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Pricing catastrophe risk in life (re)insurance

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What is the catastrophe risk a life insurance company faces? What is the correct price of a catastrophe cover? During a review of the current standard model, due to Strickler, we found that this model has some serious shortcomings. We therefore present a new model for the pricing of catastrophe excess of loss cover (Cat XL). The new model for annual claim cost C is based on a compound Poisson process of catastrophe costs. To evaluate the distribution of the cost of each catastrophe, we use the Peaks Over Threshold model for the total number of lost lives in each catastrophe and the beta binomial model for the proportion of these corresponding to customers of the insurance company. To be able to estimate the parameters of the model, international and Swedish data were collected and compiled, listing accidents claiming at least twenty and four lives, respectively. Fitting the new model to data, we find the fit to be good. Finally we give the price of a Cat XL contract and perform a sensitivity analysis of how some of the parameters affect the expected value and standard deviation of the cost and thus the price.

Keywords: catastrophe excess of loss; life reinsurance; catastrophe model; catastrophe data; Cat XL; POT-model; Solvency II; internal models

1. Introduction

A *catastrophic event*, claiming many lives, can have a severe impact on a life insurance company. In Solvency II, catastrophe risk is included in the calculation of the Solvency Capital Requirements (SCR) either by a standard formula or by the use of an approved internal model (Directive 2009/138/EC n.d.). Correctly assessing the catastrophe risk can affect both SCR and the choice of reinsurance cover.

To protect itself from the consequences of a catastrophe, a life insurance company can buy *catastrophe excess of loss cover* (Cat XL) from a reinsurer. A major question is how one should price a contract giving such cover. The currently applied pricing model is due to Strickler (1960). Strickler used data from the Statistical Bulletin of the Metropolitan Life Insurance Company in New York who had supplied summaries of the accidents in the USA which claimed five or more lives for the period 1946–1950.

The annual number of deaths for each million of population resulting from accidents claiming m or more lives was approximated by the function

$$A(m) = 8 \cdot 100^{1/m} \cdot m^{-1/3} \tag{1}$$

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From this equation, he derived an elegant pricing formula. Drawbacks with Strickler's model is that there is no statistical method to update $A(m)$ in accordance to new data, it assumes a constant deterministic rate of catastrophes and is limited to catastrophes claiming at most 1500 lives. There have been some smaller adjustments proposed to Stricklers model, see for example Harbitz (1992) and Alm (1990). These modifications have however not addressed the main weaknesses of the model.

Taking into account the above mentioned shortcomings, a new model is suggested in this paper.

2. A new model for life Cat XL pricing

We will present a model for catastrophes, how they will affect a life company and how to use the model to price a Cat XL contract. It is a hierarchical model, which is easy to implement in a statistical software package. Parameters can be estimated from real data. The pricing of a Cat XL contract is the primary goal. The model is also suitable for calculating the catastrophe risk exposure of a life company, hence it should be possible to implement in an internal model for calculating SCR.

We will only model death claims, not disability claims. One reason for this is purely practical, the authors' total lack of disability data connected with catastrophic events. Another is the experience of the reinsurance industry, which seems to be that in large catastrophes, it is death claims that constitutes the main part of the total claim cost.

2.1. The catastrophe excess of loss contract

The catastrophe excess of loss (Cat XL) contract is defined as follows: If M or more persons insured by the ceding company loose their lives as a result of a single event and if the corresponding aggregate net retention (the part that is not ceded under another reinsurance contract) payable by the ceding company exceeds the amount S, the excess will be paid by the reinsurer, with the understanding that the maximum amount payable by the reinsurer in respect of each such event does not exceed a specified amount L, this is an L xs S Cat XL contract (*A practical Guide to Reassurance* n.d.).

How many deaths constitute a catastrophe? In the context of insurance, the cases where at least three lives are lost in a single event are often considered to be catastrophic. Therefore, M is typically chosen between three and five. The retention S in a Cat XL contract is often chosen to be at least twice the retention the cedent has in its individual life surplus contract. The choice of M and S ultimately depends on the cedent's attitude to risk.

2.2. The approach

We approach the problem of determining the price of a Cat XL contract in the following manner: We use the *peaks over thresholds* (POT) model, see Rootzén and Tajvidi (1995), to describe the flow of catastrophes. Then we model the cost that each catastrophe will inflict on a given Cat XL contract.

This breaks the problem into four parts:

(1) To describe the number of catastrophes $K = K(T)$ that happen during a contract period of length T.
(2) To give the number X_k of deaths from the kth catastrophe.
(3) To derive the number Y_k of claims resulting from the X_k deaths.
(4) To asses the cost Z_k of the Y_k claims from the kth catastrophe.

Our goal is to calculate the expected value, variance and distribution of $C(T)$, which is the total claim cost on the Cat XL contract during its duration T. Usually T is one year, so sometimes we will drop the index T for convenience with the understanding that the contract duration T is assumed to be one. We express the total cost due to catastrophes during the contract period as

$$C(T) = C = \sum_{k=1}^{K} Z_k \tag{2}$$

Next we will specify how to model each part.

