A New Method to Prove Goldbach’s Conjecture

(Version 2)

This proof in detail for A New Method to Prove Goldbach’s Conjecture.

1 Eratosthenes sieve method

Characteristics of multiples of prime numbers: 1. It has periodicity. 2. It is a product function, which is "confused" with each other. A composite number may be the product of two or more prime numbers. It is not easy to separate screening.

Firstly, the multiples of prime numbers are classified into sets that can not be "confused" according to the **Eratosthenes** sieve method. That is, a composite number is regarded as the effective composite of the smallest prime number.

Then, natural numbers can be divided into effective multiple subsets of 2, effective multiple subsets of 3, effective multiple subsets of 5,…, effective multiple subsets of p_m. So, they will not be confused.

Doe example, when $2n$=26, tive multiple subsets of 2 are 2,4,6,8,10,12,14,16,18,20,22,24,26; tive multiple subsets of 3 are 3,9,15; tive multiple subsets of 5 are 5,25.

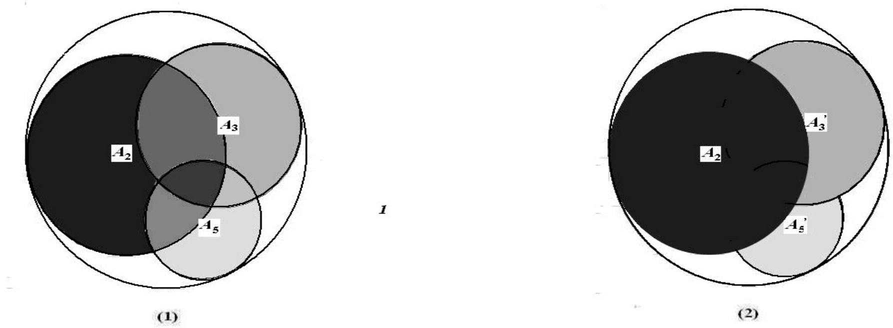


Figure 1

Eratosthenes sieve method is to scientifically divide natural numbers into large subsets: When we delete the **numbers of primes, first delete the** multiple of 2, next delete the **numbers of 3, …, finally, delete the** multiple of p_m. . See **Figure 2.**

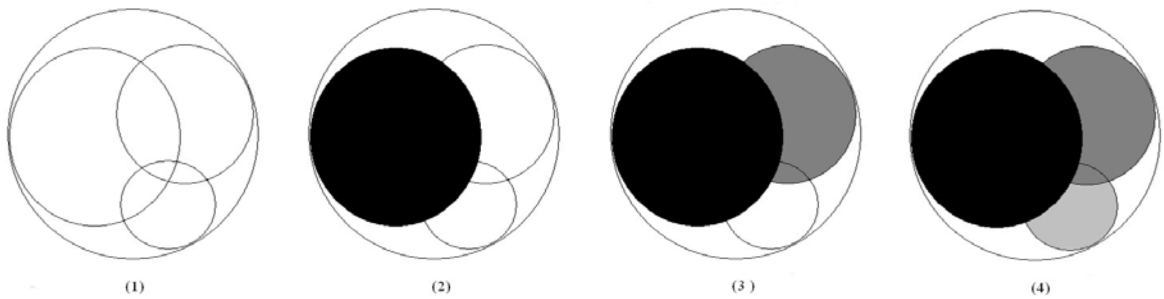


Figure 2

2 Geometry and Number

2.1 Relationship Between Geometry and Number

The most typical example of proving mathematical theorems with geometric figures is Pythagorean theorem [2][3]. Pythagorean successfully proved Pythagorean theorem by using the corresponding relationship between the area and square number of right triangle. Now, we intend to prove Goldbach's conjecture by the mapping correspondence between circle and integers [3].

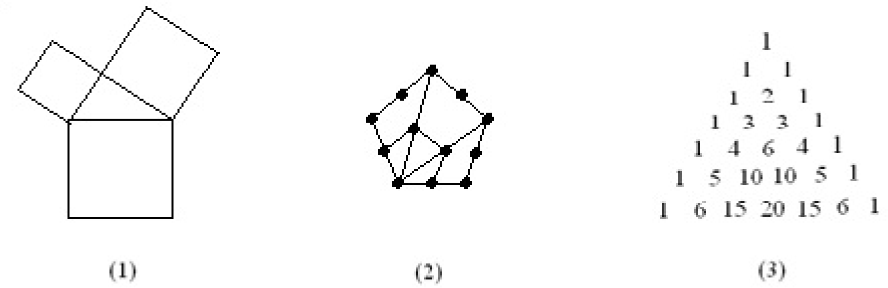


Figure 3

The relationship between rational numbers and integers can be expressed by integer points on the number axis (or line), see Figure 2.

（1）Integer points on the number axis.

(2) Euler’s pentagonal number theorem.

（3） There are 9 integer points on a straight line with a length of 9.5. There are 11 integer points on a straight line with a length of 11.5.

