《STOCHASTIC LOSS RESERVING USING BAYESIAN MCMC MODELS》模型及效果总结

一、数据说明

二、模型总览

三、模型效果

一、数据说明

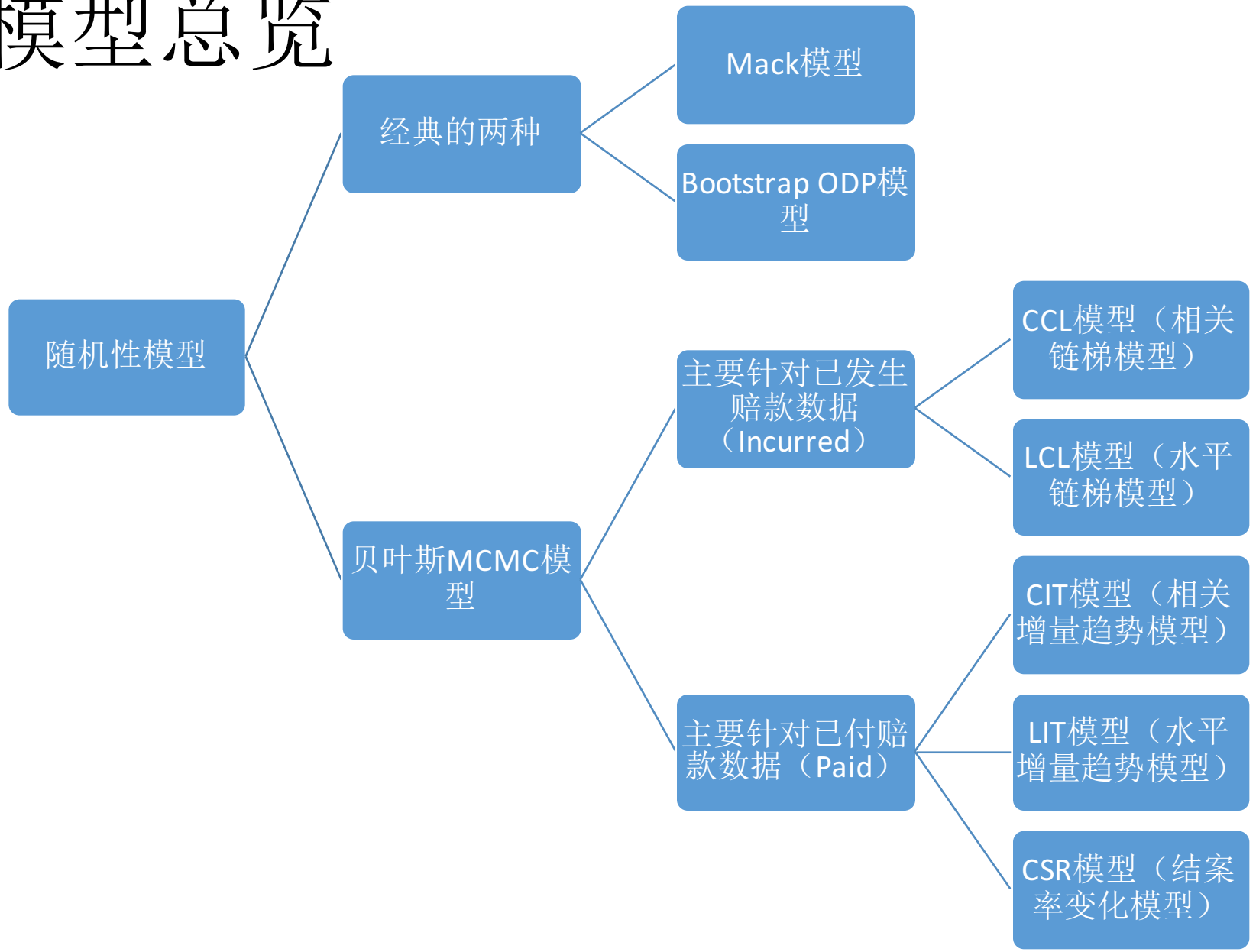
- 【数据来源】
  http://www.casact.org/research/index.cfm?fa=loss_reserves_data

- 【数据选取】

4个险种（Commercial Auto, Personal Auto, Workers Compensation, Other Liability）中，每个险种选择50家保险公司的数据。

（数据选取依据和过程详见文章Appendix A。）

二、模型总览


随机性模型
经典的两种
Mack模型
Bootstrap ODP模型
贝叶斯MCMC模型
主要针对已发生赔款数据（Incurred）
CCL模型（相关链梯模型）
LCL模型（水平链梯模型）
主要针对已付赔款数据（Paid）
CIT模型（相关增量趋势模型）
LIT模型（水平增量趋势模型）
CSR模型（结案率变化模型）