2.3. The peaks over thresholds model

The POT model can be used to study tail behaviour, events exceeding a certain threshold. Given a sequence of random variables X_1, $X_2 \ldots X_K$ and a threshold level m, only $X_k{:}X_k \geq m$ are considered. We can think of the sequence as all accidents during T years, where X_j' denotes the death toll in accident j and that we are only interested in accidents claiming at least m lives, discarding X_j' if $X_j' < m$ and putting $X_{k(j)} = X_j'$ if $X_j' \geq m$, where $k(j) = |\{i; 1 \leq i \leq j, X_i' \geq m\}|$ is the catastrophe number that accident j corresponds to.

The POT model assumes that the number K_m of X_k is Poisson distributed and that the exceedances $X_k - m$ are independent and identically Pareto distributed.

To justify the use of the POT model, the threshold parameter m must be large enough so that the exceedances are in the tail of the distribution. What constitutes large enough cannot be known a priori, one must look at data and use for example quantile-quantile plots (QQ-plots) to decide a level of m that is consistent with the model.

In the case were we study the distribution of lost lives in deadly accidents, it is known that the far majority of such events are single accidents, that is, claiming one life. Using the (perhaps outdated) formula (1) gives at hand that 97% of victims were in accidents claiming one or two lives. It is therefore reasonable to believe that the POT model can work in the life catastrophe setting with an m as low as three or four. This would be handy, since as mentioned above, the Cat XL parameter M is often chosen to be three to five.

Since the number of deaths is a discrete random variable, it could be argued that it is logical to use the discrete counterpart of the Pareto distribution, the Zeta distribution.

However, for practical purposes (e.g. parameter estimation with standard software), the generalised Pareto distribution works more smoothly.

2.4. Catastrophe rate

According to the POT model, the number of catastrophes claiming at least m lives is a Poisson process with intensity λ_m per T years. Let K_m denote the number of catastrophes claiming at least m lives during T years.

We assume

(I) $K_m(T) = K_m \sim \text{Po}(\lambda_m T)$.

We can view $\lambda_m = \lambda_1 \Pr(X_i' \geq m)$ as the intensity of a thinned Poisson process (Resnick (1992)) with a thinning mechanism that retains accidents with at least m lost lives.

2.5. Number of deaths

First, let X' denote the death toll in an arbitrary accident. Let $P_m(n) = \Pr(X' = n | X' \geq m)$ and $F_m(n) = \Pr(X' \leq n | X' \geq m)$.

Since our interest is 'catastrophes', accidents where several persons have died, we are really interested in the tail distribution of P_1. This motivates the use of the POT model.

We assume, given a threshold m (thus only studying accidents claiming at least m lives),

(II) $X_1, X_2 \ldots X_K$ are independent, identically distributed (i.i.d) as $X \sim F_m$.
(III) $X = \text{round}(\tilde{X})$, where round$(x)$ is the integer closest to x.
(IV) $\tilde{X} \sim \text{GPD}(m - \frac{1}{2}, \sigma_m, \xi_m)$, that is, $\tilde{X}$ has a *generalised Pareto distribution* (GPD) which has cumulative distribution function

$$G_{(m-\frac{1}{2},\sigma,\xi)}(x) = 1 - [1 + \xi(x - m + \frac{1}{2})/\sigma]^{-1/\xi}$$

where $m \in \Re$, $x \geq m - \frac{1}{2}$ and $\sigma > 0$.

Thus, $X \in \{m, m+1, m+2 \ldots\}$ and $\tilde{X} \in \Re$. We say that X has a *discrete generalised Pareto distribution* (DGPD), $X \sim \text{DGPD}(m, \sigma, \xi)$ where $\sigma = \sigma_m$ and $\xi = \xi_m$.

If $\tilde{X} \sim \text{GPD}\left(m - \frac{1}{2}, \sigma, \xi\right)$ then

$$E[\tilde{X}] = m - \frac{1}{2} + \frac{\sigma}{1 - \xi} \quad (\xi < 1)$$

$$\text{Var}(\tilde{X}) = \frac{\sigma^2}{(1-\xi)^2(1-2\xi)} \quad (\xi < 1/2)$$

The Pareto distribution can have a heavy tail, if $\xi \geq 1/2$ the variance does not exist, and if $\xi \geq 1$ the same holds for the expected value.

By the definition of X and the fact that G has a decreasing density function, we find that

$$E(X) \geq E(\tilde{X})$$

but the larger m is, the closer the first moment of X it to that of $\tilde{X}$, provided it exists.

2.6. Number of claims

What is the number Y' of claims that will hit a life insurer given a catastrophe with a death toll of X? We want to investigate the properties of the random variable Y'. It is clear that $0 \leq Y' \leq X$.

Define the market penetration q for a given life insurance company as

$$q = \frac{\text{Number of sold policies}}{\text{Size of total population}}$$

We assume

(V) $E[Y'|X] = qX$

The expected number of claims is proportional to the market penetration q. The more policies sold by the insurer, the likelier a claim.

We expect to see some dependence among lives for small catastrophes (think e.g. of traffic accidents), but for very large catastrophes the number of claims should be close to the expected value, that is

$$\frac{Y'}{X} \approx q \text{ for } X >> 1. \tag{3}$$

A distribution that would reflect the above mentioned properties is the Beta-binomial.

