STAT 6560 Applied Time Series Analysis

Final Project

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Data

- Monthly water usage (mL/day) in London from January 1966 to December 1988.

- This is a 23-year period, and total 276 observations

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> water
        Jan    Feb    Mar    Apr    May    Jun    Jul    Aug    Sep    Oct    Nov    Dec
1966  76.83  77.74  80.47  79.56  82.28 100.92 113.20  90.92  86.83  82.74  83.65  80.92
1967  83.19  83.65  83.65  83.65  86.83 100.47  91.38 101.38  95.92  88.19  88.19  80.47
1968  80.92  79.56  80.92  88.19  91.83  96.38  97.29 102.29  99.10  92.74  87.29  85.47
1969  91.38  92.74  89.56  88.65  93.20  99.56 109.11 124.56 115.47  96.38  92.29  86.83
1970  87.29  85.92  85.92  88.65  91.83 112.29 101.83 125.02 102.74  95.01  91.83  86.38
1971  87.29  88.19  89.10  89.10 103.65 127.75 125.47 125.47 109.11 100.01  95.01  85.01
1972  86.83  86.83  86.83  86.83 100.47 111.38 105.47 102.74 105.01  96.38  94.10  86.83
1973  92.74  93.20  95.47  96.38  99.56 120.47 123.20 114.11 120.93 102.74 101.83  95.47
1974 100.01 100.01  98.20 100.01 103.65 114.56 134.11 131.84 113.65 107.29 102.29  94.56
1975  97.29  98.20  95.47 100.47 116.38 117.29 140.93 120.02 111.38 108.65 105.92  99.10
