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New Proof of Goldbach's Conjecture

Zengyong Liang
Maternal and Child Health Care Hospital of Guangxi, Nanning 530000, China

Abstract: This paper uses graphic method to analyze the mechanism of prime number formation and the periodic characteristics of its multiple in natural numbers. The Goldbach's conjecture is proved by the analysis diagrams.

Key words: Period, prime integer-pair, effective composite numbers, icons, analysis diagrams.

1. Introduction

1.1 Origins

Goldbach's conjecture is a famous mathematical problem in the world [1]. The author's paper "Rigorous Proof of Goldbach's Conjecture" (2018) has proved that the conjecture is correct [2]. The proof method is to arrange 2n integers into the following Fig. 1, in which two integers and 2n integers form a pair of integers, such as a pair of integers is called integer-pair. That of it is proved in which 1 or more than 1 must be prime pair.

1.2 Idea of Proof Method

In nature, waves (light, radio and sound waves, etc.) are a common form of energy transmission. Inspired by the superposition effect of multi-frequency waves in different periods, I found a new method to prove the Goldbach conjecture. In natural numbers, include 1, prime and composite numbers [3, 4], the multiple of prime numbers appears periodically in the number axis. So they only have period superposition. The following Fig. 2 can be used to understand. In Fig. 2, we observe that every multiplier of prime P circularly appears on the number axis. For example, integer 6 can be regarded as the periodic superposition of multiple of primes 2 and 3, while integer 30 is the periodic superposition of multiple of 2, 3 and 5. Next, we will carry out our proof according to the nature of natural numbers.

Corresponding author: Zengyong Liang, deputy chief technician, research fields: number theory, graph theory.

2. Graphic Design

For clearer proof, we use charts to express the relationship between numbers. This is the charm of the graphic proof method [5].

2.1 The Meaning of a Single Icon

First, let us get to know the meaning of a single icon.

1 is the integer 1 itself, the starting point of odd numbers.

△ a prime 3.
☆ a prime 5.
◇ a prime 7.
□ a prime 11.
○ a prime or undefined integer.
▲ a multiple of 3.
★ a effective composite of 5.
◆ a effective composite of 7.
■ a effective composite of 11.
● a effective composite of prime number greater than 11.

2.2 Graph Representation of Continuous Odd Numbers

Definition: when an integer contains more than one prime factor, take an icon of the smallest prime factor to represent and it called effective composite. Because an icon can only be used for an integer, this is also consistent with that principle of "sieve of

Eratosthenes" [6].

Rule 1. The first effective composite number of any

1	2	3	4	···	n-4	n-3	n-2	n-1	n
2n-1	2n-2	2n-3	2n-4	···	n+4	n+3	n+2	n+1	n

Fig. 1 n pairs of integers.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	···
	2		2		2		2		2		2		2		2		2		2		2		2	···
		3			3			3			3			3			3			3			3	···
				5					5					5					5					···
						7							7							7				···

Fig. 2 The multiple of prime numbers in natural numbers.

1△☆◇▲○○▲○○▲○★▲○○▲★○▲○○▲○◆

Fig. 3 A graph of 25 continuous odd integers.

1	3	5	7	9	11	13	15	17	19	21	23	25
49	47	45	43	41	39	37	35	33	31	29	27	25

Fig. 4 13 pairs of odd integers.

1 △ ☆ ◇ ▲ ○ ○ ▲ ○ ○ ▲ ○ ★ (25)

◆ ○ ▲ ○ ○ ▲ ○ ★ ▲ ○ ○ ▲ ★

Fig. 5 Analysis diagram when n = 25.

prime p_i is p_i^2, and there after appears by form with $p_i p_{i+1}$, $p_i p_{i+1}$, … .

Proof. By the definition of effective composite, before p_i^2, all multiples of p_i must be effective composite of primes smaller than pi, which can be proved by the rule 1.

In Fig. 3, it is corresponding to 25 continuous odd integers as:

1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49

The prime number greater than 7 is expressed by "○".

And Fig. 5 is an analysis diagrams which is corresponding to Fig. 4.