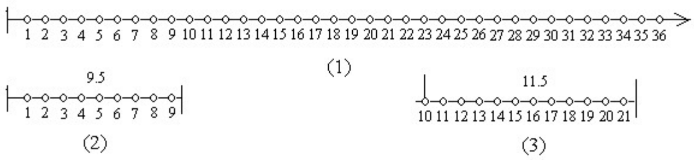


Figure 4. It is the relationship between rational numbers and integers.

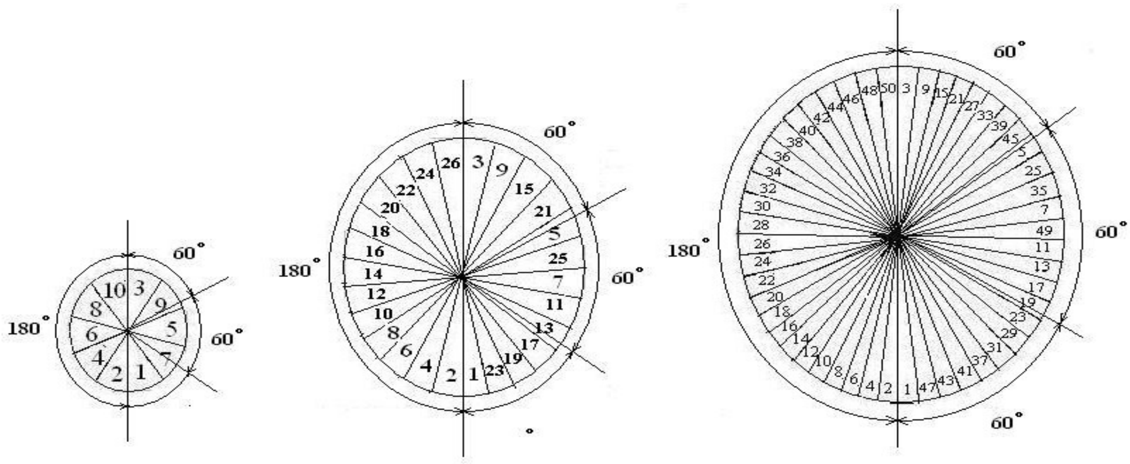


Figure 5.

We can use many sectors of a circle to represent a set of integers (integers with a region of [1,2n]), as shown (1)in Figure 3. So, a small sector to represent an integer, and interior angle of sector is θ(θ=360 / 2n). In addition, 2n integers have a one-to-one integer mapping relationship with the internal angle of the sector. In this way, we can use geometry to understand the complex relationship of various numbers and solve problems. For example,2n=26, 26 small sectors represent 26 integers, let γ Is its internal angle of a small sector, then γ= 360/26≈13.846° ·

Let set of 2n integers be A, and set of small sectors be S, set A has a one-to-one surjection relationship with set S, i.e., $A \Leftrightarrow S$. At the same time, 2n integers have a one-to-one integer mapping relationship with the circumference (arc length) and the interior angle (or area) of the sector in the circle [4]. n and θ The functional relationship is

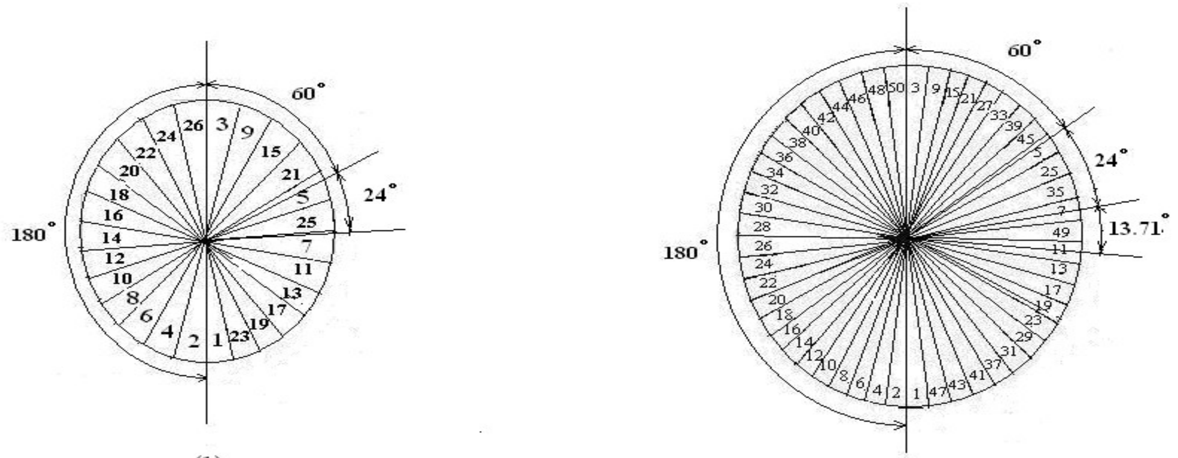
$$\theta = f(n), \quad \theta = \frac{360}{2n}, \ n>0, n \in (0,Z].$$

and

$$\theta = f(a), \quad \theta = \frac{360}{a}, \text{ a is rational number, } a>0, a \in (0,Q].$$