1976 101.83 102.74 102.74 105.47 108.65 139.57 110.47 118.65 120.02 109.11 108.20 101.38
1977 106.38 108.65 107.74 105.92 129.56 139.11 125.93 123.65 118.65 110.47 110.02 100.47
1978 104.10 106.60 105.50 107.50 117.90 136.30 156.80 135.80 130.00 117.50 115.80 105.50
1979 111.60 113.20 113.10 112.50 120.00 147.60 149.90 131.20 134.60 122.20 117.70 106.80
1980 111.50 111.30 109.50 112.10 127.00 135.90 150.40 135.60 134.90 124.10 120.80 112.80
1981 117.40 118.60 119.20 119.70 128.60 142.80 170.00 145.90 140.10 128.70 123.40 114.60
1982 120.20 122.00 121.30 123.20 141.10 129.70 152.40 141.90 137.00 129.00 124.60 117.30
1983 122.70 121.00 122.00 122.00 126.30 158.10 164.90 143.30 151.40 136.80 133.10 124.80
1984 132.60 130.20 129.60 129.70 133.70 148.30 155.10 157.20 147.20 142.70 135.90 123.80
1985 132.30 132.70 130.70 129.90 145.50 156.60 161.70 156.00 146.10 136.80 132.50 129.50
1986 129.50 134.70 136.60 138.40 149.60 159.50 171.40 162.10 163.10 152.40 145.50 133.90
1987 136.60 139.40 141.20 144.90 181.40 187.00 211.40 178.10 168.00 154.40 150.40 139.40
1988 144.70 143.00 148.30 152.70 173.30 226.30 218.20 184.60 174.90 161.40 161.40 145.80
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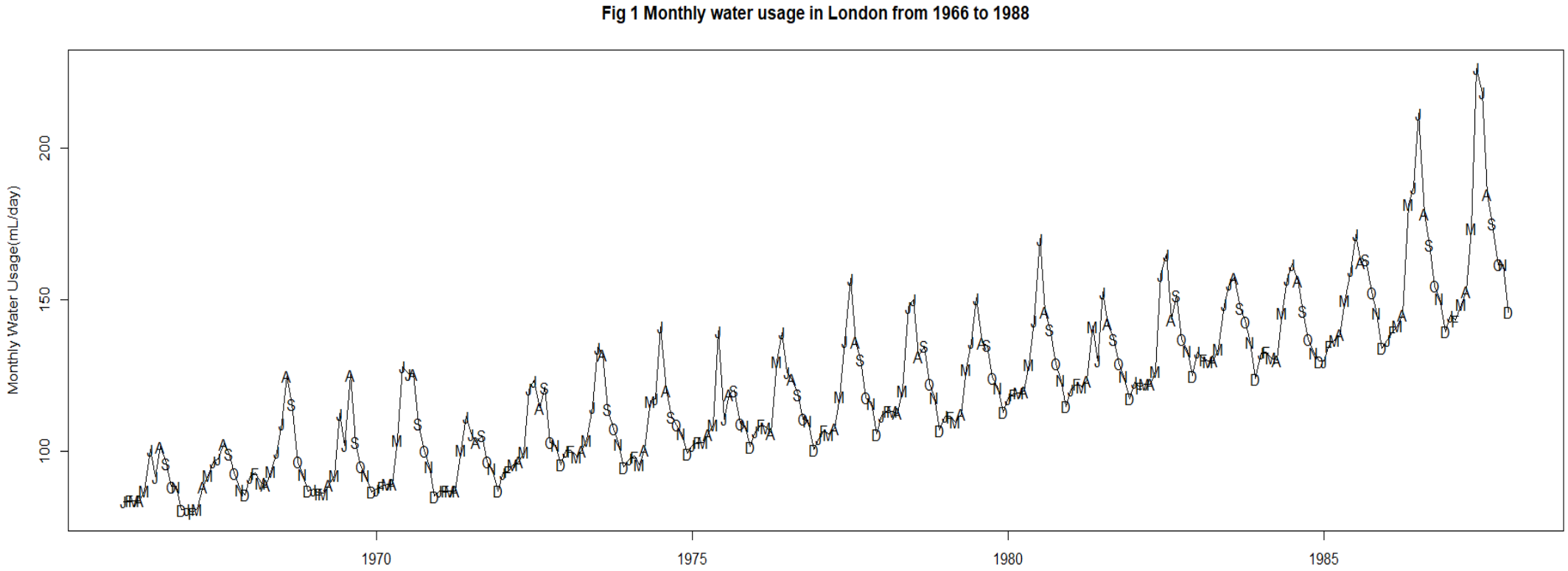
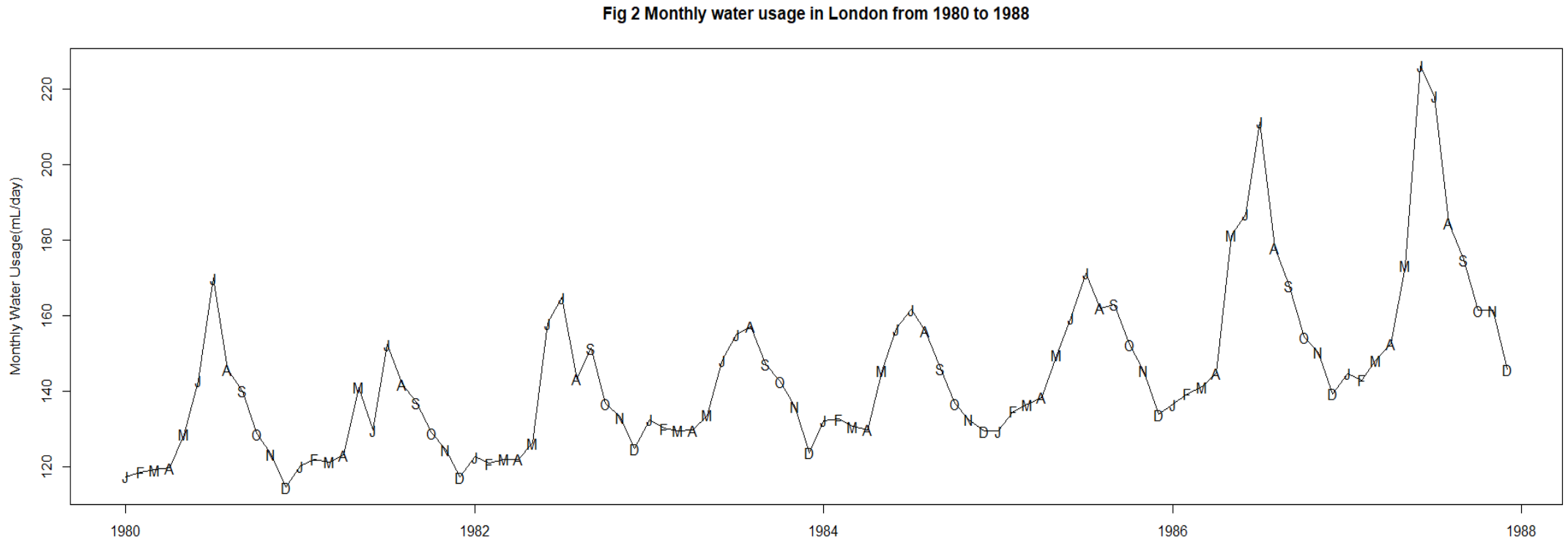
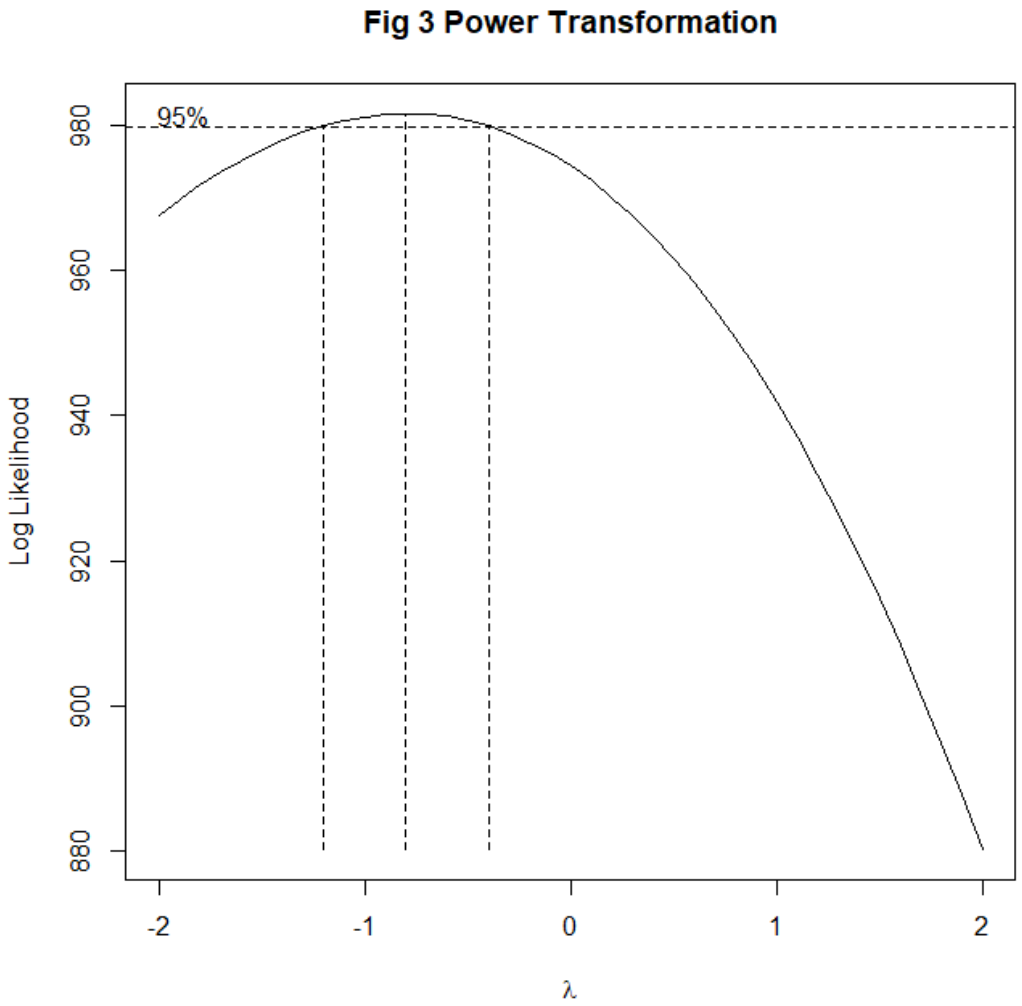
Time Series Plot