2.3 Analysis Diagrams

This proof method is to arrange n odd integers into a chart, and obtain n/2 or (n+1)/2 pairs of integers. Goldbach's conjecture can be solved by proving that one or more pairs of integers must be prime pairs. For example, Fig. 4 can be turned into a graph in Fig. 5: The number in brackets is the integer position of the last icon, or the integer value represented by the icon. Fig. 5 clearly shows the 25 consecutive odd numbers arranged from 1 where there are 4 prime integer-pairs. This graph is called analysis diagram.

With a analysis graph, we can find and prove the existence of prime number pairs. For example, in Fig. 6, graph (1) there are 2 prime pairs, that graph (2) has 3 prime pairs primes that graph (3) has 3 prime pairs.

2.4 Using of Sieve

Rule 2. When $2n \le p_m^2$, in order we calculate all the composite numbers, we have only to need to calculate the composite number of 2, 3, …, p_m. So other remnants are 1 and primers.

Proof. According to the principle of the "sieve of Eratosthenes" [7], when $2n \le p_m^2$, in order we calculate all the composite numbers, we have only to need to calculate the composite number of 2, 3, …, p_m. So other remnants are 1 and primes. For example: if consider m = 4 , p_m = 7, we only test 26 ≤ 2n ≤ 120, See Fig. 7.

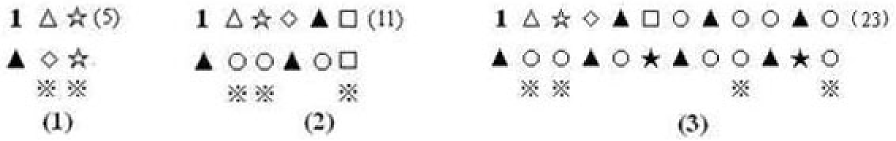

Fig. 6 Three analysis diagram.

Fig. 7 An analysis diagram when n = 59.

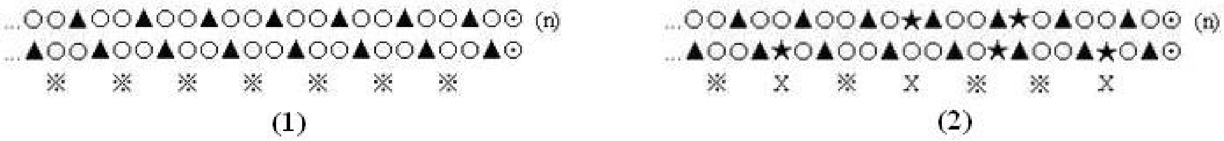

Fig. 8 An analysis diagram with only has multiples of 3, 5 or 3, 5, 7.

Fig . 9 An analysis diagram with only has multiples of 3, 5, …, 17.

Rule 3. When n is not odd prime or multiple of small odd prime, the chance of to form prime integer-pairs is the least.

Proof . (1) When n is prime, n + n = 2n.

(2) When n is multiple of odd prime p, all of multiple of odd prime p are opposite. Hence when n is not odd prime or multiple of small odd prime, the chance to form prime integer-pairs is the least.

3. Proof of Proposition

Proposition 3.1. Any even number greater than 4 can be expressed as the sum of two prime numbers.

Proof. To prove the use of graphics is to prove that in Fig 2, no matter how large 2n is, there exists a pair or more than 1 prime pairs. According to Rule 3, we only need to prove that when n is not a multiple of small odd prime numbers .

Method 1: test in real case.

(1) When 2n ≤ 24, only 3 of the composite numbers by Rule 1. As Fig. 6 (2) above, Note: "※" means that there is a prime number here. There are tree prime integers.

(2) When the composite number of 5 appears, e.g., 2n ≤ 48, only the effective composite of 3 and 5. See Fig. 6 (3).

(3) When the composite number of 7 appears, e.g., 50 ≤ 2n ≤ 120, only the effective composite of 3, 5 and 7. See Fig. 7.

(4) For the case where even numbers are greater than 122, we use method 2 to analyze and discuss:

Method 2: We artificially add multiples of prime numbers one by one to see that if we can succeed in making no prime pairs appear near n.

