std::accumulate

EXPLORING AN ALGORITHMIC EMPIRE

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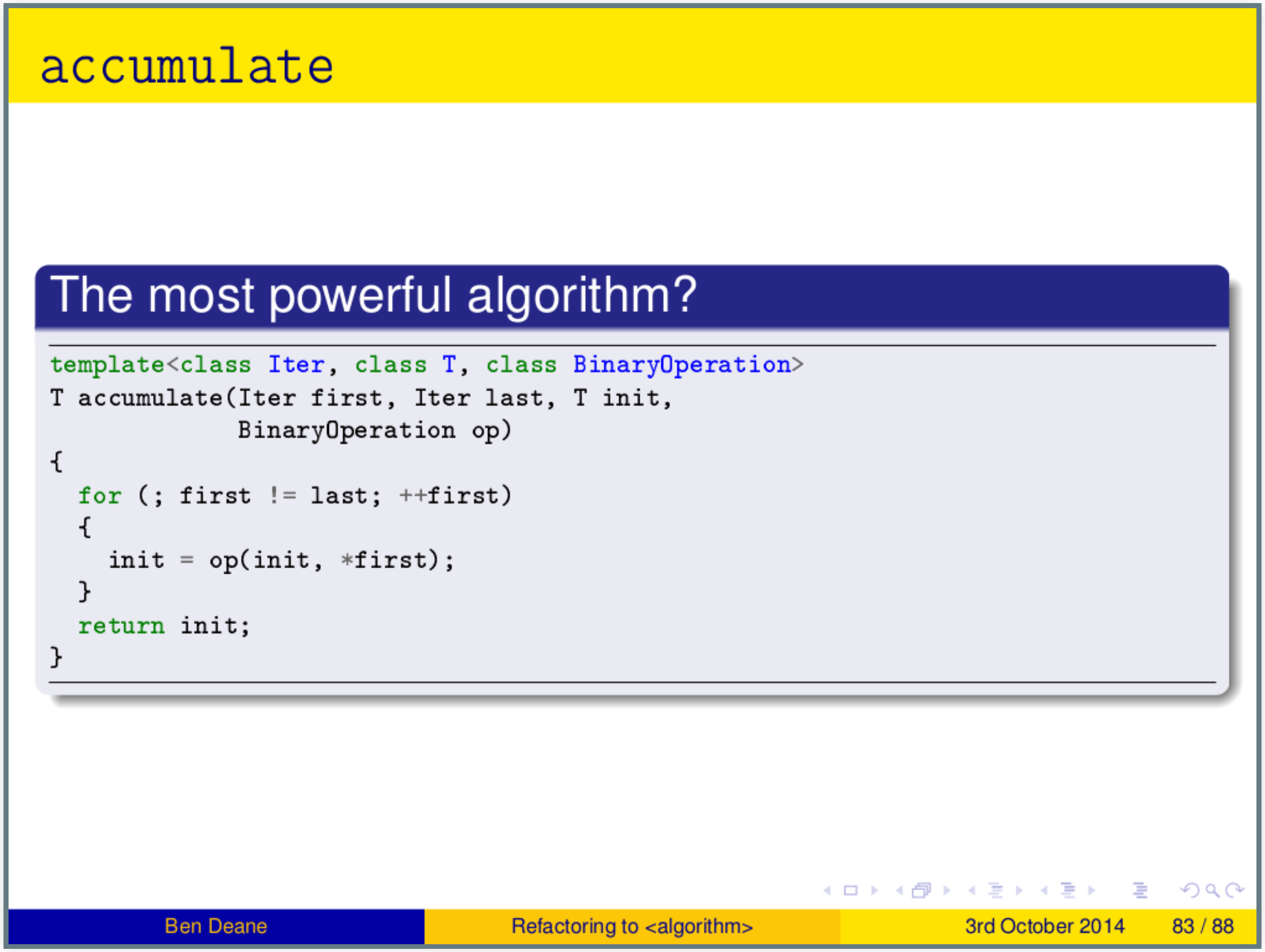
PART 0

The Most Powerful Algorithm in the World?

