CS 4649/7649 Robot Intelligence: Planning

Constraints I: Constraint Programs; Arc Consistency

CS 4649/7649 – Asst. Prof. Matthew Gombolay

Slides adapted from:
6.034 Tomas Lozano Perez
16.410 Sertac Karaman
Russell and Norvig AIMA

Assignments

- Due Today, 1/20
  - Reading: Ch. 10
  - PSet 2 due at 11:59 PM EST
- Due Tuesday, 1/25
  - Read Ch. 11
- Due Thursday, 1/27
  - PSet32 due at 11:59 PM EST

MCTS - Algorithm Recap

- Applied to solve Multi-Arm Bandit problem in a tree structure
  - UCT = UCB1 applied at each subproblem
- Due to tree structure same move can have different rewards in different subtrees
- Weight to go to a given node:
  - Mean value for paths involving node
  - Visits to node
  - Visits to parent node
  - Constant balancing exploration vs exploitation
- Determines values from Default Policy
- Determines how to choose child from Tree Policy
- Once you have acomplete tree - number of ways to pick moves during game - Max, Robust, Max-Robust, etc.

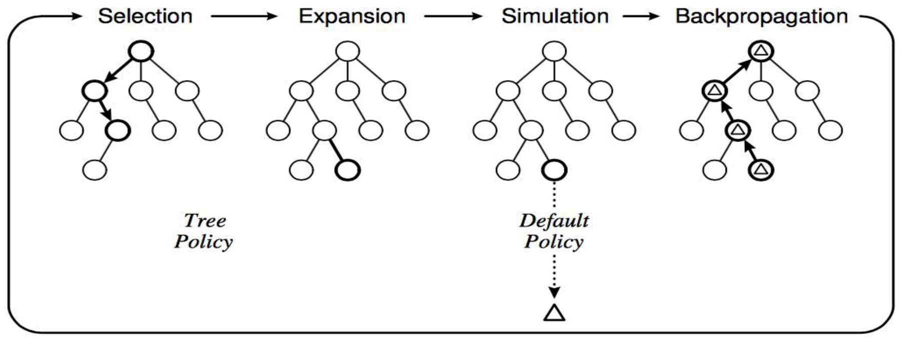


Fig. 2. One iteration of the general MCTS approach.

MCTS Case Study: AlphaGo

Go

- 2 player
- Zero-sum
- 19x19 board
- Very large search tree
    - Breadth ≈ 250, depth ≈ 150
    - Unlike chess
- No good heuristics
    - Human intuition hard to replicate
- Great candidate for applying MCTS
    - Vanilla MCTS not good enough

MCTS Case Study: AlphaGo

Idea 1) Use supervised/reinforcement to learn a <u>simulation policy</u>

→ Intelligently play game to completion to get a sense of whether the outcome will be favorable

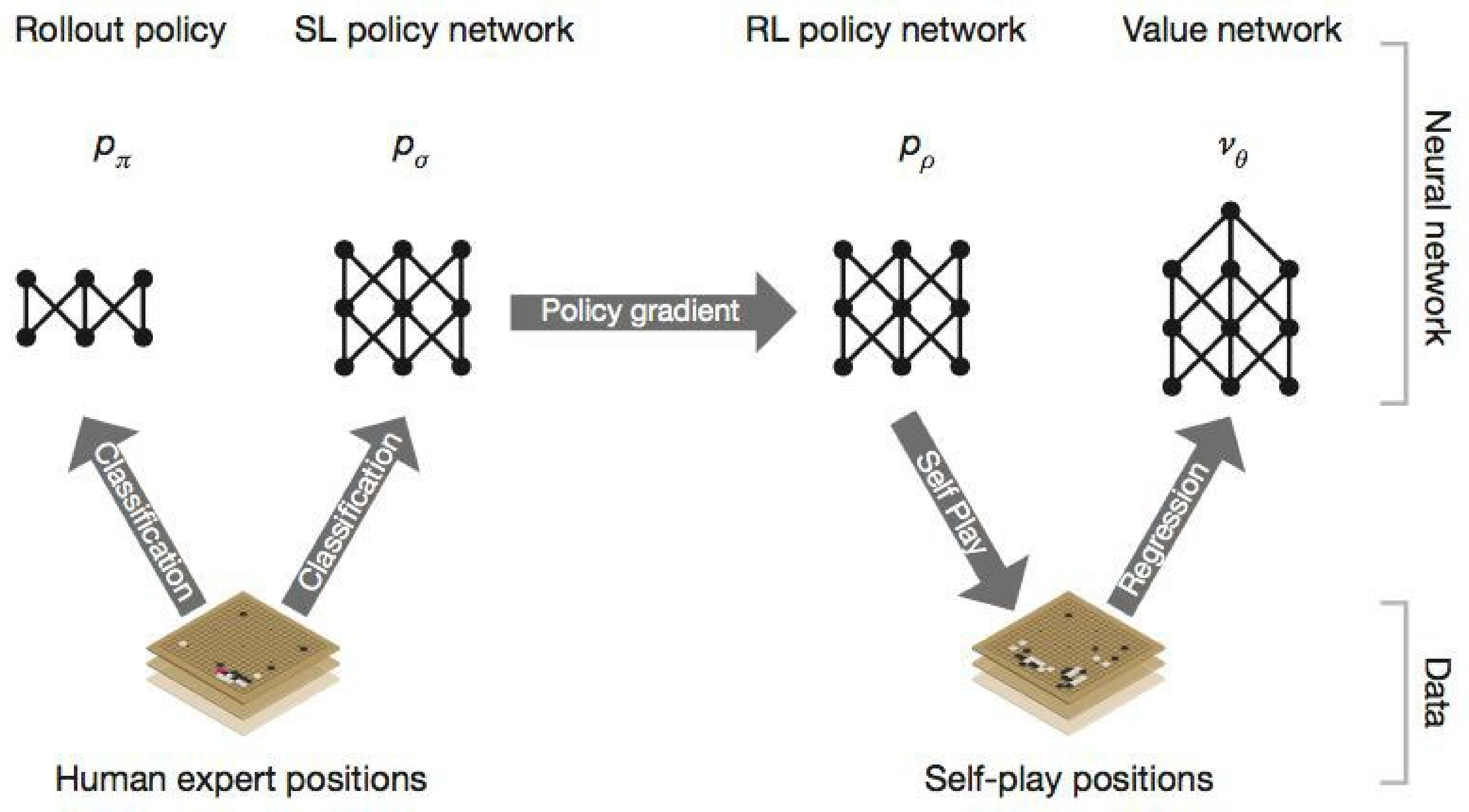
Idea 2) Use statistical inference (i.e., q-learning) to learn a cost-to-go-function (i.e., a <u>value function</u>) that can augment the <u>simulation policy</u>

→ Estimate the outcome of playing rest of game without actually playing rest of game

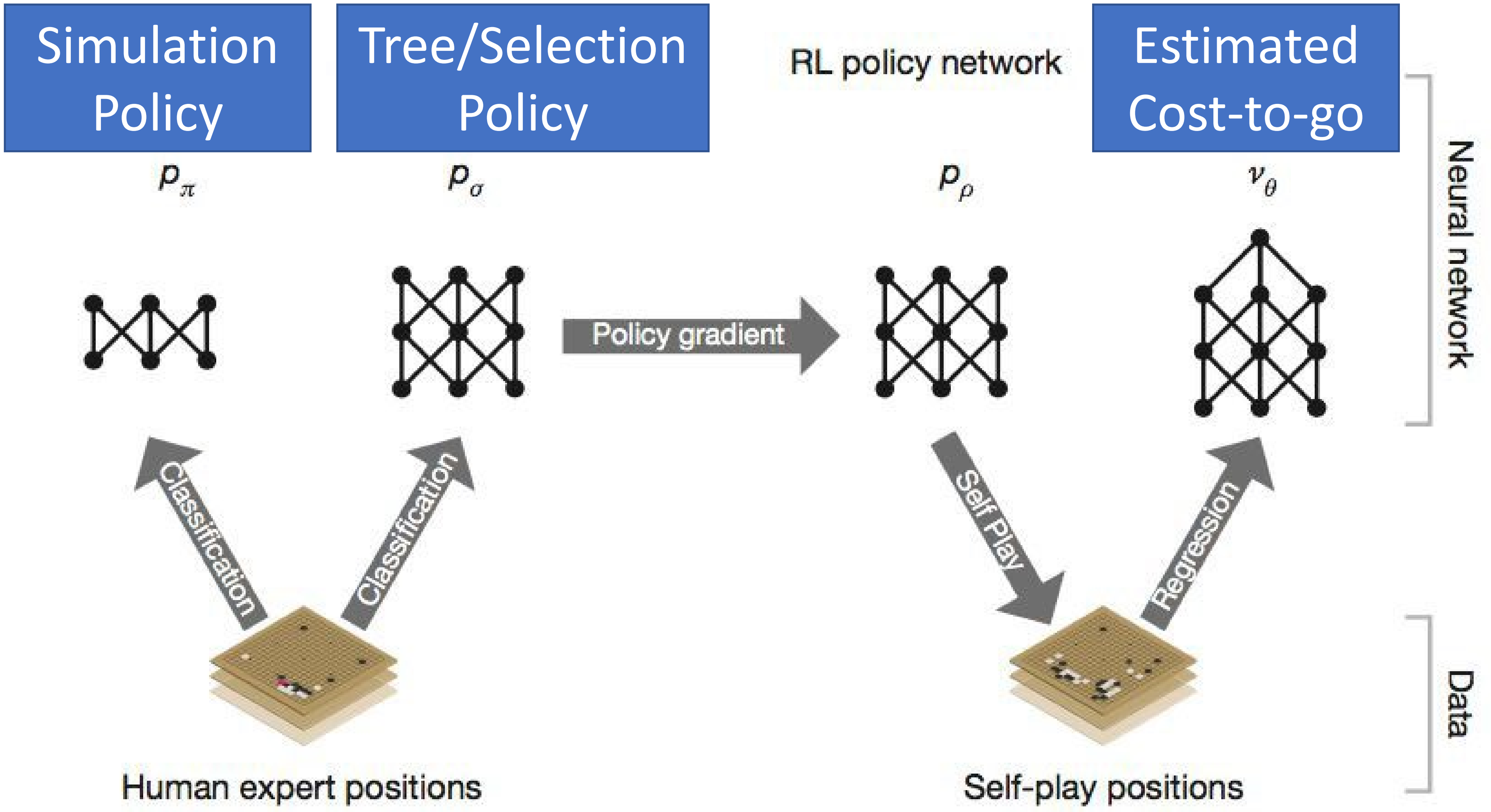
Idea 3) Use supervised/reinforcement learning to learn a <u>tree policy</u>

→ Try to make the most of each action opportunity

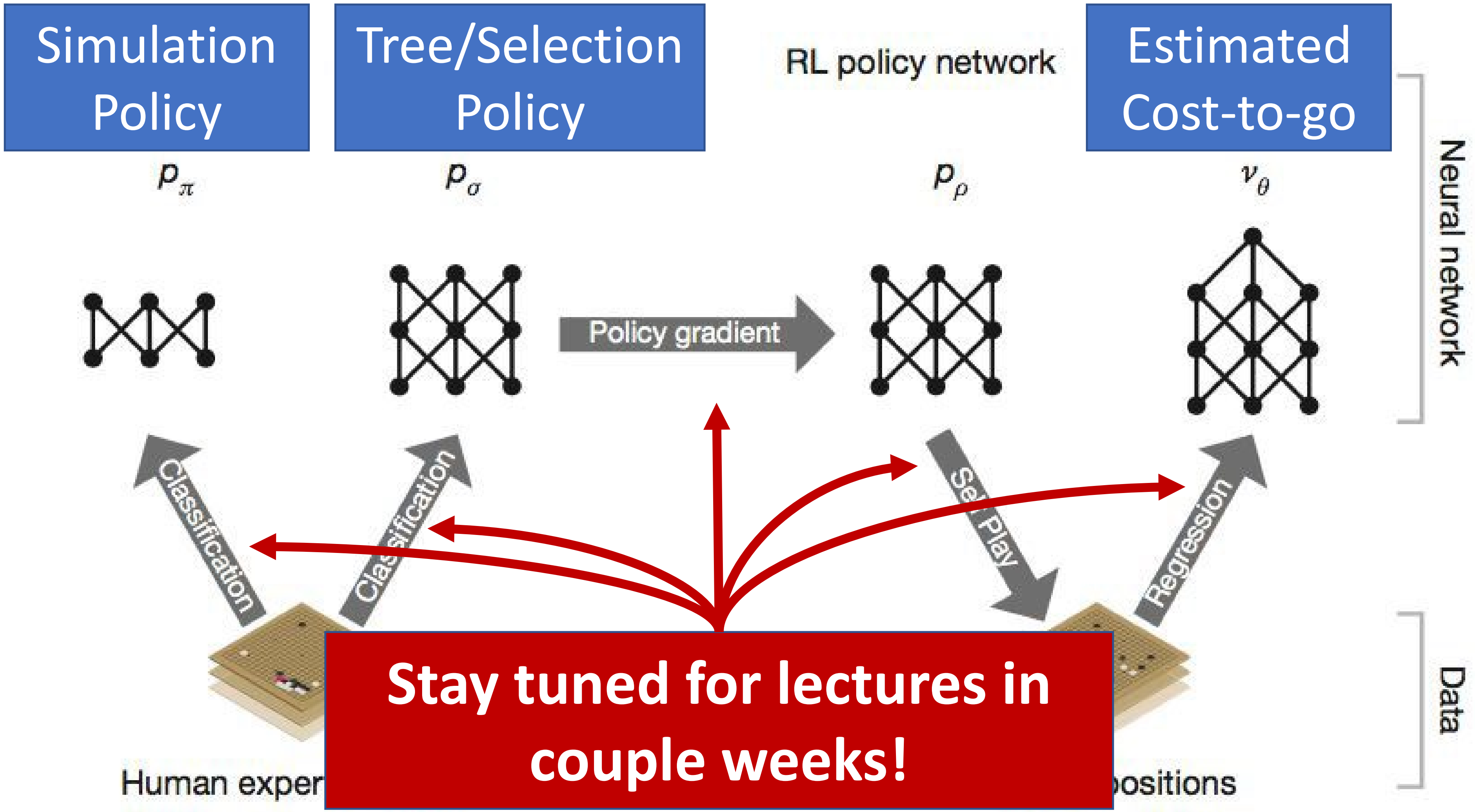
AlphaGo Architecture



AlphaGo Architecture



AlphaGo Architecture



AlphaGo: Selection/Tree Policy

$$a_t = \text{argmax}_a(Q(s_t, a) + u(s_t, a))$$

$$u(s, a) \propto \frac{P(s, a)}{1 + N(s, a)}$$

- a_t - action chosen for time step t given state s_t
- Q(s_t, a) – Average reward for playing a in state s_t (exploitation term)
  - **Not** the “Q-function” from q-learning, but rather an Alpha-Go specific term
- *P(s, a)* - prior expert probability of playing moving *a*
  - For AlphaGo, this was done via Supervised Learning (“Mimicry”)
- *N(s, a)* - number of times we have visited parent node
- $u(s_t, a)$ acts as a bonus value that decays with repeated visits

