 786us/step - loss: 0.7526 - accuracy: 0.6792
313/313 - 0s - loss: 0.8555 - accuracy: 0.6418

```

Test accuracy: 0.6417999863624573

注意改成 15。区别也不大。tf.keras.layers.Dense(88, activation='relu')，是重要的。试着 128 改成 88。得到了 Test accuracy: 0.824999988079071。128 时，是 0.879800021648407。28 时，是 0.7229999899864197。是不是越大越好，然而当改成 256 时，是 Test accuracy: 0.8409000039100647。这不禁让我们思考 loss 和 accuracy 的含义。

```

probability_model = tf.keras.Sequential([model,
                                         tf.keras.layers.Softmax()])

```

接下来预测一下。注意到 Sequential 和上面的一样。注意到参数 model 和 tf.keras.layers.Softmax()。

```

probability_model = tf.keras.Sequential([model,
                                         tf.keras.layers.Softmax()])
predictions = probability_model.predict(test_images)

```

```

def plot_image(i, predictions_array, true_label, img):
    true_label, img = true_label[i], img[i]
    plt.grid(False)
    plt.xticks([])
    plt.yticks([])

    plt.imshow(img, cmap=plt.cm.binary)

    predicted_label = np.argmax(predictions_array)
    if predicted_label == true_label:
        color = 'blue'
    else:
        color = 'red'

    plt.xlabel("{} {:2.0f}% ({})".format(class_names[predicted_label],
                                         100*np.max(predictions_array),
                                         class_names[true_label]),
              color=color)

def plot_value_array(i, predictions_array, true_label):
    true_label = true_label[i]
    plt.grid(False)
    plt.xticks(range(10))
    plt.yticks([])
    thisplot = plt.bar(range(10), predictions_array, color="#777777")
    plt.ylim([0, 1])
    predicted_label = np.argmax(predictions_array)

    thisplot[predicted_label].set_color('red')
    thisplot[true_label].set_color('blue')

i = 0
plt.figure(figsize=(6,3))
plt.subplot(1,2,1)
plot_image(i, predictions[i], test_labels, test_images)
plt.subplot(1,2,2)

```

```
plot_value_array(i, predictions[i], test_labels)
plt.show()
```

这说明这个图片 99% 的可能是 Ankle boot。注意到 plot_image 是显示左边的图。plot_value_array 是输出右边的图。

```
num_rows = 5
num_cols = 3
num_images = num_rows*num_cols
plt.figure(figsize=(2*2*num_cols, 2*num_rows))
for i in range(num_images):
    plt.subplot(num_rows, 2*num_cols, 2*i+1)
    plot_image(i, predictions[i], test_labels, test_images)
    plt.subplot(num_rows, 2*num_cols, 2*i+2)
    plot_value_array(i, predictions[i], test_labels)
plt.tight_layout()
plt.show()
```

注意到这里只是显示更多的测试结果。所以使用流程我们大致很清楚。所以我们还不知道背后怎么计算的。但我们知道如何使用它们。它们背后是微积分。如何理解微积分呢。

比如说有个数字，1 到 100 让你猜。每次你猜多少。我告诉你小了还是大了。你猜 50。我说小了。你猜 80。我说大了。你猜 65。我说大了。你猜 55。我说小了。你猜 58。我说，嗯，猜对了。

机器学习，就是在背后模拟类似的过程。只不过复杂一些。可能是很多的 1 到 100，要猜很多数。同时每次猜都要进行很多运算。以及每次判断是否大了还是小了，要计算很多。

PyTorch

安装一下。这个支持 3.9 版本的 Python。

```
$ pip install torch torchvision
```

```
Collecting torch
```

```
  Downloading torch-1.8.0-cp39-none-macosx_10_9_x86_64.whl (120.6 MB)
```

```
    |                               | 120.6 MB 224 kB/s
```

```
Collecting torchvision
```

```
  Downloading torchvision-0.9.0-cp39-cp39-macosx_10_9_x86_64.whl (13.1 MB)
```

```
    |                               | 13.1 MB 549 kB/s
```

```
Requirement already satisfied: numpy in /usr/local/lib/python3.9/site-packages (from torch) (1.20.1)
```

```
Collecting typing-extensions
```

```
  Downloading typing_extensions-3.7.4.3-py3-none-any.whl (22 kB)
```

```
Requirement already satisfied: pillow>=4.1.1 in /usr/local/lib/python3.9/site-packages (from torchvision)
Installing collected packages: typing-extensions, torch, torchvision
Successfully installed torch-1.8.0 torchvision-0.9.0 typing-extensions-3.7.4.3
```

检验一下。

```
import torch
x = torch.rand(5, 3)
print(x)
```

出错了。

Traceback (most recent call last):