A LONG TIME AGO, IN A GALAXY ETC...



accumulate

The most powerful algorithm?

```
template<class Iter, class T, class BinaryOperation>
T accumulate(Iter first, Iter last, T init,
             BinaryOperation op)
{
  for (; first != last; ++first)
  {
    init = op(init, *first);
  }
  return init;
}
```

Ben Deane Refactoring to <algorithm> 3rd October 2014 83 / 88

PART 1

Accumulatable Things

std::accumulate

```
template <class InputIt, class T, class BinaryOp>
T accumulate(InputIt first, InputIt last,
             T init, BinaryOp op)
{
  for (; first != last; ++first) {
    init = op(init, *first);
  }
  return init;
}
```

TYPICAL USES

```
vector<int> v = {1,2,3,4,5};

int sum = accumulate(v.cbegin(), v.cend(), 0, plus<>{});
cout << sum << '\n';

int product = accumulate(v.cbegin(), v.cend(), 1, multiplies<>{});
cout << product << '\n';
```

Of course, this is not why you're here. What else can we accumulate?

HOW ABOUT FINDING A MIN OR MAX?

```
vector<unsigned> v = {1,2,3,4,5};

unsigned max_val = accumulate(v.cbegin(), v.cend(), 0,
    [] (unsigned a, unsigned b) { return a > b ? a : b; });
cout << max_val << '\n';
```

HOW ABOUT FINDING A MIN OR MAX?

Value-based `min_element` or `max_element`

```
template <typename It, typename Compare,
          typename T = typename iterator_traits<It>::value_type>
T min_element(It first, It last, Compare cmp)
{
  // precondition: first != last
  auto init = *first;
  return accumulate(
      ++first, last, init,
      [&] (const T& a, const T& b) {
        return cmp(b, a) ? b : a;
      });
}
```

WHAT ABOUT `bool` VALUES?

```
bool results[] = {true, false, true, true, false};

bool all_true = accumulate(cbegin(results), cend(results),
                           true, logical_and<>{});
bool some_true = accumulate(cbegin(results), cend(results),
                            false, logical_or<>{});
bool none_true = !accumulate(cbegin(results), cend(results),
                             false, logical_or<>{});
```

Not that interesting yet...

THE SIGNATURE OF THE FUNCTION

```
Type1 fun(const Type1 &a, const Type2 &b);
```

So far, we've looked at `Type1` and `Type2` being the same.

Things get more interesting when they differ.

A MORE INTERESTING `bool` CASE

```
map<int, weak_ptr<thing>> cache;

shared_ptr<thing> get_thing(int id) {
  auto sp = cache[id].lock();
  if (!sp) make_async_request(id);
  return sp;
}

void load_things(const vector<int>& ids)
{
  bool all_cached = accumulate(
    ids.cbegin(), ids.cend(), true,
    [] (bool cached, int id) {
      return get_thing(id) && cached;
    });
  if (!all_cached)
    service_async_requests();
}
```

`bool` AS THE RESULT

We use many function results as boolean values in control flow.

- actual `bool`
- pointers
- zero-result of a comparison trichotomy
- anywhere else we want to write `if (x)`

This means we can use `accumulate` to collect these function values. (Similar to `all_of`, `any_of`, `none_of`, but where we don't want the short-circuiting behavior.)

MORE THINGS...

- joining strings
- building requests from key-value pairs
- merging JSON objects
- multiplying matrices

What do all of these have in common?

YOU ALL REMEMBER MONOIDS?

A set of objects and an operation such that:

- The operation is closed over the set
- The operation is associative
- There is an identity element

BUILDING HTTP HEADERS: BEFORE

```
curl_slist* curl_headers = NULL;
for (auto it = headers.begin();
     it != headers.end(); ++it)
{
  curl_headers = curl_slist_append(curl_headers,
    (format("%s: %s") % it->first % it->second).str().c_str());
}
```

BUILDING HTTP HEADERS: AFTER

```
curl_slist* curl_headers = accumulate(
    headers.cbegin(), headers.cend(), static_cast<curl_slist*>(nullptr),
    [] (curl_slist* h, const auto& p) {
        return curl_slist_append(h,
          (format("%s: %s") % p.first % p.second).str().c_str());
      });
```

MONOIDS ARE EVERYWHERE!

Monoids are everywhere, and any monoid can be accumulated:

- addition on integers
- concatenation on strings
- union on sets
- "merging" objects of all kinds
- max, min, and, or, ...
- parsing
- many more things...