We assume

(VI) $Y'|X, p \sim \text{Bin}(X, p)$,

(VII) $p|X \sim \text{Beta}(d(X)q, d(X)(1-q))$, $0 < d(X) < \infty$.

Taken together (VI) to (VII) imply that $Y'|X \sim \{\text{Betabin}(X, q, d(X))$. Since $E(p|X) = q$, we have in particular that (V) holds.

For every catastrophe, a $p \in [0; 1]$ is drawn from a beta distribution with mean q. This p is the probability that a life in this catastrophe was insured by the cedent, and hence Y' the total number of insured lives lost is *Bin(X,p)*.

How does $d(X)$ affect the distribution? By Equation (3) and the discussion above, we find that the two limits for $d(X)$ are

$$\begin{array}{lll} \lim\ d(X) \to \infty & \Rightarrow & Y'|X \sim \text{Bin}(X, q) \\ \lim\ d(X) \to 0 & \Rightarrow & \Pr(Y' = 0|X) = 1 - q, \Pr(Y' = X|X) = q \end{array}$$

correspond to two extremes, independence and total dependence between lives.

Hence $d(X)$ should be chosen so that $d(X) \to \infty$ as $X \to \infty$ and that $d(X)$ is small for small X.

We assume

(VIII) $d(X) = \theta \cdot \log(X), \theta \in \Re^+$.

The choice of $\log(X)$ in (VIII) is made to get a certain degree of dependence for smaller catastrophes and a slow growth towards independence for the really large catastrophes.

This is because even large catastrophes, for example, involving airplanes and ferries, tend to exhibit a large dependence among lost lives.

We notice that the variance

$$\mathrm{Var}(Y'|X) = q(1-q)(X + X(X-1)/(d(X)+1)),$$

is a decreasing function of $d(X)$, with $\mathrm{Var}(Y'|X) = Xq(1-q)$ for complete independence ($d(X) = \infty$) and $\mathrm{Var}(Y'|X) = X^2q(1-q)$ for complete dependence ($d(X) = 0$).

Remember that the Cat XL contract states that at least M insured persons have to die in order to be a valid catastrophe claim.

Let $Y'_k \sim \text{Beta bin}(X_k, q, d(X_k))$ be the number of insured lives lost in the kth catastrophe, and put

$$Y_k = \begin{cases} Y'_k, & \text{if} \quad Y'_k \geq M, \\ 0, & \text{if} \quad Y'_k < M. \end{cases} \tag{4}$$

Hence, Y_k is the number of dead in a valid catastrophe claim.

It is worth to note that the beta distribution is known to be used in non-life catastrophe modelling in a similar manner, see for instance Woo (1999), where the percentage of damage done to a building due to a natural peril (storm, flood, earthquake) is modelled as being beta distributed.

2.7. *Distribution of claims*

What is the size of a claim Z_k? Denote the individual claims in the kth catastrophe by Z_{ki}. We will use standardised amounts so that $E(Z_{ki}) = 1$. We have to consider S and L, the retention and maximal liability of the Cat XL contract. If $Z'_k = \sum_{i=1}^{Y_k} Z_{ki}$ is the actual claim amount from the Y_k insured lives lost in catastrophe k, we set

$$Z_k = \begin{cases} 0, & \text{if} \quad Z'_k < S \\ Z'_k - S, & \text{if} \quad S \leq Z'_k < S + L \\ L, & \text{if} \quad S + L \geq Z'_k \end{cases}$$

With modern information technology, it is often possible to get hold of all the individual risk sums of the life portfolio. In that case the empirical distribution of Z_{ki} can be used in numerical calculations.

In the case where the Cat XL covers a group life policy where all assureds' sum are the same, there is no randomness so that $Z_{ki} = 1$ and Z'_k equals the number of lives lost, Y_k.

Otherwise it is often a good approximation to assume that a single claim is exponentially distributed with mean value 1, that is, $\Pr(Z'_k \leq z | Y_k = 1) = 1 - e^{-z}$. Assuming independence between individual claims, this gives a gamma distribution $\Gamma(Y_k, 1)$ for the total cost Z'_k of Y_k claims.

2.8. *Total annual claim*

Now we are ready to address the question, what the total cost C in Equation (2) is. We have assumed (I), that catastrophes arrive according to a Poisson flow. If, in addition, claim amounts Z_k are independent and identically distributed, it follows that $T \to C(T)$ is a marked Poisson process. As stated above, we will mainly consider $C = C(1)$, the cost during the fixed time horizon $T = 1$.

As we have seen, C will depend on the contract parameters M, S and L as well as model parameters $\lambda_M, \sigma_M, \xi_M, q, \theta$ and the choice of claim distribution function.

Thus, for a given set of parameters, we can use Monte Carlo simulations to compute expected value, variance and even the distribution of C. Given these properties of C, we can set the price of the Cat XL contract.

3. Catastrophe data

To be able to set the correct technical price on an insurance, a theoretical model is not enough, one also needs statistical data in order to estimate the parameters of the model. An insurance company can rely on its own claim experience for estimation in most cases. However, since catastrophic events are almost rare by definition, even for a reinsurer with a large Cat XL portfolio, the use of claim experience as the only source for pricing the contracts would be unsatisfactory.