AlphaGo: Simulation/Value Policy

$$V(s_L) = (1 - \lambda)v_\theta(s_L) + \lambda z(s_L)$$

- The point estimate of the value of a node (representing state s) is given by the convex combination, controlled by λ, of two terms:
  - Cost-to-go-function, $v_\theta(s)$
    - Learned via reinforcement learning (technically regression)
  - Win/loss: The result from playing game to completion starting from state s using simulation policy z
    - This simulation policy is learned via supervised learning to “mimic” expert players.
    - The network is “small” to enable it to act quickly

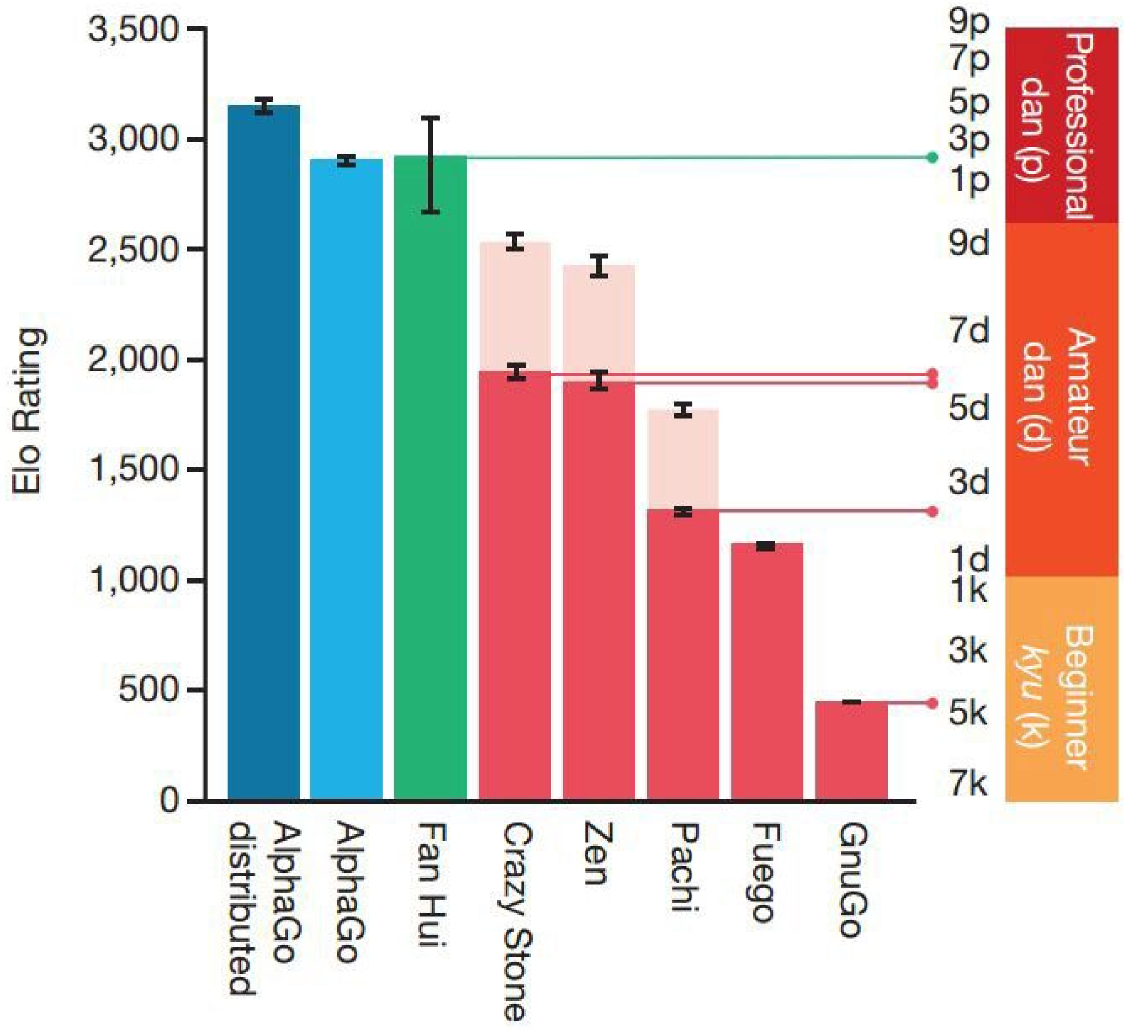
AlphaGo: Backpropagation

$$Q(s,a) = \frac{\sum_{i=1}^{n} 1_{(s,a,i)} V\left(s_L^i\right)}{\sum_{i=1}^{n} 1_{(s,a,i)}}$$

- Extra index i is to denote the $\mathrm{i^{th}}$ simulation with n total simulations
- Update visit count, mean reward of simulations passing through node

Once MCTS completes, the algorithm chooses the most-taken move from the root position

AlphaGo: Results



Takeaway

- A* search is an optimal, heuristic, search algorithm that works by relying on an admissible heuristic
  - Challenge: Good, admissible heuristics are hard to come by

  →In a few lectures, we will see how reinforcement learning can help!

- MCTS is key to modern (stochastic-)state-space search
  - Search heuristic provided (UCT: Apply UCB1 at each step)
  - Caveat: Need model of the opponent

- Key difference:
  - We are moving from a deterministic world (BFS, DFS, IDS, A, A*) vs. non-deterministic and random worlds

Outline

- Constraint Satisfaction Problems
- Solving CSPs
- Case Study: Scheduling

Constraint Satisfaction Problems (CSP)

Input: A CSP is a 3-tuple (i.e., triple) $\langle V, D, C \rangle$ where:

- V is a set of variables V_i
- D is a set of variable domains,
  - The domain of variable V_i is denoted D_i
- C is the set of constraints on assignments to V
  - Each constraint $C_j = \langle S_j, R_j \rangle$ specifies allowed variable assignments
  - S_j, the constraint's scope, is a subset of variables V
  - R_j, the constraint's relation, is a set of assignments to S_j

Output: A full assignment to V from elements of D such that all constraints C are satisfied.

Constraint Satisfaction Problems (CSP)

Example: “Provide one A and two B’s.”

- $V = \{A, B\}$, each with domains $D_i = \{1,2\}$
- $C = \left\{ \begin{array}{l} \langle\{A, B\}, \{\langle 1,2\rangle, \langle 1,1\rangle\}\rangle, \\ \langle\{A, B\}, \{\langle 1,2\rangle, \langle 2,2\rangle\}\rangle \end{array} \right\}$ “one A” “two B’s”
- Output $\langle 1,2\rangle$

Conventions

- List scope in subscript
- Specify one constraint per scope

Example: “Provide one A and two B’s.”

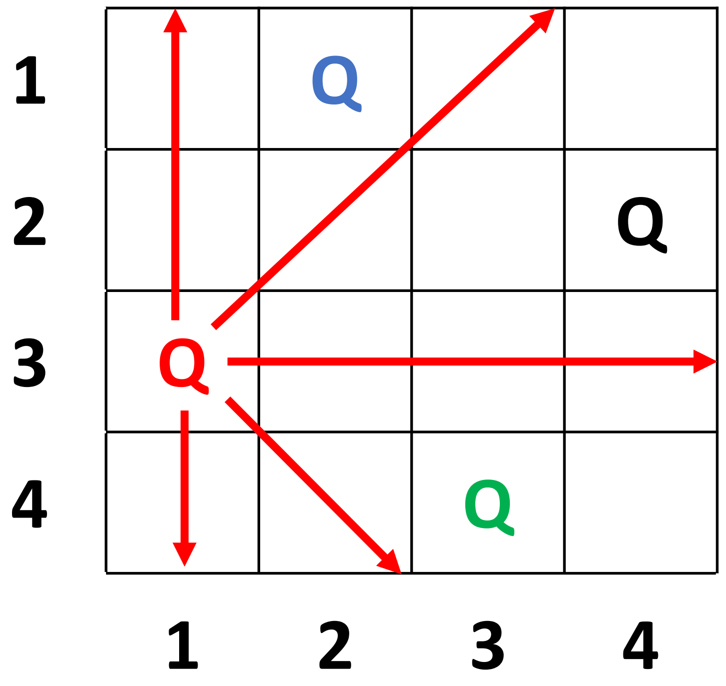
- $C = \{C_{AB}\}$
  $C_{AB} = \{\langle 1,2 \rangle\}$

- $C = \{C_A, C_B\}$
  $C_A = \{\langle 1 \rangle\}$
  $C_B = \{\langle 1 \rangle\}$

Good Encodings are Essential: 4 Queens

4 Queens Problem:

- Place 4 queens on a 4x4 chessboard so that no other queen can attack another



How big is the encoding?

Variables Chessboard positions

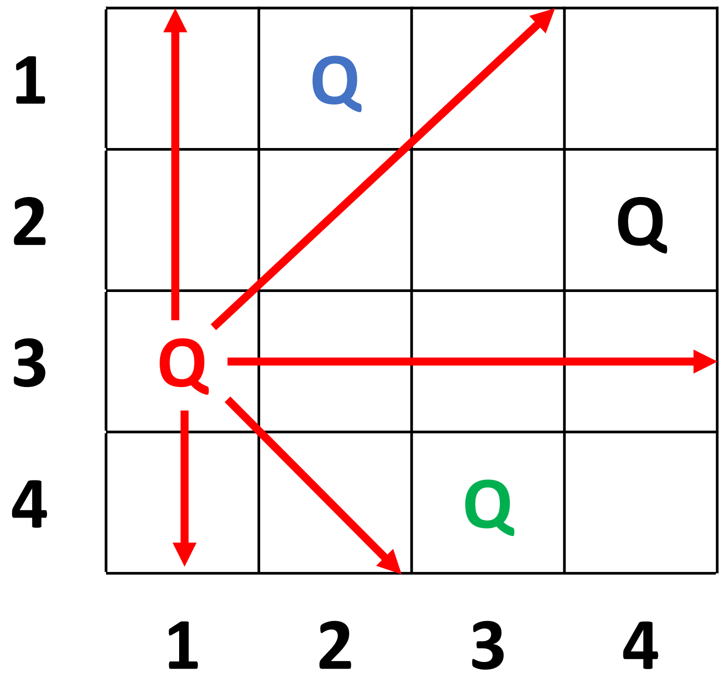
Domains Queen 1-4 or blank

Constraints Two positions on a line (vertical, horizontal, diagonal) cannot both be queens.

Good Encodings are Essential: 4 Queens

What is a better encoding?

- Assume one queen per column
- Determine what row each queen should be in



Variables: Q_1, Q_2, Q_3, Q_4

Domains: $\{1,2,3,4\}$

Constraints: $Q_i \neq Q_j$ "On different rows"

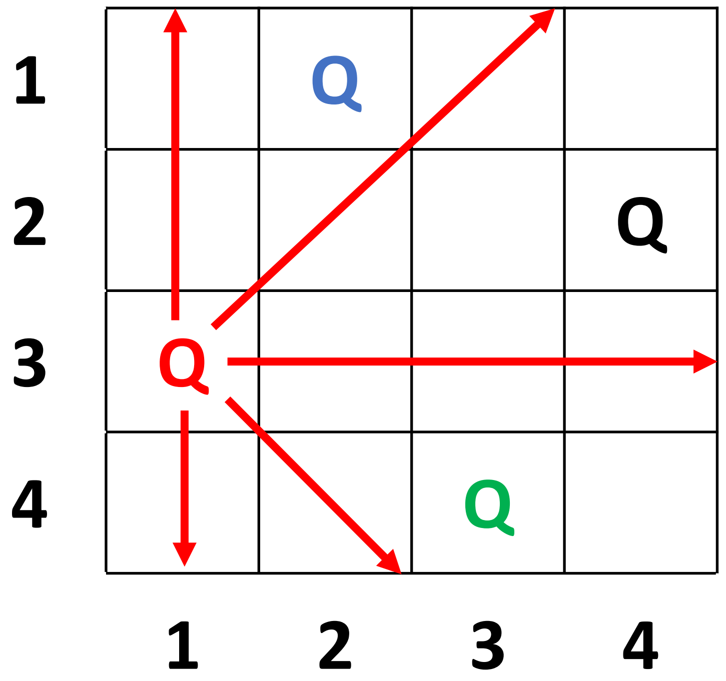
$|Q_i - Q_j| \neq |i - j|$ "Stay off diagonals"

Example: $C_{1,2} = \{(1,3), (1,4), (2,4), (3,1), (4,1), (4,2)\}$

Good Encodings are Essential: 4 Queens

What is a better encoding?