MORE MONOID OBSERVATIONS

A type may be a monoid in more than one way (under more than one operation).

A function that returns a monoid is a monoid.

An aggregate of monoids is a monoid. (e.g. `map<K,V>` where `V` is a monoid)

WHY NOT JUST WRITE A LOOP?

Some advantages to `accumulate`

- No declaration/initialization split
- It's often easier to write a binary function
  - Or a unary function with monoidal output
  - Simplifies an API
- Incremental computation
  - Can accumulate by parts
- Potential for parallel computation

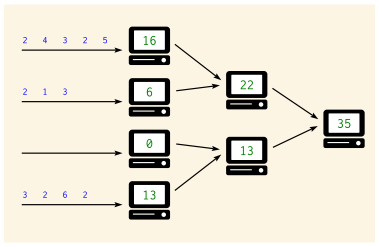
WHAT `accumulate` CAN DO

- Turn binary functions into n-ary functions
- Collect results of functions whose outputs are monoidal
- Allow part-whole hierarchies to be treated uniformly
  - which unlocks parallel computation

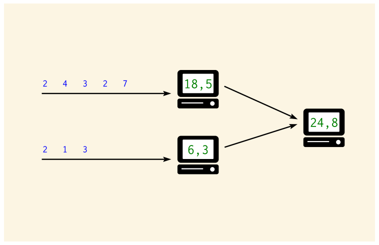
PART 2

Aside: Parallel Computations and Monoids

DISTRIBUTED ACCUMULATE



DISTRIBUTED ACCUMULATE



std::reduce

```
template <class InputIt, class T, class BinaryOp>
T reduce(InputIt first, InputIt last,
         T init, BinaryOp op);
```

The same as `accumulate`, except the sequence may be processed in any order (and perhaps in parallel according to policy).

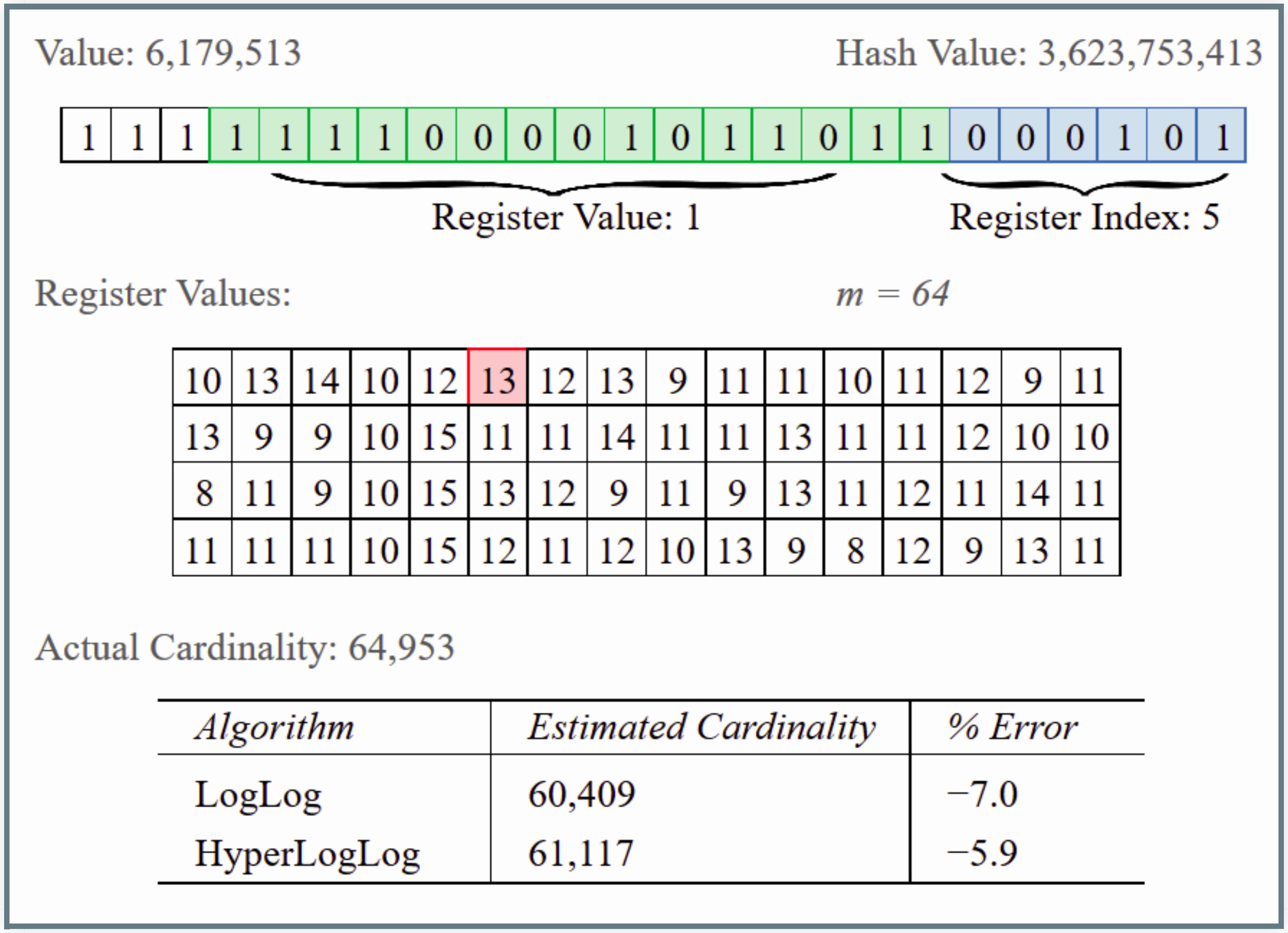
This works because of associativity (semigroup property).