To be able to estimate the parameters of this model, two data sets were collected and compiled.

Swiss Re's yearly publication, '*sigma*, – Natural catastrophes and man-made disasters' (Swiss Re (1983–1991, 1994–1999, 2002–2004)), lists catastrophic events from all over the world that have 'at least 20 dead or missing'. Complete data sets were available from the years 1983–1991, 1994–1999 and 2002–2004. Only data from those years were compiled. Some well-known catastrophes (and a lot of unknown) such as 9/11 2001 are therefore missing. Data were sorted after continent and region, as well as the cause of the disaster. Only events that fit the standard Cat XL contracts 72-hour rule were taken into account. Therefore, long lasting 'conditions' such as heat waves, cold spells and floods were excluded, even if they took many lives. Acts of war and military accidents are not accounted for since they are excluded from the insurance contracts. In total, there were 3055 observations from this source. For population data, see U.S. Census Bureau (2004).

The Swedish Rescue Services Agency (Räddningsverket) keeps a record over Swedish accidents claiming at least four lives. Data from 1970–2004, a total of 189 observations, were used in this data set.

The international data set has many observations but the quality varies with different regions of the world. The numbers from Western Europe and North America are probably the most accurate. For example, regimes in the non-free world have a reputation of trying to cover the true extent of catastrophes. Getting accurate data can be hard in some circumstances, the frequent occurrences of '50', '100' and '200' in data from some regions, see for example Figure 2, suggest that some of the observations are mere estimates. Another drawback with the international data is the fact that it only contains data from 20 dead and upward. The Swedish data is much better in this regard, ranging from four and above. The size of the data set is however limited and one can question which conclusions that can be drawn from it about circumstances in other countries.

3.1. Catastrophe intensity

We can check the validity of the Poisson assumption for each region by comparing the sample mean $\hat{\lambda}_m$ and sample variance of the yearly number of accidents.

The sample mean is

$$\hat{\lambda}_m = \frac{K_m}{T}$$

and the sample variance is

$$\widehat{\mathrm{Var}}(\hat{\lambda}_{20}) = \frac{1}{T-1} \sum_{t=1}^{T} \left(K_{m,t} - \hat{\lambda}_m \right)^2$$

where T is number of years we have observed, and $K_{m,t}$ is the number of catastrophes claiming at least m lives during the tth year.

Sample mean, sample mean normalised with the number of inhabitants and sample variance from the international data set during the periods 1983–1991 and 1994–2004 respectively, are shown in Tables 1 and 2. Sample mean and sample variance from the Swedish data are given in Table 3.

For most regions and the Swedish data, the mean and the variance are fairly close to each other, in accordance with the Poisson assumption. But for some regions such as South East Asia (SEA) and South Asia the variance is larger than twice the mean, showing an overdispersion. This suggests that an improved model could be built by incorporating more clustering of catastrophes, for example, by allowing for a time varying and/or stochastic claim intensity.

Table 1. Annual incidence rates $\hat{\lambda}_{20}$ of catastrophes for various regions.

	Average number per year		Per 100 million inhabitants	
Region	1983–1991	1994–2004	1983–1991	1994–2004
South America (SAM)	18.9	14.8	6.8	4.5
North America (NAM)	6.3	5.3	2.3	1.8
Caribbean (CAR)	2.1	3.7	5.7	8.8
Central America (CAM)	6.1	4.9	5.7	3.7
Western Europe (WEU)	8.9	5.9	2.4	1.5
Eastern Europe (EEU)	5.0	1.7	4.2	1.4
Former Soviet (SUN)	3.4	10.1	1.2	3.5
South Asia (SAS)	40.6	41.3	3.9	3.2
South East Asia (SEA)	18.4	16.6	4.3	3.2
Middle East (MIE)	6.6	12.6	3.4	4.9
Far East (FAE)	7.4	3.0	3.9	1.5
Central Asia (CAS)	17.1	26.4	1.5	2.1
Oceania (OCE)	1.0	0.8	3.9	2.6
Northern Africa (NAF)	2.6	6.6	2.3	4.6
Middle Africa (MAF)	12.7	22.3	2.9	3.8
South Africa (SAF)	2.9	3.7	7.5	8.2

Table 2. Catastrophe intensities $\hat{\lambda}_{20}$ and $\widehat{\mathrm{Var}}(\hat{\lambda}_{20})$.

	1983–1991			1994–2004		
Region	Mean	Var	Var/mean	Mean	Var	Var/mean
SAM	18.9	48.9	2.59	14.8	17.2	1.16
NAM	6.3	10.3	1.62	5.3	8.0	1.5
CAR	2.1	2.9	1.36	3.7	4.5	1.23
CAM	6.1	10.1	1.65	4.9	5.6	1.15
WEU	8.9	9.4	1.05	5.9	13.9	2.35
EEU	5.0	12.8	2.55	1.7	1.0	0.6
SUN	3.4	6.5	1.9	10.1	9.9	0.98
SAS	40.6	314.0	7.74	41.3	161.5	3.91
SEA	18.4	150.8	8.17	16.6	35	2.12
MIE	6.6	15.8	2.41	12.6	33.5	2.67
FAE	7.4	4.3	0.57	3.0	3.3	1.08
CAS	17.1	93.4	5.46	26.4	25.3	0.96
OCE	1.0	0.5	0.5	0.8	0.7	0.89
NAF	2.6	2.3	0.89	6.6	3.5	0.54
MAF	12.7	54.8	4.32	22.3	90.3	4.04
SAF	2.9	3.4	1.16	3.7	8.3	2.25

3.2. Cat size

Is the DGPD a good model for the number of lost lives in a catastrophe? The available data were used to estimate the parameters of the DGPD with the maximum likelihood (ML) method, see for example Pawitan (2001). All estimates are obtained by using the statistical software *R* and the package POT. Estimates are given in Table 4.

