MAT110 - Obligatorisk innlevering 1

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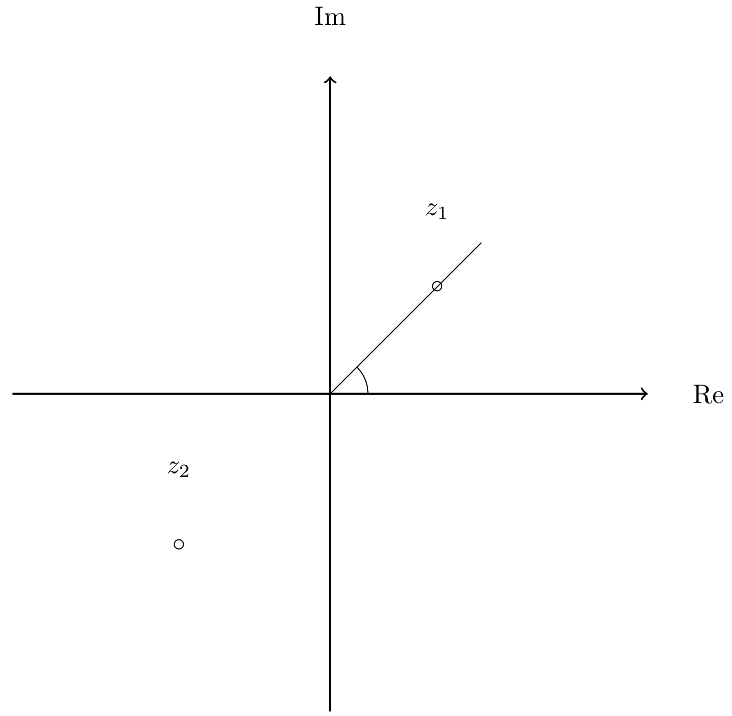
Oppgave 1



a)

b)

$$
\begin{aligned}
z_1 &= e^{i\frac{\pi}{4}} \quad \text{(eksponentialform)} \\
&e^{i\frac{\pi}{4}} \rightarrow r = 1, \quad \theta = \frac{\pi}{4} \\
&= 1(\cos\theta + i \cdot \sin\theta) \\
&= cos\frac{\pi}{4} + i \cdot sin\frac{\pi}{4} \\
&= x_1 + i \cdot y_1 \\
\Rightarrow x_1 &= \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \\
\Rightarrow y_1 &= \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \\
z_1 &= \frac{\sqrt{2}}{2} + i \cdot \frac{\sqrt{2}}{2} \quad \text{(standardform)}
\end{aligned}
$$

$$
\begin{aligned}
z_2 &= -1 - i \quad \text{(standardform)} \\
&= x_2 + i \cdot y_2 \rightarrow x_2 = -1, \quad y_2 = -1 \\
&r = \sqrt{x_2^2 + y_2^2} = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2} \\
&\theta = \tan^{-1}(|\frac{y}{x}|) \quad \text{(Generelt)} \\
&\text{Men ettersom det komplekse tallet er i 3. kvadrant} \rightarrow \theta = \alpha + \pi \\
&\alpha = \tan^{-1}(|\frac{-1}{-1}|) = \tan^{-1}(1) = \frac{\pi}{4} \\
&\theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4} \\
z_2 &= r(\cos\theta + i \cdot \sin\theta) \\
z_2 &= \sqrt{2}(\cos\frac{5\pi}{4} + i \cdot \frac{5\pi}{4}) \\
z_2 &= \sqrt{2} \cdot e^{i\frac{5\pi}{4}} \quad \text{(eksponentialform)}
\end{aligned}
$$

c)

$$
\begin{aligned}
z_1 + z_2 &= (\frac{\sqrt{2}}{2} + \sqrt{2}2 \cdot i) + (-1 - i) \\
&= (\frac{\sqrt{2}}{2} - 1) + (\frac{\sqrt{2}}{2} - 1) \cdot i \\
&= (0.7071 - 1) + (0.7071 - 1) \cdot i \\
&= -0.29 - 0.29i
\end{aligned}
$$

$$
\begin{aligned}
\frac{z_1}{z_2} &= \frac{e^{i\frac{\pi}{4}}}{\sqrt{2}e^{i\frac{5\pi}{4}}} = \frac{1}{\sqrt{2}}e^{i\frac{\pi}{4} - i\frac{t\pi}{4}} = \frac{1}{\sqrt{2}}e^{-i\pi} \\
&= \frac{1}{\sqrt{2}}(\cos(-\pi) + i\sin(-\pi)) = \frac{1}{\sqrt{2}}(-1 + i(0)) = -\frac{1}{\sqrt{2}} + i0 \\
&= -\frac{\sqrt{2}}{2}
\end{aligned}
$$

$$
\begin{aligned}
z_1 &= \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2} \\
\overline{z_1} &= \frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2} \\
-\overline{z_1} &= -1(\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}) = -\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}
\end{aligned}
$$

d) $z^6 = 1 + i$. La $w = 1 + i$, da må vi finne den 6. roten av w.
Først konverterer vi w til eksponentialform.
$r = \sqrt{1^2 + 1^2} = \sqrt{2}$
$\theta = \tan^{-1}(1) = \frac{\pi}{4}$
$w = 1 + i = \sqrt{2}e^{i\frac{\pi}{4}}$