We lose the type variation, but gain parallelism.

BIG DATA MONOIDS EVERYWHERE

- averages (regular or decayed)
- top-N calculations
- histograms
- bloom filters
- Gaussian distributions
- count-min sketch
- Hyperloglog

HYPERLOGLOG



Register Values: $m = 64$

10	13	14	10	12	13	12	13	9	11	11	10	11	12	9	11
13	9	9	10	15	11	11	14	11	11	13	11	11	12	10	10
8	11	9	10	15	13	12	9	11	9	13	11	12	11	14	11
11	11	11	10	15	12	11	12	10	13	9	8	12	9	13	11

Actual Cardinality: 64,953

Algorithm	*Estimated Cardinality*	*% Error*
LogLog	60,409	−7.0
HyperLogLog	61,117	−5.9

From http://content.research.neustar.biz/blog/hll.html

ALGEBRAIC STRUCTURES IN BIG DATA

- monoids and semigroups are the key to parallelism
- the ability to combine "summary data"
- expensive training happens once

PART 3

Nonlinear Structures

`accumulate` WORKS ON LINEAR SEQUENCES

How would we make it work on multi-dimensional structures?

Maybe we can define a linear traversal on the structure (in-order, pre-order, post-order)...

But the nodes are still homogeneous...

What if it's a bit more complex? (Like say, a JSON object?)

RECALL `std::accumulate`

```cpp
template <class InputIt, class T, class BinaryOp>
T accumulate(InputIt first, InputIt last,
             T init, BinaryOp op);
```

The `T` here deals with an empty sequence.

The `BinaryOp` deals with a non-empty sequence.

RECURSIVE DEFINITION OF A `vector`

We can view "sequence accumulation" as handling two cases:

- an empty `vector`
- a `vector` consisting of an element plus another `vector`

This is the sort of recursive definition we find in functional languages. And it's the key to accumulating other data structures.

`std::accumulate` VIEWED RECURSIVELY

```
template <typename It, typename EmptyOp, typename NonEmptyOp>
auto recursive_accumulate(It first, It last,
                          EmptyOp op1, NonEmptyOp op2)
{
  if (first == last) return op1();
  --last;
  return op2(recursive_accumulate(first, last, op1, op2), *last);
}
```

`T` (here `EmptyOp`) is really a function from empty `vector` to `T`

`BinaryOp` (here `NonEmptyOp`) is really a function from (element, `vector`) to `T`

ACCUMULATING A `variant`

```
struct JSONWrapper;
using JSONArray = vector<JSONWrapper>;
using JSONObject = map<string, JSONWrapper>;
using JSONValue = variant<bool,
                          double,
                          string,
                          nullptr_t,
                          JSONArray,
                          JSONObject>;
struct JSONWrapper
{
  JSONValue v;
  operator JSONValue&() { return v; }
  operator const JSONValue&() const { return v; }
};
```