Since $\hat{\xi} > 1/2$ for all regions, the variance does not exist, this indicates a heavy tail of the distribution, that is, according to this model, very large catastrophes can be expected. For some regions, $\hat{\xi} > 1$, the tail is so heavy that not even the expected value exists!

We use QQ-plots to determine whether the DGDP fits the data. Plots for North America and SEA are presented in Figures 1 and 2. The dashed lines indicate 95% confidence intervals. We conclude that the fit is good.

The ML estimates for the Swedish data are $\hat{\sigma}_4 = 1.37(0.16)$ and $\hat{\xi}_4 = 0.66(0.010)$, with estimated standard deviations in brackets. Looking at the corresponding QQ-plot, Figure 3, we find that the fit is good and the use of the POT model with $m = 4$ seems justified, although the tail seems to be a bit underestimated. It should be noted that the two largest catastrophes, the ferry Estonia and the tsunami, are extreme in the modern Swedish history. They would have been the largest catastrophes even if we had data from the whole twentieth century. In light of this fact, we conclude that the DGPD gives a good fit for the Swedish data. Given the good fit for $m = 4$, it would have been interesting to try with $m = 3$, but the lack of data unfortunately makes this impossible.

Table 3. Swedish catastrophe intensity $\hat{\lambda}_4$ and $\widehat{\mathrm{Var}}(\hat{\lambda}_4)$.

Year	Mean	Var	Var/mean
1970–1979	6.1	7.66	1.26
1980–1989	6.6	9.38	1.42
1990–1999	3.8	4.18	1.1
2000–2004	4.8	1.7	0.35
1990–2004	4.13	3.41	0.82

Table 4. Estimated parameters for the $DGPD(20, \sigma_{20}, \xi_{20})$ model. Standard errors are in brackets.

Region	$\hat{\sigma}_{20}$ (std err $\hat{\sigma}_{20}$)	$\hat{\xi}_{20}$ (std err $\hat{\xi}_{20}$)
SAM	15.5 (1.7)	0.83 (0.10)
NAM	17.2 (3.1)	0.68 (0.17)
CAR	18.4 (4.8)	0.98 (0.26)
CAM	13.2 (2.6)	0.98 (0.20)
WEU	14.8 (2.6)	0.84 (0.17)
EEU	13.6 (3.1)	0.63 (0.21)
SUN	20.2 (3.3)	0.79 (0.15)
SAS	20.2 (1.4)	1.00 (0.07)
SEA	19.8 (2.4)	1.15 (0.12)
MIE	18.3 (2.9)	1.38 (0.18)
FAE	17.9 (3.9)	1.12 (0.22)
CAS	20.6 (2.0)	0.76 (0.09)
OCE	17.1 (8.0)	1.13 (0.49)
NAF	15.7 (3.4)	1.02 (0.22)
MAF	25.6 (2.7)	0.66 (0.10)
SAF	10.0 (2.2)	0.61 (0.20)

3.3. Distribution of insured lives

There is no available data for the distribution of insured lives $Y_k'|X_k$ in catastrophes. What we can do is to let θ vary between two extremes, having either a binomially distributed number of insureds from the company of interest in each catastrophe ($\theta = \infty$), or all/no claims with probabilities q and $1-q$, respectively ($\theta = 0$) and see how different θ values affects the distribution of lost lives, as will be further discussed in the following section.

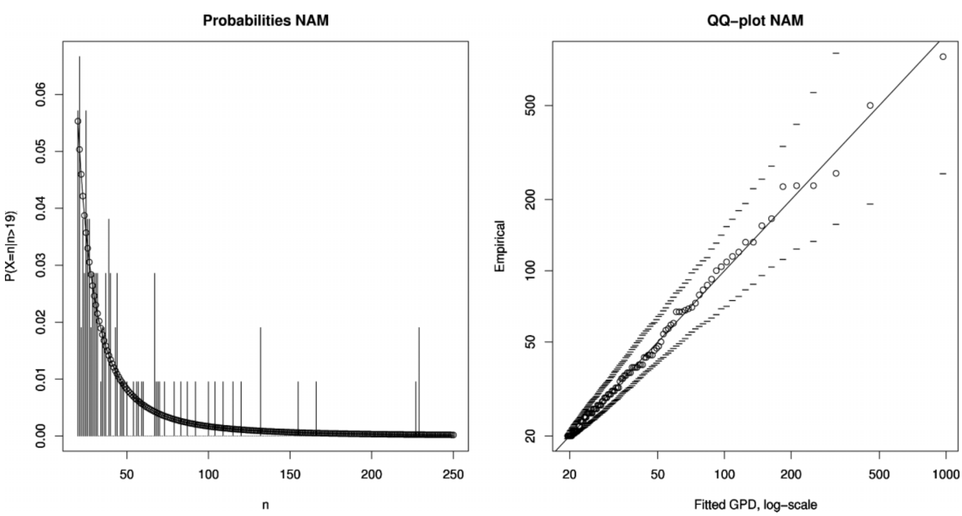

Figure 1. Empirical probabilities, fitted discrete generalised Pareto distribution (DGPD) and quantile-quantile plot NAM, at least 20 dead.