Mack模型 - Mack (1993, 1994)

- 【模型假设】

假设$\tilde{C}_{w,d}$是随机变量：

1. $\mathrm{E}\left[\tilde{C}_{w,d+1}\middle|C_{w,1},\ldots,C_{w,d}\right]=C_{w,d}\cdot f_d$
2. For any given d, the random variables $\tilde{C}_{v,d}$ and $\tilde{C}_{w,d}$ are independent for $v\neq w$.
3. $\mathrm{Var}\left[\tilde{C}_{w,d+1}\middle|C_{w,1},\ldots,C_{w,d}\right]=C_{w,d}\cdot\alpha_d^2$

- 【参数估计】 The Mack estimate for $\mathrm{E}\left[\tilde{C}_{w,K}\right]$ for $w=2,\ldots,K$ is given by

$$\hat{C}_{w,K}=C_{w,K+1-w}\cdot\hat{f}_{K+1-w}\cdot\cdots\cdot\hat{f}_{K-1}$$

where

$$\hat{f}_d=\frac{\sum_{w=1}^{K-d}C_{w,d+1}}{\sum_{w=1}^{K-d}C_{w,d}}$$

Given his assumptions above, Mack then derives expressions for the standard deviations $\mathrm{SD}\left[\tilde{C}_{w,K}\right]$ and $\mathrm{SD}\left[\sum_{w=2}^{K}\tilde{C}_{w,K}\right]$

Bootstrap ODP模型 - England and Verrall (2002)

- 【模型假设】

A key assumption made by this model is that the incremental losses are described by the overdispersed Poisson distribution with

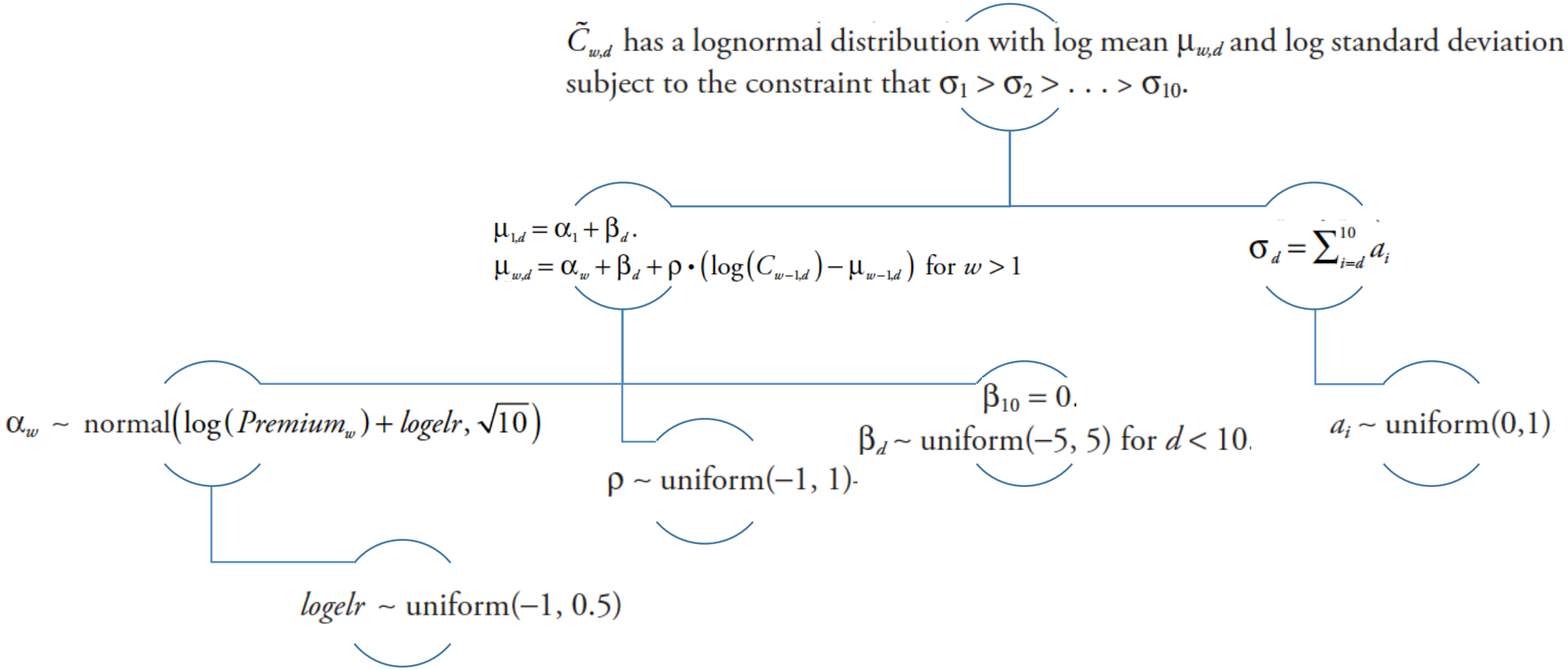
$$E\left[\tilde{I}_{w,d}\right] = \alpha_w \cdot \beta_d \quad \text{and} \quad Var\left[\tilde{I}_{w,d}\right] = \phi \cdot \alpha_w \cdot \beta_d$$