EXAMPLE: RENDER A `JSONValue` AS A `string`

We need a function for each distinct type that can be inside the `variant`.

```
string render_json_value(const JSONValue& jsv);

string render_bool(bool b) { return b ? "true" : "false"; };
string render_double(double d) { return to_string(d); };
string render_string(const string& s)
{
  stringstream ss;
  ss << quoted(s);
  return ss.str();
}
string render_null(nullptr_t) { return "null"; }
```

EXAMPLE: RENDER A `JSONValue` AS A `string`

We need a function for each distinct type that can be inside the `variant`.

```
string render_array(const JSONArray& a)
{
  return string{"["}
    + join(a.cbegin(), a.cend(), string{","},
           [] (const JSONValue& jsv) {
             return render_json_value(jsv);
           })
    + "]";
}
```

EXAMPLE: RENDER A `JSONValue` AS A `string`

We need a function for each distinct type that can be inside the `variant`.

```
string render_object(const JSONObject& o)
{
  return string{"{"}
    + join(o.cbegin(), o.cend(), string{","},
           [] (const JSONObject::value_type& jsv) {
             return render_string(jsv.first) + ":"
               + render_json_value(jsv.second);
           })
    + "}";
}
```

EXAMPLE: RENDER A `JSONValue` AS A `string`

We need a function for each distinct type that can be inside the `variant`.

```
string render_json_value(const JSONValue& jsv)
{
  return fold(jsv,
              render_bool, render_double, render_string,
              render_null, render_array, render_object);
}
```

A GENERIC `fold` FOR `variant`

```
template <typename... Ts, typename... Fs>
auto fold(const variant<Ts...>& v, Fs&&... fs)
{
  static_assert(sizeof...(Ts) == sizeof...(Fs),
                "Not enough functions provided to variant fold");
  return fold_at(
      v, v.index(),
      forward<Fs>(fs)...);
}
```

A `variant`, and N functions (one for each case of the `variant`).

Recall that the "zero value" is implicit in the functions if required.

A GENERIC `fold` FOR `variant`

```
template <typename T, typename F, typename... Fs>
static auto fold_at(T&& t, size_t n, F&& f, Fs&&... fs)
{
  using R = decltype(f(get<0>(t)));
  return apply_at<0, sizeof...(Fs)+1>::template apply<R, T, F, Fs...>(
      forward<T>(t),
      n,
      forward<F>(f),
      forward<Fs>(fs)...);
}
```

A GENERIC fold FOR variant

```cpp
template <size_t N, size_t Max>
struct apply_at
{
  template <typename R, typename T, typename F, typename... Fs>
  static auto apply(T&& t, size_t n, F&& f, Fs&&... fs)
  {
    if (n == N)
      return forward<F>(f)(get<N>(forward<T>(t)));
    else
      return apply_at<N+1, Max>::template apply<R, T, Fs...>(
          forward<T>(t),
          n,
          forward<Fs>(fs)...);
  }
};
```

A GENERIC `fold` FOR `variant`

```cpp
template <size_t Max>
struct apply_at<Max, Max>
{
  template <typename R, typename T, typename... Fs>
  static auto apply(T, size_t, Fs...)
  {
    assert("Variant index out of range" && false);
    return R{};
  }
};
```

GENERIC `variant` ACCUMULATION

```
template <typename... Ts, typename... Fs>
auto fold(const variant<Ts...>& v, Fs&&... fs)
```

Hmm, this looks a lot like visitation.

RECURSIVE REDUCTION

Any recursively-specified data structure can be accumulated using "visitation" to produce a monoidal value which is accumulated.

- tree -> string rendering
- depth, fringe of trees
- lighting contributions
- scene graph operations
- etc, etc

This is a useful alternative when dealing with heterogeneous hierarchies that it is difficult to define a linear traversal for.

PART 4

Heterogeneous Sequences

BEYOND MONOIDS

```
template <class InputIt, class T, class BinaryOp>
T accumulate(InputIt first, InputIt last,
             T init, BinaryOp op);

Type1 fun(const Type1 &a, const Type2 &b);
```

`Type1` and `Type2` can be different: this is saying that we know how to "fold" values of `Type2` into values of `Type1`.

HETEROGENEOUS FOLDING

```
template <typename T>
Accumulator fun(const Accumulator &a, const T &b);
```

What if `T` varied all the time? We could have cases where we know how to "fold" lots of different types into an accumulated value.

