

## 本节主题



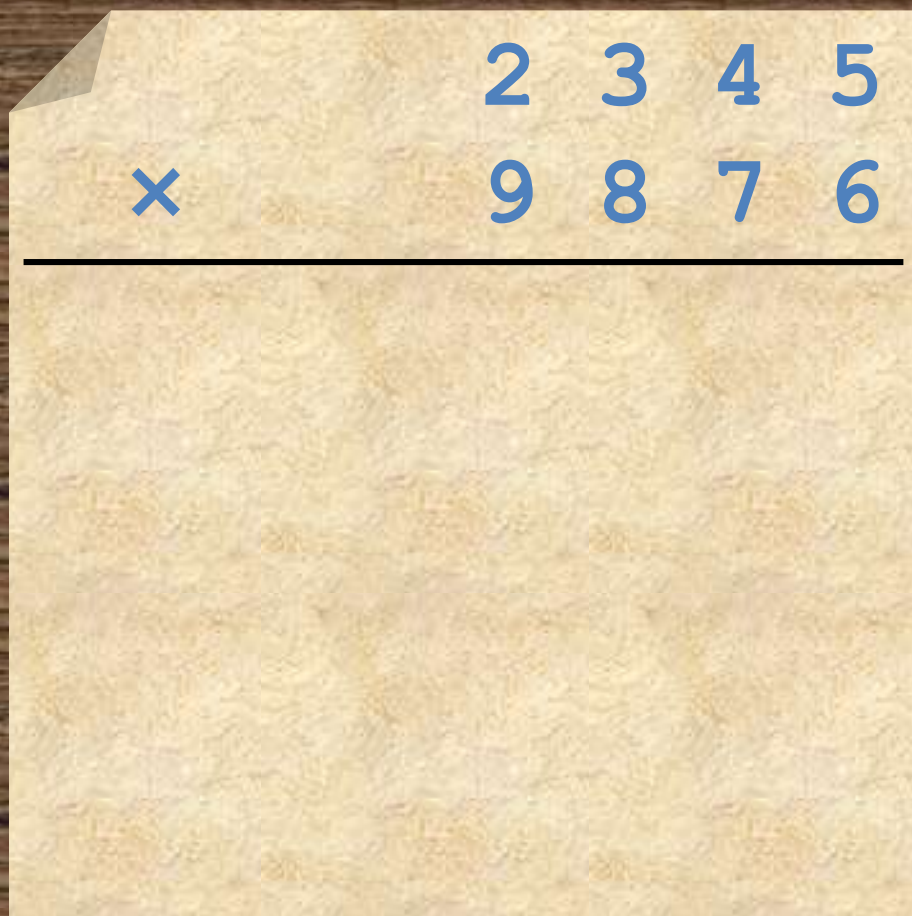
# 乘法的运算过程

真题未考过，但仍需重视！  
此节为运算过程，理解即可。

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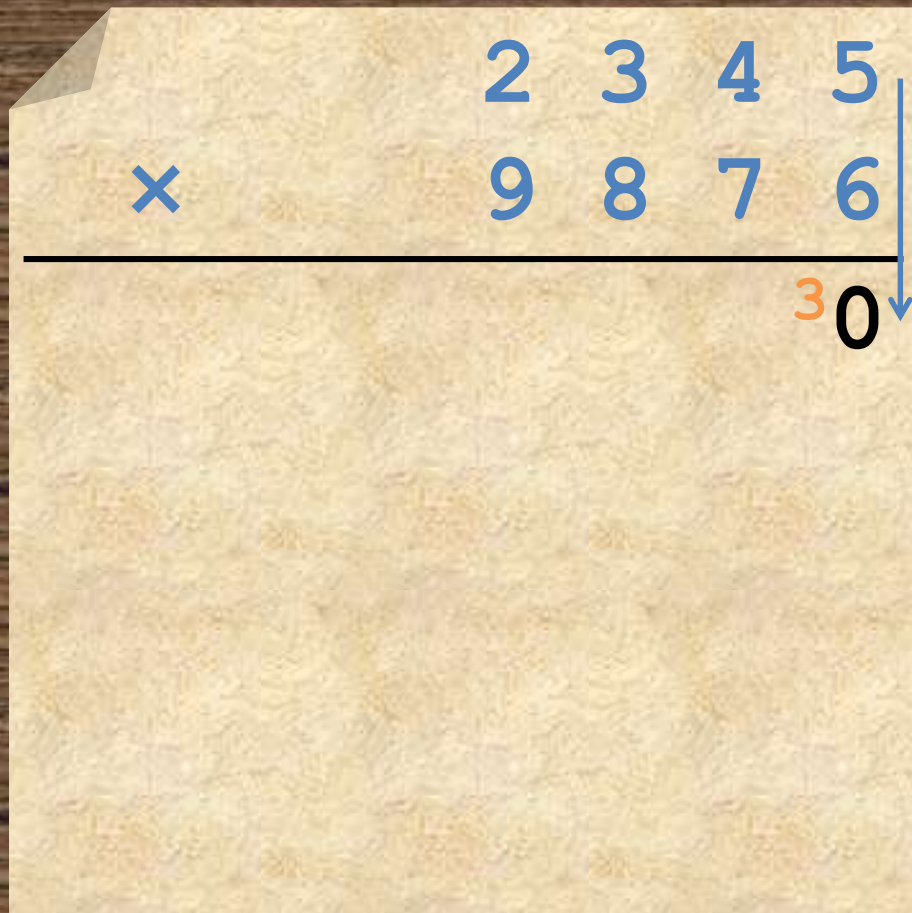
# 手工进行乘法运算