- Assume one queen per column
- Determine what row each queen should be in



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Constraints: $Q_i \neq Q_j$ "On different rows"

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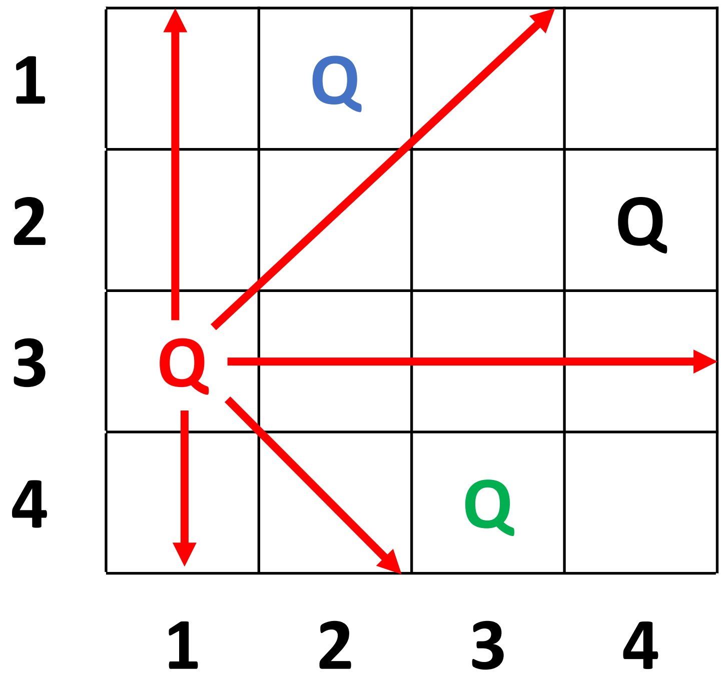
Example: $C_{1,3} = ?$

Good Encodings are Essential: 4 Queens



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Example: $C_{1,3} = \{(1,2), (1,4), (2,1), (2,3), (3,2), (3,4), (4,1), (4,3)\}$

General class of CSPs

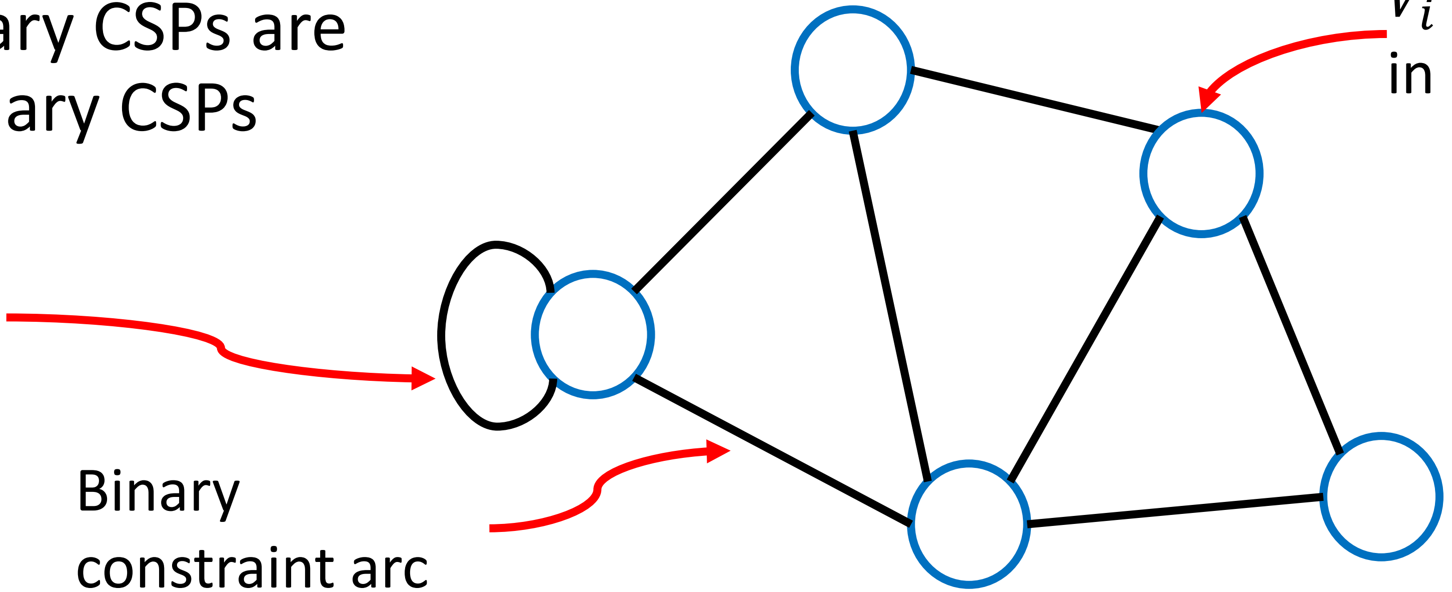
<u>Finite Domain, Binary CSPs</u>

- Each constraint relates at most two variables
- Each variable domain is finite

<u>Property:</u> all n-ary CSPs are reducible to binary CSPs

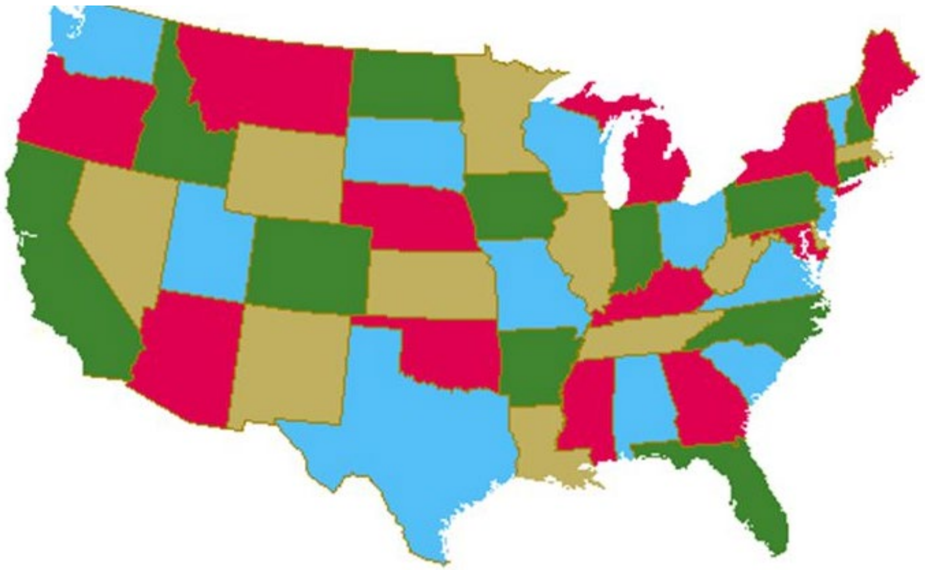
<u>Depict as a Constraint Graph</u>

- Nodes (vertices) are variables
- Arcs (edges) are binary constraints.



Example: Graph Coloring

Pick colors for map regions without coloring adjacent regions with the same color



Credit: Berniece Houston

Variables regions

Domains allowed colors

Constraints adjacent regions must have different colors

Outline

- Constraint Satisfaction Problems
- Solving CSPs
  - Arc-consistency and propagation
  - Analysis of constraint propagation (next lecture)
  - Search student (next lecture)
- Case Study: Scheduling

Good News / Bad News

Good news:

- Very general & interesting family of problems
- Problem formulation used extensively in autonomy and decision-making applications.

Bad new:

- Includes NP-Hard problems

Algorithmic Design Paradigm

Solving CSPs involves a combination of:

Today

1. Inference:
   - Solve partially by eliminating values that cannot be part of any solution (constraint propagation)
   - Make implicit constraints explicit

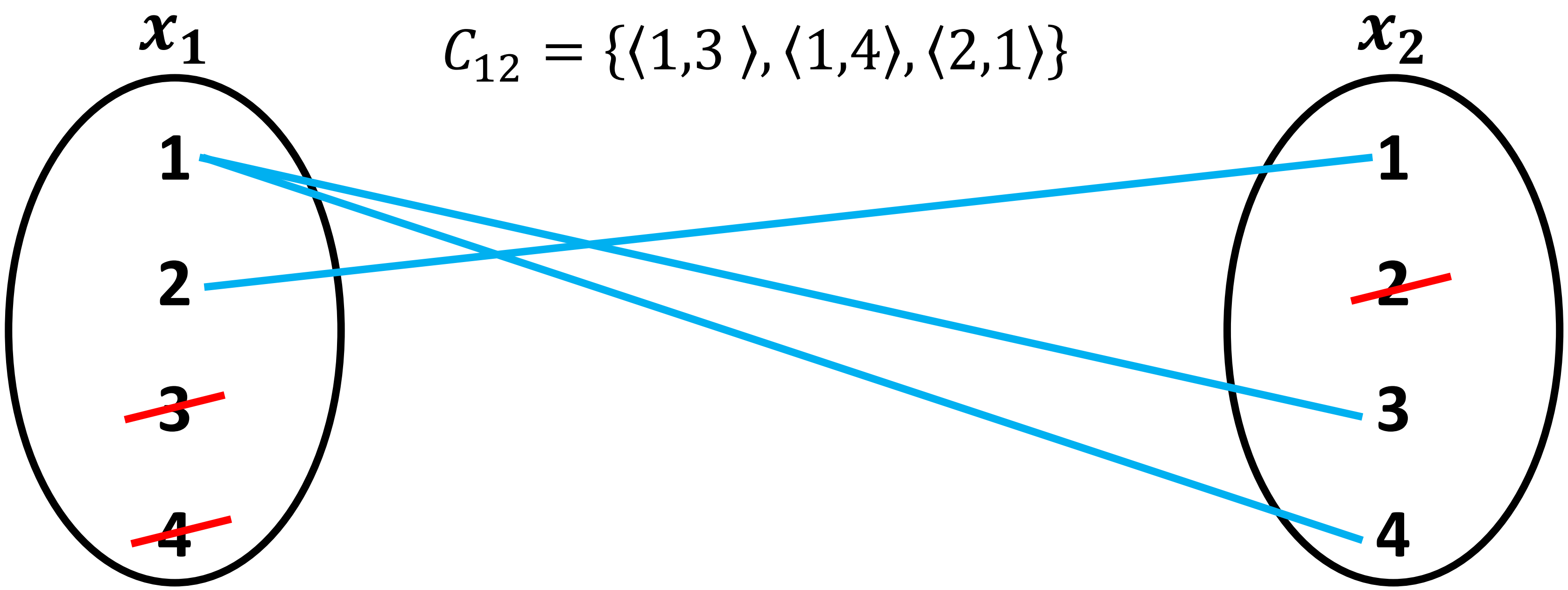
Next Time

2. Search:
   - Try alternative assignments against constraints

Directed Arc Consistency

Idea: Eliminate values of variable domain that can never satisfy a specified constraint (an arc)

$C_{12} = \{\langle 1,3 \rangle, \langle 1,4 \rangle, \langle 2,1 \rangle\}$

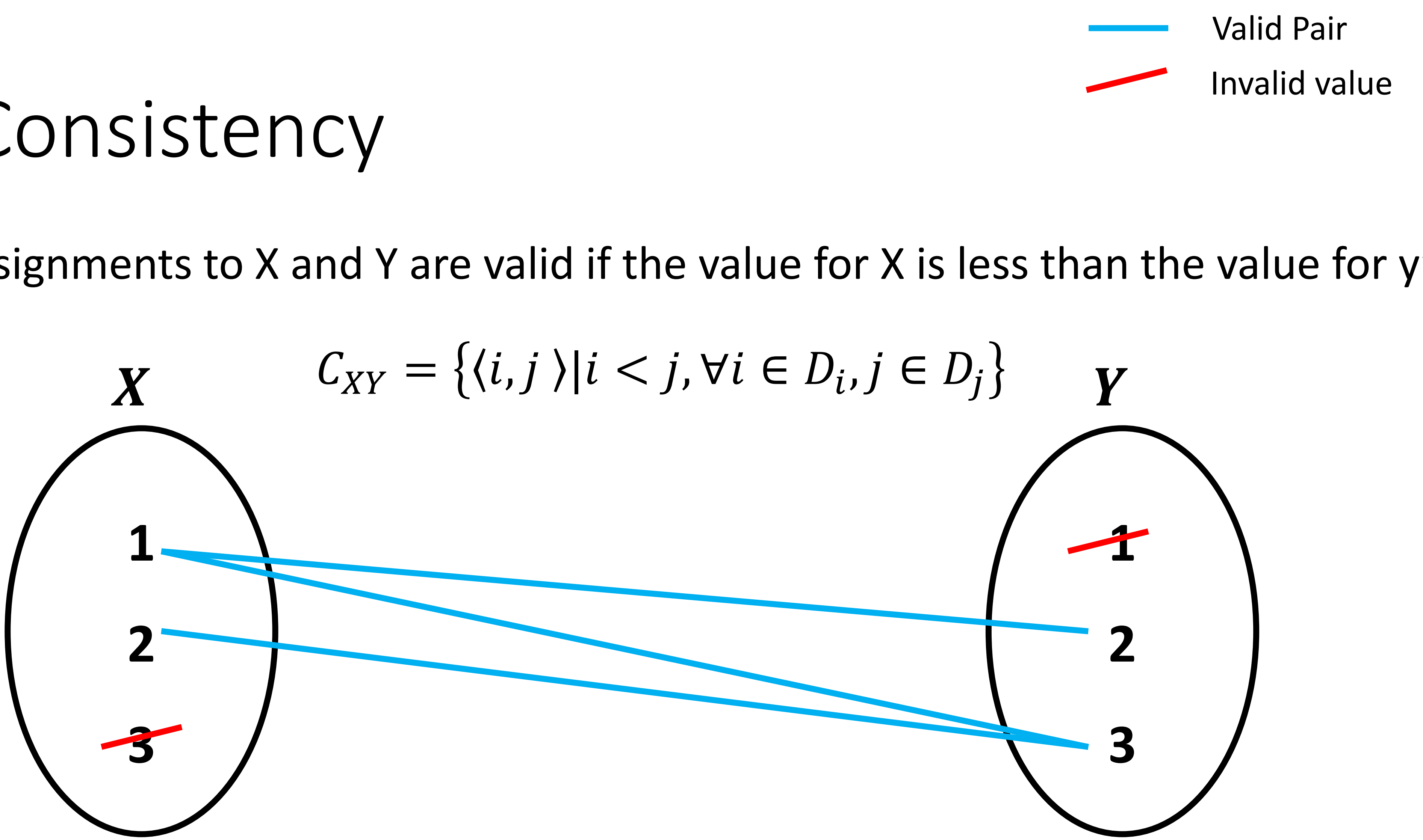


Definition: arc $\langle x_i, x_j \rangle$ is arc consistent if $\langle x_i, x_j \rangle$ and $\langle x_j, x_i \rangle$ are directed arc consistent, i.e. $\forall a_i \in D_i, \exists\, a_j \in D_j | \langle a_i, a_j \rangle \in C_{ij}$.