Videre tar vi i bruk De Moivres thm for røtter:
Hvis $z^n = w = re^{i\theta}$, da er røttene gitt ved $z_k = \sqrt[n]{r}e^{i(\frac{\theta + 2\pi k}{n})}$, for $k = 0, 1, 2, \ldots, n-1$.

I dette tilfellet blir $n = 6, r = \sqrt{2}$ og $\theta = \frac{\pi}{4}$.
$\sqrt[6]{r} = \sqrt[6]{\sqrt{2}} = \sqrt[6]{2^{\frac{1}{2}}} = 2^{(\frac{1}{2})(\frac{1}{6})} = 2^{\frac{1}{12}} = \sqrt[12]{2}$.
Videre finner vi argumentene for $k = 0, 1, 2, 3, 4, 5$:
Det generelle argumentet er $\phi_k = \frac{\frac{\pi}{4} + 2\pi k}{6} = \frac{\pi}{24} + \frac{2\pi k}{6} = \frac{\pi}{24} + \frac{\pi k}{3}$.

$$
\begin{aligned}
k = 0 &: \phi_0 = \frac{\pi}{24} \\
k = 1 &: \phi_1 = \frac{\pi}{24} + \frac{\pi}{3} = \frac{9\pi}{24} \\
k = 2 &: \phi_2 = \frac{\pi}{24} + \frac{2\pi}{3} = \frac{17\pi}{24} \\
k = 3 &: \phi_3 = \frac{\pi}{24} + \pi = \frac{25\pi}{24} \\
k = 4 &: \phi_4 = \frac{\pi}{24} + \frac{4\pi}{3} = \frac{33\pi}{24} \\
k = 5 &: \phi_5 = \frac{\pi}{24} + \frac{5\pi}{3} = \frac{41\pi}{24}
\end{aligned}
$$

Det er seks løsninger:
$z_k = 2^{\frac{1}{12}} \cdot e^{i\phi_k}$, for $k = 0, 1, 2, 3, 4, 5$.

Oppgave 2

a) $\begin{bmatrix} 1 & 0 & 2k & k & 2 \\ 1 & 1 & (2k-1) & 0 & 1 \\ -2 & 0 & -4k & k & 5 \end{bmatrix} -R_1 + R_2 \begin{bmatrix} 1 & 0 & 2k & k & 2 \\ 0 & 1 & -1 & -k & -1 \\ -2 & 0 & -4k & k & 5 \end{bmatrix} 2R_1 + R_3 \begin{bmatrix} 1 & 0 & 2k & k & 2 \\ 0 & 1 & -1 & -k & -1 \\ 0 & 0 & 0 & 3k & 5 \end{bmatrix}$

(1) $x_1 + 2kx_3 + kx_4 = 2$

(2) $x_2 - x_3 - kx_4 = -1$

(3) $3kx_4 = 9$

b) Fra (3) $3kx_4 = 9$
Case 1:
$k \neq 0 \Rightarrow x_4 = \frac{3}{k}$
Vi setter $x_4 = \frac{4}{k}$ inn i (2):
$x_2 = x_3 + k\frac{3}{k} - 1 = x_3 + 2$
Så vi trenger verdien til x_3 for å beregne x_2.

Vi setter $x_4 = \frac{3}{k}$ inn i (1):
$x_1 + 2kx_3 + k\frac{3}{k} = x_1 + 2kx_3 + 3 = 2$
$\Rightarrow x_1 = -1 - 2kx_3$

For $x \neq$, x_1 og x_2 er uttrykt med bruk av den frie variablen x_3.
Derfor er det uendelig mange løsninger for alle $k \neq 0$.

Case 2:
$k = 0 \Rightarrow$ (3) blir $0 \cdot x_4 = 9 \Rightarrow 0 = 9$.

Dette er umulig og vi kan konkludere at det er ingen løsning for $k = 0$.