Some definition:

- Interior angle of sector (or circle) is noted by θ° .
- A small sector represents an integer , and its interior angle is noted by γ° , $\gamma = \frac{360}{2n}$.
- An interior angle of sector with | Ap | is noted by α_p , $\alpha_p = |A\text{p}| \times \gamma^\circ$.
- Component of p in a sector (primitive or remaining) is noted by β_p. For example, if in a circle, $\beta_p = \frac{360}{p}$ $^\circ$;in a semicircle, $\beta'_3 = \frac{180}{3}$; in a sector of 60 $^\circ$, $\beta'_5 = \frac{60}{5}$.



(1) (2)

Figure 6

Some definition:

- Interior angle of sector (or circle) is noted by θ° .
- A small sector represents an integer , and its interior angle is noted by γ° , $\gamma=\dfrac{360}{2n}$.
- An interior angle of sector with $|A\mathrm{p}|$ is noted by α_{p} , $\alpha_{\mathrm{p}}=|A\mathrm{p}|\times\gamma^{\circ}$.
- Component of p in a sector (primitive or remaining) is noted by β_{p}. Foe example,

Definition: the remaining area without deletion or deletion is called the original area.

β_p is defined as the component function of the original region to p. For example, the area of a circle (360°) component function of pair 2 is $\beta_2=\dfrac{360}{2}$.

Delete the multiples of 2, there is another semicircle (odd number) left. Then the component function of 3 iv the odd semicircle is $\beta'_3=\dfrac{180}{3}$.
By definition, we establish α、β and γ with the relationship between the relevant angles.

This paper will use the above mapping relationships to transform each other (without explaining and declaring their relationships) to directly transform roles, and measure, compare and judge and prove through the size of the angle.

3 Sieve Method

Steps for using the emerald sieve method:

3.1 Sieving of Composites

Therefore, to screen out the composites of primes 2, 3, 5,..., p_{m} in $2n$ by using method of the 'sieve of Eratosthenes', it is only necessary to delete the component of each prime number to $2n$,and according to the following steps [5]. The specific operations are as follows:

1) First, we delete the even component $\beta_2 = 360 / 2$ (paint the semicircle representing even number with black), which the remaining is 180° . The semicircle (white) of represents the odd component, see (2) in Figure 3.

$$f(2n) = 360(1-\frac{1}{2})=360-180=180 .$$

2) Delete this integer pairs of multiples of 3 by deleting a sector of β'_3 (painted dark grey) in the sector of 180° . Since $\beta'_3=\dfrac{180}{3}=60^{\circ}$, then

$$f(2n) =360(1-\frac{1}{2})(1-\frac{1}{3})= 180-\frac{180}{3}=120.$$

3) Delete component of 5 by deleting a sector of β'_5 (painted light gray) in the sector of 120°.

Since $\beta'_5 = \dfrac{120}{5} = 24^\circ$, $f(n)$ corresponding to this operation is

$$f(2n) = 360(1-\frac{1}{2})(1-\frac{1}{3})(1-\frac{1}{5}) = 360-180-60-24 = 96\,,$$

see (3) in Figure 3.

And so on …, finally get

$$f(2n) = 360(1-\frac{1}{2})(1-\frac{1}{3})(1-\frac{1}{5})\ldots(1-\frac{1}{p_m})\,. \tag{4}$$

Refer to Figure 3 for this operation. This specific method is as follows:

（1）26 integers (or pairs of integers) are represented by a circle. Each interior angle of sector is γ°. A small sector represents an integer. $\gamma = 360/26 = 13.846^\circ$.

（2）Even and odd numbers account for 180 respectively, which are two semicircles (or even and odd pairs).

（3）There are 4 multiples of 3，$\alpha_3 = 4\gamma = 4*13.846 = 55.384^\circ$, it is less than $\beta'_3 = 60^\circ$. Deleted multiples of 3 (by β'_3), interior angle of remaining sector is x°, $x = 180 - 60 = 120^\circ$. $\beta'_5 = 120/5 = 24^\circ$. Then, we delete component of 5 (by β'_5) finally get a remaining sector, its interior angle is $120 - 24 = 96^\circ$. There are 6 of multiples of 3 and 5. $\alpha_3 + \alpha_5 = 6\gamma = 6*13.846 = 83.076^\circ$, it is less than $\beta'_3 + \beta'_5 = 60 + 24 = 84^\circ$. When is deleted component of β'_3 and β'_5, there's still left 96° of interior angle remaining sector (white). Since $96/\gamma = 96/13.846 = 6.933$, it is show that there are at least 6 complete integers, excluding integer 1, there are at least 5 odd primes.