Arc Consistency

“Assignments to X and Y are valid if the value for X is less than the value for y”

$$C_{XY} = \{\langle i, j \rangle | i < j, \forall i \in D_i, j \in D_j\}$$



Revise: A directed arc consistency procedure

Definition: $\langle x_i, x_j \rangle$ is arc consistent if $\forall a_i \in D_i, \exists\ a_j \in D_j | \langle a_i, a_j \rangle \in C_{ij}$

Revise(x_i, x_j)

Input: Variables x_i and x_j with domains D_i and D_j and constraint relation R_{ij}

Output: Pruned D_i such that x_i is directed arc-consistent relative to x_j

1. FOR each $a_i \in D_i$
2. IF there is no $a_j \in D_j$ such that $\langle a_i, a_j \rangle \in R_{ij}$ THEN
3. Delete a_i from D_i
4. ENDIF
5. ENDFOR

Full Arc Consistency over all Constraints via Constraint Propagation

Definition: $\langle x_i, x_j \rangle$ is arc consistent if $\forall a_i \in D_i, \exists\ a_j \in D_j | \langle a_i, a_j \rangle \in C_{ij}$

Constraint Propagation:

To achieve (directed) arc consistency over CSP:

1. For every arc C_{ij} in CSP, with tail domain D_i, call Revise(x_i, x_j)
2. Repeat until quiescence:
    - If an element was deleted from D_i, then repeat Step 1 //(AC-1)

Full Arc-Consistency via AC-1

AC-1 (CSP)
Input: CSP = $\langle X, D, C \rangle$
Output: CSP', the largest arc-consistent subset of CSP

1. WHILE (domains are being changed)
2. FOR every $C_{ij} \in C$
3. Revise(x_i, x_j)
4. Revise(x_j, x_i)
5. ENDFOR
6. ENDWHILE

Full Arc Consistency over all Constraints via Constraint Propagation

Definition: $\langle x_i, x_j \rangle$ is arc consistent if $\forall a_i \in D_i, \exists\, a_j \in D_j | \langle a_i, a_j \rangle \in C_{ij}$

Constraint Propagation:

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2. Repeat until quiescence:
   - If an element was deleted from D_i, then repeat Step 1 //(AC-1)

Mid-lecture Break

Full Arc Consistency over all Constraints via Constraint Propagation

Definition: $\langle x_i, x_j \rangle$ is arc consistent if $\forall a_i \in D_i, \exists\, a_j \in D_j | \langle a_i, a_j \rangle \in C_{ij}$

Constraint Propagation:

To achieve (directed) arc consistency over CSP:

1. For every arc C_{ij} in CSP, with tail domain D_i, call Revise(x_i, x_j)
2. Repeat until quiescence:
   - If an element was deleted from D_i, then repeat Step 1 //(AC-1)
   - OR call Revise on each arc with head D_i //(AC-3)
     (use FIFO Q, remove duplicates)

Full Arc-Consistency via AC-3 (Waltz CP)

AC-3 (CSP)

Input: CSP = $\langle X, D, C \rangle$

Output: CSP', the largest arc-consistent subset of CSP

FOR every $C_{ij} \in C$

  $Q \leftarrow Q \cup \{\langle x_i, x_j \rangle, \langle x_j, x_i \rangle\}$

ENDFOR

While $Q \neq \emptyset$

  Select and delete arc $\langle x_i, x_j \rangle$ from Q

  Revise(x_i, x_j)

  IF Revise(x_i, x_j) caused a change to D_i

    $Q \leftarrow Q \cup \{\langle x_k, x_i \rangle | k \neq i, k \neq j\}$

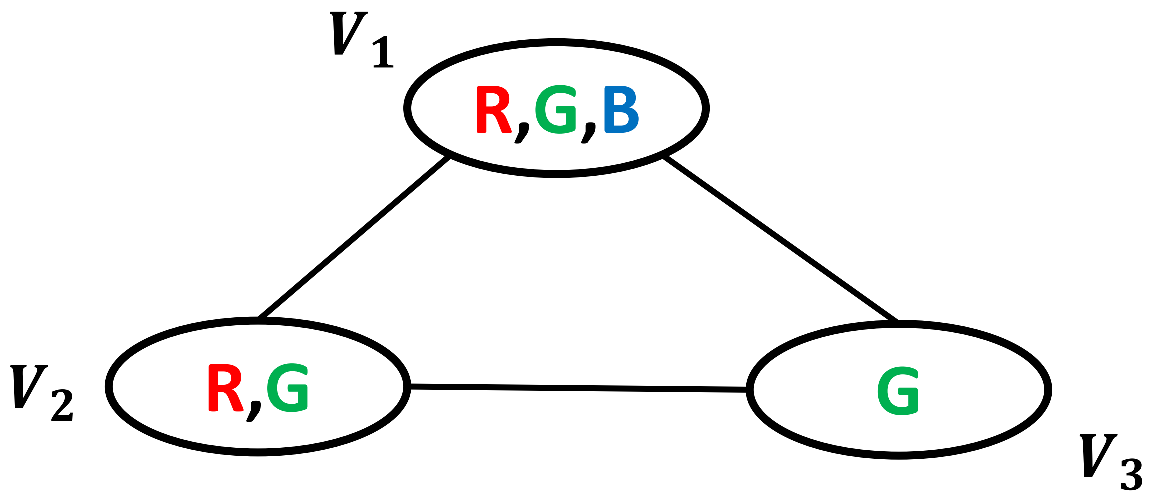
  ENDIF

ENDWHILE

Constraint Propagation Example AC-3

Graph Coloring

(initial domains)

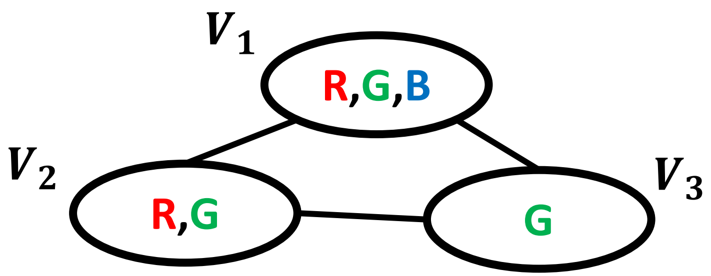


Each undirected arc denotes two directed arcs

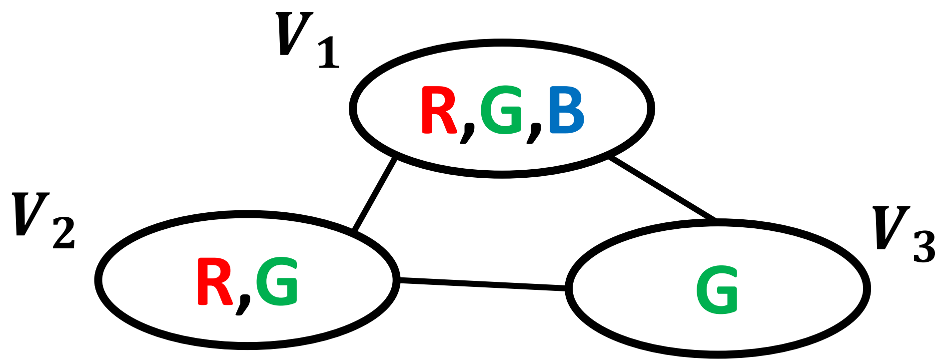
Constraint Propagation Example AC-3

Graph Coloring

(initial domains)



Arc examined	Value Deleted



Arcs to examine

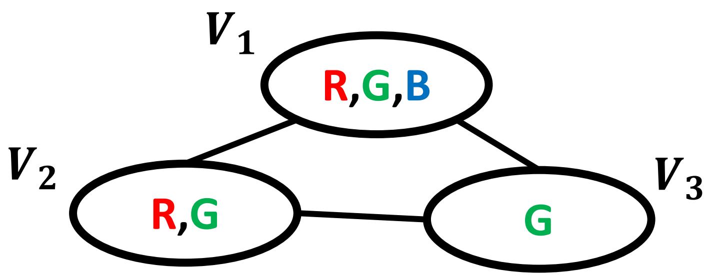
$a_{1-2}, a_{1-3}, a_{2-3}$

- Introduce queue of arcs to be examined
- Start by adding all arcs to the queue

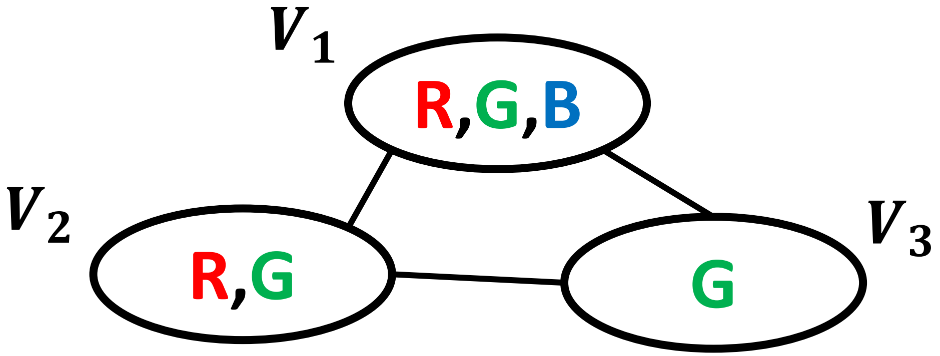
Constraint Propagation Example AC-3

Graph Coloring

(initial domains)



Arc examined	Value Deleted
$a_{1\to 2}$	



Arcs to examine

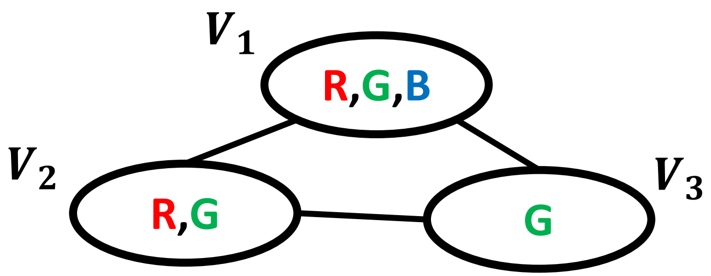
$a_{2\to 1}, a_{1-3}, a_{2-3}$

- Delete unmentioned tail values

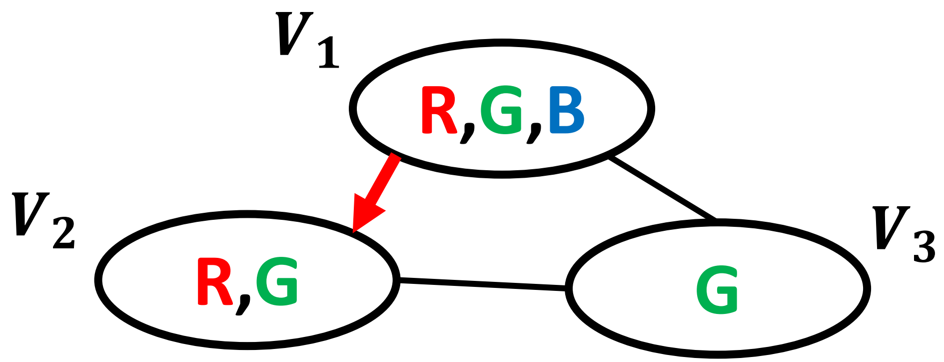
Constraint Propagation Example AC-3

Graph Coloring

(initial domains)