Utifra dette kan vi konkludere at det kan ikke være akkurat en eller akkurat to løsninger i dette tilfelle.

c)

(3) $(3)(6)x_4 = 9 \Rightarrow 18x_4 = 9 \Rightarrow x_4 = \dfrac{1}{2}$

(2) $x_2 - x_3 - 6(\dfrac{1}{2}) = -1 \Rightarrow x_3 - x_3 - 3 = -1 \Rightarrow x_2 = x_3 + 2$

(1) $x_1 + 2(6)x_3 + 6(\dfrac{1}{2}) = 2 \Rightarrow x_1 + 12x_3 + 3 = 2 \Rightarrow x_1 = -1 - 12x_3$

$\therefore \quad (x_1, x_2, x_3, x_4) = (-1 - 12t, t + 2, t, \dfrac{1}{2}), t \in \mathbb{R}$ (Uendelig mange løsninger)

Oppgave 3

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -1 \\ 2 & 3 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ -1 & 2 \end{bmatrix}$$

a) $A + B$ er ikke mulig ettersom matrisene har ulike dimensjoner.

$$3A = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 6 & -3 \\ 6 & 9 & 9 \end{bmatrix}, \quad B^T = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & -1 \end{bmatrix}$$

$3A - B^T$ er ikke mulig ettersom matrisene har ulike dimensjoner.

$$AB = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -1 \\ 2 & 3 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1*1+1*1+1(-1) & 1*1+1(-1)+1*2 \\ 1*1+2*1-1(-1) & 1*1+2(-1)-1*2 \\ 2*1+3*1+3(-1) & 2*1+3(-1)+3*2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 4 & -3 \\ 2 & 5 \end{bmatrix}$$

BA er ikke mulig ettersom dimensjonene på matrisene må være $m \times p \cdot p \times n$ for å gjennomføre multiplikasjon.

$$\mathrm{B}^T A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -1 \\ 2 & 3 & 3 \end{bmatrix} = \begin{bmatrix} 1(1)+1(1)-1(2) & 1(1)+1(2)-1(3) & 1(1)+1(-1)-1(3) \\ 1(1)-1(1)+2(2) & 1(1)-1(2)+2(3) & 1(1)-1(-1)+2(3) \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 4 & 5 & 8 \end{bmatrix}$$

b) $$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -1 \\ 2 & 3 & 3 \end{bmatrix}$$

$$\det(A) = 1 \cdot \begin{vmatrix} 2 & -1 \\ 3 & 3 \end{vmatrix} - 1 \cdot \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} + 1 \cdot \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix}$$

$$= 1 \cdot (6+3) - 1(3+2) + 1(3-4)$$

$$= 9 - 5 - 1 = 3$$

c) $$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & -1 & 0 & 1 & 0 \\ 2 & 3 & 3 & 0 & 0 & 1 \end{array}\right] \sim -R_1 + R_2 \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & -1 & 1 & 0 \\ 2 & 3 & 3 & 0 & 0 & 1 \end{array}\right] \sim_{-2R_1 + R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & -1 & 1 & 0 \\ 0 & 1 & 1 & -2 & 0 & 1 \end{array}\right]$$

$$\sim_{-R_2 R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & -1 & 1 & 0 \\ 0 & 0 & 3 & -1 & -1 & 1 \end{array}\right] \sim_{\frac{1}{3}R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & -1 & 1 & 0 \\ 0 & 0 & 1 & -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \end{array}\right] \sim 2R_3 + R_2 \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -\frac{5}{3} & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 1 & -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \end{array}\right]$$

$$\sim^{-R_2 + R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & \frac{8}{3} & -\frac{1}{3} & -\frac{2}{3} \\ 0 & 1 & 0 & -\frac{5}{3} & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 1 & -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \end{array}\right] \sim^{-R_3 + R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 0 & -1 \\ 0 & 1 & 0 & -\frac{5}{3} & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 1 & -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \end{array}\right]$$

$$\mathrm{A}^{-1} = \begin{bmatrix} 3 & 0 & -1 \\ -\frac{5}{3} & \frac{1}{3} & \frac{2}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

d) $A\vec{x} = \vec{b}$, der $\vec{b} = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$

$$\Rightarrow \vec{x} = A^{-1}\vec{b} = \begin{bmatrix} 3 & 0 & -1 \\ -\frac{5}{3} & \frac{1}{3} & \frac{2}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$$

$$\begin{aligned}
&3(1) + 0(4) - 1(2) = 3 - 2 = 1 \\
&-\frac{5}{3}(1) + \frac{1}{3}(4) + \frac{2}{3}(2) = -\frac{5}{3} + \frac{4}{3} + \frac{4}{3} = 1 \\
&-\frac{1}{3}(1) - \frac{1}{3}(4) + \frac{1}{3}(2) = -\frac{1}{3} - \frac{4}{3} + \frac{2}{3} = -1 \\
&x_1 = 1, x_2 = 1, x_3 = -1
\end{aligned}$$

e)