A New Discussion on Waeing Problem

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Abstract: in this paper, we prove the result of *g* (2) = 4 for the Waring problem by using algebraic equations and modules. Create a new way for the study of Hualin problem.

Keywords :algebraic equation, determinant, module

1 Preface

Walling problem and Goldbach's conjecture are world mathematical problems that compare with each other. It is assumed that *g* (2) = 4*, g* (3) = *9*, g (4) = 19.But except that *g* (2) = 4 is proved ;by; *g*(3)=9, *g*(4)=19. there is no reliable proof yet[1][2]. This paper will propose a new method to prove *g* (2) = 4 by using algebraic equations and modules.

2 Big Data

Table 1 is the data provided by the computer, which can prove that G (2) = 4 is correct in the range of [1120]. If you continue to search, you can see that G (2) = 4 is correct in the range of [11000]. But the data of computer can only provide us with a direction of efforts, and can not replace mathematical proof.

3 Formula Proof

Theorem 1.

$$(a+b)^2+(a-b)^2+(c+d)^2+(c-d)^2==2(a^2+b^2+c^2+d^2) \quad (1)$$

Proof .

$$(a+b)^2+(a-b)^2+(c+d)^2+(c-d)^2=a^2+2ab+b^2+a^2-2ab+b^2+c^2+2cd+d^2+c^2-2cd+d^2$$

$$(a+b)^2+(a-b)^2+(c+d)^2+(c-d)^2=2(a^2+b^2+c^2+d^2)$$

By analogy, it is still not easy to prove that it can be extended to all natural numbers.

n	a_1	a_2	a_3	a_4	$f(0^2)$	n	a_1	a_2	a_3	a_4	$f(0^2)$	n	a_1	a_2	a_3	a_4	$f(0^2)$
1	0	0	0	1	3	41	0	0	16	25	2	81	0	0	81	0	3
2	0	0	1	1	2	42	0	1	16	25	1	82	0	0	81	1	2
3	0	1	1	1	1	43	0	9	9	25	1	83	0	1	81	1	1
4	0	0	0	4	3	44	0	4	4	36	1	84	0	4	64	16	1
5	0	0	1	4	2	45	0	0	9	36	2	85	0	0	49	36	2
6	0	1	1	4	1	46	0	1	9	36	1	86	0	1	49	36	1
7	1	1	1	4	0	47	1	1	9	36	0	87	1	1	49	36	0
8	0	0	4	4	2	48	0	16	16	16	1	88	0	16	36	36	1
9	0	0	0	9	3	49	0	0	49	0	3	89	0	0	64	25	2
10	0	0	1	9	2	50	0	0	25	25	2	90	0	0	81	9	2
11	0	1	1	9	1	51	0	1	25	25	1	91	0	1	81	9	1
12	0	4	4	4	1	52	0	0	16	36	2	92	1	1	81	9	0
13	0	0	4	9	2	53	0	0	49	4	2	93	0	4	64	25	1
14	0	1	4	9	1	54	0	1	49	4	1	94	0	4	81	9	1
15	1	1	4	9	0	55	1	1	49	4	0	95	1	4	81	9	0
16	0	0	0	16	3	56	0	4	16	36	1	96	0	16	64	16	1
17	0	0	1	16	2	57	0	4	49	4	1	97	0	0	81	16	2
18	0	0	9	9	2	58	0	0	49	9	2	98	0	1	81	16	1
19	0	1	9	9	1	59	0	1	49	9	1	99	0	9	81	9	1
20	0	0	4	16	2	60	1	1	49	9	0	100	0	0	64	36	2
21	0	1	4	16	1	61	0	0	25	36	2	101	0	0	100	1	2
22	0	4	9	9	1	62	0	1	25	36	1	102	0	1	100	1	1
23	1	4	9	9	0	63	1	1	25	36	0	103	1	1	100	1	0
24	0	4	4	16	1	64	0	0	64	0	3	104	0	0	100	4	2
25	0	0	0	25	3	65	0	0	49	16	2	105	0	1	100	4	1
26	0	0	1	25	2	66	0	1	49	16	1	106	0	0	81	25	2
27	0	1	1	25	1	67	0	9	49	9	1	107	0	1	81	25	1
28	1	1	1	25	0	68	0	0	64	4	2	108	0	4	100	4	1
29	0	0	4	25	2	69	0	1	64	4	1	109	0	0	100	9	2
30	0	1	4	25	1	70	0	9	25	36	1	110	0	1	100	9	1
31	1	1	4	25	0	71	1	9	25	36	0	111	1	1	100	9	0
32	0	0	16	16	2	72	0	0	36	36	2	112	4	4	100	4	0
33	0	1	16	16	1	73	0	0	64	9	2	113	0	4	100	9	1
34	0	0	9	25	2	74	0	0	49	25	2	114	0	25	64	25	1
35	0	1	9	25	1	75	0	1	49	25	1	115	0	9	81	25	1
36	0	0	0	36	3	76	0	4	36	36	1	116	0	0	100	16	2
37	0	0	1	36	2	77	0	4	64	9	1	117	0	0	81	36	2
38	0	1	1	36	1	78	0	4	49	25	1	118	0	1	81	36	1
39	1	1	1	36	0	79	1	4	49	25	0	119	1	1	81	36	0
40	0	0	4	36	2	80	0	0	64	16	2	120	0	4	100	16	1