Arc examined	Value Deleted
$a_{1\to2}$	none



Arcs to examine

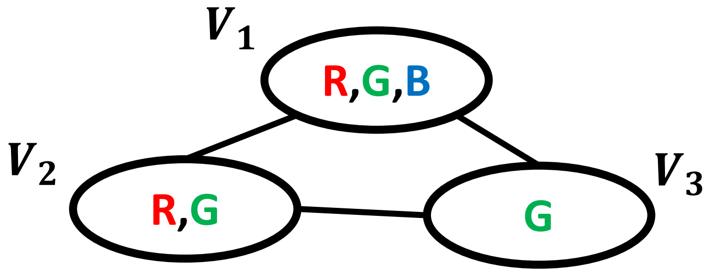
$a_{2\to1}, a_{1-3}, a_{2-3}$

- Delete unmentioned tail values

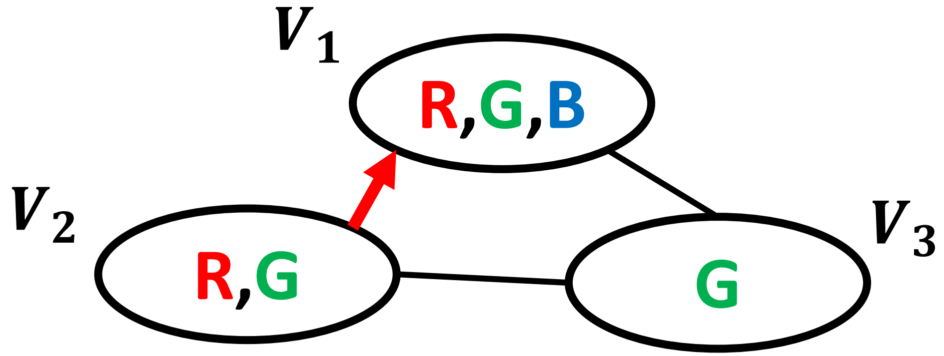
Constraint Propagation Example AC-3

Graph Coloring

(initial domains)



Arc examined	Value Deleted
$a_{1\to 2}$	none
$a_{2\to 1}$	



Arcs to examine

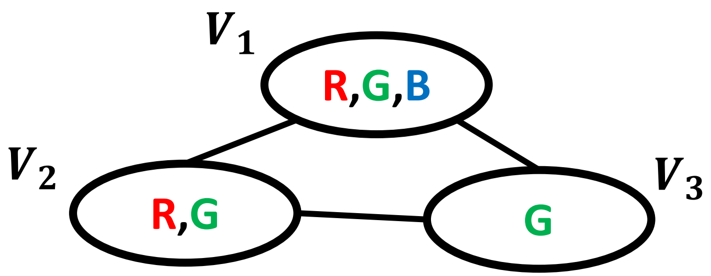
a_{1-3}, a_{2-3}

- Delete unmentioned tail values

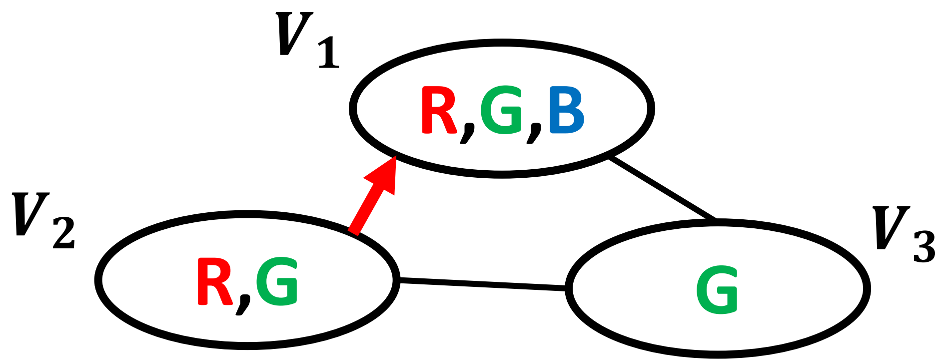
Constraint Propagation Example AC-3

Graph Coloring

(initial domains)



Arc examined	Value Deleted
$a_{1\to2}$	none
$a_{2\to1}$	none



Arcs to examine

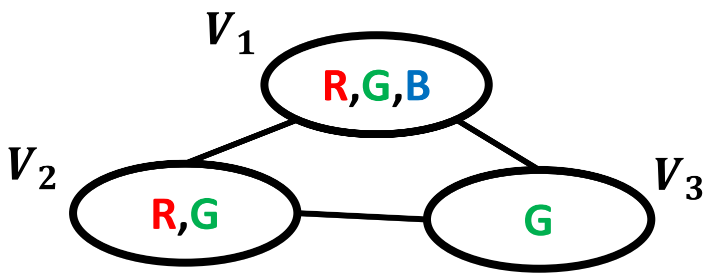
a_{1-3}, a_{2-3}

- Delete unmentioned tail values

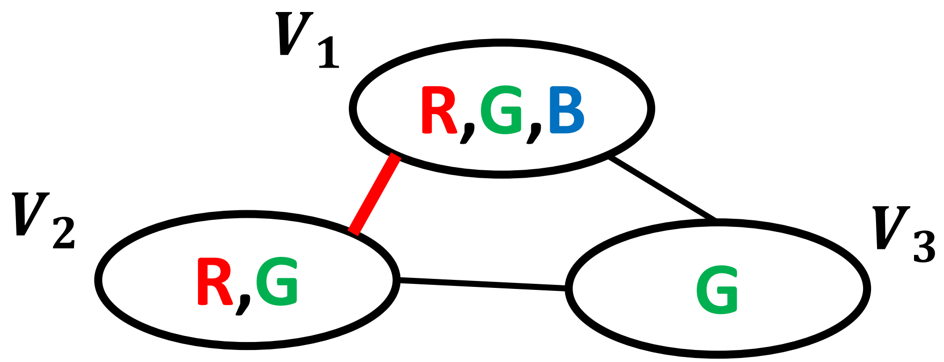
Constraint Propagation Example AC-3

Graph Coloring

(initial domains)



Arc examined	Value Deleted
a_{1-2}	none



Arcs to examine

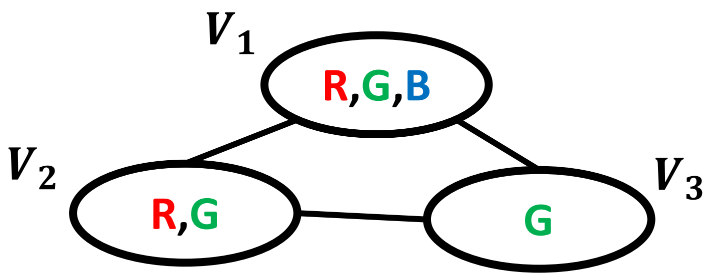
a_{1-3}, a_{2-3}

- Delete unmentioned tail values

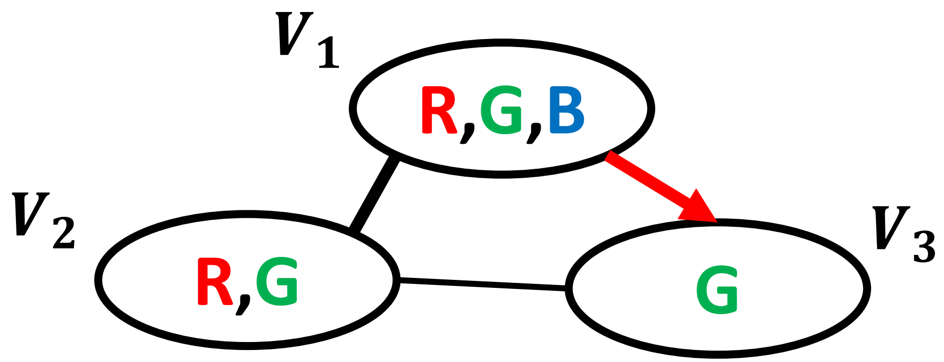
Constraint Propagation Example AC-3

Graph Coloring

(initial domains)



Arc examined	Value Deleted
a_{1-2}	none
$a_{1\to3}$	



Arcs to examine

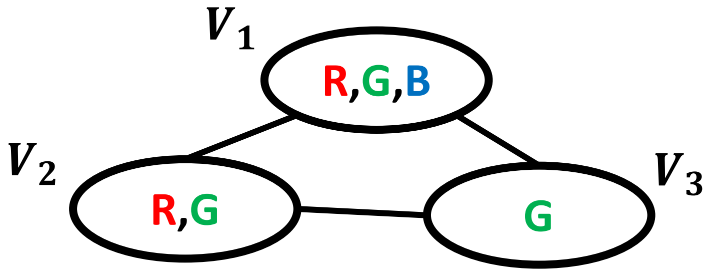
$a_{3\to1}, a_{2-3}$

- Delete unmentioned tail values

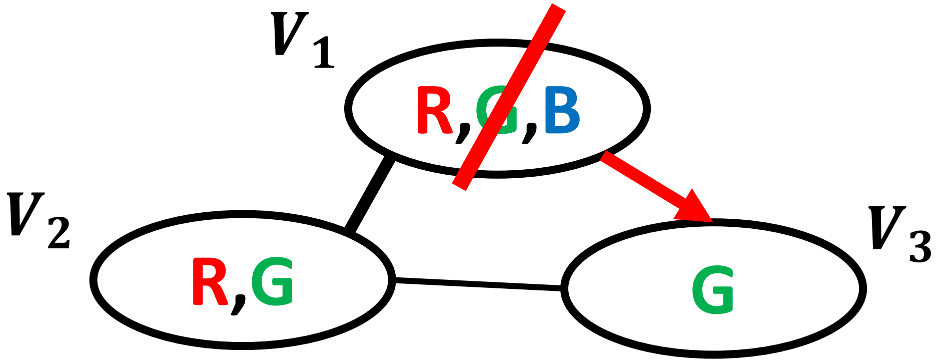
Constraint Propagation Example AC-3

Graph Coloring

(initial domains)



Arc examined	Value Deleted
a_{2-1}	none
$a_{1\to 3}$	$V_1(G)$



Arcs to examine

$a_{3\to 1}, a_{2-3}, a_{2\to 1}, a_{3\to 1}$

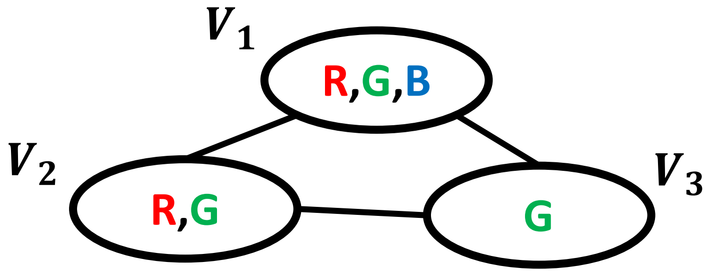
IF An element of a variable's domain is removed

THEN Add all arcs to that variable to the examination queue

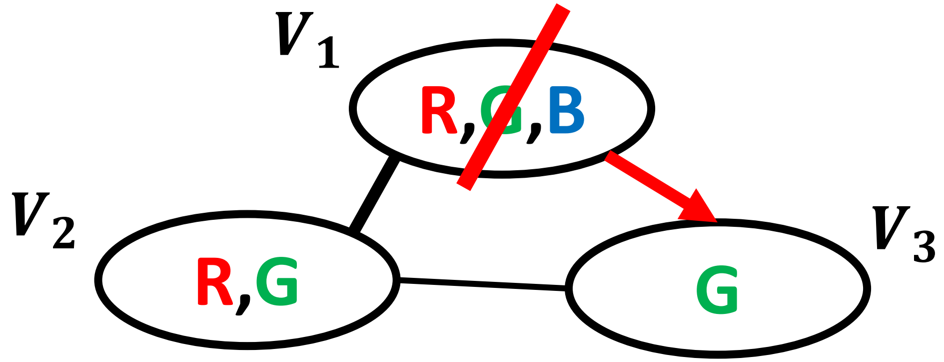
Constraint Propagation Example AC-3

Graph Coloring

(initial domains)



Arc examined	Value Deleted
a_{2-1}	none
$a_{1\to3}$	$V_1(G)$



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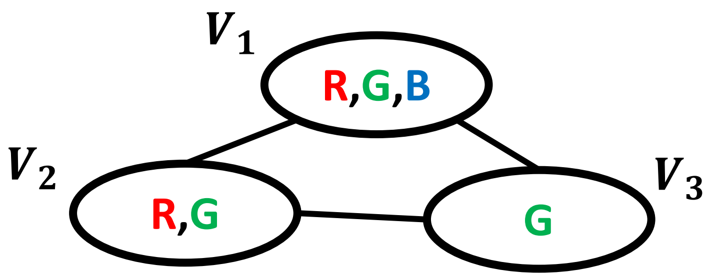
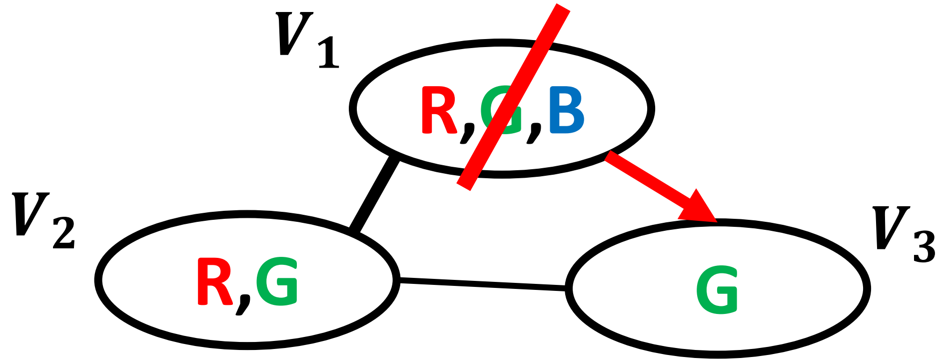
(initial domains)