A piece of yellow paper with a folded top-left corner, placed on a dark wood-grain background. The paper contains a handwritten multiplication problem in blue ink. The problem is arranged in two rows: the top row contains the numbers 2, 3, 4, and 5; the bottom row contains a multiplication symbol (×) followed by the numbers 9, 8, 7, and 6. A horizontal line is drawn below the second row.

$$\begin{array}{r} 2 \ 3 \ 4 \ 5 \\ \times \ 9 \ 8 \ 7 \ 6 \\ \hline \end{array}$$

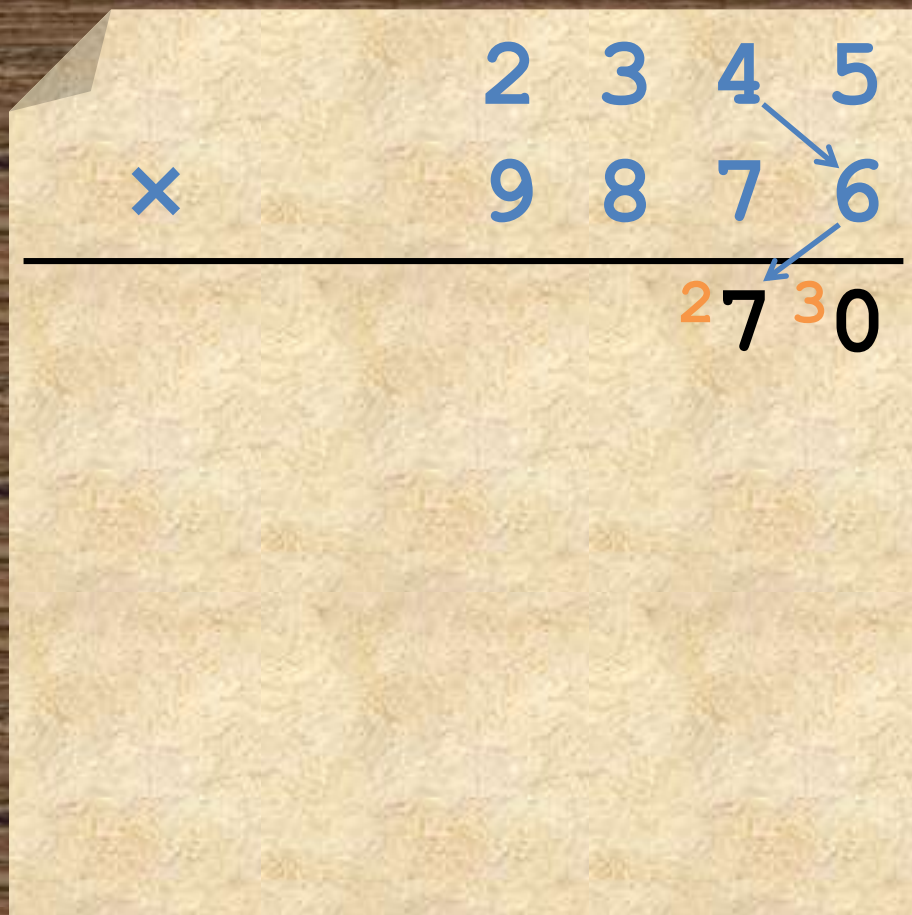
# 手工进行乘法运算



A piece of yellow paper with a folded top-left corner, showing a handwritten multiplication problem. The numbers are written in blue ink. The multiplier 2345 is on the top line, and the multiplicand 9876 is on the line below it. A horizontal line separates the two. Below the line, the product 30 is written, with the '3' in orange and the '0' in black. A blue arrow points downwards from the right side of the numbers.

$$\begin{array}{r} 2345 \\ \times 9876 \\ \hline 30 \end{array}$$

# 手工进行乘法运算



A handwritten multiplication problem is shown on a piece of yellow paper with a folded top-left corner. The problem is:

$$\begin{array}{r} \times \quad 2345 \\ 9876 \\ \hline \end{array}$$

Below the horizontal line, the result  $2730$  is written. The digits  $2$  and  $3$  are orange, while  $7$  and  $0$  are black. Two blue arrows point from the digit  $4$  in the second row to the digit  $6$  in the second row, and from the digit  $6$  in the second row to the digit  $0$  in the result.



