电磁波传输类型

- 区域
  - 无限空间
    - 一种媒质
      - 传播
        - 理想介质 无衰减
        - 导电媒质 有衰减
    - 两种媒质
      反射、折射
      - 垂直入射
        - 理想导体 反射
        - 理想介质 反射、透射
      - 斜入射
        - 理想导体 反射
        - 理想介质 反射、折射
  - 有限空间 → 波导 → 矩形波导 无衰减

非限定形式

$$\begin{cases}\nabla\times\vec{H}=\vec{J}_e+\partial\vec{D}/\partial t\\ \nabla\times\vec{E}=-\partial\vec{B}/\partial t\\ \nabla\cdot\vec{B}=0\\ \nabla\cdot\vec{D}=\rho\end{cases}$$

$$\nabla\cdot\vec{J}+\partial\rho/\partial t=0$$

电流连续性方程

特性方程

$$\vec{D}=\varepsilon\vec{E}\qquad \vec{B}=\mu\vec{H}\qquad \vec{J}=\sigma\vec{E}$$

限定形式

$$\begin{cases}\nabla\times\vec{H}=\sigma\vec{E}+\varepsilon\,\partial\vec{E}/\partial t\\ \nabla\times\vec{E}=-\mu\,\partial\vec{H}/\partial t\\ \nabla\cdot\vec{H}=0\\ \nabla\cdot\vec{E}=\rho/\varepsilon\end{cases}$$

无源

$$\rho=0\qquad \vec{J}=0$$

波动方程

$$\nabla^2\vec{E}-\mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}=0$$

$$\nabla^2\vec{H}-\mu\varepsilon\frac{\partial^2\vec{H}}{\partial t^2}=0$$

$$\nabla\cdot\left(\vec{E}\times\vec{H}\right)=-\frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon E^2+\frac{1}{2}\mu H^2\right)-\sigma E^2$$

坡印廷定律

$$\vec{S}=\vec{E}\times\vec{H}$$

$\vec{S}$ 坡印廷矢量

积分形式

$$\begin{cases}\oint_C\vec{H}\cdot d\vec{l}=\int_S(\vec{J}_e+\frac{\partial\vec{D}}{\partial t})\cdot d\vec{S}\\ \oint_C\vec{E}\cdot d\vec{l}=-\int_S\frac{\partial\vec{B}}{\partial t}\cdot d\vec{S}\\ \oint_S\vec{B}\cdot d\vec{S}=0\\ \oint_S\vec{D}\cdot d\vec{S}=\int_V\rho dV=Q\end{cases}$$

边界条件

$$\hat{n}\times(\vec{H}_1-\vec{H}_2)=\vec{J}_S$$
$$\hat{n}\times(\vec{E}_1-\vec{E}_2)=0$$
$$\hat{n}\cdot\left(\vec{B}_1-\vec{B}_2\right)=0$$
$$\hat{n}\cdot(\vec{D}_1-\vec{D}_2)=\rho_s$$

$$\vec{H}_2=0\qquad \vec{E}_2=0$$

理想导体边界条件

$$\hat{n}\times\vec{H}=\vec{J}_S$$
$$\hat{n}\times\vec{E}=0$$
$$\hat{n}\cdot\vec{B}=0$$
$$\hat{n}\cdot\vec{D}=\rho_s$$

$$\nabla\cdot\vec{B}=0$$

$$\vec{B}=\nabla\times\vec{A}$$

$\vec{A}$ 矢量位

$$\nabla\times\vec{E}=-\partial\vec{B}/\partial t$$

$$\nabla\times\left(\vec{E}+\partial\vec{A}/\partial t\right)=0$$

$$\vec{E}+\partial\vec{A}/\partial t=-\nabla\varphi$$

φ 标量位

$$\vec{E}=-\partial\vec{A}/\partial t-\nabla\varphi$$

$$\nabla\cdot\vec{A}+\mu\varepsilon\frac{\partial\varphi}{\partial t}=0$$

罗伦兹条件

$$\nabla^2\vec{A}-\mu\varepsilon\frac{\partial^2\vec{A}}{\partial t^2}=-\mu\vec{J}$$

$$\nabla^2\varphi-\mu\varepsilon\frac{\partial^2\varphi}{\partial t^2}=-\frac{\rho}{\varepsilon}$$