Arc examined	Value Deleted
a_{2-1}	none
$a_{1\to3}$	$V_1(G)$



Arcs to examine

$a_{3\to1}, a_{2-3}, a_{2\to1}, \cancel{a_{3\to1}}$

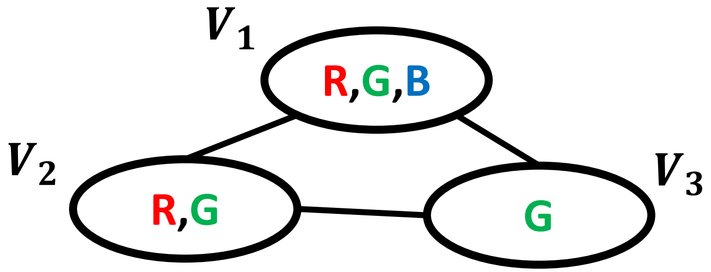
IF An element of a variable's domain is removed

THEN Add all arcs to that variable to the examination queue

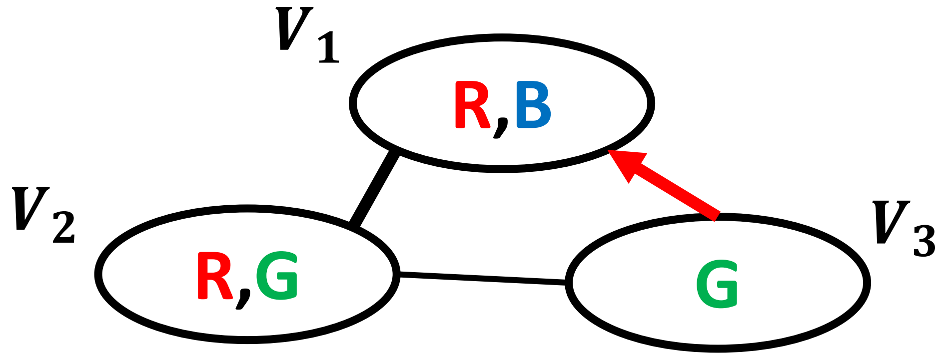
Constraint Propagation Example AC-3

Graph Coloring

(initial domains)



Arc examined	Value Deleted
a_{2-1}	none
$a_{1\to3}$	$V_1(G)$
$a_{3\to1}$	



Arcs to examine

$a_{2-3}, a_{2\to1}$

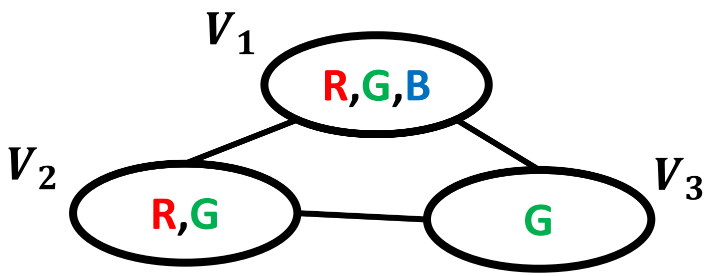
IF An element of a variable's domain is removed

THEN Add all arcs to that variable to the examination queue

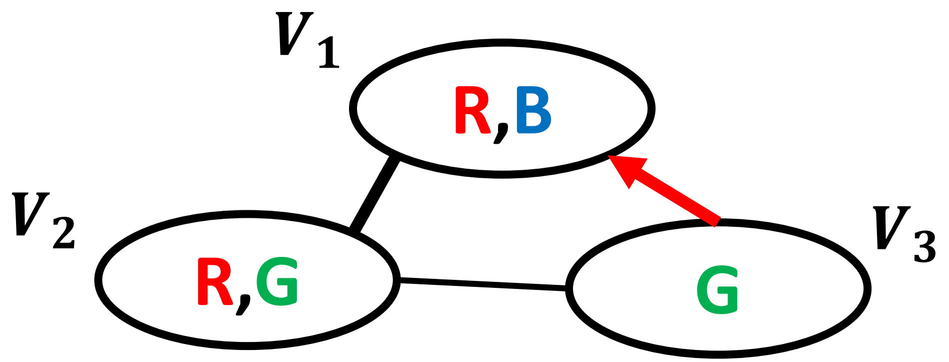
Constraint Propagation Example AC-3

Graph Coloring

(initial domains)



Arc examined	Value Deleted
a_{2-1}	none
$a_{1\to 3}$	$V_1(G)$
$a_{3\to 1}$	none



Arcs to examine

$a_{2-3}, a_{2\to 1}$

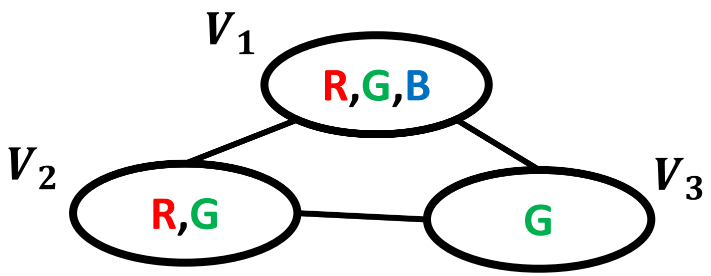
IF An element of a variable's domain is removed

THEN Add all arcs to that variable to the examination queue

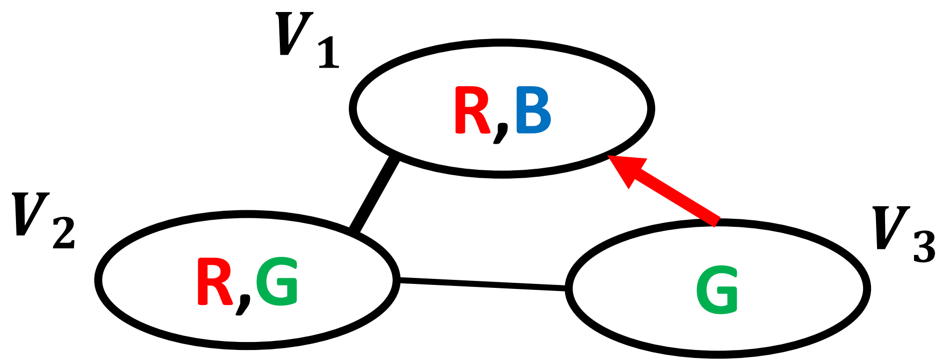
Constraint Propagation Example AC-3

Graph Coloring

(initial domains)



Arc examined	Value Deleted
a_{2-1}	none
a_{1-3}	$V_1(G)$



Arcs to examine

$a_{2-3}, a_{2\to1}$

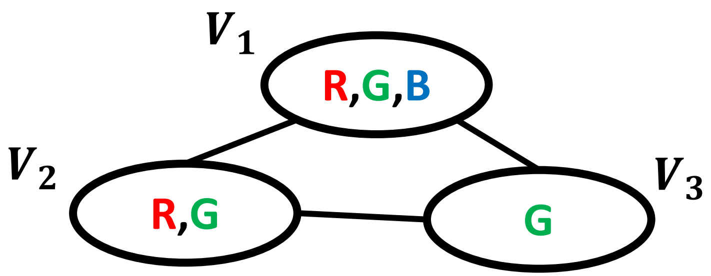
IF An element of a variable's domain is removed

THEN Add all arcs to that variable to the examination queue

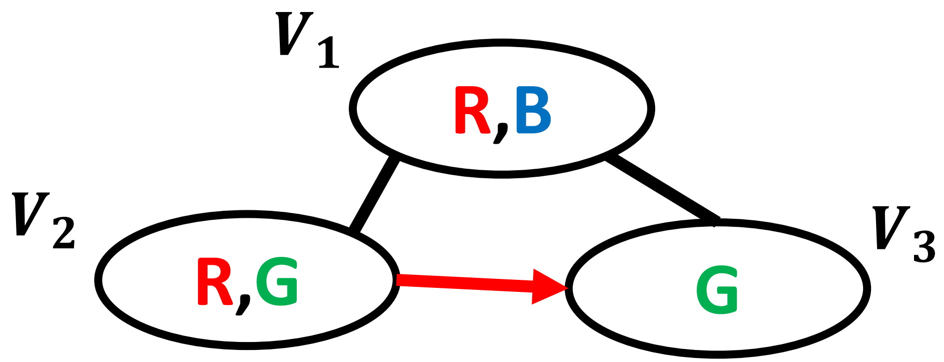
Constraint Propagation Example AC-3

Graph Coloring

(initial domains)



Arc examined	Value Deleted
a_{2-1}	none
a_{1-3}	$V_1(G)$
$a_{2\rightarrow 3}$	



Arcs to examine

$a_{3\rightarrow 2}, a_{2\rightarrow 1}$

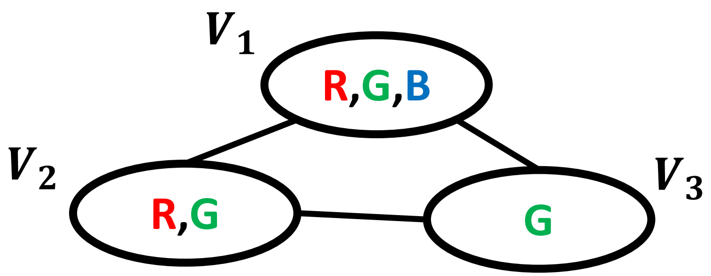
IF An element of a variable's domain is removed

THEN Add all arcs to that variable to the examination queue

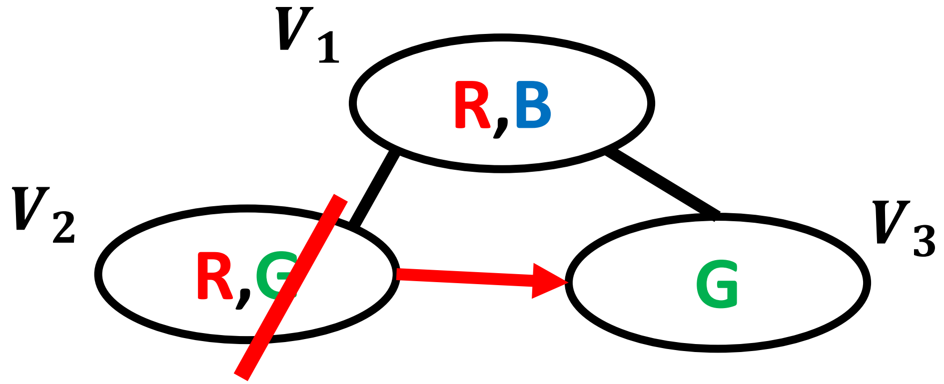
Constraint Propagation Example AC-3

Graph Coloring

(initial domains)



Arc examined	Value Deleted
a_{2-1}	none
a_{1-3}	$V_1(G)$
$a_{2\to3}$	



Arcs to examine

$a_{3\to2}, a_{2\to1}$

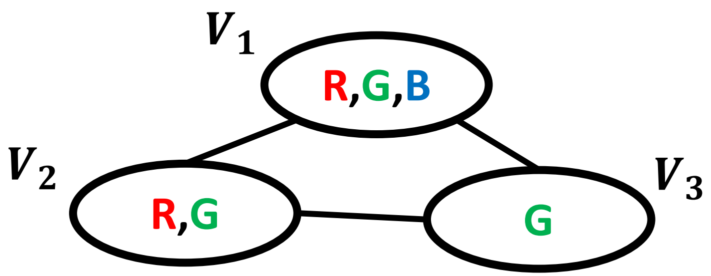
IF An element of a variable's domain is removed

THEN Add all arcs to that variable to the examination queue

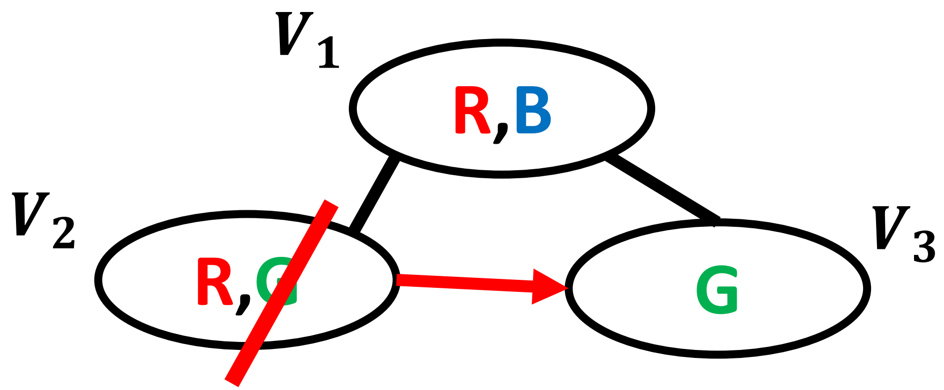
Constraint Propagation Example AC-3

Graph Coloring

(initial domains)



Arc examined	Value Deleted
a_{2-1}	none
a_{1-3}	$V_1(G)$
$a_{2\to 3}$	$V_2(G)$



Arcs to examine

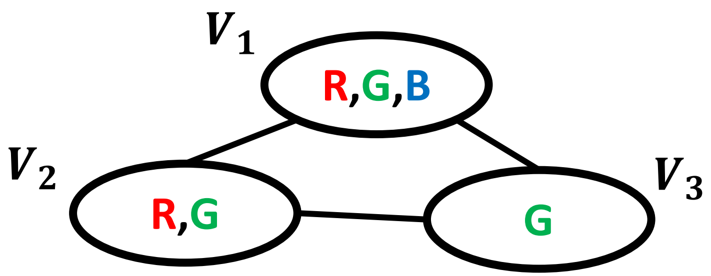
$a_{3\to 2}, a_{2\to 1}, a_{1\to 2}$

IF An element of a variable's domain is removed
THEN Add all arcs to that variable to the examination queue