# 手工进行乘法运算

A piece of yellow paper with a folded top-left corner, showing a handwritten multiplication problem. The numbers are written in blue ink. The multiplier 2345 is on the top line, and the multiplicand 9876 is on the bottom line, separated by a horizontal black line. Below the line, the partial products are written in orange ink: 20, 27, and 30. Blue arrows indicate the alignment of the partial products: one arrow points from the '3' in 2345 to the '2' in 20, another from the '4' to the '2' in 27, and a third from the '5' to the '3' in 30.

$$\begin{array}{r} 2345 \\ \times 9876 \\ \hline 20 \\ 27 \\ 30 \end{array}$$

# 手工进行乘法运算

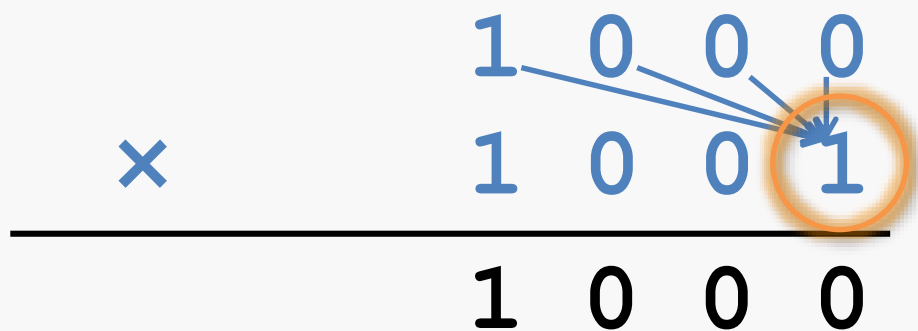
A handwritten multiplication problem on a piece of paper. The problem is  $2345 \times 9876$ . The result  $2310730$  is written below a horizontal line. Blue arrows indicate the calculation steps: from the '2' in the thousands place of the multiplier to the '6' in the units place of the multiplicand, and from the '6' in the units place of the multiplier to the '0' in the tens place of the product.

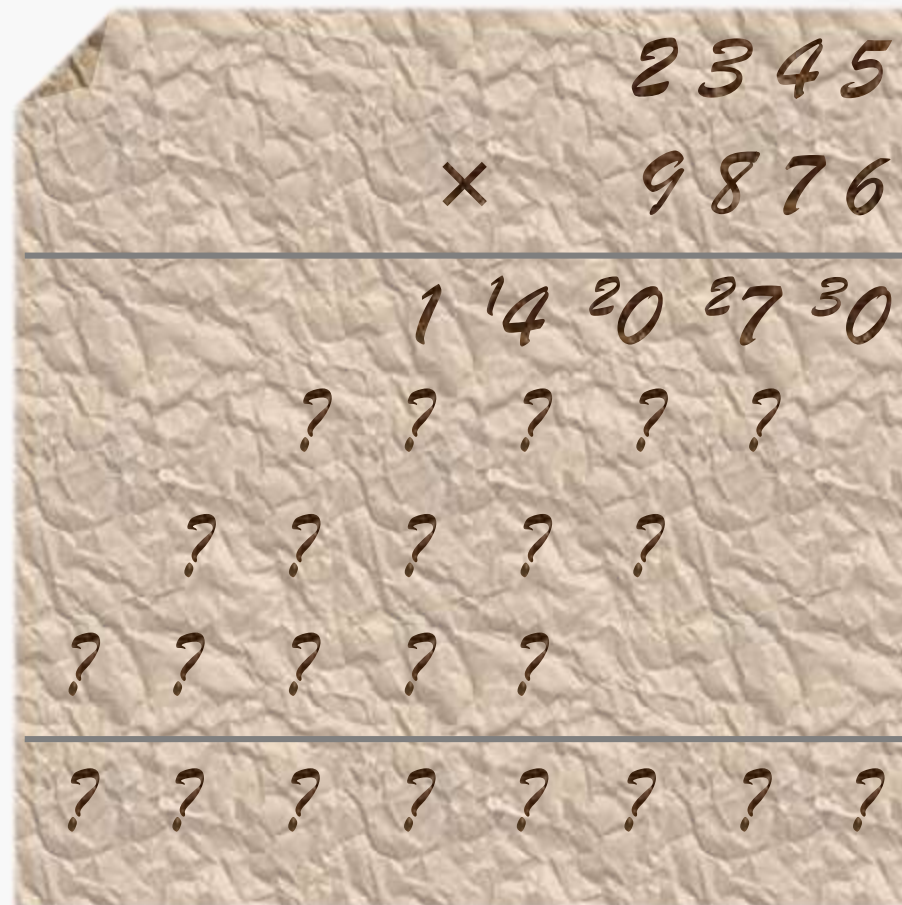
$$\begin{array}{r} 2345 \\ \times 9876 \\ \hline 2310730 \end{array}$$

# 手工进行乘法运算

				2	3	4	5
×				9	8	7	6
<hr/>							
			1	4	0	7	0
		?	?	?	?	?	
	?	?	?	?	?		
?	?	?	?	?			
<hr/>							
?	?	?	?	?	?	?	?