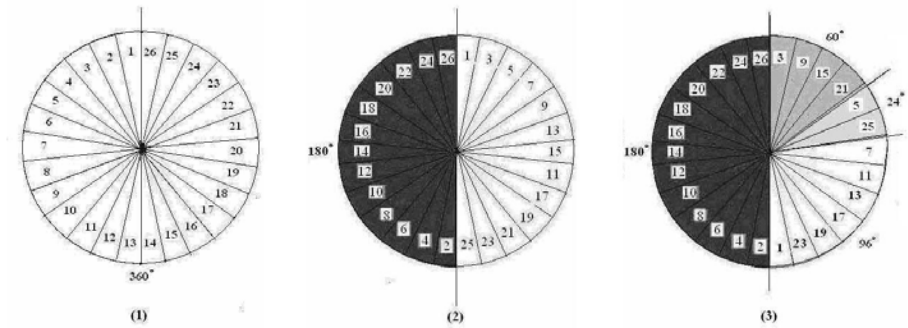


Figure7 . It is show that of mapping relationship between circle and integers .

We can also use the **Figure 8** to represent their relationship.

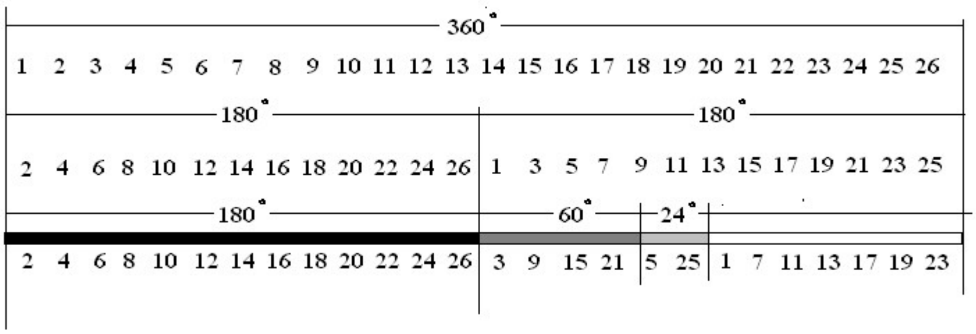


Figure 8

3.2 Sieving of Integer Pairs

The following is the case of using graphics to screen out integer pairs. Now the circle represents $2n$ integer pairs**.**

Theorem 2. *When 2n is a multiple of odd prime p, all multiples of p are relative; when 2n is not a multiple of prime p, all multiples of p are not relative.*

Proof. When $2n$ is a multiple of odd prime p, that is, n is a multiple of prime p, and kp away from n is also a multiple of p. So it is that all multiples of p with above and below are relative. Conversely, when $2n$ is not a multiple of prime p, all multiples of p are not relative. □

From Theorem 2, it can be seen that when $2n$ is not a multiple of prime p, all integer pairs containing odd multiples of p are not relative, which is the worst case for formation of prime pairs. In this way, when we discuss the elimination of integer pairs with composite numbers, we take the latter case which is most unfavorable to the generation of prime pairs as the condition. When we screen integer pairs with composite numbers, we screen out integer pairs with odd multiples of $p$at the top and bottom once at the same time. The formula is as follows:

$$g(2n) = 360(1-\frac{1}{2})(1-\frac{2}{3})(1-\frac{2}{5})\ldots(1-\frac{2}{p_m}) . \tag{5}$$

The specific methods are as follows:

1) First, we delete even pairs by deleting a semicircle of β_2..

$$g(n) = 360-180=180.$$

2) Delete this integer pairs of multiples of 3 by β'_3 in semicircle on the right , $2\times\beta'_3=2\times\frac{180}{3}=120$, as

$$g(n) = 360-180-120=180$$

or

$$g(2n) = 360(1-\frac{1}{2})(1-\frac{2}{3}) = 180-120==60 .$$

3) Similarly, screen out the composites of 5 in the remaining sector(60°) (painted light gray) $\beta'5=\frac{60}{5}=12$. $\theta=2\times 12=24$. Finally, the remaining interior angle of sector is 36° . See (4) in Figure 2. The $g(n)$ corresponding to this operation is

$$g(2n) = 360(1-\frac{1}{2})(1-\frac{2}{3})(1-\frac{2}{5}) =180-2\times 60-2\times 12=36 .$$

And so on …, finally get

$$g\,(2n)=360(1-\frac{1}{2})(1-\frac{2}{3})\,(1-\frac{2}{5})\ldots(1-\frac{2}{p_m})\,. \qquad (6)$$

Refer to Figure 2 for this operation, the Figure 3 is shows that:

（1）26 integers (or pairs of integers) are represented by a circle. Each small sector represents an integer pair, which a interior angle is γ, γ= 360/26=13.846 .

（2）Even and odd numbers account for 180° respectively, which are two semicircles (or even and odd pairs).