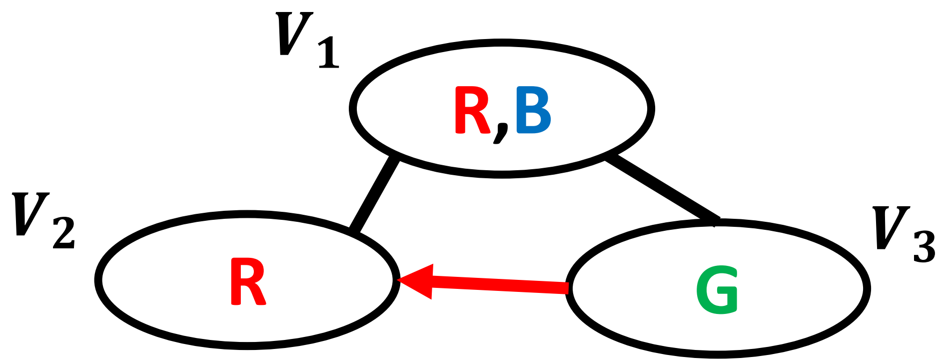
Constraint Propagation Example AC-3

Graph Coloring

(initial domains)



Arc examined	Value Deleted
a_{2-1}	none
a_{1-3}	$V_1(G)$
$a_{2\to3}$	$V_2(G)$
$a_{3\to2}$	



Arcs to examine

$a_{2\to1}, a_{1\to2}$

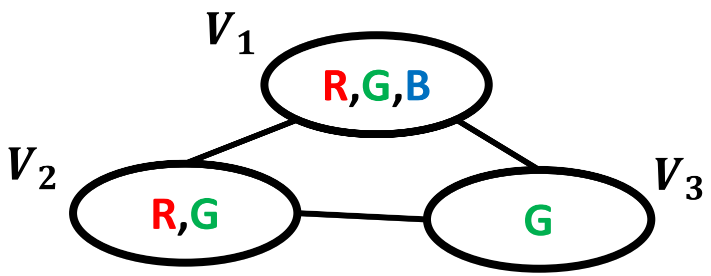
IF An element of a variable's domain is removed

THEN Add all arcs to that variable to the examination queue

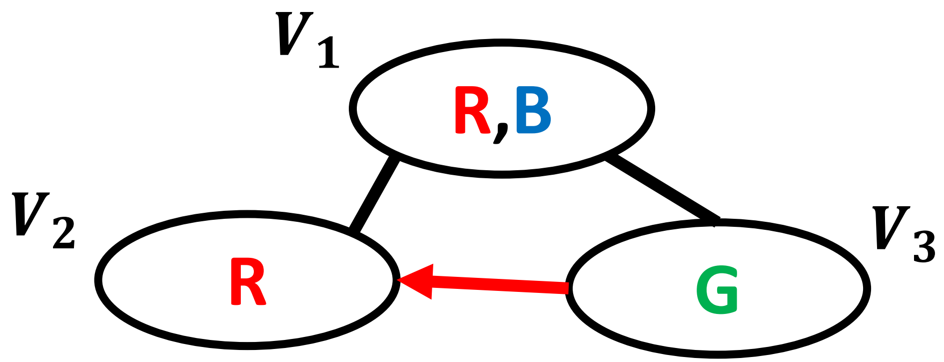
Constraint Propagation Example AC-3

Graph Coloring

(initial domains)



Arc examined	Value Deleted
a_{2-1}	none
a_{1-3}	$V_1(G)$
$a_{2\to 3}$	$V_2(G)$
$a_{3\to 2}$	none



Arcs to examine

$a_{2\to 1}, a_{1\to 2}$

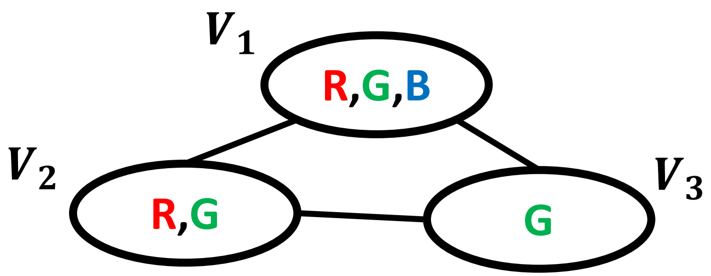
IF An element of a variable's domain is removed

THEN Add all arcs to that variable to the examination queue

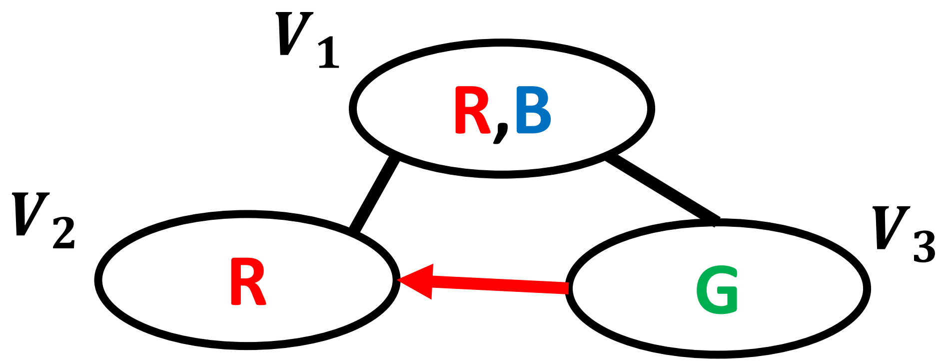
Constraint Propagation Example AC-3

Graph Coloring

(initial domains)



Arc examined	Value Deleted
a_{2-1}	none
a_{1-3}	$V_1(G)$
a_{2-3}	$V_2(G)$



Arcs to examine

$a_{2\to1}, a_{1\to2}$

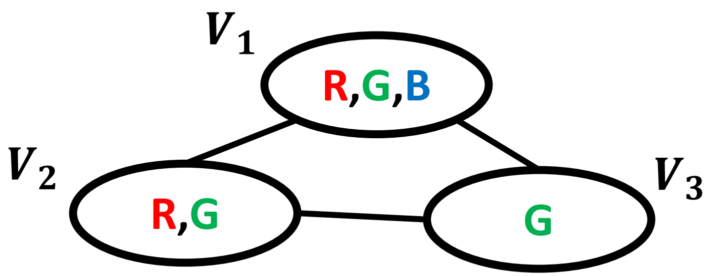
IF An element of a variable's domain is removed

THEN Add all arcs to that variable to the examination queue

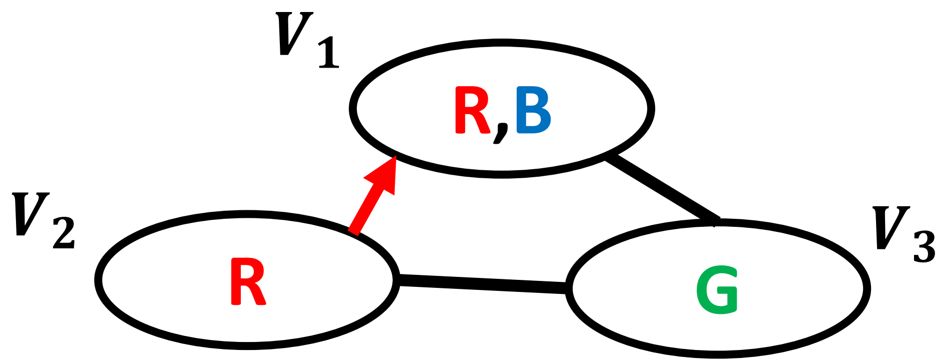
Constraint Propagation Example AC-3

Graph Coloring

(initial domains)



Arc examined	Value Deleted
a_{2-1}	none
a_{1-3}	$V_1(G)$
a_{2-3}	$V_2(G)$
$a_{2\to1}$	



Arcs to examine

$a_{1\to2}$

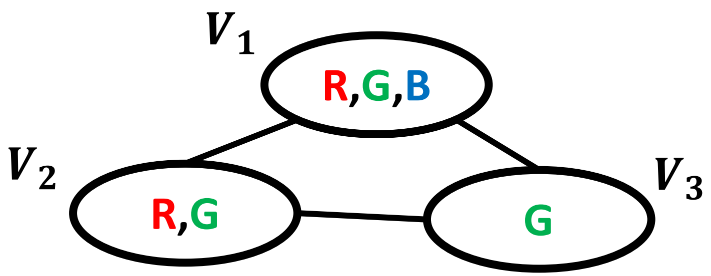
IF An element of a variable's domain is removed

THEN Add all arcs to that variable to the examination queue

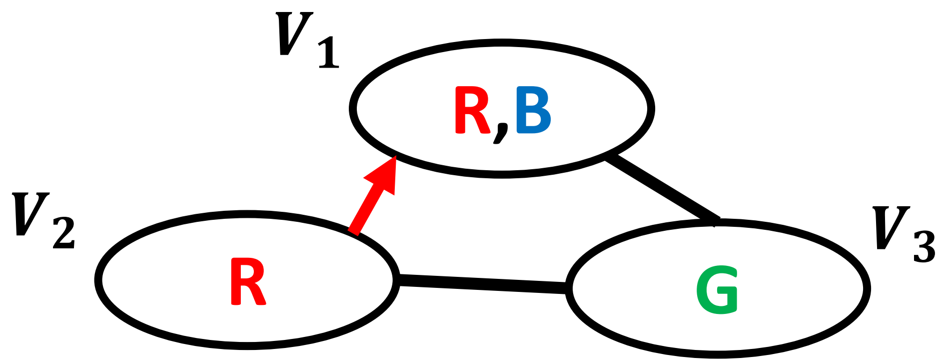
Constraint Propagation Example AC-3

Graph Coloring

(initial domains)



Arc examined	Value Deleted
a_{2-1}	none
a_{1-3}	$V_1(G)$
a_{2-3}	$V_2(G)$
$a_{2\to1}$	none



Arcs to examine

$a_{1\to2}$

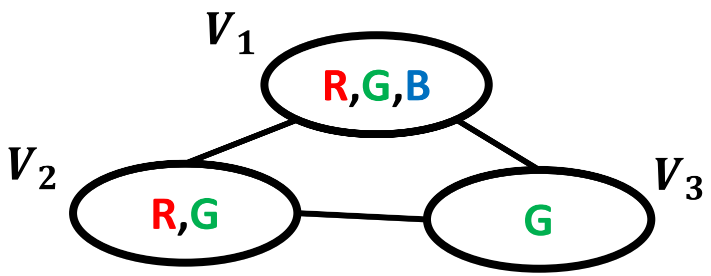
IF An element of a variable's domain is removed

THEN Add all arcs to that variable to the examination queue

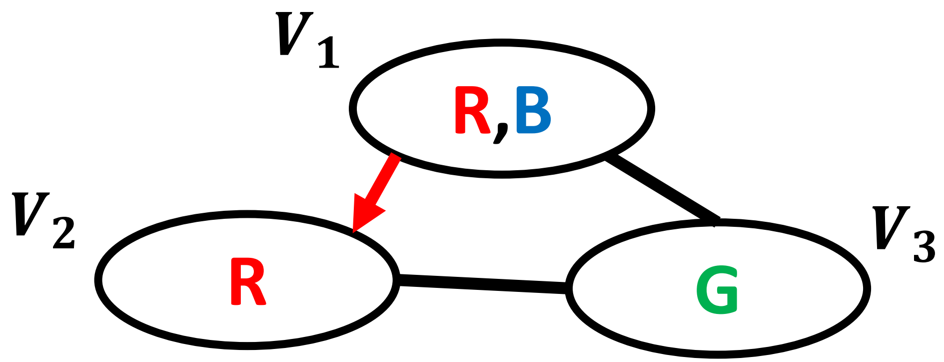
Constraint Propagation Example AC-3

Graph Coloring

(initial domains)



Arc examined	Value Deleted
a_{2-1}	none
a_{1-3}	$V_1(G)$
a_{2-3}	$V_2(G)$
$a_{2\to1}$	none
$a_{1\to2}$	



Arcs to examine

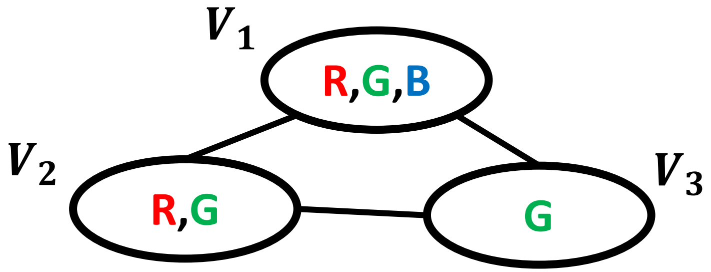
IF An element of a variable's domain is removed

THEN Add all arcs to that variable to the examination queue

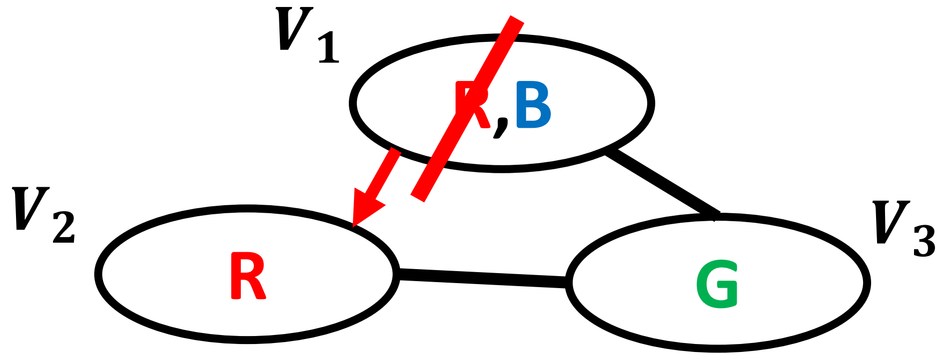
Constraint Propagation Example AC-3

Graph Coloring

(initial domains)



Arc examined	Value Deleted
a_{2-1}	none
a_{1-3}	$V_1(G)$
a_{2-3}	$V_2(G)$
$a_{2\to1}$	none
$a_{1\to2}$	$V_1(R)$