# 较为简单的数字

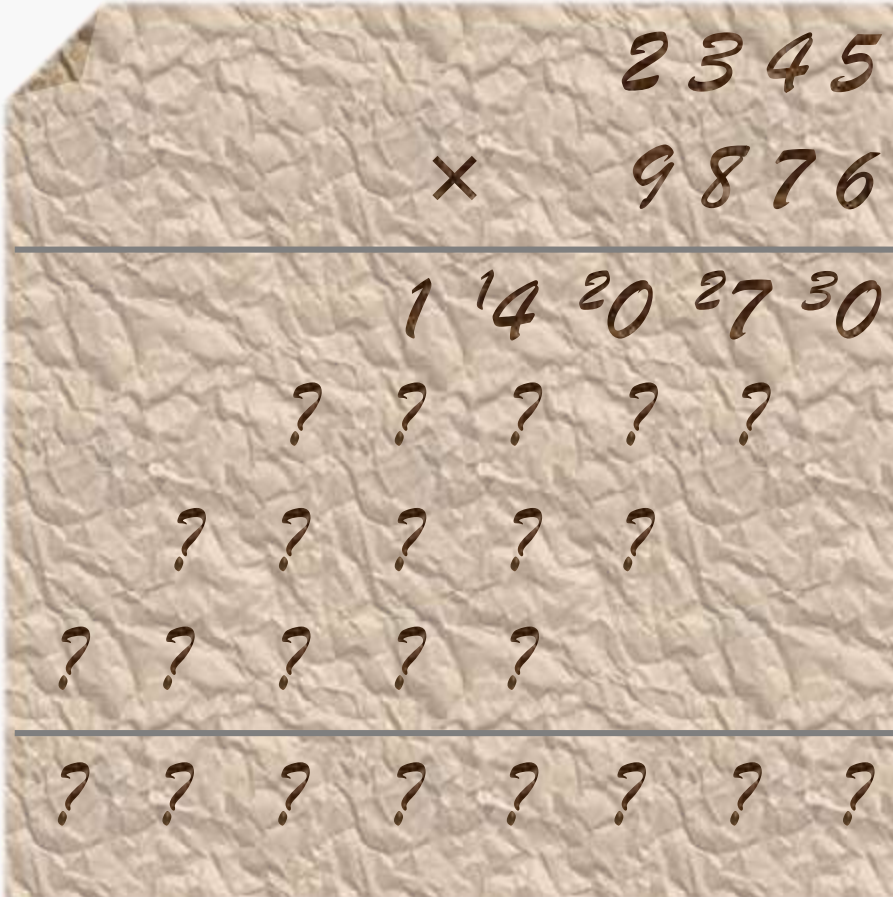
$$\begin{array}{r} \times \quad 1000 \\ 1000 \\ \hline 1000 \end{array}$$



$$\begin{array}{r} 2345 \\ \times 9876 \\ \hline 1\ 14\ 20\ 27\ 30 \\ ?\ ?\ ?\ ?\ ? \\ ?\ ?\ ?\ ?\ ? \\ ?\ ?\ ?\ ?\ ? \\ ?\ ?\ ?\ ?\ ? \end{array}$$



# 较为简单的数字

$$\begin{array}{r} \times \quad \quad \quad 1 \ 0 \ 0 \ 0 \\ 1 \ 0 \ 0 \ 1 \\ \hline 1 \ 0 \ 0 \ 0 \\ 0 \ 0 \ 0 \ 0 \end{array}$$



A photograph of a piece of crumpled, light-brown paper with handwritten mathematical content in dark ink. The paper is folded at the top-left corner. The content includes a multiplication problem and a grid of numbers and question marks.

$$\begin{array}{r} \times \quad \quad \quad 2 \ 3 \ 4 \ 5 \\ 9 \ 8 \ 7 \ 6 \\ \hline 1 \ 14 \ 20 \ 27 \ 30 \\ ? \ ? \ ? \ ? \ ? \\ ? \ ? \ ? \ ? \ ? \\ ? \ ? \ ? \ ? \ ? \\ ? \ ? \ ? \ ? \ ? \end{array}$$