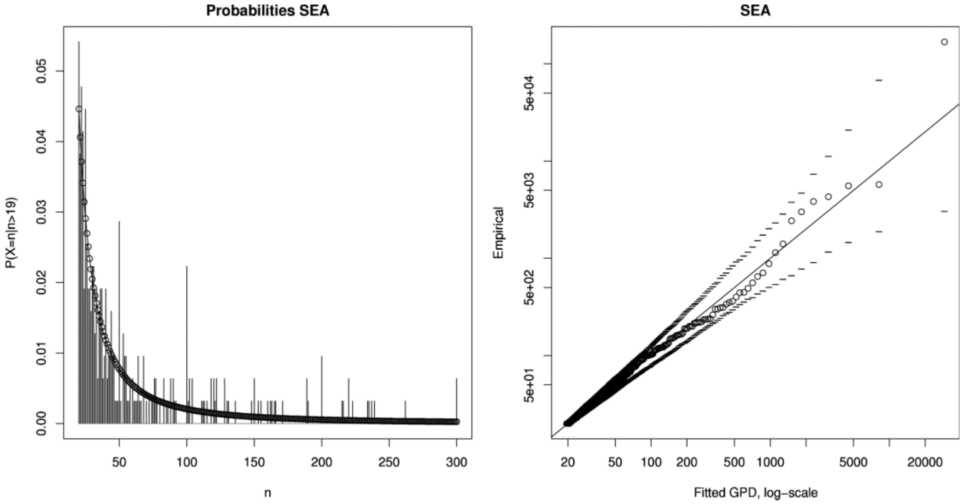

Figure 2. Empirical probabilities, fitted discrete generalised Pareto distribution (DGPD) and quantile-quantile plot SEA, at least 20 dead.

4. Pricing

4.1. The pricing principle for a Cat XL contract

In non-proportional reinsurance, it is often not possible to acquire a portfolio with a large number of independent contracts. The dependence between contracts is an important reason why insurers want and need reinsurance. A reinsurance portfolio can be subject to major fluctuations, that is, there is a lot of systematic risk involved. This in turn requires more regulatory capital. Apart from the expected claims, the pure premium, the reinsurer

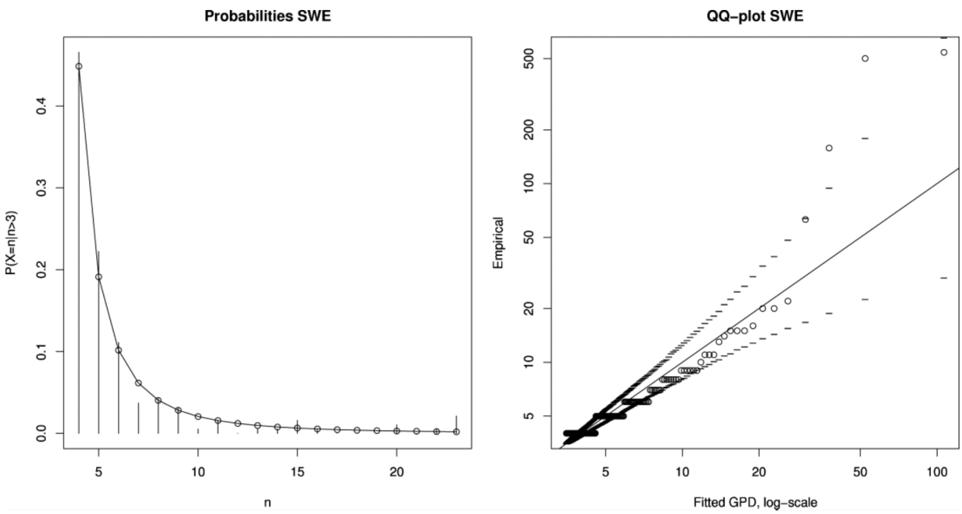

Figure 3. Empirical probabilities, fitted discrete generalised Pareto distribution (DGPD) and quantile-quantile plot SWE, at least four dead.

will charge for its capital costs and, therefore, typically adds a percentage of the standard deviation to the pure premium. This gives the pricing formula

$$P = E[C] + \alpha * \mathrm{SD}(C), \tag{5}$$

where typically $\alpha \in [0.1; 0.5]$.

Our model gives us the possibility to use an even more sophisticated pricing since we will in fact get hold of the complete distribution of C. This gives the possibility to use, for example, the quantiles of C in the pricing or in the determination of contract parameters.