```
File "torch.py", line 1, in <module>
    import torch
File "torch.py", line 2, in <module>
    x = torch.rand(5, 3)
```

AttributeError: partially initialized module 'torch' has no attribute 'rand' (most likely due to a circular dependency)

谷歌一下这个错误信息。原来是因为我们的文件也叫 torch。重名了。改一下然后就正确了。

```
tensor([[0.5520, 0.9446, 0.5543],
        [0.6192, 0.0908, 0.8726],
        [0.0223, 0.7685, 0.9814],
        [0.4019, 0.5406, 0.3861],
        [0.5485, 0.6040, 0.2387]])
```

找到一个例子。

```
# -*- coding: utf-8 -*-
```

```
import torch
import math
dtype = torch.float
device = torch.device("cpu")
# device = torch.device("cuda:0") # Uncomment this to run on GPU

# Create random input and output data
x = torch.linspace(-math.pi, math.pi, 2000, device=device, dtype=dtype)
y = torch.sin(x)
```

```

# Randomly initialize weights
a = torch.randn((), device=device, dtype=dtype)
b = torch.randn((), device=device, dtype=dtype)
c = torch.randn((), device=device, dtype=dtype)
d = torch.randn((), device=device, dtype=dtype)

learning_rate = 1e-6
for t in range(2000):
    # Forward pass: compute predicted y
    y_pred = a + b * x + c * x ** 2 + d * x ** 3

    # Compute and print loss
    loss = (y_pred - y).pow(2).sum().item()
    if t % 100 == 99:
        print(t, loss)

    # Backprop to compute gradients of a, b, c, d with respect to loss
    grad_y_pred = 2.0 * (y_pred - y)
    grad_a = grad_y_pred.sum()
    grad_b = (grad_y_pred * x).sum()
    grad_c = (grad_y_pred * x ** 2).sum()
    grad_d = (grad_y_pred * x ** 3).sum()

    # Update weights using gradient descent
    a -= learning_rate * grad_a
    b -= learning_rate * grad_b
    c -= learning_rate * grad_c
    d -= learning_rate * grad_d
print(f'Result: y = {a.item()} + {b.item()} x + {c.item()} x^2 + {d.item()} x^3')

```

运行一下。

```

99 1273.537353515625
199 849.24853515625
299 567.4786987304688
399 380.30291748046875
499 255.92752075195312

```

```

599 173.2559814453125
699 118.2861328125
799 81.72274780273438
899 57.39331817626953
999 41.198158264160156
1099 30.41307830810547
1199 23.227672576904297
1299 18.438262939453125
1399 15.244369506835938
1499 13.113286972045898
1599 11.690631866455078
1699 10.740333557128906
1799 10.105220794677734
1899 9.6804780960083
1999 9.39621353149414

```

Result: $y = -0.011828352697193623 + 0.8360244631767273 x + 0.002040589228272438 x^2 + -0.09038365632295 x^3$

看看只用 numpy 库的代码。

```

# -*- coding: utf-8 -*-
import numpy as np
import math

# Create random input and output data
x = np.linspace(-math.pi, math.pi, 2000)
y = np.sin(x)

# Randomly initialize weights
a = np.random.randn()
b = np.random.randn()
c = np.random.randn()
d = np.random.randn()

learning_rate = 1e-6
for t in range(2000):
    # Forward pass: compute predicted y
    #  $y = a + b x + c x^2 + d x^3$ 

```

```

y_pred = a + b * x + c * x ** 2 + d * x ** 3

# Compute and print loss
loss = np.square(y_pred - y).sum()
if t % 100 == 99:
    print(t, loss)

# Backprop to compute gradients of a, b, c, d with respect to loss
grad_y_pred = 2.0 * (y_pred - y)
grad_a = grad_y_pred.sum()
grad_b = (grad_y_pred * x).sum()
grad_c = (grad_y_pred * x ** 2).sum()
grad_d = (grad_y_pred * x ** 3).sum()

# Update weights
a -= learning_rate * grad_a
b -= learning_rate * grad_b
c -= learning_rate * grad_c
d -= learning_rate * grad_d

print(f'Result: y = {a} + {b} x + {c} x^2 + {d} x^3')

```

注意到这是两种方式来计算。

这两个例子，是先生成了一组 x 和 y 。接着假设是三次方程。再接着用些方法把系数迭代计算出来。这些算法是怎样的呢。注意到是循环了 2000 次。每次拟合地精确一些。这里先不细究。

最后

目前，我们不懂机器学习背后是怎么计算的。然而，暂时不重要。我们用上面类似的知识已经可以来干很多事情了。还可以用机器学习来处理文本、音频等的。等我们试探了几十个例子，再学原理也不迟。

练习

- 学生像上面那样探索一遍。