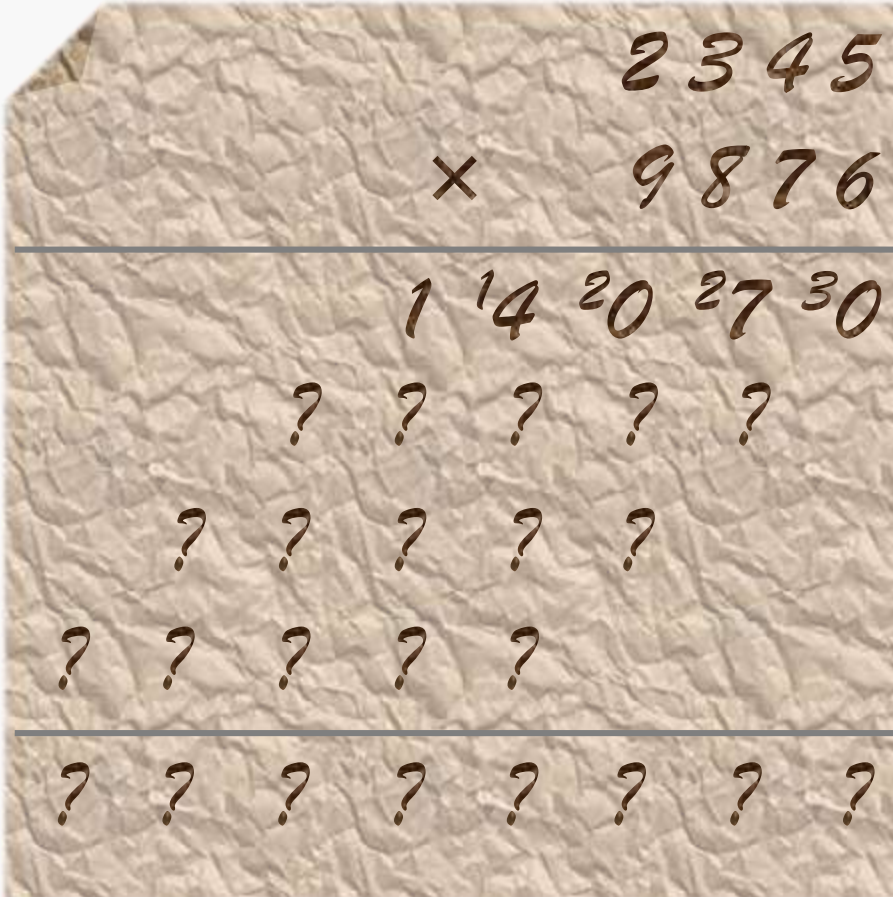
# 较为简单的数字

		1	0	0	0
×		1	0	0	1
<hr/>					
		1	0	0	0
	0	0	0	0	
0	0	0	0		

								2	3	4	5		
								×		9	8	7	6
<hr/>													
								1	14	20	27	30	
								?	?	?	?	?	
								?	?	?	?	?	
								?	?	?	?	?	
								?	?	?	?	?	
<hr/>													
								?	?	?	?	?	?

# 较为简单的数字

$$\begin{array}{r} \phantom{\times} \phantom{000} 1000 \\ \times \phantom{000} \textcircled{1}001 \\ \hline \phantom{000} 1000 \\ \phantom{00} 0000 \\ \phantom{0} 0000 \\ 1000 \end{array}$$



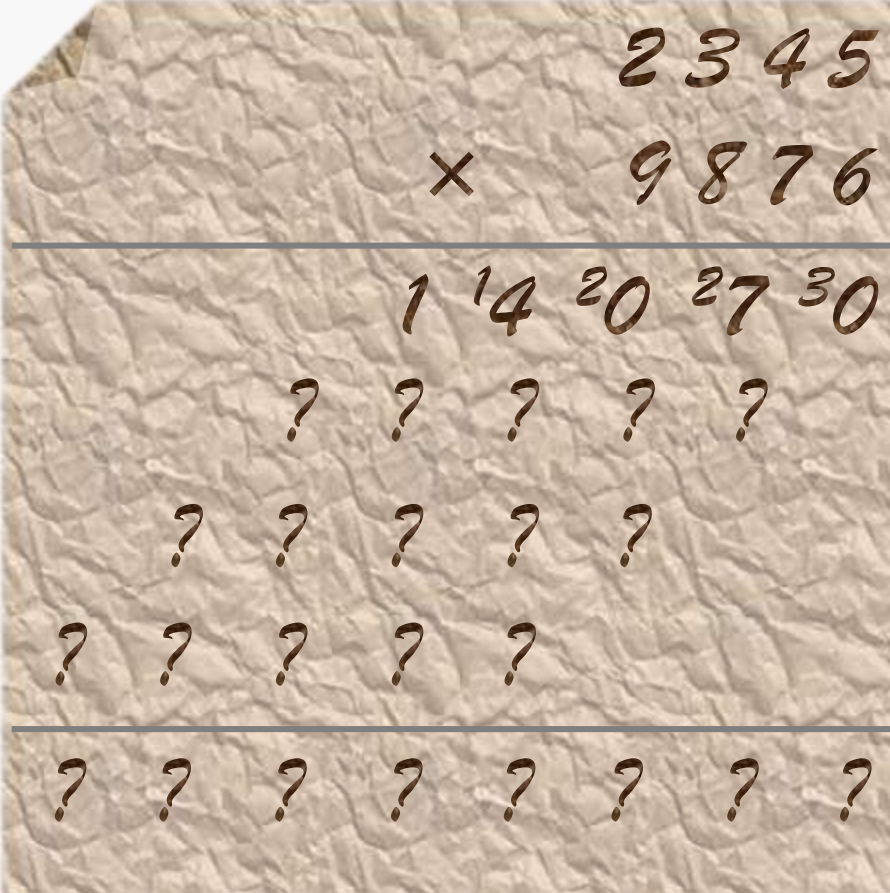
A photograph of a piece of crumpled, light-brown paper with handwritten numbers in dark ink. The paper is folded at the top-left corner. The handwriting is in a cursive style. The multiplication problem is as follows:

$$\begin{array}{r} \phantom{\times} \phantom{0000} 2345 \\ \times \phantom{0000} 9876 \\ \hline \phantom{0000} 114202730 \\ \phantom{000} ? ? ? ? ? \\ \phantom{00} ? ? ? ? ? \\ \phantom{0} ? ? ? ? ? \\ ? ? ? ? ? \end{array}$$

The first line shows the numbers 2345 and 9876. The second line shows the product 114202730. The third line shows a row of five question marks. The fourth line shows a row of five question marks. The fifth line shows a row of five question marks. The sixth line shows a row of five question marks.

# 较为简单的数字

$$\begin{array}{r} \phantom{0000}1000 \\ \times \phantom{0000}1001 \\ \hline \phantom{0000}1000 \\ \phantom{000}0000 \\ \phantom{00}0000 \\ 1000 \\ \hline 1001000 \end{array}$$



A photograph of a piece of crumpled, aged paper with handwritten numbers and a multiplication problem. The numbers are written in a dark ink, possibly with a pen. The multiplication is set up with the multiplier 2345 on the right and the multiplicand 9876 on the left, separated by a horizontal line. Below the line, the first row of the product is written as 1 14 20 27 30. The subsequent rows are filled with question marks, indicating a step-by-step calculation process.

$$\begin{array}{r} \phantom{0000}2345 \\ \times \phantom{0000}9876 \\ \hline \phantom{0000}1\phantom{0}14\phantom{0}20\phantom{0}27\phantom{0}30 \\ \phantom{000}?\phantom{0}?\phantom{0}?\phantom{0}?\phantom{0} \\ \phantom{00}?\phantom{0}?\phantom{0}?\phantom{0}?\phantom{0} \\ ?\phantom{0}?\phantom{0}?\phantom{0}?\phantom{0}?\phantom{0} \\ \hline ?\phantom{0}?\phantom{0}?\phantom{0}?\phantom{0}?\phantom{0}?\phantom{0}?\phantom{0} \end{array}$$





# 简化后的运算过程

				1	0	0	0	被乘数 Multiplicand
×				1	0	0	1	乘数 Multiplier
<hr/>								
				1	0	0	0	
			0	0	0	0		
		0	0	0	0			
	1	0	0	0				
<hr/>								
	1	0	0	1	0	0	0	乘积 Product



# 简化后的运算过程

				1	0	0	0	被乘数 Multiplicand
×				1	0	0	1	乘数 Multiplier
<hr/>								
				1	0	0	0	
			0	0	0	0	0	
		0	0	0	0			
	1	0	0	0				
<hr/>								
	1	0	0	1	0	0	0	乘积 Product

如果当前参与运算的乘数位为1，  
则直接将乘数放置在对位位置上



# 简化后的运算过程

				1	0	0	0
				1	0	0	1
×				1	0	0	0
<hr/>							
				1	0	0	0
			0	0	0	0	0
		0	0	0	0		
	0	0	0	0			
1	0	0	0				
<hr/>							
1	0	0	1	0	0	0	