Prices of Cat-contracts are often related to the maximal liability of the reinsurer. They are given as a 'rate on line' (RoL) $P \div L\%$. Here we would like to cite Bostrom and Cirkovic (2008, p. 177), who expressed another important pricing principle:

> There is a saying in catastrophe reinsurance that "nothing is less than 1 on line", meaning that the vagaries of life are such that you should never price high-level risk at less than a chance of a total loss once in a hundred years (1%).

4.2. *The rating factors and the total claim cost*

As we have seen, there are many parameters that affect the price P of a Cat XL contract. The model presented in this paper includes the catastrophe rate λ, the distribution of catastrophes determined by (σ,ξ), the market penetration q, the dependence parameter θ, the contract parameters M,S and L, and the extent α to which we take the standard deviation of the claim cost C into account. This gives the price $P = P(\lambda, \sigma, \xi, q, \theta, M, S, L, \alpha)$ where

(1) λ,σ and ξ are to be estimated from data.
(2) q is determined by the size of the ceding company.
(3) M, S *and* L are determined by the Cat XL contract.
(4) α is determined by the reinsurer's risk appetite.
(5) θ is tricky in the sense that we lack data to estimate θ. We can however do a sensitivity analysis to see how it affects P, see below.

With as many variables and truncations in different steps, an analytical formula for P is not to be expected. The model is however well suited for simulation studies by means of parametric bootstrap (Efron & Tibshirani (1993)). We start by estimating the model parameters and then run numerical simulations from the so estimated distributions to simulate the distribution of the total claim cost C. With the distribution of C known it is easy to determine a price P according to ones risk preferences.

4.3. *A numerical example*

A Swedish insurance company reinsures its portfolio of 900,000 policies. Sweden has a population of 9 million people, this yields $q = 0.1$. The other parameter values are, according to our previous findings, $\hat{\lambda} = 4.13$, (Table 3), $\hat{\sigma} = 1.37$, $\hat{\xi} = 0.66$ (see Section 3.2).

We assume that $\theta = 0.1$ and that the sum insured is 1 million Swedish Krona (MSEK) for each policyholder. The contract parameters are set to $M = 4$, $S = 5$ MSEK, $L = 100$ MSEK which is a realistic choice for a Cat XL contract. To determine the price we simulate claims for 100,000 years and find that $E[C] = 1.08$ MSEK and $SD(C) = 5.41$ MSEK. Assuming $\alpha = 0.20$, Equation (5) gives the price $P = 2.16$ MSEK corresponding to a rate on line of $P/100 = 2.16\%$.

We find that the probability of a claim is 0.15 per year. In Figure 4, we present the conditional claim distribution $C|C > 0$ and the corresponding size biased distribution of the cost. With a density proportional to $x \cdot f_C(x)$, where f_C is the density of C, we can see which claims that actually will cost the most. It is the relatively small claims that will cost the most due to their frequency. Claims ranging from 0 to 5 correspond to 25% of the total claims, those from 0 to 14 correspond to 50%. However, the large claims, limited in size by $L = 100$, contribute with 9% of the expected claim cost, even if the risk of such an event is just one in a thousand years.

4.4. Sensitivity analysis for the Cat XL contract

4.4.1. The effect of catastrophe intensity λ

How does C depend on λ? According to Equation (2), we have $C = \sum_{k=1}^{K} Z_k$. Let $E[Z] = \mu, \mathrm{Var}(Z) = \kappa^2$. Then Equation (2) and (I) imply

$$E[C] = \mu \cdot E[K] = \mu \cdot \lambda$$

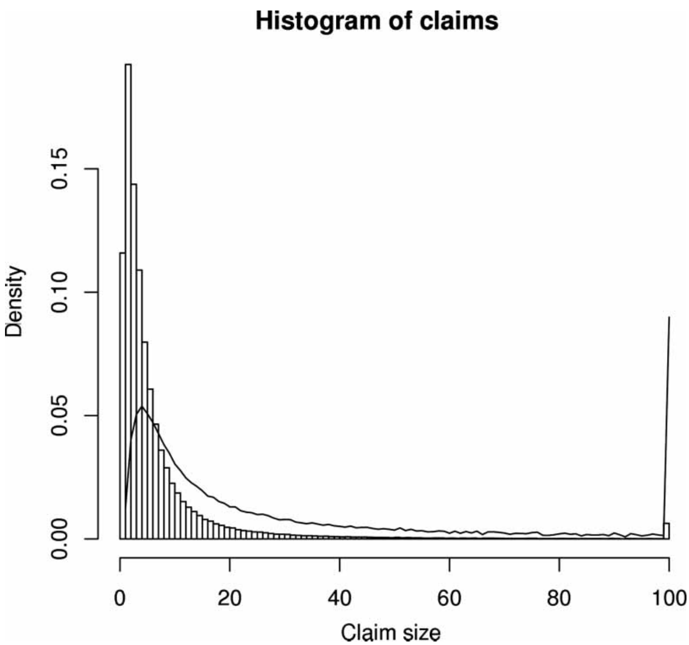

Figure 4. Histogram, claim distribution and claim cost distribution.