达朗贝尔方程

麦克斯韦方程组

波动方程

时变电磁场 $\xrightarrow{e^{j\omega t}}$ **时谐电磁场** $e^{j\omega t}$

麦克斯韦方程组

$\vec{E}(\vec{r},t)$

瞬时

$$\begin{cases} \nabla\times\vec{H} = \vec{J}_e + \partial\vec{D}/\partial t \\ \nabla\times\vec{E} = -\partial\vec{B}/\partial t \\ \nabla\cdot\vec{B} = 0 \\ \nabla\cdot\vec{D} = \rho \end{cases}$$

$e^{j\omega t}$ → $\dfrac{\partial}{\partial t} \rightarrow j\omega$

时谐场

麦克斯韦方程组（时谐场）

$\vec{E}(\vec{r})$

复数

$$\begin{cases} \nabla\times\vec{H} = \vec{J} + j\omega\vec{D} \\ \nabla\times\vec{E} = -j\omega\vec{B} \\ \nabla\cdot\vec{B} = 0 \\ \nabla\cdot\vec{D} = \rho \end{cases}$$

无源

$\rho = 0$

$\vec{J} = 0$

$$\nabla^2\vec{E} - \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} = 0$$

$$\nabla^2\vec{H} - \mu\varepsilon\frac{\partial^2\vec{H}}{\partial t^2} = 0$$

波动方程

$e^{j\omega t}$ → $\dfrac{\partial^2}{\partial t^2} \rightarrow -\omega^2$

$$k = \omega\sqrt{\mu\varepsilon}$$

波数

无源

$\rho = 0$

$\vec{J} = 0$

$$\nabla^2\vec{E} + k^2\vec{E} = 0$$

$$\nabla^2\vec{H} + k^2\vec{H} = 0$$

亥姆霍兹方程

电磁波在无损介质中传播

$k=\omega\sqrt{\mu\varepsilon}$

$$\nabla^2\vec{E}+k^2\vec{E}=0$$
$$\nabla^2\vec{H}+k^2\vec{H}=0$$

?

$$\frac{d^2E_x}{dz^2}+k^2E_x=0$$

$$E_x=E_m e^{-jkz}$$

$$E_x=E_m\cos(\omega t-kz)$$

T, f, λ

$v=\omega/k=1/\sqrt{\mu\varepsilon}$

$v_g=d\omega/dk$

$$\vec{E}=\hat{e}_x E_m e^{-jkz}$$

电场取任意方向

$$\vec{E}=\vec{E}_0 e^{-jkz}$$

沿任意方向传播

$$\vec{E}=\vec{E}_0 e^{-j\vec{k}\cdot\vec{r}}$$

$$\nabla\times\vec{E}=-j\omega\mu\vec{H}$$

$$\vec{H}=\frac{1}{\eta}\hat{e}_n\times\vec{E}$$

$\eta=\sqrt{\dfrac{\mu}{\varepsilon}}$

本征阻抗

$\vec{E}(x,y,z)=??$

均匀平面波

(沿z方向传播)

$\vec{E}(z)$

电场取沿x轴

$\vec{E}(z)=\hat{e}_x E_x(z)$

$$E_x=E_{mx}\cos(\omega t-kz+\varphi_x)$$
$$E_y=E_{my}\cos(\omega t-kz+\varphi_y)$$

极化

线圆极化

$\varphi_x-\varphi_y=0,\pm\pi$

椭圆极化

$\varphi_x-\varphi_y=$任意

$\varphi_x-\varphi_y=\pm\pi/2$

$E_{mx}=E_{my}$

圆极化

旋转

左旋

右旋

判断法则

- 大拇指指向传播方向；
- 四指指向相位滞后分量；
- 符合左手定律则为左旋
- 符合右手定律则为右旋；

电磁波在有损介质中传播

$$\nabla\times\vec{H}=\sigma\vec{E}+j\omega\varepsilon\vec{E}=j\omega\varepsilon_c\vec{E}$$