被乘数 Multiplicand

乘数 Multiplier

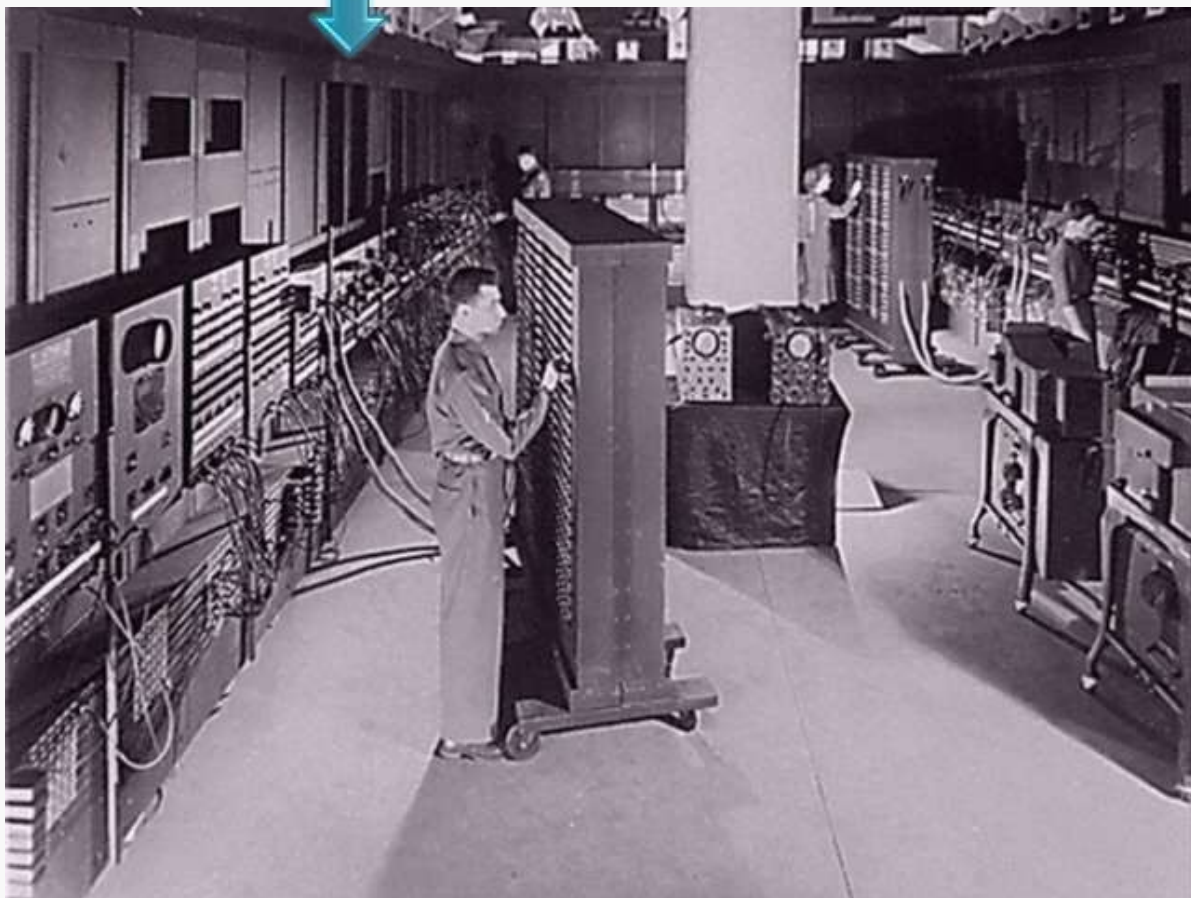
如果当前参与运算的乘数位为1，  
则直接将被乘数放置在对应位置上

如果当前参与运算的乘数位为0，  
则直接将“0”放置在对应位置上

乘积 Product

# 十进制和二进制运算的选择

采用十进制的ENIAC

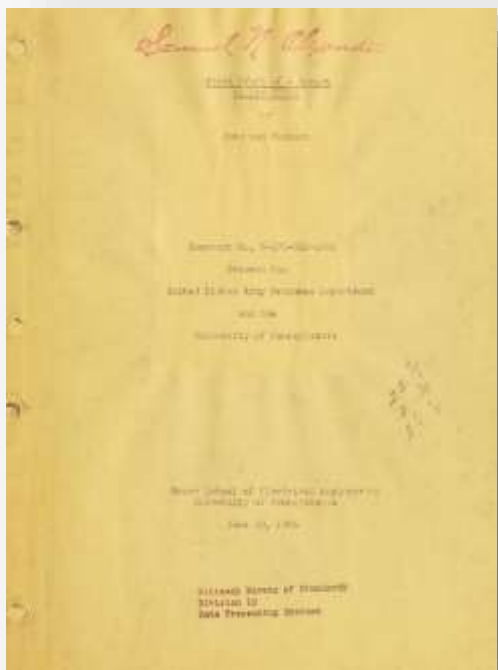


采用二进制的EDVAC





# 十进制和二进制运算的选择



关于EDVAC的  
报告草案  
1945

- ❏ 电子管是一种“全或无”设备（all-or-none），适合表示只有两个数值的系统，即二进制。
- ❏ 二进制可以大幅度地简化乘法和除法的运算过程。尤其是对于乘法，不再需要十进制乘法表，也不再需要两轮的加法。
- ❏ **必须要记住**，十进制才是适合人使用的。因此，输入输出设备需要承担二进制和十进制之间的转换工作。



