

A New Discussion on Waring's Problem

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Abstract: in this paper, we prove the result of $g(2) = 4$ for the Waring problem by using algebraic equations and modules. It find a new way for the study of Waring's problem.

Keywords: Algebraic equation, Determinant, Module

1 Preface

Walling's problem and Goldbach's conjecture are world mathematical problems that compare with each other. It is assumed that $g(2) = 4$, $g(3) = 9$, $g(4) = 19$. But except that $g(2) = 4$ is proved; by $g(3)=9$, $g(4)=19$. there is no reliable proof yet[1][2]. This paper will propose a new method to prove $g(2) = 4$ by using algebraic equations and modules.

2 Big Data

Table 1 is the data provided by the computer, which can prove that $g(2) = 4$ is correct in the range of $[1,120]$. If you continue to search, you can see that $g(2) = 4$ is correct in the range of $[11000]$. But the data of computer can only provide us with a direction of efforts, and cannot replace mathematical proof.

n	a ₁	a ₂	a ₃	a ₄	n	a ₁	a ₂	a ₃	a ₄	n	a ₁	a ₂	a ₃	a ₄
1	0	0	0	1	41	0	0	16	25	81	0	0	81	0
2	0	0	1	1	42	0	1	16	25	82	0	0	81	1
3	0	1	1	1	43	0	9	9	25	83	0	1	81	1
4	0	0	0	4	44	0	4	4	36	84	0	4	64	16
5	0	0	1	4	45	0	0	9	36	85	0	0	49	36
6	0	1	1	4	46	0	1	9	36	86	0	1	49	36
7	1	1	1	4	47	1	1	9	36	87	1	1	49	36
8	0	0	4	4	48	0	16	16	16	88	0	16	36	36
9	0	0	0	9	49	0	0	49	0	89	0	0	64	25
10	0	0	1	9	50	0	0	25	25	90	0	0	81	9
11	0	1	1	9	51	0	1	25	25	91	0	1	81	9
12	0	4	4	4	52	0	0	16	36	92	1	1	81	9
13	0	0	4	9	53	0	0	49	4	93	0	4	64	25
14	0	1	4	9	54	0	1	49	4	94	0	4	81	9
15	1	1	4	9	55	1	1	49	4	95	1	4	81	9
16	0	0	0	16	56	0	4	16	36	96	0	16	64	16
17	0	0	1	16	57	0	4	49	4	97	0	0	81	16
18	0	0	9	9	58	0	0	49	9	98	0	1	81	16
19	0	1	9	9	59	0	1	49	9	99	0	9	81	9
20	0	0	4	16	60	1	1	49	9	100	0	0	64	36
21	0	1	4	16	61	0	0	25	36	101	0	0	100	1
22	0	4	9	9	62	0	1	25	36	102	0	1	100	1
23	1	4	9	9	63	1	1	25	36	103	1	1	100	1
24	0	4	4	16	64	0	0	64	0	104	0	0	100	4
25	0	0	0	25	65	0	0	49	16	105	0	1	100	4
26	0	0	1	25	66	0	1	49	16	106	0	0	81	25
27	0	1	1	25	67	0	9	49	9	107	0	1	81	25
28	1	1	1	25	68	0	0	64	4	108	0	4	100	4
29	0	0	4	25	69	0	1	64	4	109	0	0	100	9
30	0	1	4	25	70	0	9	25	36	110	0	1	100	9
31	1	1	4	25	71	1	9	25	36	111	1	1	100	9
32	0	0	16	16	72	0	0	36	36	112	4	4	100	4
33	0	1	16	16	73	0	0	64	9	113	0	4	100	9
34	0	0	9	25	74	0	0	49	25	114	0	25	64	25
35	0	1	9	25	75	0	1	49	25	115	0	9	81	25
36	0	0	0	36	76	0	4	36	36	116	0	0	100	16
37	0	0	1	36	77	0	4	64	9	117	0	0	81	36
38	0	1	1	36	78	0	4	49	25	118	0	1	81	36
39	1	1	1	36	79	1	4	49	25	119	1	1	81	36
40	0	0	4	36	80	0	0	64	16	120	0	4	100	16

Table 1. The data provided by the computer in $[1,120]$

3 Proof of Formula

Theorem 1.

$$(a+b)^2+(a-b)^2+(c+d)^2+(c-d)^2=2(a^2+b^2+c^2+d^2) \quad (1)$$

Proof.

$$(a+b)^2+(a-b)^2+(c+d)^2+(c-d)^2=a^2+2ab+b^2+a^2-2ab+b^2+c^2+2cd+d^2+c^2-2cd+d^2$$

$$(a+b)^2+(a-b)^2+(c+d)^2+(c-d)^2=2(a^2+b^2+c^2+d^2)$$

By analogy, it is still not easy to prove that it can be extended to all natural numbers.