Time Series Plot



Power transformation



Sample ACF

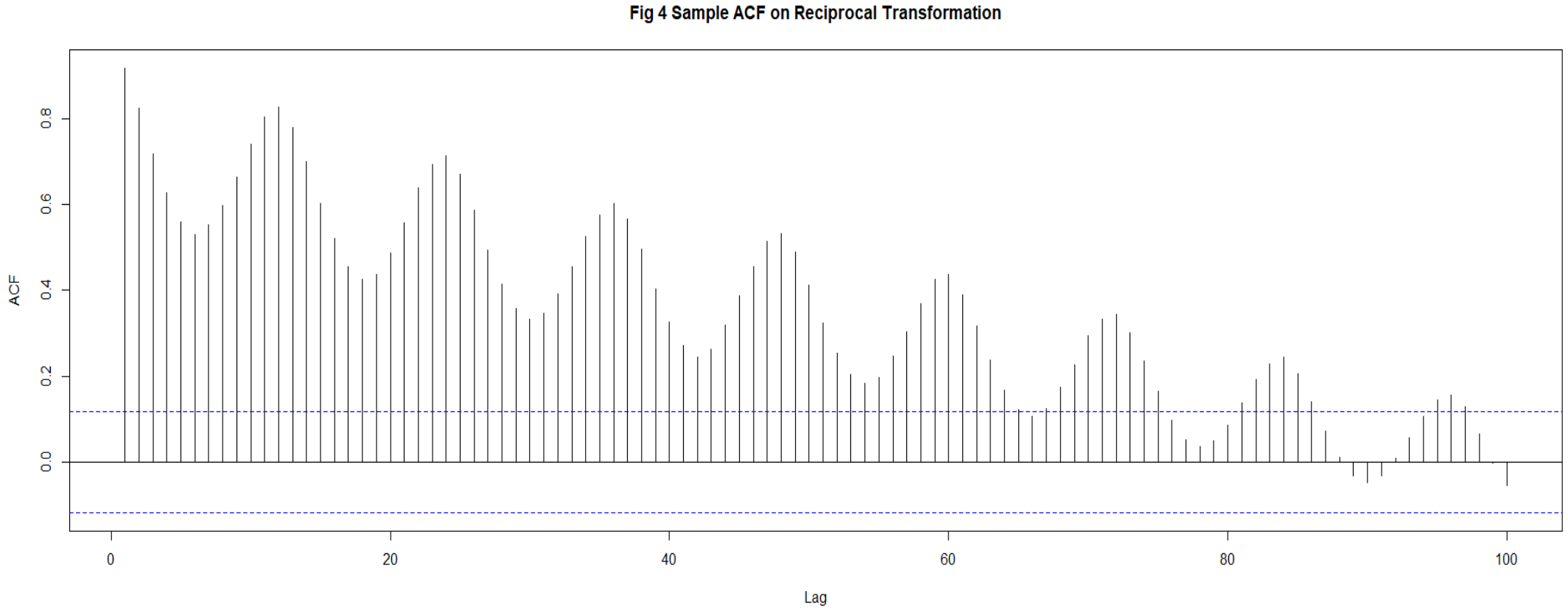


Fig 4 Sample ACF on Reciprocal Transformation
ACF
0.0
0.2
0.4
0.6
0.8
0
20
40
60
80
100
Lag