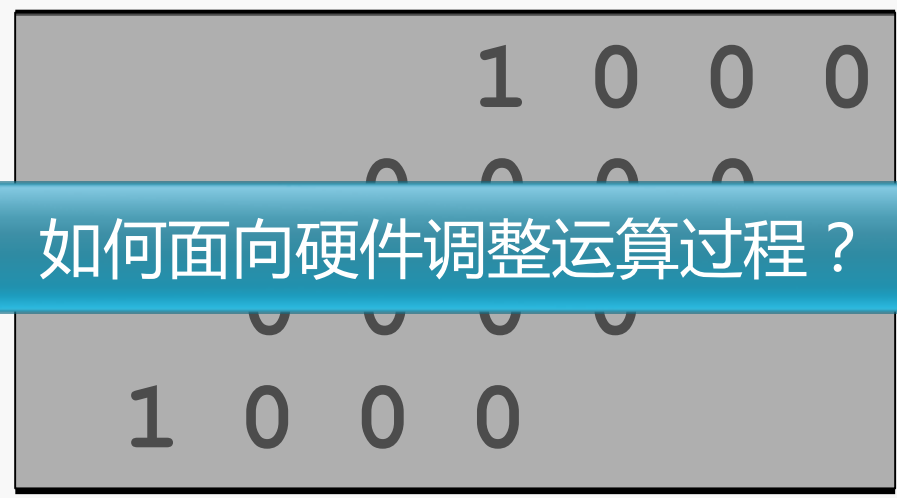
约翰·冯·诺依曼  
John Von Neumann  
1903~1957



# 二进制乘法的运算过程

×                    1 0 0 0    被乘数 Multiplicand  
                      1 0 0 1    乘数 Multiplier

如何面向硬件调整运算过程？



1 0 0 1 0 0 0    乘积 Product

# 运算过程的进一步调整



				1	0	0	0	
×					1	0	0	1
				1	0	0	0	
			0	0	0	0		
		0	0	0	0			
1	0	0	0					
1	0	0	1	0	0	0		

被乘数 Multiplicand

乘数 Multiplier

乘积 Product

# 运算过程的进一步调整



		1	0	0	0		
×		1	0	0	1		
		1	0	0	0		
		0	0	0	0		
		0	0	0	0		
1	0	0	0				
1	0	0	1	0	0	0	

被乘数 Multiplicand

乘数 Multiplier

运算开始时，乘积记为“0”

乘积 Product



# 运算过程的进一步调整



		1	0	0	0
×		1	0	0	1
		1	0	0	0
	0	0	0	0	
	0	0	0	0	
1	0	0	0		
0	0	0	0	0	0

被乘数 Multiplicand

乘数 Multiplier

运算开始时，乘积记为“0”

乘积 Product

# 运算过程的进一步调整



				1	0	0	0	
×					1	0	0	1
				<u>1</u>	0	0	0	
				0	0	0		
				0	0	0		
				1	0	0		
0	0	0	0	0	0	0	0	

被乘数 Multiplicand

乘数 Multiplier

每个中间结果产生后  
直接与当前的乘积累加

乘积 Product

# 运算过程的进一步调整



				1	0	0	0	
×					1	0	0	1
				1	0	0	0	
				0	0	0	0	
				0	0	0	0	
1	0	0	0					
0	0	0	1	0	0	0		

被乘数 Multiplicand

乘数 Multiplier

每个中间结果产生后  
直接与当前的乘积累加

每产生一个中间结果  
被乘数向左移动一位

乘积 Product

# 运算过程的进一步调整

			1	0	0	0	
×			1	0	0	1	
			1	0	0	0	
		<u>0</u>	<u>0</u>	<u>0</u>	<u>0</u>		
	0	0	0	0			
1	0	0	0				
	0	0	0	1	0	0	0

被乘数 Multiplicand

乘数 Multiplier

每个中间结果产生后  
直接与当前的乘积累加

每产生一个中间结果  
被乘数向左移动一位

乘积 Product



# 运算过程的进一步调整

$$\begin{array}{r} \phantom{\times} 1000 \\ \times \phantom{00} 1001 \\ \hline \phantom{000} 1000 \\ \phantom{00} 0000 \\ \phantom{0} 0000 \\ 1000 \\ \hline 0001000 \end{array}$$

被乘数 Multiplicand

乘数 Multiplier

每个中间结果产生后  
直接与当前的乘积累加

每产生一个中间结果  
被乘数向左移动一位

乘积 Product

# 运算过程的进一步调整

$$\begin{array}{r} 1000 \\ \times \quad 1001 \\ \hline 1000 \\ 0000 \\ 0000 \\ 0000 \\ \hline 0001000 \end{array}$$

被乘数 Multiplicand

乘数 Multiplier

每个中间结果产生后  
直接与当前的乘积累加

每产生一个中间结果  
被乘数向左移动一位

乘积 Product

# 运算过程的进一步调整

$$\begin{array}{r} 1000 \\ \times \quad 1001 \\ \hline 1000 \\ 0000 \\ 0000 \\ 0000 \\ \hline 1001000 \end{array}$$

被乘数 Multiplicand

乘数 Multiplier

每个中间结果产生后  
直接与当前的乘积累加

每产生一个中间结果  
被乘数向左移动一位

乘积 Product

# 运算过程的进一步调整



$$\begin{array}{r} 1000 \\ \times \quad 1001 \\ \hline \end{array}$$

1001000

被乘数 Multiplicand

乘数 Multiplier

适合硬件实现的运算过程！

乘积 Product

## 本节小结



# 乘法的运算过程

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计算机组成  
制作人：陆俊林