- 【参数估计】

The parameters of the model can be estimated by a standard generalized linear model (GLM) package.[9] They then use a bootstrap resampling procedure to quantify the volatility of the estimate.

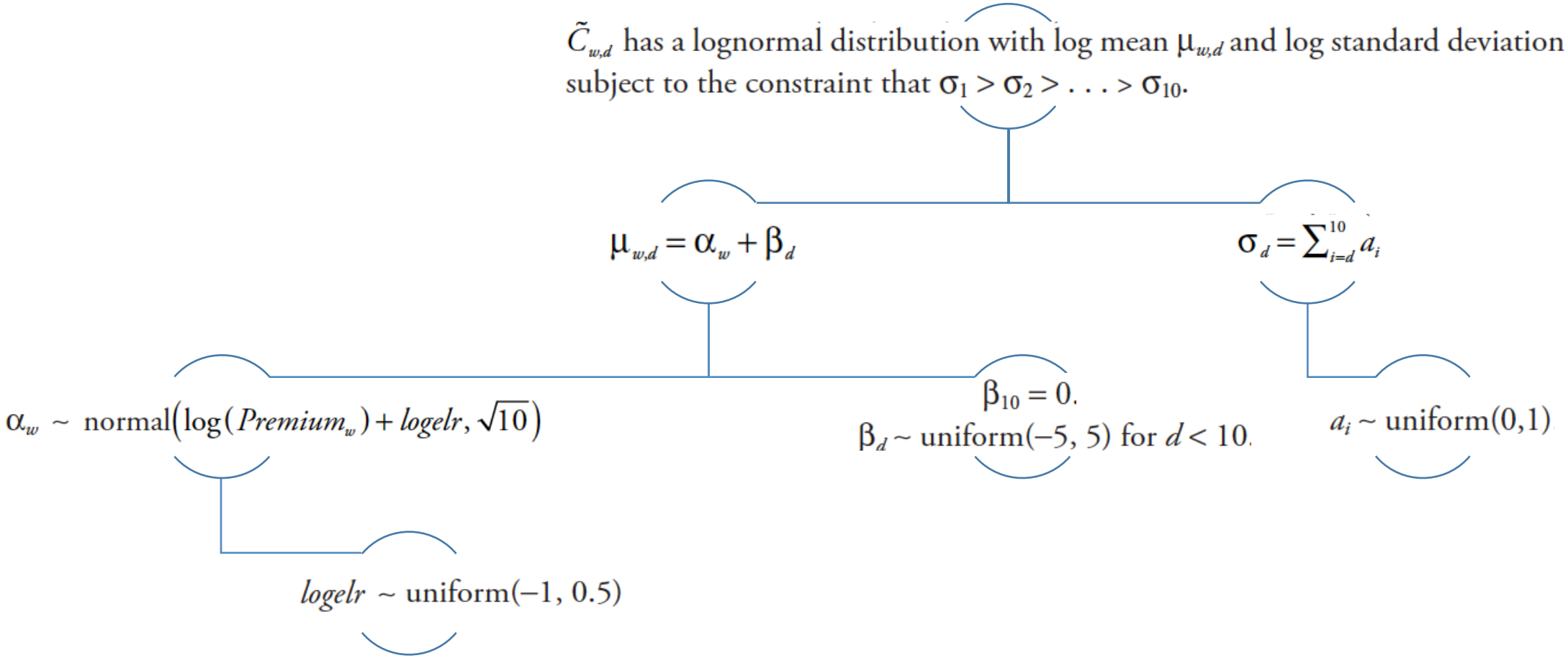