(4）There are the two circle represents two set of 26 integers. Use excessive screening is $g\,(n)$= 180－2×60－2×12=36° **.** Multiples of 3 and composites of 5 are marked here. (It is painted black and light gray respectively, and indicates that it is deleted). The last remaining sector (inner angle is 36°) . Since 36/ γ=36/13.846 =2.6 . it is that, there are remaining of at least two integer pairs.

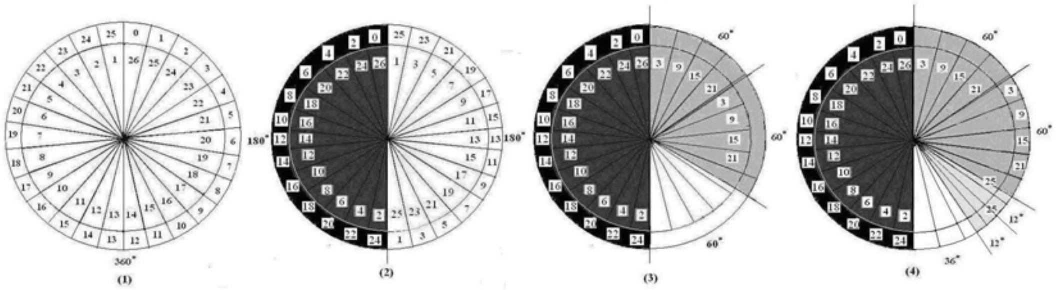
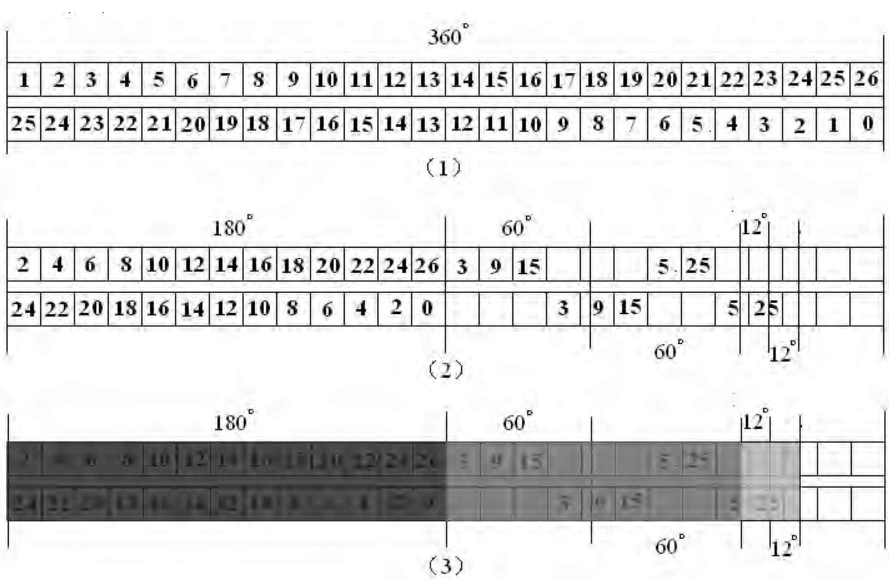


Figure 9

We can also use the **Figure 10** to represent their relationship.



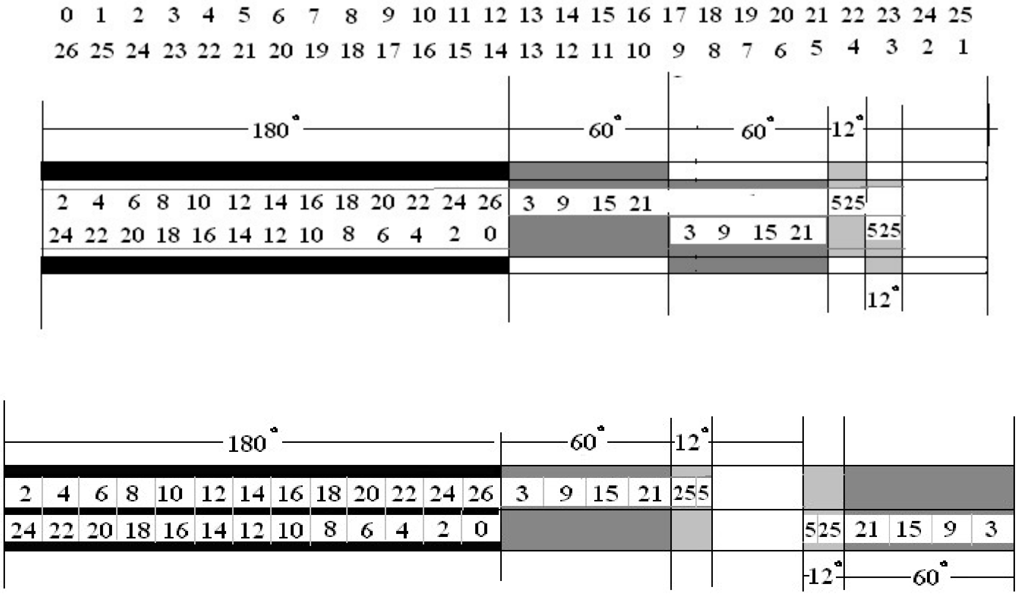


Figure 10

It can be clearly seen in the figure 10:

1） Since $2n$ is a multiple of 2, even numbers are relative to the multiples of 2 in the set of integer pairs.