THE OBVIOUS EXAMPLE

```
template <typename T>
ostream& operator<<(ostream& s, const T &t);
```

THE OBVIOUS EXAMPLE

```
auto t = make_tuple("Hello", 3.14, 1729, 'a');
auto f = [] (ostream& s, const auto& x) -> ostream& {
  return s << x << '\n';
};
fold(t, cout, f) << "done" << endl;
```

```
$ ./a.out
Hello
3.14
1729
a
done
$
```

HETEROGENEOUS FOLDING

```
template <typename F, typename Z, typename... Ts>
decltype(auto) fold(const tuple<Ts...>& t, Z&& z, F&& f);
```

(Implementation left as an exercise - link at the end)

DIFFERENT TYPES OF ACCUMULATION

- `accumulate`
  - 1 function, linear homogeneous structure
- `accumulate` with linear tree traversal
  - 1 function, multidimensional homogeneous structure
- variant-fold
  - n functions, multidimensional heterogeneous structure
- tuple-fold
  - n functions, linear heterogeneous structure

The empire so far... all of these could also be parallel, given the appropriate monoidal (or semigroup) structure.

PART 5

The Opposite of Accumulate?

FOLD? UNFOLD

If `accumulate` is folding up a data structure to produce a value…

The opposite is "unfolding" a seed value to produce a data structure.

HOW TO UNFOLD

```
template <typename InputIt, typename T, typename F>
T accumulate(InputIt first, InputIt last, T init, F f);

template <typename OutputIt, typename T, typename F>
OutputIt unfold(F f, OutputIt it, T init);
```

`F` will be repeatedly called with a "reducing" `T` value and write the result(s) to the output `it`.

- What should the signature of `F` be?
- How do we know when we're done?

SIGNATURE OF THE FUNCTION PASSED TO `unfold`

`F` is the opposite of `accumulate`'s BinaryOp

```
Type1 fun(const Type1 &a, const Type2 &b);
```

It's clear that `F` needs to return a `pair`

- result (say of type `U`) to write into the output range
- new value of `T` to feed into next invocation of `F`

In general the "result to write to the iterator" may be a range or sequence of values.

THREE CHOICES FOR `unfold` TERMINATION

```
// 1. provide a sentinel value of type T
template <typename OutputIt, typename T, typename F>
OutputIt unfold(F f, OutputIt it, T init, T term);
```

Choice 1: terminate when a termination value (of type `T`) is returned.

THREE CHOICES FOR `unfold` TERMINATION

```
// 2. provide a sentinel value of type (other thing returned by F)
template <typename OutputIt, typename T, typename F, typename U>
OutputIt unfold(F f, OutputIt it, T init, U term);
```

Choice 2: terminate when a termination value (of type `U`) is returned.

THREE CHOICES FOR `unfold` TERMINATION

```
// 3. F will return an optional
template <typename OutputIt, typename T, typename F>
OutputIt unfold(F f, OutputIt it, T init);
```

Choice 3: `F` returns an `optional`; terminate when `nullopt` is returned.

HOW TO UNFOLD

```
template <typename OutputIt, typename T, typename F>
OutputIt unfold(F f, OutputIt it, T&& init)
{
  for (auto o = f(forward<T>(init)); o;
       o = f(move(o->second))) {
    it = move(begin(o->first), end(o->first), it);
  }
  return it;
}
```

F returns `optional<pair<range, T>>`

UNFOLD EXAMPLE

```cpp
optional<pair<string, int>> to_roman(int n)
{
  if (n >= 1000) return {{ "M", n-1000 }};
  if (n >= 900) return {{ "CM", n-900 }};
  if (n >= 500) return {{ "D", n-500 }};
  if (n >= 400) return {{ "CD", n-400 }};
  if (n >= 100) return {{ "C", n-100 }};
  if (n >= 90) return {{ "XC", n-90 }};
  if (n >= 50) return {{ "L", n-50 }};
  if (n >= 40) return {{ "XL", n-40 }};
  if (n >= 10) return {{ "X", n-10 }};
  if (n >= 9) return {{ "IX", n-9 }};
  if (n >= 5) return {{ "V", n-5 }};
  if (n >= 4) return {{ "IV", n-4 }};
  if (n >= 1) return {{ "I", n-1 }};
  return nullopt;
}
```

UNFOLD EXAMPLE

```
int main()
{
  string r;
  unfold(to_roman, back_inserter(r), 1729);
  cout << r << '\n';
}
```

```
$ ./a.out
MDCCXXIX
$
```

FOLD AND UNFOLD ARE REALLY THE SAME

Just transformations on a data structure.