CCL模型（相关链梯模型）

- 【模型假设】假设事故年之间的累计赔款是相关的。



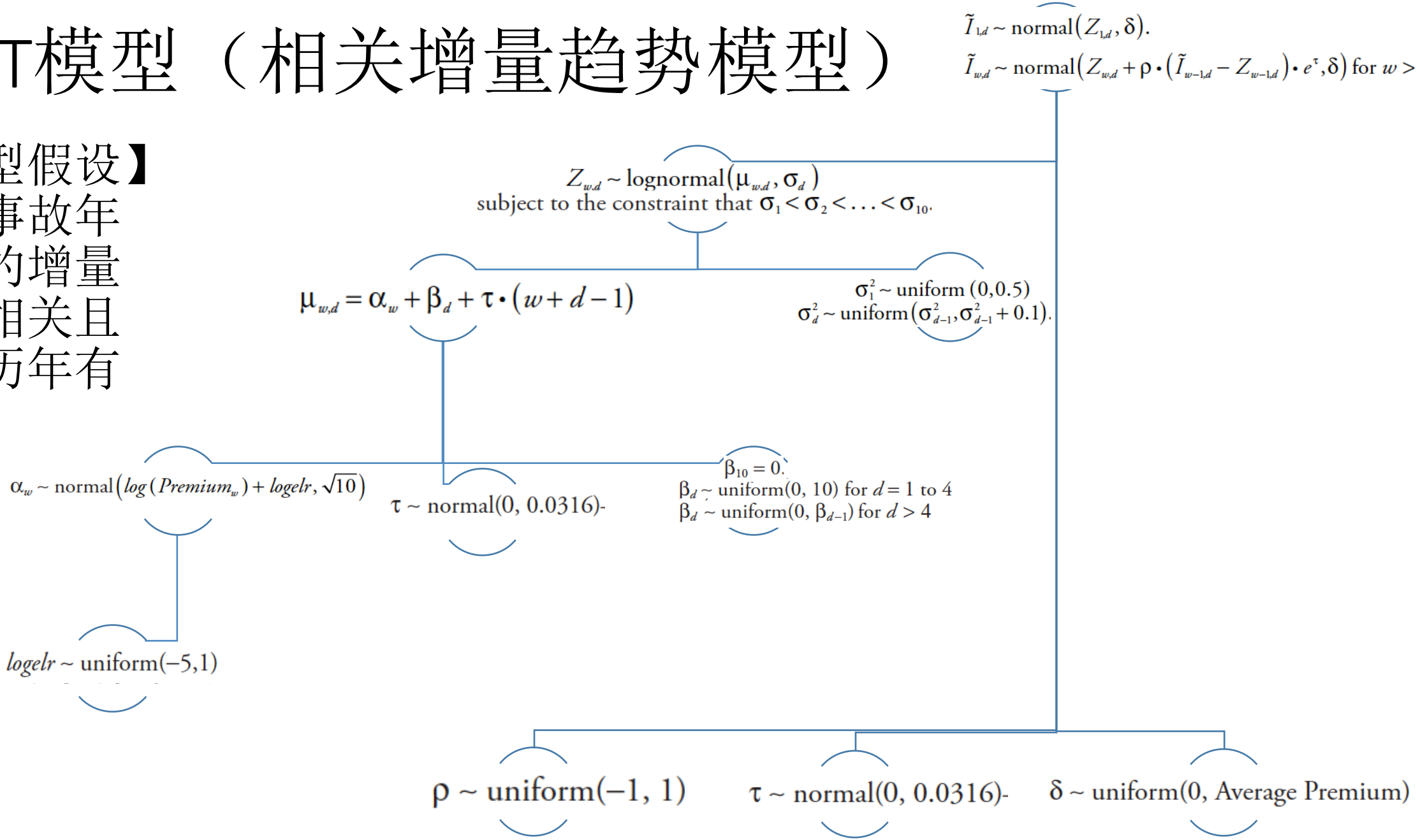
LCL模型（水平链梯模型）

- 【模型假设】CCL模型的特例：$\rho = 0$



CIT模型（相关增量趋势模型）

- 【模型假设】假设事故年之间的增量赔款相关且随日历年有趋势。

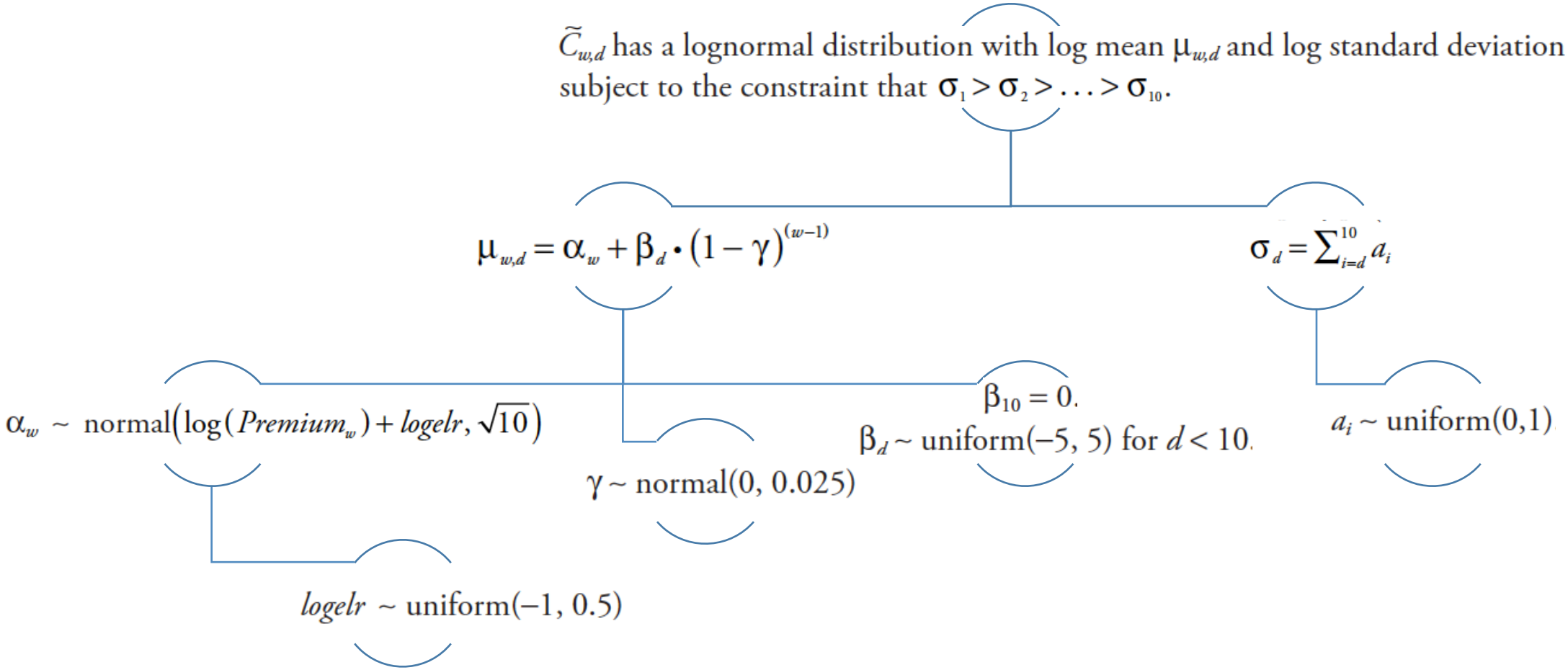