Fig 5 Time Series Plot of the First Differences of Monthly Water Usage

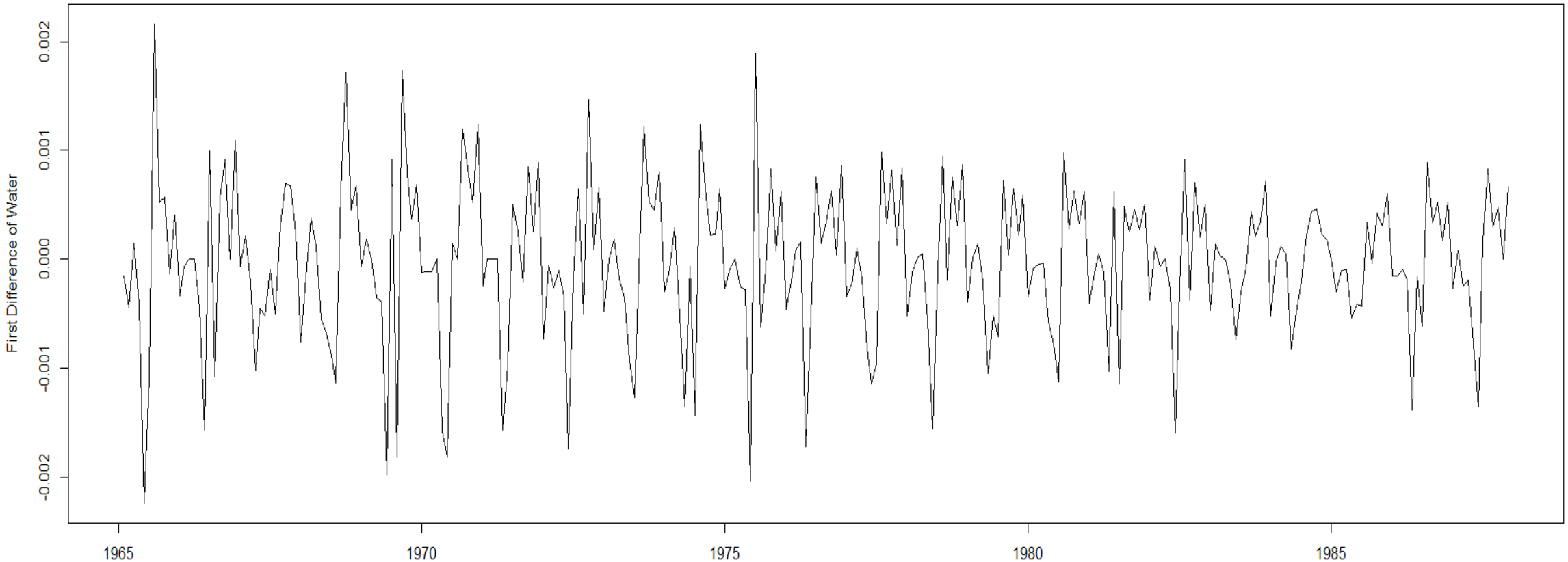


Fig 6 Sample ACF of First Differences of Monthly Water Usage

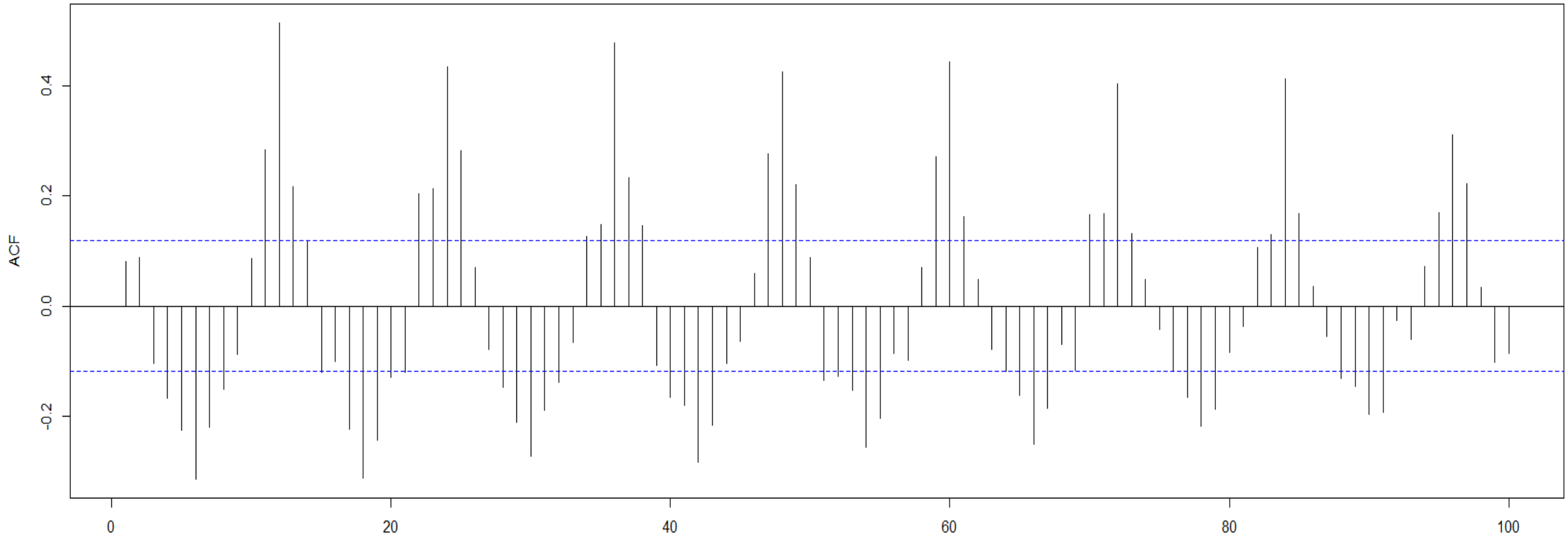