$$\varepsilon_c=\varepsilon-j\frac{\sigma}{\omega}$$

$$\nabla^2\vec{E}+k^2\vec{E}=0$$

$$k_c=\omega\sqrt{\mu\varepsilon_c}$$

$$\nabla^2\vec{E}+k_c^2\vec{E}=0$$

$$\Gamma=jk_c=\alpha+j\beta$$

传播系数

$$\nabla^2\vec{E}-\Gamma^2\vec{E}=0$$

$$\alpha=\omega\sqrt{\frac{\mu\varepsilon}{2}[\sqrt{1+(\frac{\sigma}{\omega\mu})^2}-1]}$$

$$\beta=\omega\sqrt{\frac{\mu\varepsilon}{2}[\sqrt{1+(\frac{\sigma}{\omega\mu})^2}+1]}$$

$$\frac{\gamma}{\omega\varepsilon}$$

$$\frac{\sigma}{\omega\varepsilon}\ll 1$$

弱导电媒质

$$\alpha\approx\frac{1}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

$$\beta\approx\omega\sqrt{\mu\varepsilon}$$

$$\eta_c\approx\sqrt{\frac{\mu}{\varepsilon}}\left(1+j\frac{\sigma}{2\omega\varepsilon}\right)$$

$$\frac{\sigma}{\omega\varepsilon}\gg 1$$

强导电媒质

$$\alpha\approx\sqrt{\pi f\mu\sigma}$$

$$\beta\approx\sqrt{\pi f\mu\sigma}$$

$$\eta_c\approx\sqrt{\frac{\pi f\mu}{\sigma}}(1+j)$$

$$\vec{E}=\hat{e}_xE_x(z)$$

均匀平面波；
沿z传播

$$\frac{d^2E_x}{dz^2}-\Gamma^2E_x=0$$

$$E_x=E_me^{-\Gamma z}=E_me^{-\alpha z}e^{-j\beta z}$$

$$\nabla\times\vec{E}=-j\omega\mu\vec{H}$$

$$\vec{H}=\hat{e}_y\frac{E_m}{\eta_c}e^{-\alpha z}e^{-j\beta z}=\hat{e}_y\frac{E_m}{|\eta_c|}e^{-\alpha z}e^{-j\beta z}e^{-j\varphi}$$