LIT模型（水平增量趋势模型）

- 【模型假设】CIT模型的特例：$\rho = 0$

CSR模型（结案率变化模型）

- 【模型假设】考虑案件处理速度的逐年变化。



三、模型效果

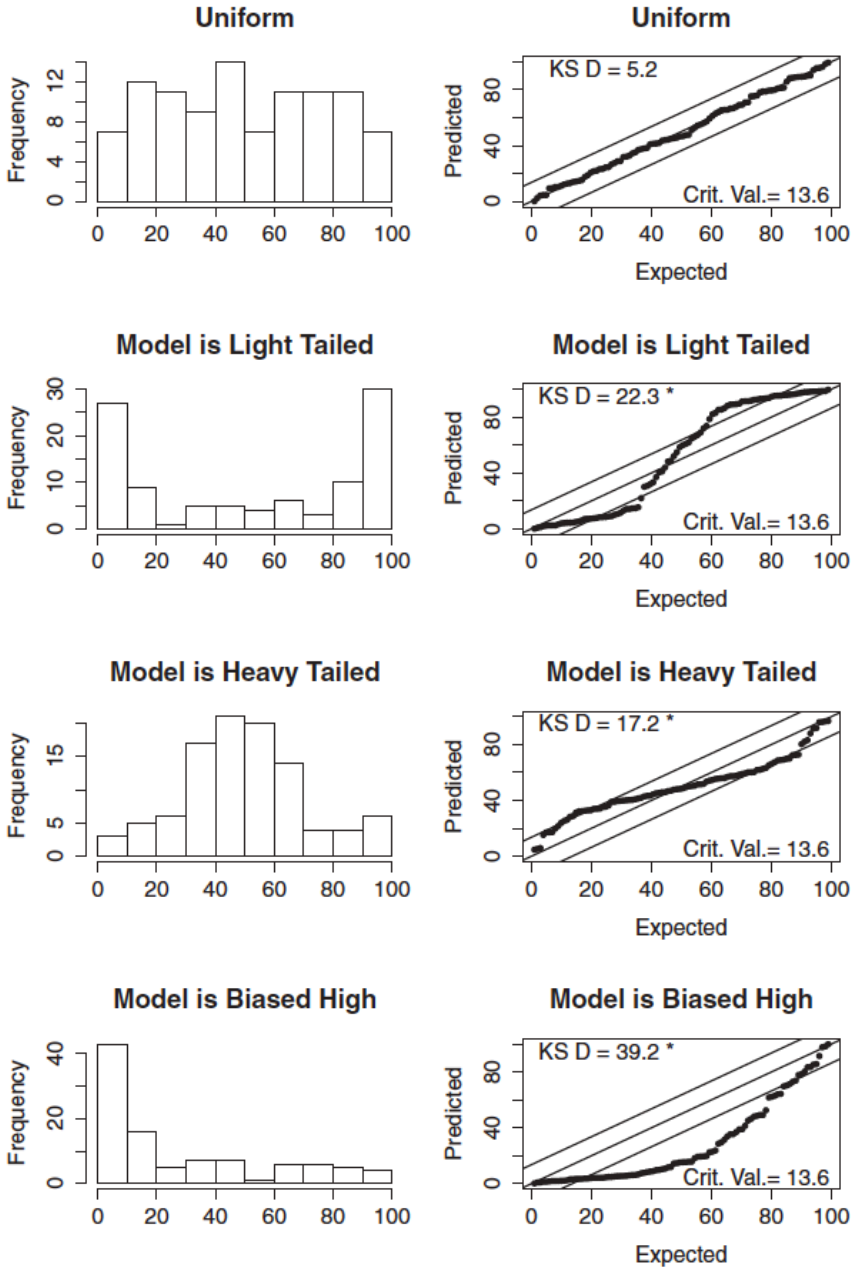
- 【评价标准】

假设$\sum_{w=1}^{10} C_{w,10}$服从对数正态分布，均值和标准差分别为通过模型得到的$\sum_{w=1}^{10} \tilde{C}_{w,10}$的均值和标准差。