Fig 7 Time Series Plot of First and Seasonal Differences of Monthly Water Usage

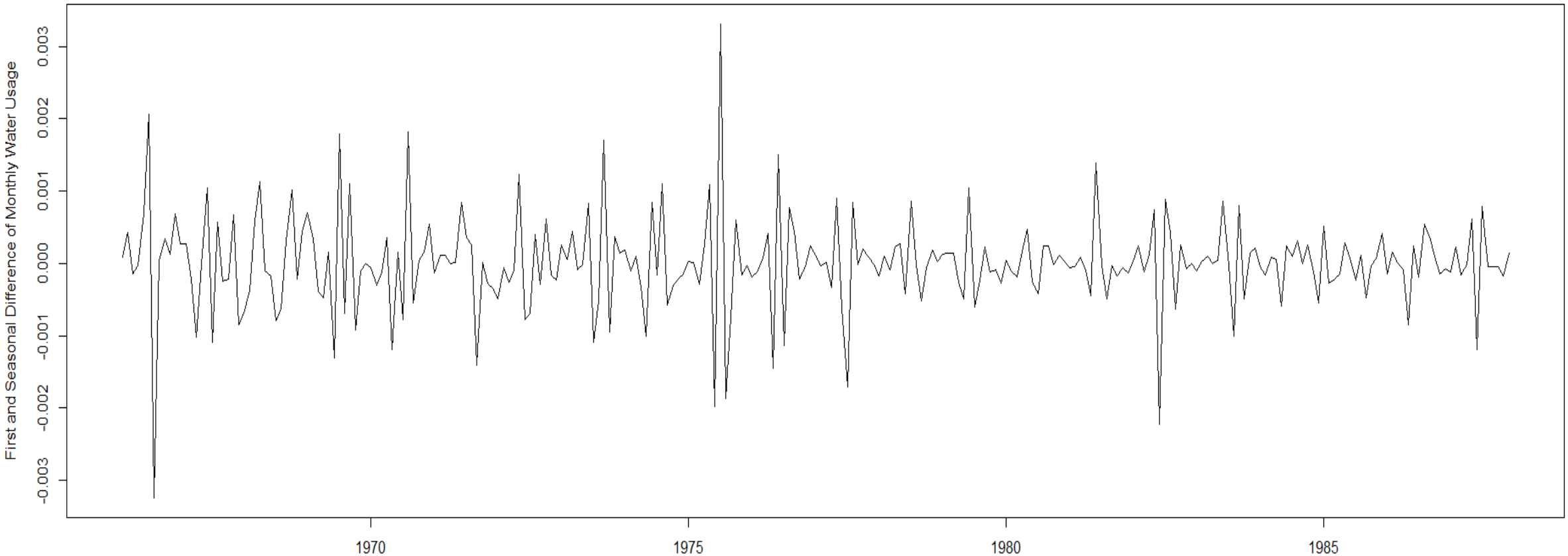
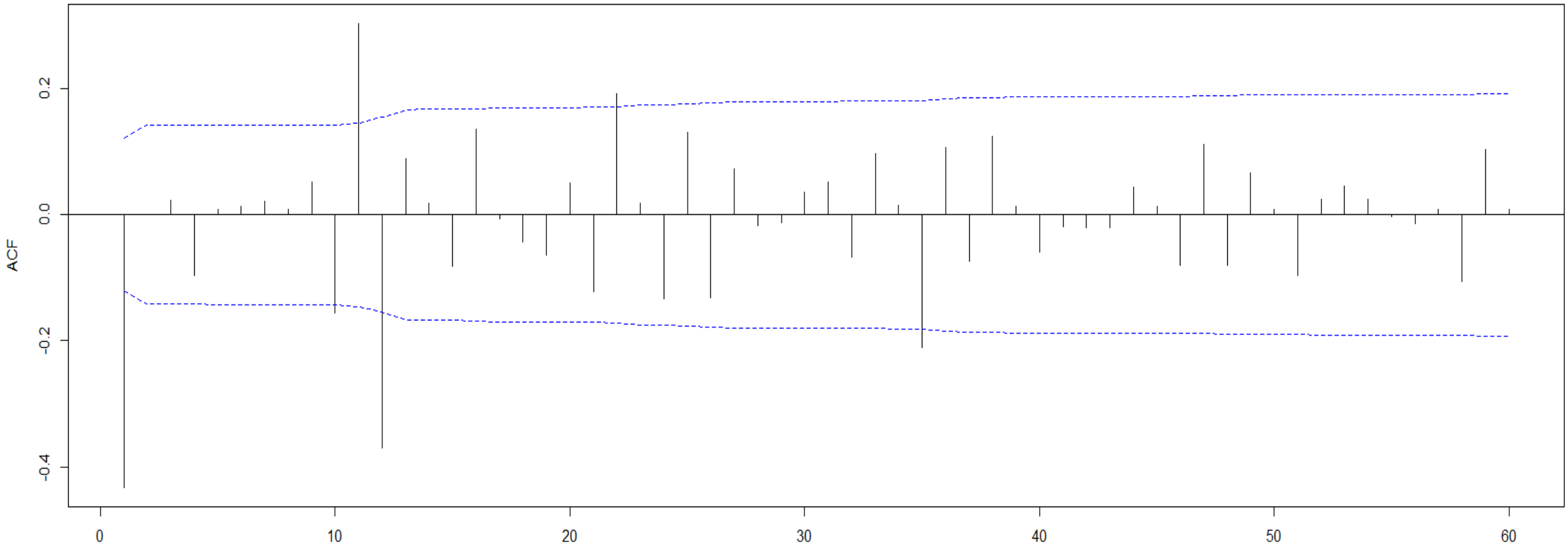
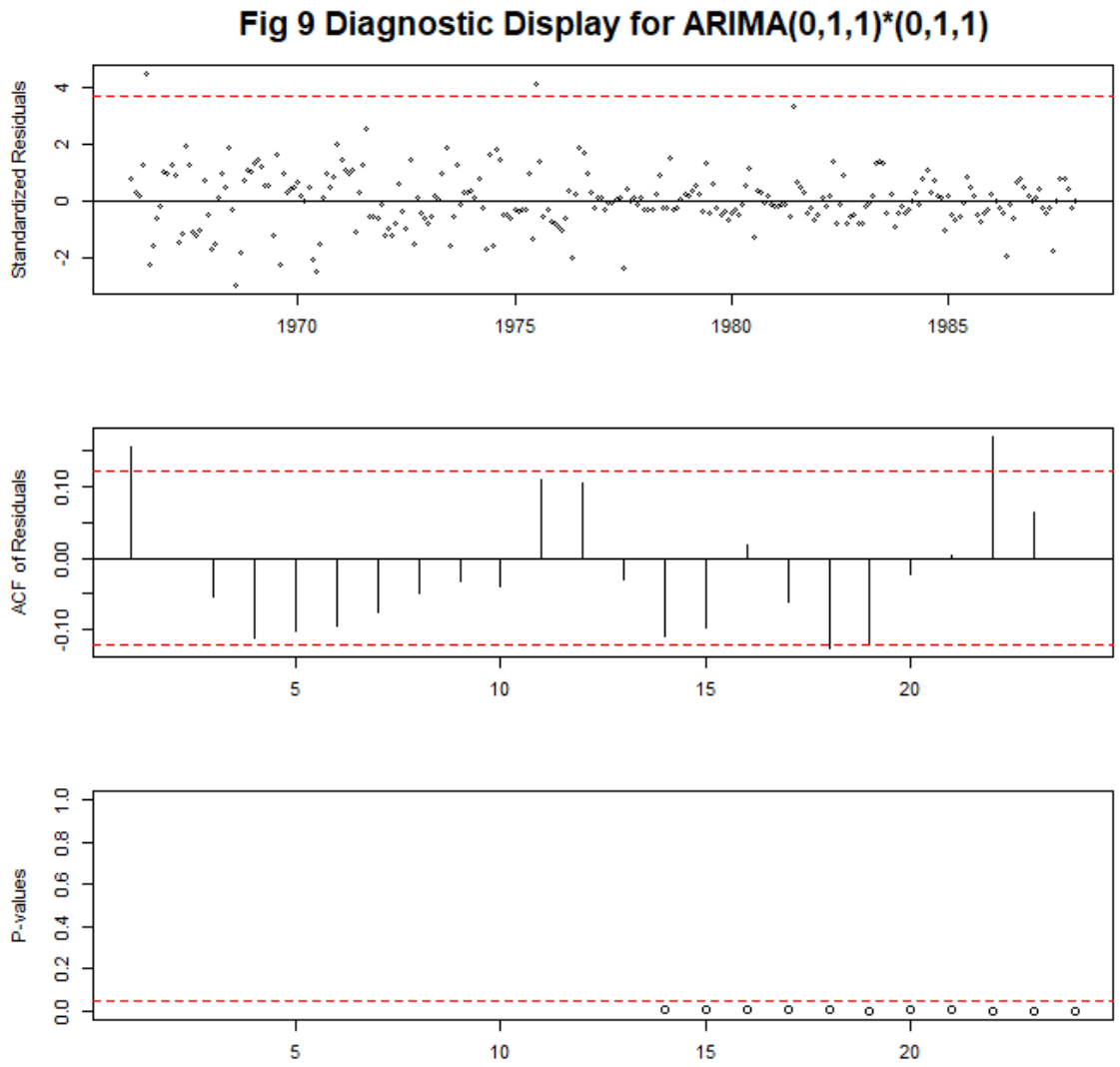