$$\eta_c=|\eta|e^{j\varphi}$$

$$E_x=E_me^{-\alpha z}\cos(\omega t-kz)$$

电磁波垂直入射－理想导体

入射波

$$E_x^+ = E_m^+ e^{-j\beta z}$$

$$H_y^+ = \frac{E_m^+}{\eta} e^{-j\beta z}$$

反射波

$$E_x^- = E_m^- e^{j\beta z}$$

$$H_y^- = \frac{E_m^-}{\eta} e^{j\beta z}$$

合波

$$E_x = E_x^+ + E_x^- = E_m^+ e^{-j\beta z} + E_m^- e^{j\beta z}$$

边界条件

$$\vec{E}_x(0) = 0$$

$$E_m^+ = -E_m^-$$

$$E_x = -2jE_m^+ \sin(\beta z)$$

$$E_x = 2E_m^+ \sin(\beta z) \sin(\omega t)$$

$$z = -n\frac{\lambda}{2}$$

$$E_x = 0$$

$$H_y^- = -\frac{E_x^-}{\eta}$$

$$H_y = H_y^+ + H_y^- = \frac{2E_m^+}{\eta} \cos\beta z$$

$$H_y = \frac{2E_m^+}{\eta} \cos(\beta z) \cos(\omega t)$$

$$z = -(2n+1)\frac{\lambda}{4}$$

$$H_y = 0$$

驻波

$$\vec{J}_s = n \times \vec{H}\Big|_{z=0} = \hat{e}_x \, 2E_m^+ / \eta$$

电磁波垂直入射－导电媒质

媒质1

入射波

$$\vec{E}_1^+ = \hat{e}_x E_{m1}^+ e^{-\Gamma_1 z}$$

$$\vec{H}_1^+ = -\hat{e}_y \frac{E_{m1}^+}{\eta} e^{-\Gamma_1 z}$$

反射波

$$E_1^- = E_{m1}^- e^{\Gamma_1 z}$$

$$H_1^- = -\frac{E_{m1}^-}{\eta} e^{\Gamma_1 z}$$

合波（媒质1）

$$\vec{E}_1 = \vec{E}_1^+ + \vec{E}_1^-$$

$$\vec{H}_1 = \vec{H}_1^+ + \vec{H}_1^-$$

媒质2

$$E_2^+ = \hat{e}_x E_{m2}^+ e^{-\Gamma_2 z}$$

$$H_2^+ = -\hat{e}_y \frac{E_{m2}^+}{\eta} e^{-\Gamma_2 z}$$

透射波（媒质2波）

边界条件：

场量切向连续

$$E_{1t}(0) = E_{2t}(0)$$

$$H_{1t}(0) = H_{2t}(0)$$

$$E_{m1}^+ + E_{m1}^- = E_{m2}^+$$

$$\frac{E_{m1}^+}{\eta_1} - \frac{E_{m1}^-}{\eta_1} = \frac{E_{m2}^+}{\eta_2}$$

$$\Gamma = \frac{E_{m1}^-}{E_{m1}^+} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\tau = \frac{E_{m2}^+}{E_{m1}^+} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

$$\Gamma + 1 = \tau$$

$$E(x,t) = \hat{e}_x E_m e^{-\alpha z} \cos(\omega t - kz)$$

$$\delta = 1/\alpha$$

导行电磁波

$$\begin{cases}\nabla\times\vec{H}=\vec{J}+j\omega\vec{D} & \nabla\cdot\vec{B}=0\\ \nabla\times\vec{E}=-j\omega\vec{B} & \nabla\cdot\vec{D}=\rho\end{cases}$$

无源

$$\begin{cases}\nabla\times\vec{H}=j\omega\vec{D}\\ \nabla\times\vec{E}=-j\omega\vec{B}\\ \nabla\cdot\vec{H}=0\\ \nabla\cdot\vec{E}=0\end{cases}$$

$\vec{E}(x,y,z)$

$$\nabla^2\vec{E}+k^2\vec{E}=0$$
$$\nabla^2\vec{H}+k^2\vec{H}=0$$

$\vec{E}(x,y)$

$$\nabla^2_{xy}\vec{E}+\left(\Gamma^2+k^2\right)\vec{E}=0$$
$$\nabla^2_{xy}\vec{H}+\left(\Gamma^2+k^2\right)\vec{H}=0$$

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}=\nabla^2_{xy}+\Gamma^2$$

沿z轴传播

$$\vec{E}(x,y,z)=\vec{E}_m(x,y)e^{-\Gamma z}$$

$$\frac{\partial^2}{\partial z^2}=\Gamma^2$$

$$\frac{\partial}{\partial z}=-\Gamma$$

纵横关系

$$E_x=-\frac{1}{\Gamma^2+k^2}\left(\Gamma\frac{\partial E_z}{\partial x}+j\omega\mu\frac{\partial H_z}{\partial y}\right)$$
$$E_y=-\frac{1}{\Gamma^2+k^2}\left(\Gamma\frac{\partial E_z}{\partial y}-j\omega\mu\frac{\partial H_z}{\partial x}\right)$$
$$H_x=-\frac{1}{\Gamma^2+k^2}\left(\Gamma\frac{\partial H_z}{\partial x}-j\omega\varepsilon\frac{\partial E_z}{\partial y}\right)$$
$$H_y=-\frac{1}{\Gamma^2+k^2}\left(\Gamma\frac{\partial H_z}{\partial y}+j\omega\varepsilon\frac{\partial E_z}{\partial x}\right)$$

$E(x,y)$

E_z, H_z

$H_z=0$, $E_z\neq 0$ — TM波

$$E_x=-\frac{\Gamma}{\Gamma^2+k^2}\frac{\partial E_z}{\partial x}\qquad H_x=\frac{j\omega\varepsilon}{\Gamma^2+k^2}\frac{\partial E_z}{\partial y}$$
$$E_y=-\frac{\Gamma}{\Gamma^2+k^2}\frac{\partial E_z}{\partial y}\qquad H_y=-\frac{j\omega\varepsilon}{\Gamma^2+k^2}\frac{\partial E_z}{\partial x}$$