Table 1. The data provided by the computer in[1,120]

3 Module Certification

3.1 Determinant

In higher algebra, the determinant is as follows [2]:

$$\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ a_{31} & a_{32} & \dots & a_{3n} \\ a_{41} & a_{42} & \dots & a_{4n} \end{vmatrix} = a_{11} \quad a_{22} \quad \dots \quad a_{nn}$$

Here, we just compare the familiar determinant with the module in form, which makes it easier for us to understand the module (there is no need to know more about determinant). Note: in the determinant, the operator in the symbol "|　|" is omitted.

3.2 Module

M_1

1	0	0	0	1
2	0	0	1	1
3	0	1	1	1
4	1	1	1	1

Figure 1 . A module

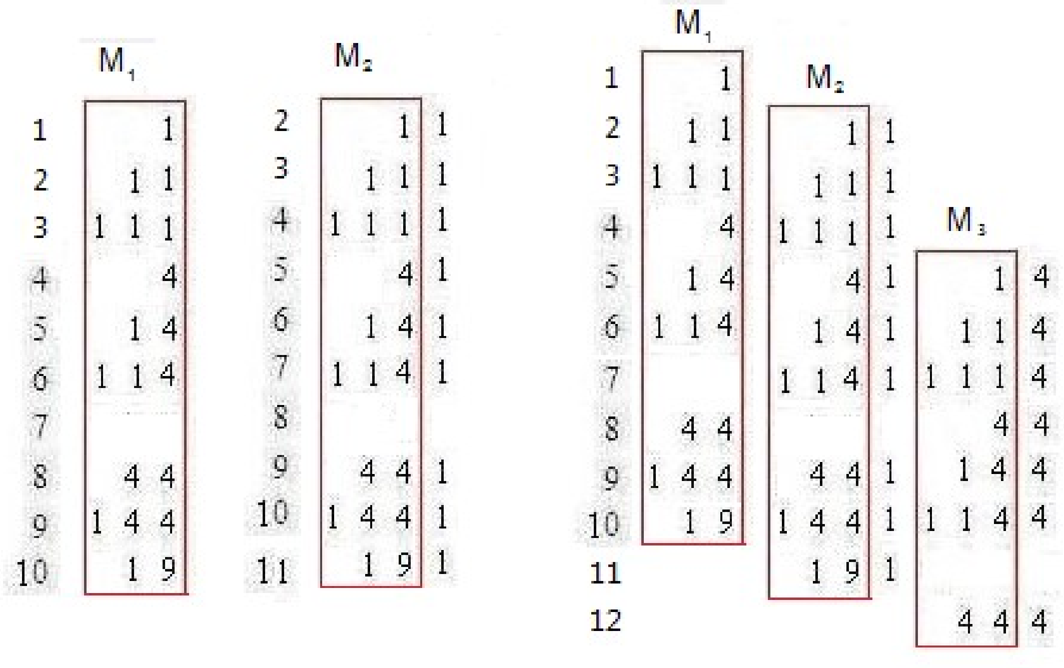
This is a module with four expressions. It is not difficult for us to find its rule: the sum of each line of numbers in the box is equal to the number on the left. Ad locum:

1） Each line in the box is an additive formula (the operation symbol is omitted). Each of its terms (addends) is a power element.

2） The number on the left is the sum of the addends corresponding to a line. Also known as ordinal number.

3） The name of this module is "M_i".

