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Incomplete Cholesky factorization

In numerical analysis, an **incomplete Cholesky factorization** of a symmetric positive definite matrix is a sparse approximation of the Cholesky factorization. An incomplete Cholesky factorization is often used as a preconditioner for algorithms like the conjugate gradient method.

The Cholesky factorization of a positive definite matrix A is $A = LL^*$ where L is a lower triangular matrix. An incomplete Cholesky factorization is given by a sparse lower triangular matrix K that is in some sense close to L. The corresponding preconditioner is KK^*.

One popular way to find such a matrix K is to use the algorithm for finding the exact Cholesky decomposition, except that any entry is set to zero if the corresponding entry in A is also zero. This gives an incomplete Cholesky factorization which is as sparse as the matrix A.

Algorithm

For i from 1 to N:

$$L_{ii} = \left(a_{ii} - \sum_{k=1}^{i-1} L_{ik}^2 \right)^{\frac{1}{2}}$$

For j from $i+1$ to N:

$$L_{ji} = \frac{1}{L_{ii}} \left(a_{ji} - \sum_{k=1}^{i-1} L_{ik} L_{jk} \right)$$

Implementation

Implementation of the incomplete Cholesky factorization in the Octave scripting language. The factorization is stored as a lower triangular matrix, with the elements in the upper triangle set to zero.

```
function a = ichol(a)
    n = size(a,1);

    for k=1:n
        a(k,k) = sqrt(a(k,k));
        for i=(k+1):n
            if (a(i,k)!=0)
                a(i,k) = a(i,k)/a(k,k);
            endif
        endfor
        for j=(k+1):n
            for i=j:n
                if (a(i,j)!=0)
                    a(i,j) = a(i,j)-a(i,k)*a(j,k);
                endif
            endfor
        endfor
    endfor

    for i=1:n
        for j=i+1:n
            a(i,j) = 0;
        endfor
```

```
    endfor
endfunction
```

References

- Incomplete Cholesky factorization (http://www.cfd-online.com/Wiki/Incomplete_Cholesky_factorization) at CFD Online wiki
- Golub, Gene H.; Van Loan, Charles F. (1996), *Matrix Computations* (3rd ed.), Johns Hopkins, ISBN 978-0-8018-5414-9. See Section 10.3.2.