Which you use is a matter of convenience.

We think of `accumulate` as working on structures and producing values, and `unfold` vice versa.

But structures are themselves values…

POSTSCRIPT

The Fruits of Algorithmic Perversions

THE QUESTION

If you were stuck on a desert island, which algorithms would you take with you?

Maybe some "building block" algorithms?

- `partition`
- `rotate`
- `reverse`

Maybe some others?

Which algorithms are the most powerful?

What if you couldn't write any loops, so you're stuck with what you have?

THE ALGORITHMS (PRE-C++17)

accumulate	adjacent_difference	adjacent_find	all_of	any_of
binary_search	copy	copy_backward	copy_if	copy_n
count	count_if	equal	equal_range	fill
fill_n	find	find_end	find_first_of	find_if
find_if_not	for_each	generate	generate_n	includes
inner_product	inplace_merge	iota	is_heap	is_heap_until
is_partitioned	is_permutation	is_sorted	is_sorted_until	
lexicographical_compare	lower_bound	make_heap	max	max_element
merge	min	min_element	minmax	minmax_element
mismatch	move	move_backward	next_permutation	none_of
nth_element	partial_sort	partial_sort_copy	partial_sum	partition
partition_copy	partition_point	pop_heap	prev_permutation	push_heap
	remove	remove_copy	remove_copy_if	remove_if
replace	replace_copy	replace_copy_if	replace_if	reverse
reverse_copy	rotate	rotate_copy	search	search_n
set_difference	set_intersection	set_symmetric_difference	set_union	shuffle
sort	sort_heap	stable_partition	stable_sort	

Of 90 total, 77 are basically "plain loops"

WHY DOESN'T `accumulate` WORK ON ITERATORS?

```cpp
template <class InputIt, class T, class BinaryOp>
T accumulate(InputIt first, InputIt last,
             T init, BinaryOp op)
{
  for (; first != last; ++first) {
    init = op(init, *first);
  }
  return init;
}
```

The function receives a value rather than an iterator…

WHY DOESN'T `accumulate` WORK ON ITERATORS?

```
template <class InputIt, class T, class BinaryOp>
T accumulate(InputIt first, InputIt last,
             T init, BinaryOp op)
{
  for (; first != last; ++first) {
    init = op(init, first);
  }
  return init;
}
```

With this formulation, we can view an iterator as an accumulator value.

ABUSING `accumulate`

I'm the first to admit that some of these algorithm implementations are… interesting.

- `find` (using exceptions for control flow)
- `reverse` (using forward iterators and a `function` as the accumulator)

BUT...

Some interesting alternatives arise.

- `find_if` -> `find_all`?
- `adjacent_find` -> `adjacent_find_all`?
- `min_element` that returns an `optional` value?
- `sort` with forward iterators?

HUNTING FOR (RAW?) LOOPS, REDUX

Almost everything can be expressed as some form of accumulation.

Should it be? That's for you to decide.

But when you get used to seeing monoids, everything is monoids.

SOME FURTHER READING

- Fold (Haskell Wiki)
- An Introduction to Recursion Schemes
- Gaussian Distributions form a Monoid
- Add ALL the Things: Abstract Algebra Meets Analytics
- Variant folding code
  - http://gist.github.com/elbeno/e5c333fff247e78990e08ab8ed56aecd
- Tuple folding code
  - http://gist.github.com/elbeno/95e710c04d56d8cc72ade2e6b2bbe9d5

ACCUMULATION POWER

- Any time you write an API, see if any of your data types form a monoid or a semigroup under any operations you provide.
- Look for opportunities where you are applying a function in a loop.
- Monoids are everywhere.
- Think about folding with multidimensional structures or heterogeneous sequences.
- Unfolds are an alternative way to think of things; can be combined with folds to produce arbitrary structural transformations.
- Algorithmic perversions can be fruitful in terms of learning…
- Accumulation: not just for adding things up!