A Simple Proof of Fermat's Last Theorem

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Fermat is a French justice and amateur mathematician .In 1637, Fermat was inadvertently attracted by the Pythagorean work described in the ancient Greek mathematician Diophantine's arithmetic. He had a whim about whether he could find a solution to the indefinite equation of the Pythagorean equation.

The original Pythagorean equation can be expressed as the search equation:

$$x^n + y^n = z^n$$

integer solution of.

Fermat then annotated in the book "arithmetic": it is impossible to divide a cubic number into two cubic numbers, a fourth power into two fourth powers, or generally divide a power higher than the second power into two powers of the same power. On this, I am sure I have found a wonderful proof. Unfortunately, the blank space here is too small to write down." This problem is then called "Fermat's Last Theorem".

For more than three centuries, the best mathematicians in history have tried to prove it, but got nothing. In 1994, Wiles indirectly proved Fermat's theorem by using modern mathematical methods such as modular form, Gushan Zhicun conjecture and the properties of elliptic curves of Galois group. But his proof is 130 pages long and quite esoteric, which is obviously not the short proof Fermat said. People are still seeking a concise Fermat's theorem as Fermat said

If x, y and z have common factor, we can eliminate them. Is that the equation becomes

$$A^n + B^n = C^n$$

where $n \geq 3$.

First Proof

Fermat's last theorem: If equation
$$A^n + B^n = C^n \tag{1}$$
there is no positive integer solution.

Proof. If equation (1) is true, let $A = C - a, \quad B = C - b,$ then
$$A^n = (C - a)^n$$
$$A^n = C^n - n C^{n-1}a + \ldots + (-1)^{n-1}nCa^{n-1} + (-1)^n a^n \tag{2}$$
$$B^n = (C - b)^n$$
$$B^n = C^n - n C^{n-1}b + \ldots + (-1)^{n-1}nCb^{n-1} + (-1)^n b^n \tag{3}$$

By (2)+ (3):

$$A^n + B^n = 2C^n - nC^{n-1}(a+b)+\ldots+(-1)^{n-1}nC(a^{n-1}+b^{n-1})+(-1)^n(a^n+b^n) \quad (4)$$

Since $A^n + B^n = C^n$, then

$$C^n = 2C^n - nC^{n-1}(a+b)+\ldots+(-1)^{n-1}nC(a^{n-1}+b^{n-1})+(-1)^n(a^n+b^n)$$

by (4).

So

$$C^n - nC^{n-1}(a+b)+\ldots+(-1)^{n-1}nC(a^{n-1}+b^{n-1})+(-1)^n(a^n+b^n)=0 \quad (5)$$

By Theorem 1, we know that if

$$C^n - nC^{n-1}q+\ldots+(-1)^{n-1}nC\,q^{n-1}+(-1)^n q^n =0 \quad, \quad (6)$$

where this asked $C=q$.

Comparison (5) and (6), we get :

If (5) is true, this asked :

$$a+b=q=C;$$
$$a^2+b^2=q^2=C^2;$$
$$\ldots\ldots$$
$$a^n+b^n=q^n=C^n.$$

This is impossible, then equation (5) is not true. Thus, the equation (1) is not true.

Theorem 1. If

$$C^n - nC^{n-1}q+\ldots+(-1)^{n-1}nC\,q^{n-1}+(-1)^n q^n =0,$$

this asked $C=q$.

Proof. Since

$$(C-q)^n = C^n - nC^{n-1}q+\ldots+(-1)^{n-1}nC\,q^{n-1}+(-1)^n q^n \quad,$$

then the equation only holds if $C=q$.

The Second Proof

Fermat's last theorem: If equation

$$A^n + B^n = C^n \quad (1)$$

there is no positive integer solution.

Proof. Let $C = A + c$,

$$A^n + B^n = (A+c)^n,$$
$$A^n + B^n = A^n + nA^{n-1}c+\ldots\ldots+nAc^{n-1}+c^n,$$
$$B^n = nA^{n-1}c+\ldots\ldots+nAc^{n-1}+c^n,$$
$$B^n = c\,(nA^{n-1}+\ldots\ldots+nAc^{n-2}+c^{n-1}) \quad. \quad (2)$$

Then B contains c, $c \mid B$.

Let $B = ck$, substitute it into (2):

$$c^n k^n = c\,(nA^{n-1}+\ldots\ldots+nAc^{n-2}+c^{n-1}),$$
$$c^{n-1}k^n = nA^{n-1}+\ldots\ldots+nAc^{n-2}+c^{n-1},$$
$$c^{n-1}k^n = nA^{n-1}+\ldots\ldots+nAc^{n-2}+c^{n-1},$$

$$c^{n-1}k^n = n A^{n-1} + \frac{n(n-1)}{2} A^{n-2}c + \ldots + n A c^{n-2} + c^{n-1} \qquad (3)$$

$$c^{n-1}k^n - c^{n-1} = n A^{n-1} + \frac{n(n-1)}{2} A^{n-2}c + \ldots + n A c^{n-2},$$

$$c^{n-1}(k^n - 1) = n A\left(A^{n-1} + \frac{n(n-1)}{2} A^{n-2}c + \ldots + c^{n-2}\right).$$

Then , this asked n contains c^{n-1}, it is impossible. Therefore equation (3) Is not true, and equation (2)and (1) are not true.