2） Since $2n$ is not a multiple of 3, 5,... p_m, the multiples of p are not relative. Therefore, when using the sieve method, the multiples containing 3, 5,... p_m above and below the odd integer pair should be sieved twice, so $\left(1-\frac{2}{p}\right)$ is used in function $w(n)$. (due to the proof conjecture, we do not use a more complex distinction when $2n$ is a multiple of a prime in 3, 5,... p_m, because in this case, there will be more prime pairs)

Schematic diagram 1 as **Figure 11.**

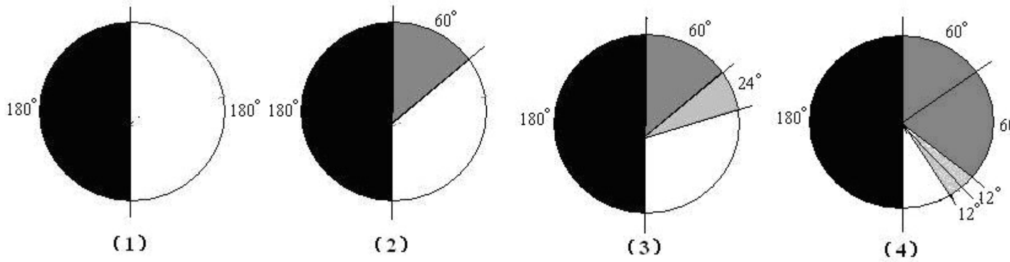


Figure 11. The process of deleting component of composites or integer pairs containing

In this way, however large $2n$ is, according to Theorem 1, we can use the same method of screening to screen out the composites of prime numbers, and finally get the lower limit of the number of prime numbers (or the number of prime pairs), as shown in Figure 4. At the same time, we get the functions $f(2n)$ and $g(n)$ as follows

$$f(2n) = 360\prod_{i=1}^{m}(1-\frac{1}{p_m}) \tag{7}$$

$$g(n) = 180\prod_{i=2}^{m}(1-\frac{2}{p_m}) \tag{8}$$

4 Key Issues of Proof

There are two Key Issues of Proof:

1) Are the composite numbers completely eliminated?

2) Finally, whether the sector of the rest in midsummer contains 3 or more complete integer pairs.

Let's analyze whether the composite number is completely eliminated?

For an integer pair containing a composite number of *p*, the integer pair containing a composite number of 2 and *p* has been screened out in the even pair; The integer pairs of prime multiples (components) containing 3 have been screened out, and the integer pairs of composite numbers containing 3 and *p* have been screened out; The integer pairs of prime multiples (components) containing *p* have been screened out, and the integer pairs of prime factors containing *p* and greater than *p* have been screened out. In this way, the composite numbers of *p* completely eliminated.

Let we see series of odd numbers:

1	**3**	**5**	**7**	**9**	**11**	**13**	**15**	**17**	**19**	**21**	**23**	**25**	**27**	**29**	**31**	**33**	**35**	**……**
	3			**9**			**15**			**21**			**27**			**33**	**35**	**……**
	5						**15**					**25**					**35**	**……**

Reason for error in certificate 1:yes ,the first period of the multiple of each odd prime is incomplete.

The solution is to adopt the method of screening the effective composite numbers of prime numbers. Using this method, the count of the effective composite number of prime numbers starts from the second the multiple (composite number) of prime number. For example 9 is the effective composite number of 3 Then, there is already a "buffer period", and the count obtained by calculating the sum of prime numbers with $[\frac{n}{p}]-1$ is positive error. That is, only more will not be less the composite numbers of prime number p.

In this way, it can be realized $\alpha' p \leqslant \beta' p$.