$E_z=0$, $H_z=0$ — TEM波

$$\Gamma^2+k^2=0\rightarrow\Gamma=jk=j\omega\sqrt{\mu\varepsilon}\rightarrow v_p=\omega/k=1/\sqrt{\mu\varepsilon}$$

$$Z_{TEM}=E_x/H_y=j\omega\mu/\Gamma=\Gamma/j\omega\varepsilon=\eta$$

$$\vec{H}=\frac{1}{Z_{TEM}}\left(\hat{e}_z\times\vec{E}\right)$$

$E_z=0$, $H_z\neq 0$ — TE波

$$E_x=-\frac{j\omega\mu}{\Gamma^2+k^2}\frac{\partial H_z}{\partial y}\qquad H_x=-\frac{\Gamma}{\Gamma^2+k^2}\frac{\partial H_z}{\partial x}$$
$$E_y=+\frac{j\omega\mu}{\Gamma^2+k^2}\frac{\partial H_z}{\partial x}\qquad H_y=-\frac{\Gamma}{\Gamma^2+k^2}\frac{\partial H_z}{\partial y}$$

矩形波导

$$\nabla_{xy}^2\vec{H}+\left(\Gamma^2+k^2\right)\vec{H}=0$$
$$\nabla_{xy}^2\vec{E}+\left(\Gamma^2+k^2\right)\vec{E}=0$$

$$\vec{E}\left(E_x,E_y,E_z\right)$$

$$\nabla_{xy}^2E_z+\left(\Gamma^2+k^2\right)E_z=0$$

分离变量

$$E_z=X\left(x\right)Y\left(y\right)$$

边界条件

$$E_z\left(0,y\right)=E_z\left(a,y\right)=0$$
$$E_z\left(x,0\right)=E_z\left(x,b\right)=0$$

$$\boxed{E_z=E_0\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)}$$

TM_{nm}

$$\begin{cases}E_x=-\dfrac{\Gamma}{h^2}\dfrac{m\pi}{a}E_0\cos\left(\dfrac{m\pi}{a}x\right)\sin\left(\dfrac{n\pi}{b}y\right)e^{-\Gamma z}\\ E_y=-\dfrac{\Gamma}{h^2}\dfrac{n\pi}{b}E_0\sin\left(\dfrac{m\pi}{a}x\right)\cos\left(\dfrac{n\pi}{b}y\right)e^{-\Gamma z}\\ H_x=j\dfrac{\omega\varepsilon}{h^2}\dfrac{n\pi}{b}E_0\sin\left(\dfrac{m\pi}{a}x\right)\cos\left(\dfrac{n\pi}{b}y\right)e^{-\Gamma z}\\ H_y=-j\dfrac{\omega\varepsilon}{h^2}\dfrac{m\pi}{a}E_0\cos\left(\dfrac{m\pi}{a}x\right)\sin\left(\dfrac{n\pi}{b}y\right)e^{-\Gamma z}\end{cases}$$

$$\boxed{H_z=H_0\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)}$$

TE_{nm}

$$E_x=j\frac{\omega\mu}{h^2}\frac{n\pi}{b}H_0\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-\Gamma z}$$
$$E_y=-j\frac{\omega\mu}{h^2}\frac{m\pi}{a}H_0\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-\Gamma z}$$
$$H_x=\frac{\Gamma}{h^2}\frac{m\pi}{a}H_0\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-\Gamma z}$$
$$H_y=\frac{\Gamma}{h^2}\frac{n\pi}{b}H_0\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-\Gamma z}$$

$$\begin{cases}\text{截止区} & 2a\sim\infty\\ \text{单模区} & a\sim 2a\\ \text{多模区} & <a\end{cases}$$

基模—TE_{10}

$$\begin{cases}E_x=0\\ E_y=-j\omega\mu\dfrac{a}{\pi}H_0\sin\left(\dfrac{\pi}{a}x\right)\\ E_z=0\\ H_x=jk_z\dfrac{a}{\pi}H_0\sin\left(\dfrac{\pi}{a}x\right)\\ H_y=0\\ H_z=H_0\cos\left(\dfrac{\pi}{a}x\right)\end{cases}$$

$$h^2=k_x^2+k_y^2=\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2$$

$$\Gamma=\sqrt{h^2-k^2}=\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2-\omega^2\mu\varepsilon}$$

$$h^2=\Gamma^2+k^2$$

$$\Gamma=0$$

$$\lambda_c=\frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}$$