We draw the information related to *g* (3) and *g* (4) into the following module diagram:



(1) (2) (3)

Figure 2. Some modules

In Figure 2:

(1) M_1 is the of g (3). about

$$A_1^2 + A_2^2 + A_3^2 = n. \qquad (1)$$

When the power element has 0^2, 1^2, 2^2 and 3^2, only $n = 7$ has no solution.

(2) M_2 is g (4). Why is an addend (power) of g (4) written outside the box? This table M_2 is composed of M_1 plus an addend (power). about

$$A_1^2 + A_2^2 + A_3^2 + A_4^2 = n \qquad (2)$$

When the power element has 0^2, 1^2, 2^2 and 3^2,, add a power term (1^2) based on (1^2) to get the result of M_2, because here, using the method of adding 1^2, $n = 8$ has no solution.

(3) The module group is composed of three modules (sharing one n mark).Through the combination of modules, all n have solutions in [1,12], where g '(2) = 4.

Note: the solution of g' (4) is moved back by an integer position than the solution of g' (3). M_3 is to change the added power of M_2 into 2^2, and the solution ratio of g (4) moves back three integer positions than m_2. The purpose of using the module is to continuously expand the scope of the solution, so that all n can find the solutions.

Here, we call the module using g (3) composed of power elements 0^2, 1^2, $2^2$and 3^2 as the basic module, and the additional power term is called the addition term or (addition power). It will be discussed later that the modules for the purpose of $g(2) = 4$ are in the form of M_3.

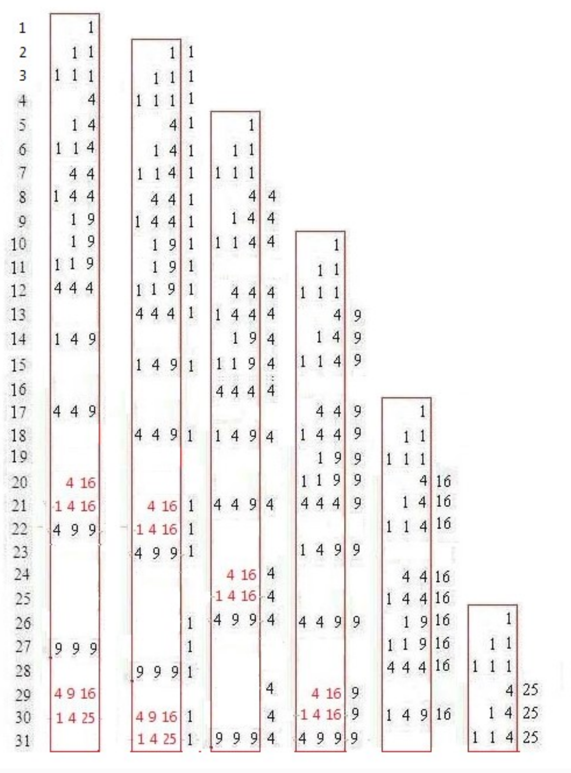


Figure 3 . The $g'(2)$=4 in [1,31]

3.4 proposition proof

Proposition. $g(2)$=4.

Proof. If we continue to transform and add the range of powers, from 0^2、1^2、2^2 to 6^2, we will get the information in Figure 4:

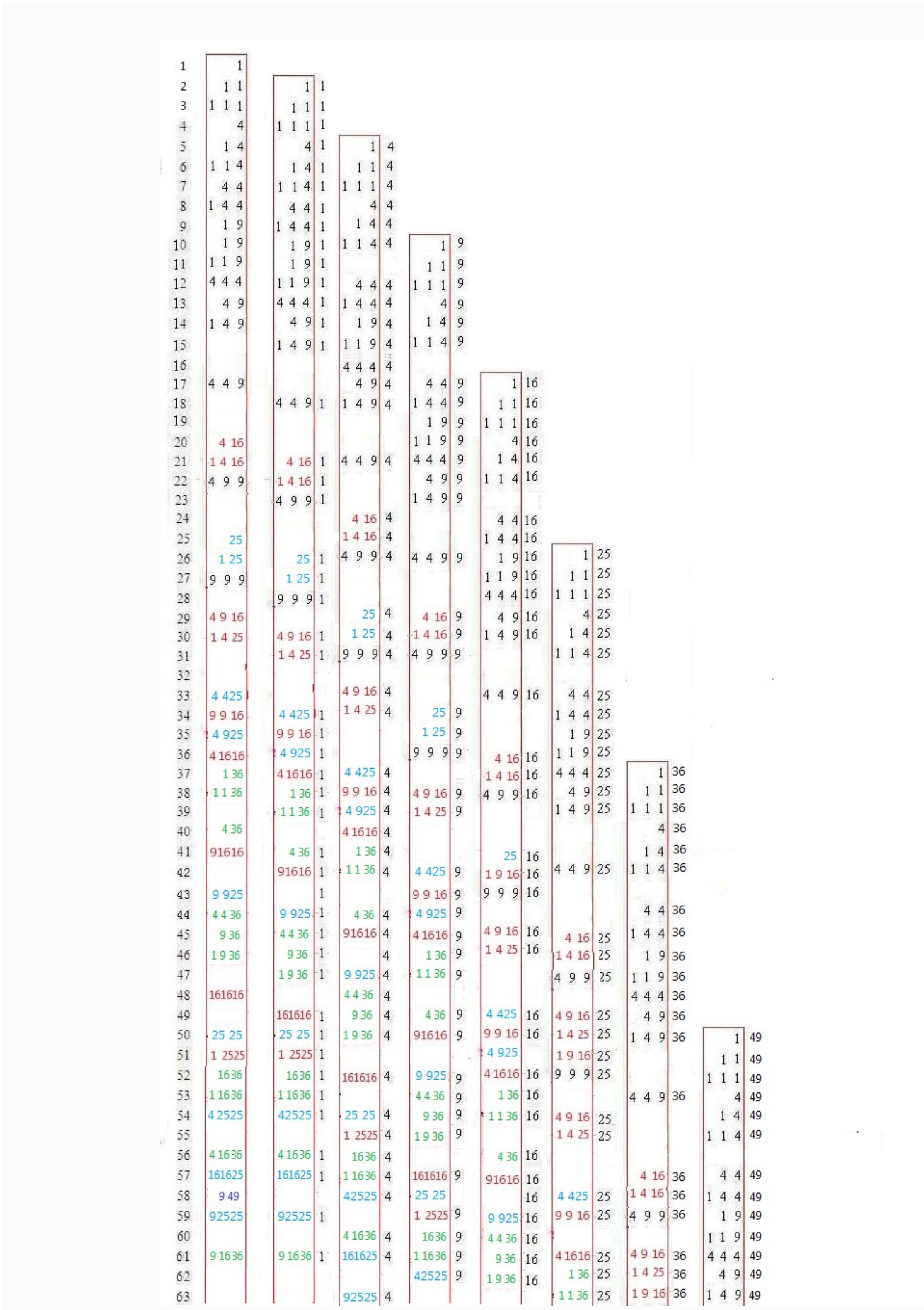


Figure 4. The $g'(2)=4$ in [1,63]

As can be seen from Figure 3, when n is [1,31], only 0^2、1^2、2^2,..., 4^2, $g'(2) = 4$ is valid for the basic module. Because $p_{l+1}^2 - p_l^2 = p_l^2 + 2p + 1 - p_l^2 = 2p + 1$, the starting point of each module is an odd series.

We continue to add n and power elements. After careful observation, there are the following situations:

1） The distance between each new power and the previous power is 2n + 1;

2) The solution of $g'(2) = 3$ of the basic module M_1 increases;

3) The de overlap of transverse modules M_1, M_2, M_3,..., M_7, $g'(2) = 4$ increases.Due to the reasons of 1) 2) 3), the solution of $g'(2) = 4$ remains continuous when n is greater than 31.

Theorem 2. $g(2) = 4$.

Proof. 1) Since this is irreplaceable by $n = 7$, $1^2 + 1^2 + 1^2 + 2^2 = 7$, so $g'(2)$ is at least 4. 2) It is proved from Fig. 4 that when n is greater than 7, there are continuous solutions with $g'(2) = 4$. Therefore, when n is between 1 and infinity, $g(2) = 4$ holds.

Now, we completed proof with the $g(2) = 4$, The proof about $g(3) = 9$ and $g(4) = 15$ will be supplemented later.

4 Conclusion

This paper vividly proves the proposition of G (2) = 4 in the form of module. Morphological mathematics is a very useful theoretical tool to solve integer problems. After more than ten years of painstaking study and research, the author has solved the proof of Goldbach conjecture, Fermat's theorem, Four-color theorem, odd perfect number, higher-order indefinite equation and Waring problem. Through the application of morphological mathematics, many outstanding mathematical problems about integers in number theory are solved one by one, which shows the power of morphological mathematics. As long as we can master their application skills flexibly and rigorously, we can solve more pure mathematics and applied mathematics problems.

Reference

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