We can also analyze in more detail:

Proof 2.

$$v(n)=\prod_{i=2}^{t-1}(1-\frac{1}{p_i})$$

$$\beta'_{p_t}=\frac{1}{p_t}\times 180\prod_{i=2}^{t-1}(1-\frac{1}{p_i})=\frac{180}{p_t}\prod_{i=2}^{t-1}(1-\frac{1}{p_i}). \qquad (4)$$

$$\alpha'_{p_t}=(\frac{n}{p_t}\prod_{i=2}^{t-1}(1-\frac{1}{p_i})-1)\ \frac{360}{2n},$$

$$\alpha'_{p_t}=(\frac{n}{p_t}\prod_{i=2}^{t-1}(1-\frac{1}{p_i}))\times\frac{360}{2n}-\frac{360}{2n}$$

$$=(\frac{1}{p_t}\prod_{i=2}^{t-1}(1-\frac{1}{p_i}))\times\frac{180}{1}-\frac{360}{2n}$$

$$=(\frac{180}{p_t}\prod_{i=2}^{t-1}(1-\frac{1}{p_i}))-\frac{360}{2n} \qquad (5)$$

Compare （4）and （5），we get $\alpha'_{p_t}<\beta'_{p_t}$ 。

Proof 3.

$$v(n)=\prod_{i=2}^{t-1}(1-\frac{1}{p_i})$$

$$\beta'_{p_t}=\frac{1}{p_t}\times 180\prod_{i=2}^{t-1}(1-\frac{1}{p_i})=\frac{180}{p_t}\prod_{i=2}^{t-1}(1-\frac{1}{p_i})$$

$$\beta' p = 180\times\mathbf{(1-\frac{1}{3})(1-\frac{1}{5})}$$

$$\beta'_{p_t}=\frac{n}{5}-\frac{n}{3*5}>\ \alpha'_{p_t}=(\left[\frac{n}{5}\right]-\left[\frac{n}{3*5}\right])*\frac{360}{2n}$$

When 5|*n*, $\beta'_{p_t}=\alpha'_{p_t}$ **;otherwise,** $\beta'_{p_t}>\alpha'_{p_t}$ 。

But in fact, in the odd series, the first period of prime *p* is less than *p,* so it leads to the error of screening out prime multiples. See table 1

In order to avoid this situation of positive error, we adopt the method of screening out the prime number composite, that is, we define α'_{p_t} is the inner angle of the sector represented by the composites of prime number. Then, = (|ap| - 1)* γ， Make > = (|ap| - 1)* γ Established.

We can also prove it by the following actual parameters:

2n	γ	β	|Ap|	f(|Ap|)	f(|Ap|-1)
10	36	60	2	72	36
2n	γ	β	|Ap|	f(|Ap|)	f(|Ap|-1)
26	13.8461538461538	60	4	55.3846153846154	41.5384615384615
		24	2	27.6923076923077	13.8461538461538
2n	γ	β	|Ap|	f(|Ap|)	f(|Ap|-1)
50	7.2	60	8	57.6	50.4
		24	3	21.6	14.4
		13.7142857142857	2	14.4	7.2
2n	γ	β	|Ap|	f(|Ap|)	f(|Ap|-1)
122	2.95081967213115	60	20	59.016393442623	56.0655737704918
		24	8	23.6065573770492	20.655737704918
		13.7142857142857	5	14.7540983606557	11.8032786885246
		7.48051948051948	2	5.9016393442623	2.95081967213115

Table 1.

Where, $|A_p|$ is actual the number of the effective composite of P

$f(|A_p|)$ is the component of the effective multiple of P.

$f(|A_p|-1)$ is the component of the effective composite of P.

It can be seen from the table that when $2n=26$, $f(|A_5|)=27.6923$, $\beta'_5=24$, $f(|A_5|)>\beta'_5$, the screening is not enough to screen out the components of the effective multiple of 5.

When 2n = 122, $f(|A_7|)=14.754$, $\beta'_7=13$, $f(|A_7|)>\beta'_7$, the component that is not enough to screen out the effective multiple of 7.

The concept of sifting out the composite of prime number is adopted. When $2n=26$, $f(|A_5|-1)=13.846$, $\beta'_5=24$, $f(|A_5|-1)<\beta'_5$, the component sufficient to sift out the effective composite of 5 (that is, the effective composite of 5 can be completely sifted out).

When 2n = 122, $f(|A_7|-1)=11.804$, $\beta'_7=13$,, $f(|A_7|)<\beta'_7$, the component sufficient to screen out the component of effective multiple of 7 (that is, the effective composite of 7 can be completely screened out).

3 Prove of Proposition

3.1 Prove of Proposition

Proposition: any even number greater than 4 can be expressed as the sum of two prime numbers.

The proof is known from subsections 1 and 2. If it can be proved that after over screening all composite numbers, there are at least one or more prime pairs in the last remaining sector, the proposition can be proved.

Next, we use mathematical induction for m to prove [6]:

1) When m=4, $p_4=7$, $2n=7^2+1$, $\gamma=360/50=7.2^\circ$, deleted the components of all composites, the last remaining internal angle of the sector is α°, $\alpha^\circ=180-2*60-2*12-2*5.14=25.72^\circ$, $25.72/7.2\approx 3.572$, namely $\alpha/\gamma=\alpha/[360/(p_4^2+1)]\geqslant 3$.