4 Module Certification

4.1 Determinant

In higher algebra, the determinant is as follows [2]:

$$\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ a_{31} & a_{32} & \dots & a_{3n} \\ a_{41} & a_{42} & \dots & a_{4n} \end{vmatrix} = a_{11} a_{22} \dots a_{nn}$$

Here, we just compare the familiar determinant with the module in form, which makes it easier for us to understand the module (there is no need to know more about determinant). Note: in the determinant, the operator in the symbol " $\begin{vmatrix}$ " is omitted.

4.2 Modules

	M_1			
1	0	0	0	1
2	0	0	1	1
3	0	1	1	1
4	1	1	1	1

Figure 1. A module

This is a module with four expressions. It is not difficult for us to find its rule: the sum of each line of numbers in the box is equal to the number on the left. And:

1) Each line in the box is an additive formula (the operation symbol is omitted). Each of its terms (addends) is a power element.

2) The number on the left is the sum of the addends corresponding to a line. Also known as ordinal number.

3) The name of this module is "M_i".

We draw the information related to $g(3)$ and $g(4)$ into the following module diagram:

M ₁		M ₂		M ₁		M ₂		M ₃	
1	1	2	1 1	1	1	2	1 1		
2	1 1	3	1 1 1	2	1 1 1	3	1 1 1		
3	1 1 1	4	1 1 1 1	3	1 1 1	4	1 1 1 1		
4	4	5	4 1	4	4	5	4 1		
5	1 4	6	1 4 1	5	1 4	6	1 4 1		
6	1 1 4	7	1 1 4 1	6	1 1 4	7	1 1 4 1		
7		8		7		8			
8	4 4	9	4 4 1	8	4 4	9	4 4 1		
9	1 4 4	10	1 4 4 1	9	1 4 4	10	1 4 4 1		
10	1 9	11	1 9 1	10	1 9	11	1 9 1		
				11		12			
				12					

Figure 2. Some modules

In Figure 2:

(1) M₁ is the of $g(3)$. about

$$A_1^2 + A_2^2 + A_3^2 = n. \quad (1)$$

When the power elements are 0^2 , 1^2 , 2^2 and 3^2 , it is only $n = 7$ has no solution.

(2) M₂ is $g(4)$. Why is an addend (power) of $g(4)$ written outside the box? This table M₂ is composed of M₁ plus an addend (power). about

$$A_1^2 + A_2^2 + A_3^2 + A_4^2 = n \quad (2)$$

When the power elements have 0^2 , 1^2 , 2^2 and 3^2 , add a power term (1^2) into M₁ to get the result of M₂. There when $n = 8$ has no solution.

(3) The module group is composed of three modules (sharing one n mark). Through the combination of modules, all n have solutions in $[1, 12]$, so $g'(2) = 4$.

Note: the solution of $g'(4)$ is moved back by an integer position than the solution of $g'(3)$. M₃ is to change the added power of M₂ into 2^2 , and the solution of $g(4)$ moves back three integer positions than m_2 . The purpose of using the module is to continuously expand the scope of the solution, so that all n can find the solutions.

Here, we call the module using $g(3)$ composed of power elements 0^2 , 1^2 , 2^2 and 3^2 as the basic module, and the additional power term is called the addition term or (addition power). It will be discussed later that the modules for the purpose of $g(2) = 4$ are in the form of M₃.

2) The solution of $g'(2) = 3$ of the basic module M_1 increases;
 3) The de overlap of transverse modules $M_1, M_2, M_3, \dots, M_7$, overlapping solutions($g'(2) = 4$) increases. Due to the reasons of 1) 2) 3), the solution of $g'(2) = 4$ remains continuous when n is greater than 31.

Theorem 2. $g(2) = 4$.

Proof. 1) Since this is irreplaceable by $n = 7$, $1^2 + 1^2 + 1^2 + 2^2 = 7$, so $g'(2)$ is at least 4.
 2) It is proved from Fig. 4 that when n is greater than 7, there are continuous solutions with $g'(2) = 4$. Therefore, when n is between 1 and infinity, $g(2) = 4$ holds.

Now, we completed proof with the $g(2) = 4$, The proof about $g(3) = 9$ and $g(4) = 15$ will be supplemented later.

5 Conclusion

This paper vividly proves the proposition of $g(2) = 4$ in the form of module. Morphological mathematics is a very useful theoretical tool to solve integer problems. After more than ten years of painstaking study and research, the author has solved the proof of Goldbach conjecture, Fermat's theorem, Four-color theorem, odd perfect number, higher-order indefinite equation and Waring's problem. Through the application of morphological mathematics, many outstanding mathematical problems about integers in number theory are solved one by one, which shows the power of morphological mathematics. As long as we can master their application skills flexibly and rigorously, we can solve more pure mathematics and applied mathematics problems.

Reference

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