Thus $E[C]$ is linear in λ. What about Var(C)? Using a well-known conditioning formula for the variance we find

$$\begin{aligned}\mathrm{Var}(C) &= \mathrm{Var}(E[C|K]) + E[\mathrm{Var}(C|K)]\\ &= \mathrm{Var}(\mu K) + E[\kappa^2 K]\\ &= \mu^2\lambda + \kappa^2\lambda\\ &= (\mu^2 + \kappa^2)\lambda\end{aligned}$$

so that Var(C) is also linear in λ.

4.4.2. The effect of q and θ

Recall from Section 2.6 that in this model $E[Y'|X] = q \cdot X$, hence $E[C]$ is approximately linear in q due to the truncation in Equation (4). Considering the binomial distribution and that q typically is small, Var(C) is also approximately linear in q.

In Section 2.6, we also saw that a small θ implies large dependence and a large θ implies more of independence. The expected number of insured lives lost, $E[Y']$, does not depend on θ. However, the expected number of lost lives hitting the Cat XL contract, $E[Y]$, will depend on θ due to the truncation in Equation (4). Computing $E[C]$ and $SD(C)$ as functions of θ (with the other parameters as in Section 4.3), see Figure 5, reveal that they are decreasing in θ, going from $\theta = 10$ to $\theta = 0.1$ triples $E[C]$ and increases SD(C) by a factor of 1.8, a significant effect. Ones belief about the dependence among the insured life will therefore heavily influence ones view on the catastrophe risk.

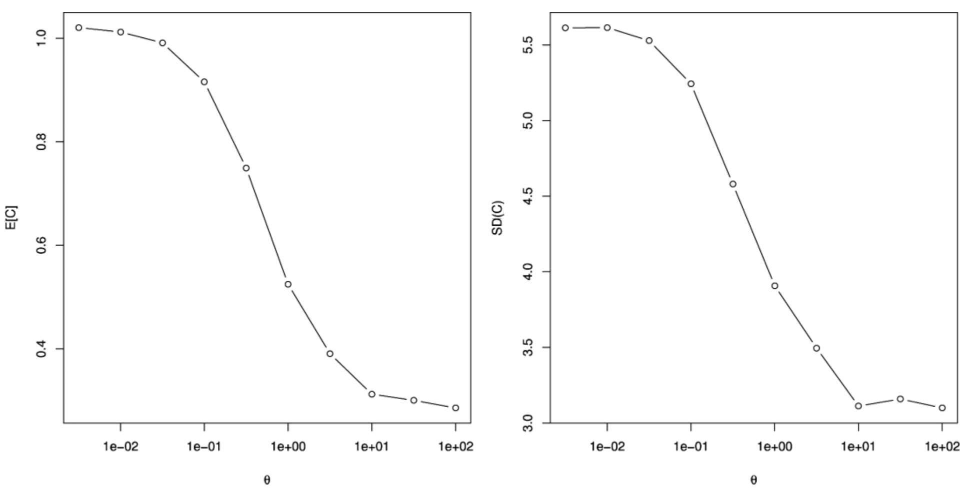

Figure 5. $E[C]$ and SD(C) as functions of θ.

5. Conclusion

In this paper, we have provided a new model for assessing life catastrophe risk, especially pricing catastrophe excess of loss (Cat XL) contracts in life reinsurance, but also for Solvency II purposes.

We first studied Strickler's well-known model for pricing. Although Strickler's model has its merits, it is inflexible – there is no statistically motivated way how to estimate the model parameters – and to some extent unrealistic, for example, the deterministic catastrophe rate. Some modifications of the model have made it more up to date but still not corrected these basic problems.

To get a statistically satisfying way of pricing a Cat XL contract, we construct a new model in the following way. In Equation (2), we express the total cost due to catastrophes during a contract period as $C = \sum_{k=1}^{K} Z_k$ where K is the number of catastrophes assumed to be Poisson distributed and Z_k is the cost inflicted on the Cat XL contract by the kth catastrophe. To obtain Z_k we start with X_k, the number of lost lives in the kth catastrophe is assumed to have a generalised Pareto distribution. The number of insured lives lost, Y_k, is assumed to follow a truncated Beta-binomial distribution conditional on X_k in order to reflect the possible dependence among lost insured lives. The loss in the kth catastrophe, Z_k, is the sum of the insured for each of the Y_k lost lives minus the retention stated in the Cat XL contract. The sum insured for each life can have a, possibly truncated, exponential distribution or be deterministic in case of a group policy.

In order to use the model for actuarial purposes, we need data for parameter estimation. We work with two data sets, one international with catastrophes claiming at least 20 lives and a Swedish data set with data from accidents claiming at least four lives. With those data sets, we were able to estimate parameters for both catastrophe intensity and size. Comparing the fitted model with the catastrophe data, we found the fit to be good. For a more detailed review of the data sets, see Ekheden (2008).

The modular structure of the model would make it possible to extend it to take disability claims into account. But this would require collection of data not at hand for the moment.

By using the estimated parameters together with the parameters defining a Cat XL contract, we can now calculate its price. We do it by running computer simulations (a parametric bootstrap) to find the claim distribution of the contract. We also conduct a sensitivity analysis, varying some of the parameters and observe how they affect the expected value and standard deviation of the claim distribution. We find that the assumption of to which degree insured lives are dependent in catastrophic events has a significant effect on the risk and hence the price of a Cat XL contract.

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