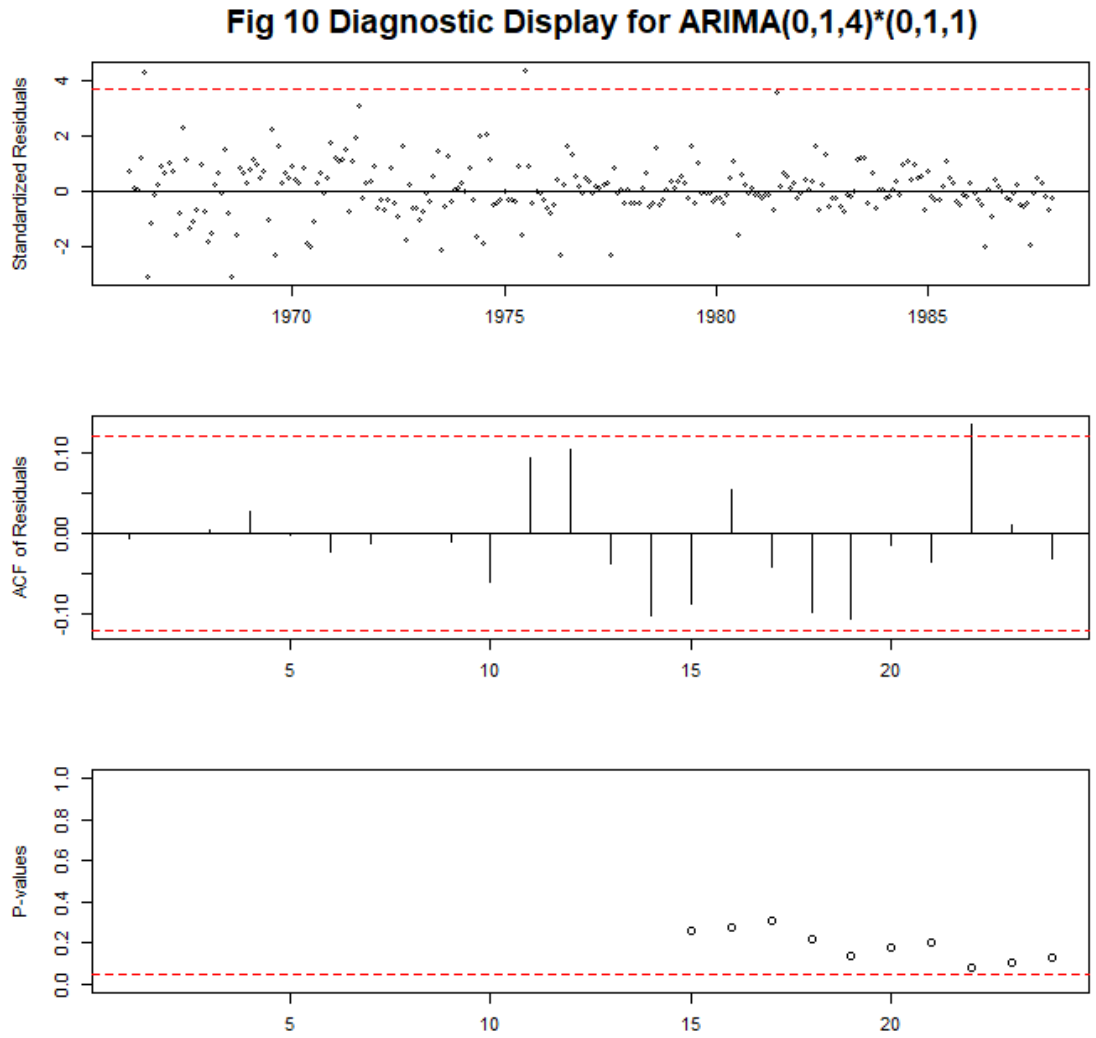
Fig 8 Sample ACF of First and Seasonal Differences of Monthly Water Usage



ARIMA(0,1,1) X $(0,1,1)_{12}$



ARIMA(0,1,4) X $(0,1,1)_{12}$



Detect outlier

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> model

Call:
arima(x = Water, order = c(0, 1, 4), seasonal = list(order = c(0, 1, 1), period = 12),
    io = c(19, 44, 55, 56, 80, 127, 151, 198))