2) When $m=k$, $2n=p_k^2+1$, here, $\gamma'=360/(p_k^2+1)^\circ$. Suppose that the last remaining internal angle of the sector is x° and $x/\gamma'\geqslant 3$. Namely

$$x/\gamma'=x/[360/(p_k^2+1)]=x(p_k^2+1)/360\geqslant 3. \tag{9}$$

So, when $m=k+1$, $2n=p_{k+1}^2+1$, and $\gamma''=360/(p_{k+1}^2+1)$, let that the last remaining internal angle of the sector is y°, we have

$$y/\gamma''=y/[360/(p_{k+1}^2+1)]. \tag{10}$$

Because $\beta'_{k+1}=x/p_{k+1}$,

$$y^\circ=x-2\beta'_{k+1}=x-2x^\circ/p_{k+1}=x^\circ(1-2/p_{k+1}), \tag{11}$$

By (10) and (11) get

$$\begin{aligned} y/\gamma'' &= y/[360/(p_{k+1}^2+1)] \\ &=[x^\circ(1-2/p_{k+1})]/[360/(p_{k+1}^2+1)] \\ &=x(p_{k+1}^2+1)(1-2/p_{k+1})/360 \\ &=x(p_{k+1}^2+1)(p_{k+1}-2)/p_{k+1}/360 \\ &>x\,p_{k+1}^2(p_{k+1}-2)/p_{k+1}/360 \\ &>x\,p_{k+1}(p_{k+1}-2)/360. \end{aligned} \tag{12}$$

Let $p_{k+1}=p_k+d$, $d\geqslant 2$, then

$$x\,p_{k+1}(p_{k+1}-2)/360=x(p_k+d)(p_k+d-2)/360,$$

$$x\,p_{k+1}(p_{k+1}-2)/360\geqslant x(p_k^2+1)/360. \tag{13}$$

In this way, by (9), (12) and (13), we get

$$y/\gamma''>x\,p_{k+1}(p_{k+1}-2)/360\geqslant x(p_k^2+1)/360\geqslant 3. \qquad \square$$

Because the remaining sector may be contain two integer pairs containing 1 ({1, $2n$-1}, {$2n$-1, 1}), the other integer pair must be a prime pair. Namely there is least one of prime pair.

Now, we have been proved the case when $2n=p_m^2+1$. For $2n$ by the case from p_m^2+1 to p_{m+1}^2+1 is also valid because m remains unchanged but $2n$ increases. Therefore, when $2n$ is greater than 50, we can prove it graphically Goldbach's conjecture holds. (As for the solution of $2n$ in [4, 48], it is easy to prove by list.)

When n is a multiple of odd prime P, all multiples of P are relative. In other words, it has the smallest effect on the number of prime pairs. With the congruence of n pairs of odd primes, the values of prime pairs will vary from large to small in an average. In other words, it is a discrete function, not a monotonically increasing function, but its general trend is increasing. With the infinity of PM, the number of prime pairs will be infinite. But Lim d (n) / n = 0. Therefore, it is difficult (even impossible) to prove Goldbach conjecture from the perspective of explaining number theory.

In paper of Rigorous Proof of Goldbach's Conjecture1:

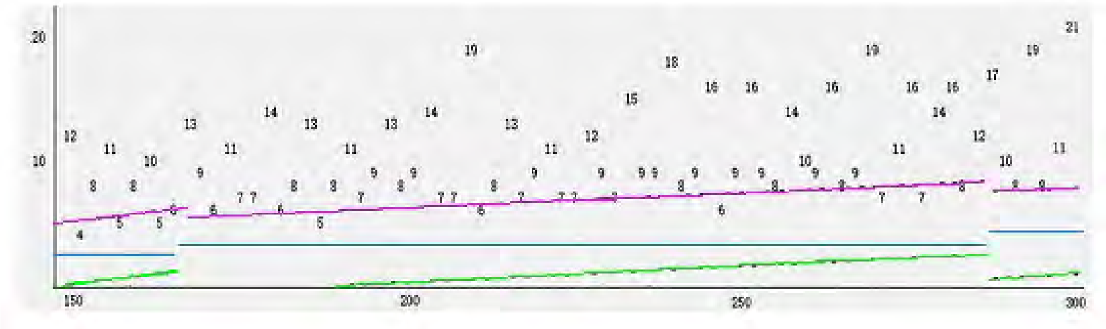


Figure 9. Diagram of the number of prime integer-pairs.

Figure 9 is shows that:

1) An integer number represents the number of actual prime pair of $2n$.

2) The red line represents the function $d(n)$. And $d(n)$ - 1 is always below the integer numbers. It is proved that $d(n)$ - 1 is less or equals than the number of actual prime pairs.

3) The blue line indicates $P_m/4$. Obviously, it is absolutely less than the actual number of prime pairs.

4) The green line indicates $d(n) - m$, which indicates that the problem of error can be completely solved.

5) All lines have an upward trend, which means that when $2n$ is larger, the overall trend is that the number of actual prime pairs will be more and more.

This can also be used trend chart to prove that Goldbach's conjecture is correct.