Firstly, we make a basic analysis graph, which has only multiples of 3 and undetermined integers. According to rule 3, n is not a prime number and is expressed by "⊙", as Fig. 8 (1).

Then we will consider only the integer pair near n. Artificially increase the density of prime multiplier to counteract all possible prime pairs.

(a) First, add 5 effective composite number in the basic analysis diagram, as Fig. 8 (2). Obviously, there are two fewer prime pairs (using "X" to represent). But there are 4 prime pairs. In the same times, we seen that $25 \leq n \geq 59$. Obviously, there are at least 2 pairs of prime numbers.

(b) Next, add 7 effective composite number, …, this increases the effective multiple of primes until the prime number is 17. See Fig. 9.

Note that 2n will also increase to 17^2 or less than 19^2. Obviously, though there are 39 continuous odd integer-pairs that are not prime integer-pairs, but there are still two prime integer-pairs at left side. The reason is obvious that the increase of 2n is much larger than the ability of the effective composite of primes 13 and 17 to eliminate prime number pairs. So analogous …. No matter how to increase the effective composite points of new prime numbers, the occurrence of prime number pairs cannot be eliminated. Inversion there will be more and more prime number pairs. In other words, no matter how large the 2n is, there is at least 1 prime pair.

Method 3: Consider the worst condition when 2n is equal to infinity.

When 2n is infinite, the prime density from 1 to 8 is 2/3, and the density limit at 2n is 0. Since the density of a prime is the opposite of the density of a composite number, and the density of multiple of a prime is a linear function, then the density of prime numbers is quasi-linear, and the density at n is Therefore, the number of upper and lower descending parts near 1/2 is relatively loose [7]. By using this method, the multiples of primes are artificially increased. When the density of primes is increased to use the multiples of prime 17, the ratio of the primes reached 1/3. See Fig. 13 before, there is the density of primes is $31/90 \approx 1/3$, and has two prime integer-pairs. Therefore, when 2n tends to infinity, there will be the prime number pairs, and the prime number pairs will be more and more.

In addition, let card (D^n) be the number of the prime integer-pairs when prime density is 1/3, we can use the formula of Ref. [3] to calculate as below:

$$\text{card}(D^n) = \frac{n}{4} \times \frac{1}{3} \left(1 - \frac{2}{5}\right) \dots \left(1 - \frac{2}{p_m}\right)$$

$$= \frac{n}{2} \times \frac{1}{3} \times \frac{3}{5} \times \frac{5}{7} \times \dots \times \frac{p_m - 2}{p_m}$$

$$\geq \frac{n}{2p_m} = \frac{p_m^2}{4p_m} = \frac{p_m}{4}.$$

$$\textit{And } p_m{}^2 \div \frac{p_m}{4} = 4p_m.$$

That is to say, when $2n = p_m^2$, there must be one prime integer-pair in $4p_m$ integer-pair near n.

So the Proposition 3.1 is proved.

Now, we have proved the Goldbach's conjecture that is correct.

4. Data of Computer

Data of Table 1 are given by a computer. It shows the number of 2pm integer-pair that near n (f(2*pm*)) is more than 2.

5. Data of Computer

From the above proof, we can see that prime numbers are infinite in natural numbers, because the multiples of primes cannot fill the natural number position in the number axis, and the multiples of odd primes cannot fill the possible prime space in Fig. 1. So no matter how large the even 2n is, there is a prime number pair certainly. It shows that any even number greater than or equal to 4 can be expressed as the sum of two prime numbers, which completes the proof of that the Goldbach's conjecture.

Table 1 The number of prime pairs.

m	P_m	2n	$f(2P_m)$
5	11	122	2
6	13	170	4
7	17	290	5
8	19	362	3
9	23	530	4
10	29	842	2

11	31	962	3
12	37	1,370	8
13	41	1,682	3
14	43	1,850	7
15	47	2,210	6
16	53	2,810	7
17	59	3,482	7
18	61	3,722	4
19	67	4,490	9
20	71	5,042	7

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