计算原始数据中$\sum_{w=1}^{10} C_{w,10}$在该对数正态分布中的百分位数（percentile）。

如果所有选取样本得到的百分位数的分布接近均匀分布，则说明模型效果较好。

Figure 1. *p–p* Plots Test for Uniformity



For Incurred Loss Data： CCL＞LCL＞Mack

Figure 2. *p–p* Plots for the Mack Model on Incurred Loss Triangles

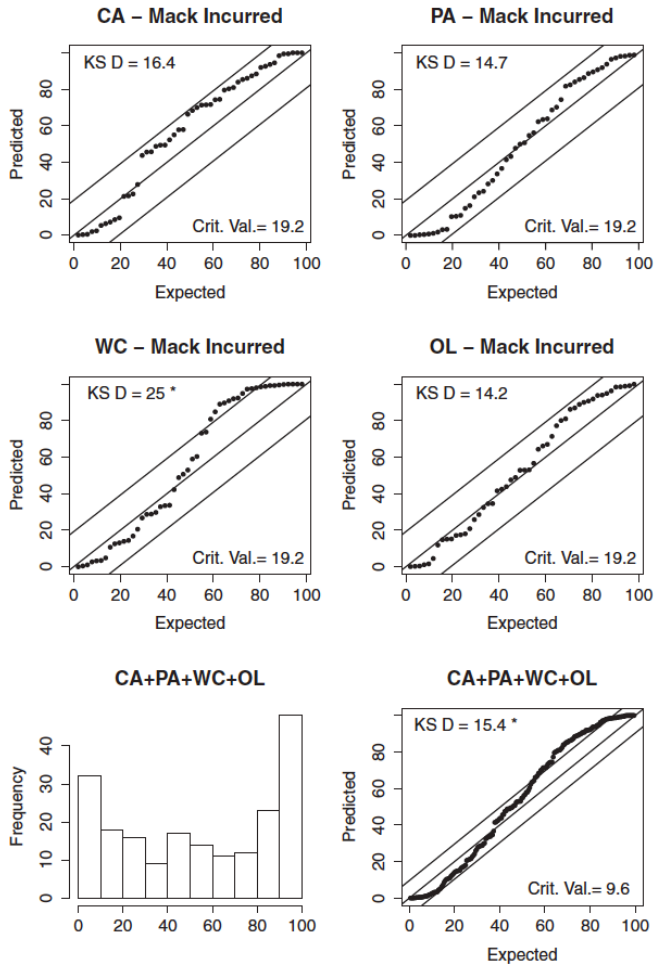


Figure 8. *p–p* Plots for the LCL Model on the Incurred Loss Triangles

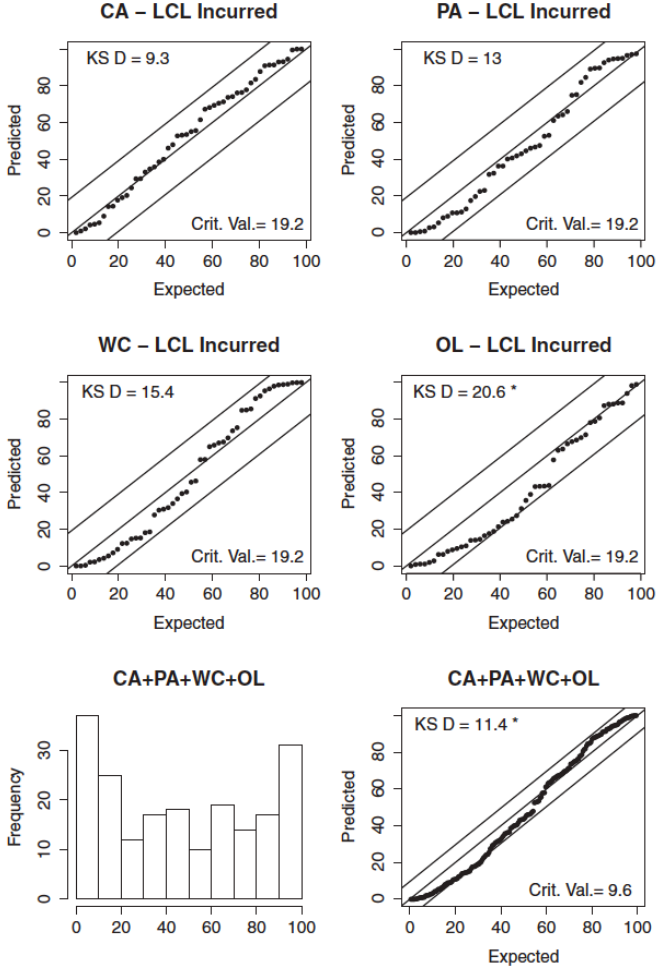
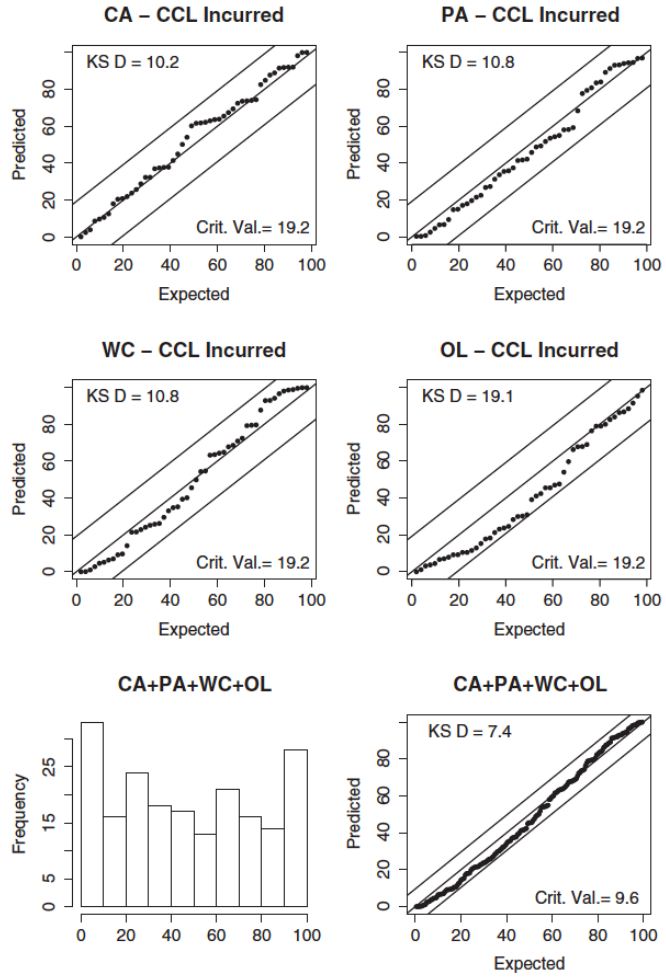


