Table 3: **Properties of the z-Transform**

Property	Sequence	Transform	ROC
	$x[n]$	$X(z)$	R
	$x_1[n]$	$X_1(z)$	R_1
	$x_2[n]$	$X_2(z)$	R_2
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	At least the intersection of R_1 and R_2
Time shifting	$x[n - n_0]$	$z^{-n_0} X(z)$	R except for the possible addition or deletion of the origin
Scaling in the z-Domain	$e^{j\omega_0 n} x[n]$	$X(e^{-j\omega_0} z)$	R
	$z_0^n x[n]$	$X\left(\frac{z}{z_0}\right)$	$z_0 R$
	$a^n x[n]$	$X(a^{-1} z)$	Scaled version of R (i.e., $|a|R =$ the set of points $\{|a|z\}$ for z in R)
Time reversal	$x[-n]$	$X(z^{-1})$	Inverted R (i.e., R^{-1} = the set of points z^{-1} where z is in R)
Time expansion	$x_{(k)}[n] = \begin{cases} x[r], & n = rk \\ 0, & n \neq rk \end{cases}$ for some integer r	$X(z^k)$	$R^{1/k}$ (i.e., the set of points $z^{1/k}$ where z is in R)
Conjugation	$x^*[n]$	$X^*(z^*)$	R
Convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	At least the intersection of R_1 and R_2
First difference	$x[n] - x[n-1]$	$(1 - z^{-1})X(z)$	At least the intersection of R and $|z| > 0$
Accumulation	$\sum_{k=-\infty}^{n} x[k]$	$\frac{1}{1-z^{-1}} X(z)$	At least the intersection of R and $|z| > 1$
Differentiation in the z-Domain	$nx[n]$	$-z\frac{dX(z)}{dz}$	R

Initial Value Theorem
If $x[n] = 0$ for $n < 0$, then
$x[0] = \lim_{z\to\infty} X(z)$

Table 4: **Some Common z-Transform Pairs**

Signal	Transform	ROC
1. $\delta[n]$	1	All z
2. $u[n]$	$\frac{1}{1-z^{-1}}$	$|z| > 1$
3. $-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$|z| < 1$
4. $\delta[n-m]$	z^{-m}	All z except 0 (if $m > 0$) or ∞ (if $m < 0$)
5. $\alpha^n u[n]$	$\frac{1}{1-\alpha z^{-1}}$	$|z| > |\alpha|$
6. $-\alpha^n u[-n-1]$	$\frac{1}{1-\alpha z^{-1}}$	$|z| < |\alpha|$
7. $n\alpha^n u[n]$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$|z| > |\alpha|$
8. $-n\alpha^n u[-n-1]$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$|z| < |\alpha|$
9. $[\cos\omega_0 n]u[n]$	$\frac{1-[\cos\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$|z| > 1$
10. $[\sin\omega_0 n]u[n]$	$\frac{[\sin\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$|z| > 1$
11. $[r^n\cos\omega_0 n]u[n]$	$\frac{1-[r\cos\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2 z^{-2}}$	$|z| > r$
12. $[r^n\sin\omega_0 n]u[n]$	$\frac{[r\sin\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2 z^{-2}}$	$|z| > r$