Coefficients:
          ma1      ma2     ma3      ma4     sma1   IO-19   IO-44   IO-55    IO-56  IO-80  IO-127  IO-151  IO-198
      -0.4843  -0.3522  0.0487  -0.1864  -0.8806  0.0015  -1e-03  0.0012  -0.0015  4e-04  0.0018  -6e-04  0.0012
s.e.   0.0677   0.0741  0.0805   0.0664   0.0446  0.0003   3e-04  0.0003   0.0003  3e-04  0.0003   3e-04  0.0003

sigma^2 estimated as 1.198e-07:  log likelihood = 1710.97,  aic = -3395.95
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> model3

Call:
arima(x = Water, order = c(0, 1, 4), seasonal = list(order = c(0, 1, 1), period = 12),
    fixed = c(NA, NA, 0, rep(NA, 10)), io = c(19, 44, 55, 56, 80, 127, 151,
        198))

Coefficients:
          ma1      ma2  ma3      ma4     sma1   IO-19   IO-44   IO-55    IO-56  IO-80  IO-127  IO-151  IO-198
      -0.5036  -0.3529    0  -0.1707  -0.8832  0.0015  -1e-03  0.0012  -0.0015  3e-04  0.0017  -6e-04  0.0012
s.e.   0.0760   0.0733    0   0.0553   0.0445  0.0003   3e-04  0.0003   0.0003  3e-04  0.0003   3e-04  0.0003

sigma^2 estimated as 1.164e-07:  log likelihood = 1710.82,  aic = -3397.64
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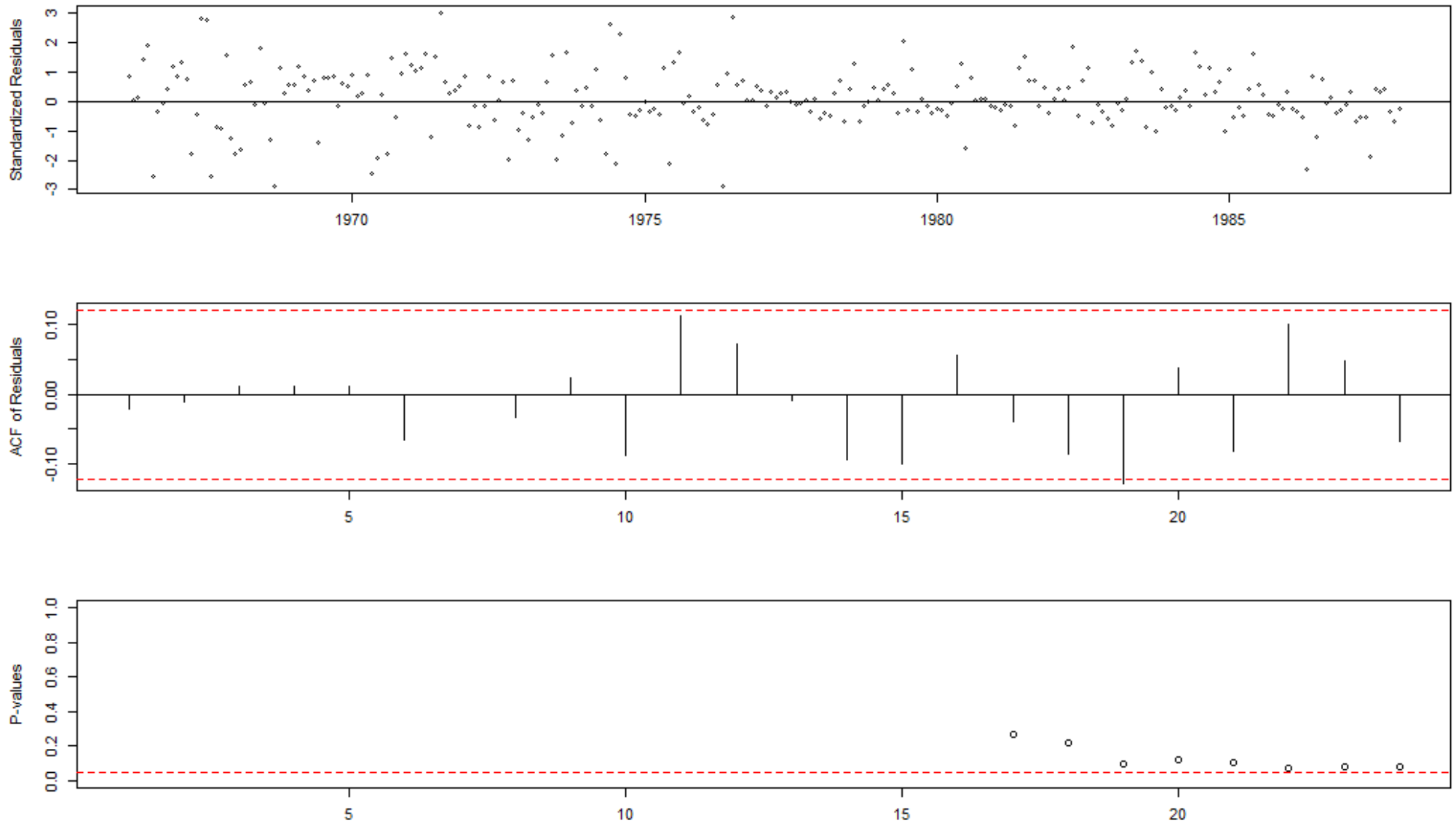
Fig 13 Diagnostic Display for ARIMA(0,1,4)*(0,1,1) with io=19,44,55,56,80,127,151,198 and ma3 = 0