Figure 9. *p–p* Plots for the CCL Model on the Incurred Loss Triangles



For Paid Loss Data：
CSR＞CIT≈LIT＞Bootstrap ODP＞Mack

Figure 4. ***p–p*** **Plots for the Mack Model on Paid Loss Triangles**

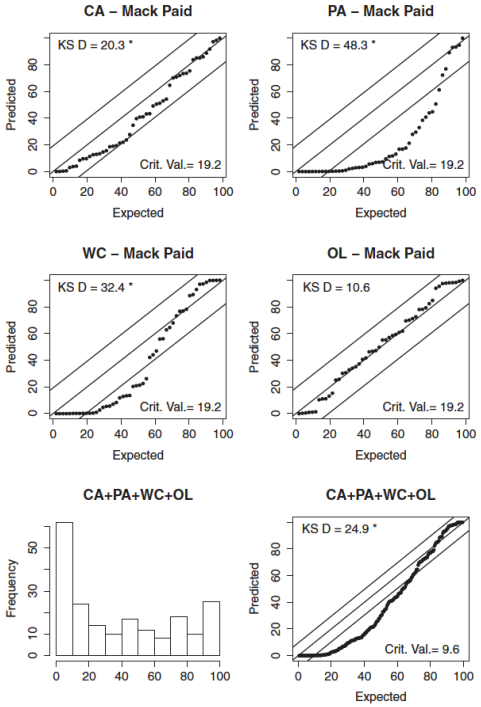


Figure 3. ***p–p*** **Plots for the Bootstrap ODP Model on Paid Loss Triangles**

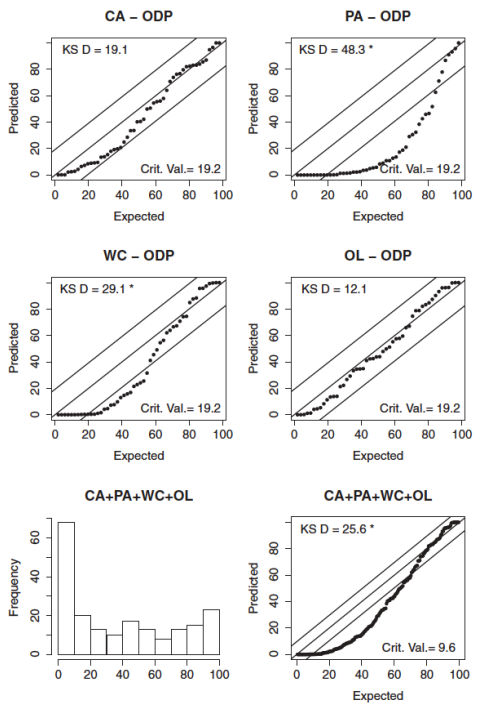


Figure 20. ***p–p*** **Plots for the LIT Model**

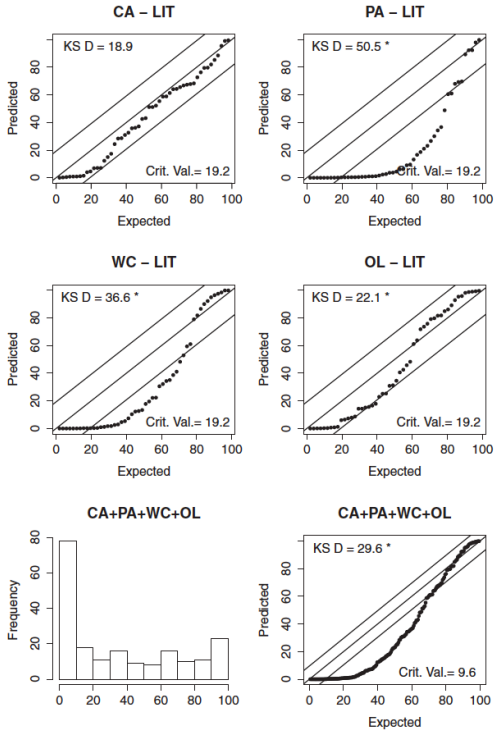


Figure 19. ***p–p*** **Plots for the CIT Model**

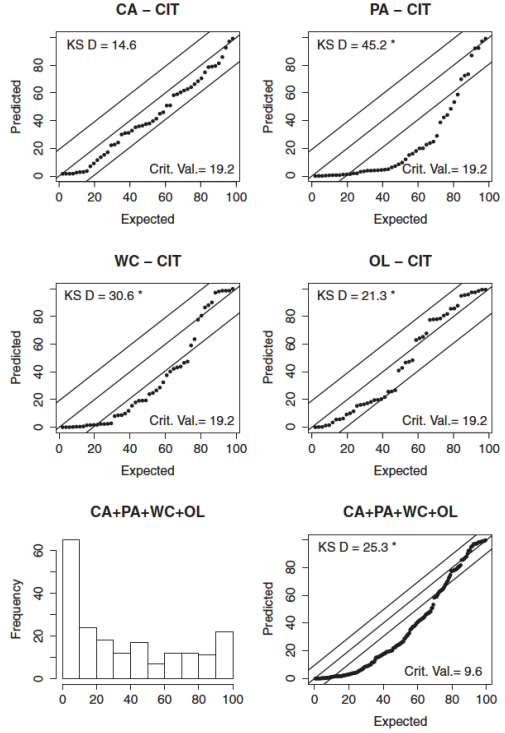


Figure 22. ***p–p*** **Plots for the CSR Model**