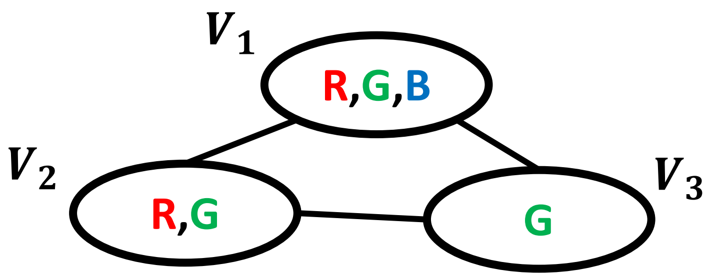
Arcs to examine

IF An element of a variable's domain is removed
THEN Add all arcs to that variable to the examination queue

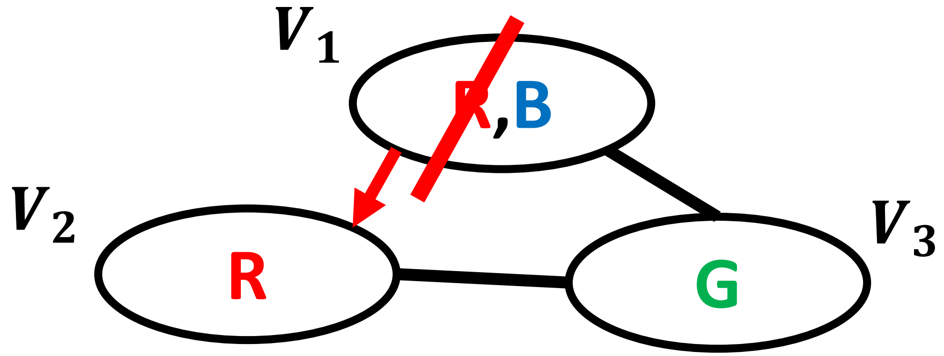
Constraint Propagation Example AC-3

Graph Coloring

(initial domains)



Arc examined	Value Deleted
a_{2-1}	none
a_{1-3}	$V_1(G)$
a_{2-3}	$V_2(G)$
$a_{2\to1}$	none
$a_{1\to2}$	$V_1(R)$



Arcs to examine

$a_{2\to1}, a_{3\to1}$

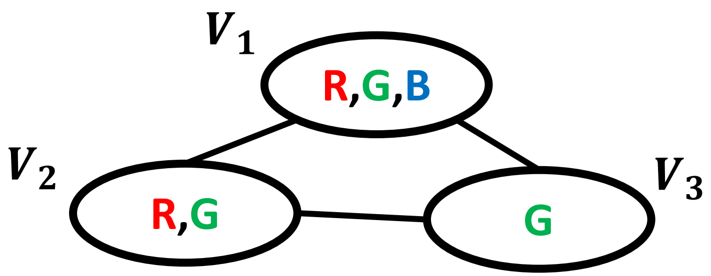
IF An element of a variable's domain is removed

THEN Add all arcs to that variable to the examination queue

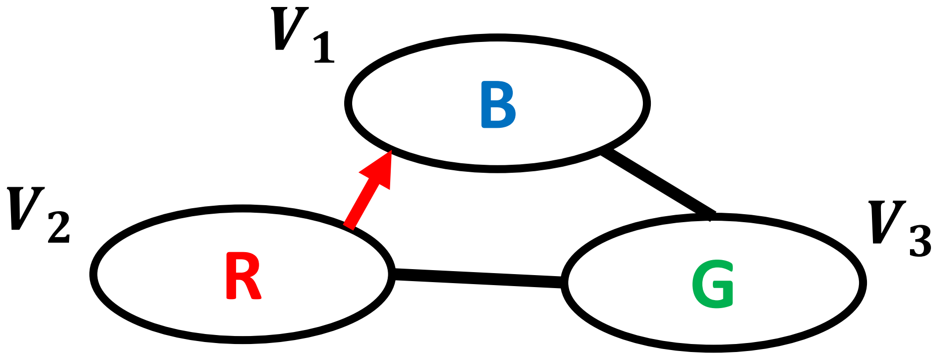
Constraint Propagation Example AC-3

Graph Coloring

(initial domains)



Arc examined	Value Deleted
a_{2-1}	none
a_{1-3}	$V_1(G)$
a_{2-3}	$V_2(G)$
a_{2-1}	$V_1(R)$
$a_{2\rightarrow 1}$	



Arcs to examine

$a_{3\rightarrow 1}$

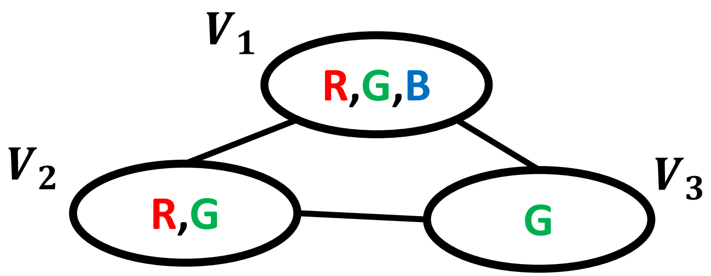
IF An element of a variable's domain is removed

THEN Add all arcs to that variable to the examination queue

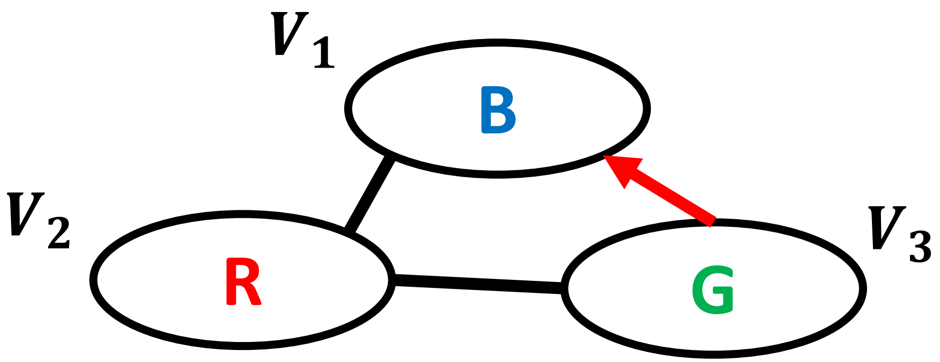
Constraint Propagation Example AC-3

Graph Coloring

(initial domains)



Arc examined	Value Deleted
a_{2-1}	none
a_{1-3}	$V_1(G)$
a_{2-3}	$V_2(G)$
a_{2-1}	$V_1(R)$
$a_{2\to1}$	none
$a_{3\to1}$	



Arcs to examine

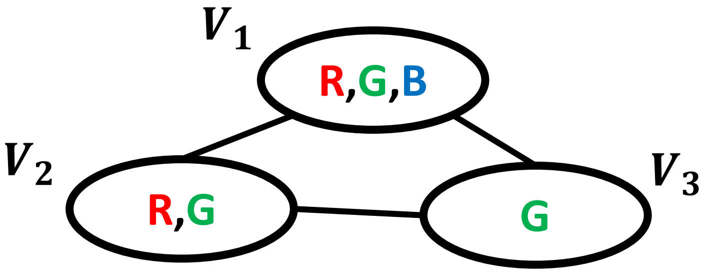
IF An element of a variable's domain is removed

THEN Add all arcs to that variable to the examination queue

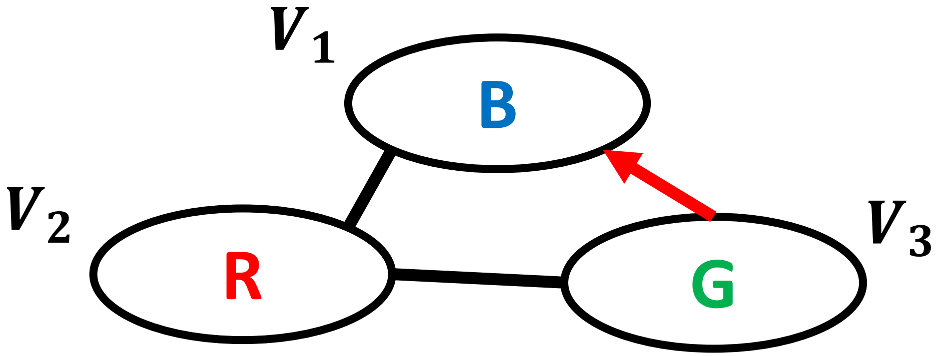
Constraint Propagation Example AC-3

Graph Coloring

(initial domains)



Arc examined	Value Deleted
a_{2-1}	none
a_{1-3}	$V_1(G)$
a_{2-3}	$V_2(G)$
a_{2-1}	$V_1(R)$
$a_{2\rightarrow1}$	none
$a_{3\rightarrow1}$	none



Arcs to examine

IF examination of queue is empty
THEN arc (pairwise) consistent

To solve CSPs, we combine arc consistency and search

Today

1. Arc consistency (constraint propagation),
   - Eliminates values that are shown locally to not be a part of any solution
2. Search
   - Explores consequences of committing to particular assignments
   - Methods incorporating search:
     - Standard Search
     - Backtrack Search (BT)
     - BT with Forward Checking (FC)
     - Dynamic Variable Ordering (DVO)
     - Iterative Repair
     - Back jumping (BJ)

Next Time

Outline

- Constraint Satisfaction Problems
- Solving CSPs
  - Arc-consistency and propagation
  - Analysis of constraint propagation (next lecture)
  - Search student (next lecture)
- Case Study: Scheduling

Real World Example: Scheduling as a CSP

Choose time of activities:

- Shifts of nurses in the Labor & Delivery Ward
- Classes taken for a degree
- Tennis matches for Wimbledon

Variables	activities
Domains	possible start times (or “chunks” of time)
Constraints	1) Activities that use the same resource cannot overlap in time
	2) Prerequisites are satisfied (round of 128 before 64)

Case Study: Course Scheduling

Given:

- 32 required courses
- 8 terms (Fall 2018, Spring 2019, ...)

Find: a feasible schedule

Constraints:

- Pre-requisites are satisfied
- Courses offered only during certain times
- Limited number of courses can be taken per term (e.g., 4) and
- Avoid time conflicts between courses

Note: traditional CSPs are not for expressing (soft) preferences

e.g., minimize difficulty, balance subject areas, etc.

But see recent research on valued CSPs!

Alternate formulations for variables, domains

Variables

A. 1 variable per Term

(Fall ’18), (Spring ’19), …

Domains

All legal combinations of 4 courses, all offered during that term

A. 1 variable per Term-Slot

Subdivide each term

into 4 course slots

(Fall ‘18, 1), (Fall ‘18, 2),

(Fall ‘18, 3), (Fall ‘18, 4)

All courses offered during that term.

A. 1 variable per Course

Terms or term-slots

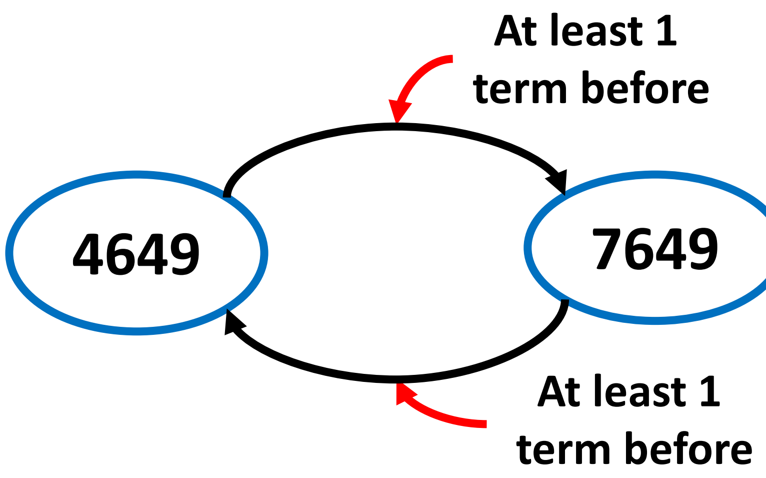
Term-slots make it easier to express the constraint limiting the number of courses per term

Encoding Constraints

Assume: Variables = Courses, Domains = term-slots

Constraints:

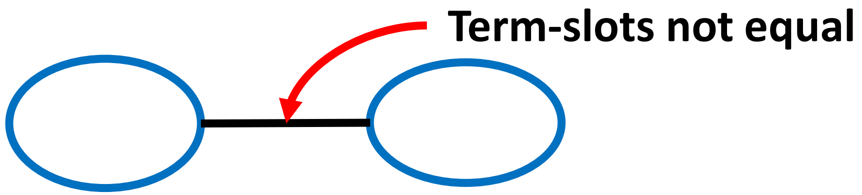
Pre-requisite →



For pairs of courses that must be ordered

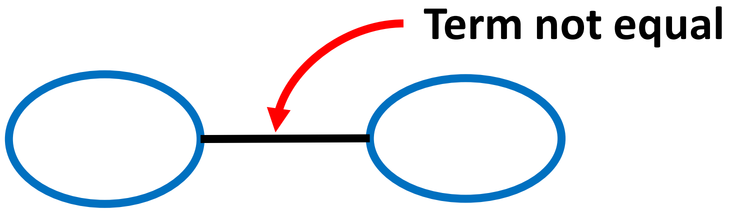
Courses offered only during certain terms → Filter domain

Limit # courses →



Use term-slots once

Avoid time conflicts →



For course pairs offered at same time or overlapping times

What you should be able to do…

- Provide the definition of a CSP as a 3-tuple
- Manually follow (i.e., by hand) and implement (i.e., write a computer program to perform) AC-1 and AC-3
- Model real-world problems as a CSP