Model Checking

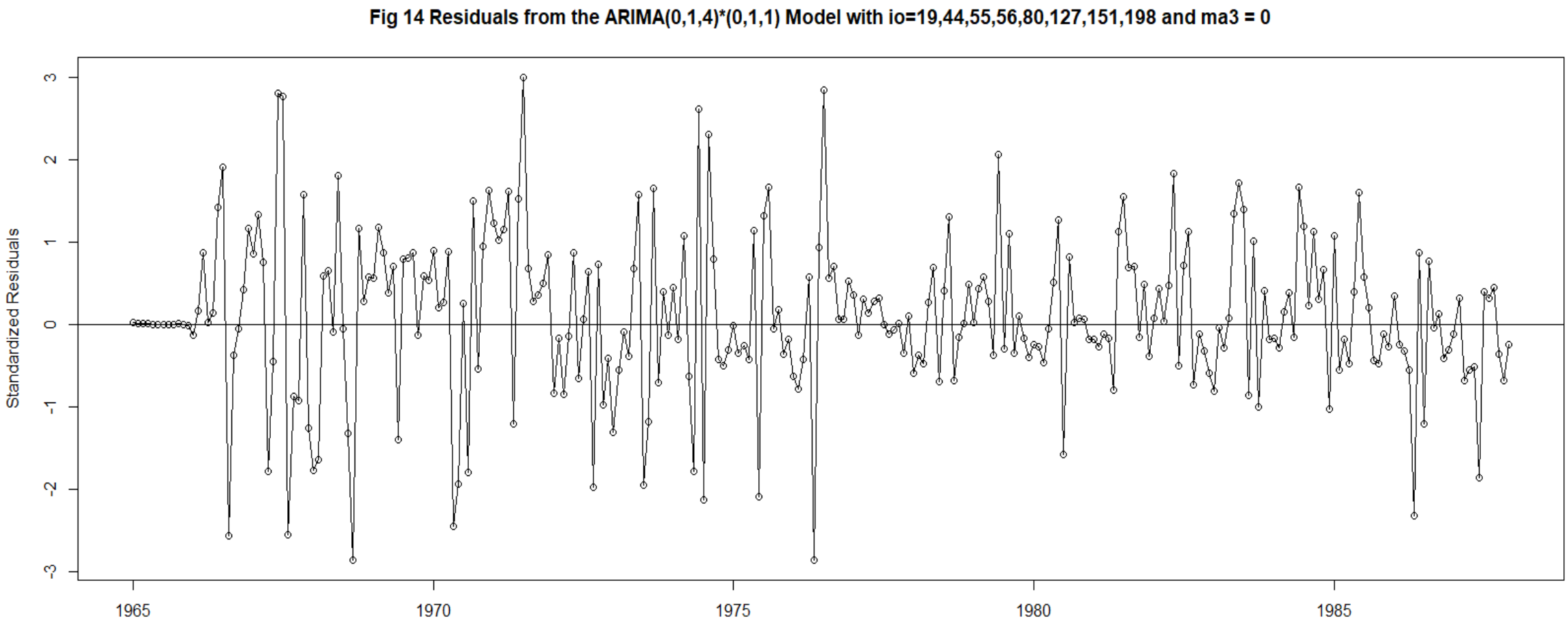
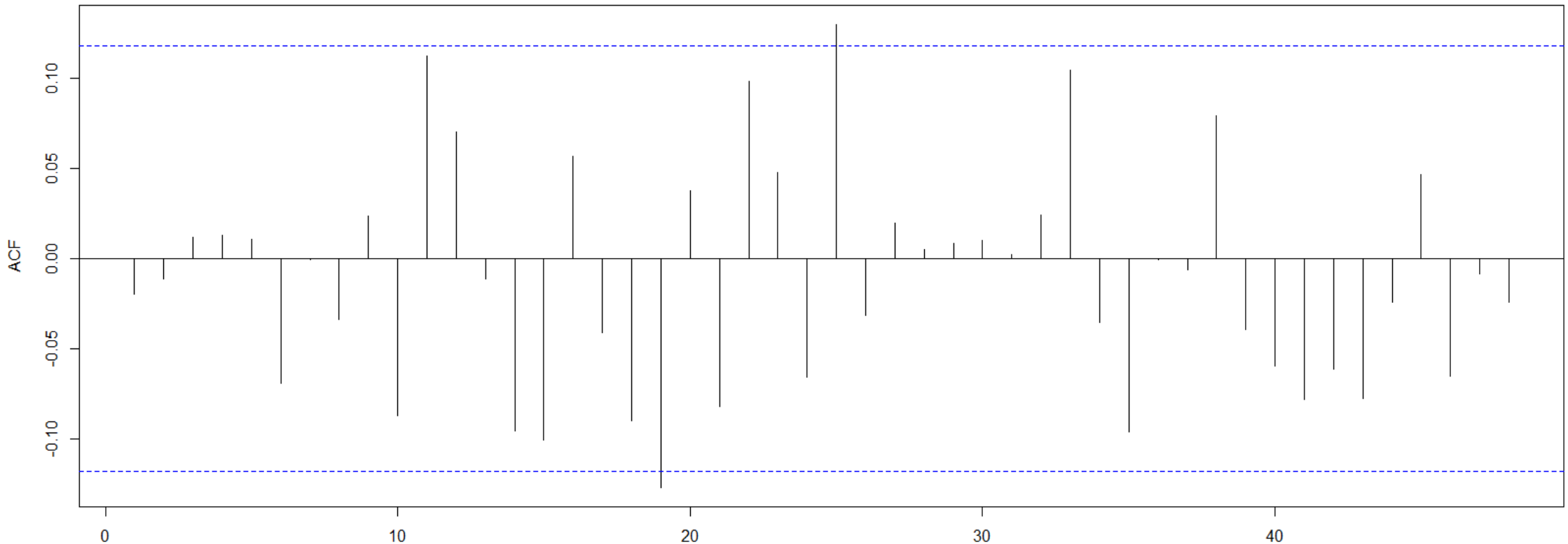


Fig 15 ACF of Residuals from the ARIMA(0,1,4)*(0,1,1) Model with io=19,44,55,56,80,127,151,198 and ma3 = 0



Conclusion

- Time series analysis of 23 years of monthly water usage shows a strong upward trend and a seasonal pattern.

- ARIMA $(0, 1, 4) \times (0, 1, 1)_{12}$ model with IO = 19, 44, 55, 56, 80, 127, 151, 198 and ma3 = 0.

- Compared with winter months, people usually take more showers and play more water games in summer, an as a result, the demand for water is higher during the summer months and lower during the winter time.