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Due Date: 20 JAN 2022 at 11:59 Eastern Standard Time

PSet 2 – CS 4649/7649

CS 4649/7649 Robot Intelligence: Planning
Instructor: Matthew Gombolay

Instructions:

- You may work with one or more classmates on this assignment. However, all work must be your own, original work (i.e., no copy+pasting code). You must list all people you worked with and sources you used on the document you submit for your homework
- All final solutions to written problem must be enclosed by a box to make it easy and unambiguous for the graders what your final answer is. If your answer is illegible, you will not receive credit. If you answer is not boxed, you will not receive credit.

Problem 1:

Define the following terms:

- Soundness: It is when the algorithm, if it returns an answer, it will be true.
- Completeness: It is when the algorithm is able to terminate, if there exists a solution.

Problem 2:

Prove the following holds true: $\sum_{k=0}^{n-1} r^k = \frac{1-r^n}{1-r}$

Let $\lambda = r^0 + r^1 + \dots + r^{n-2} + r^{n-1}$

$-\quad \lambda r = \quad r^1 + \dots + r^{n-2} + r^{n-1} + r^n$

$\lambda - \lambda r = r^0 - r^n$

$\lambda(1-r) = 1 - r^n$

$\therefore \lambda = \frac{1-r^n}{1-r}$

$\therefore \sum_{k=0}^{n-1} r^k = \frac{1-r^n}{1-r}$

	State	g	h	f
	<S>	0	10	10
	<A, S>	7	4	11
	<B, S>	6	6	12
	<B, S>	6	6	12
	<C, A, S>	10	2	12
	<D, A, S>	10	4	14
	<D, B, S>	7	4	11
	<C, A, S>	10	2	12
	<G, B, S>	13	0	13
	<D, A, S>	10	4	14
	<C, D, B, S>	8	2	10
	<G, D, B, S>	12	0	12
	<C, A, S>	10	2	12
	<G, B, S>	13	0	13
	<D, A, S>	10	4	14
	<E, C, D, B, S>	10	1	11
	<G, D, B, S>	12	0	12
	<C, A, S>	10	2	12
	<G, B, S>	13	0	13
	<D, A, S>	10	4	14
	<G, E, C, D, B, S>	11	0	11
	<G, D, B, S>	12	0	12
	<C, A, S>	10	2	12
	<G, B, S>	13	0	13
	<D, A, S>	10	4	14
Final Path	<G, E, C, D, B, S>			

PAGE No.
DATE / /

A). No. of paths DFS examines.

if goal G

nodes examined

G = $2 = 2\times 1 \Rightarrow$ 3 $= \cancel{2\times 0}\ 1 + 2$

$6 = 2\times 3 \Rightarrow$ 5 $= \cancel{2\times}\ 3 + 2$

$10 = 2\times 5 \Rightarrow$ 7 $= \cancel{2\times}\ 5 + 2$

$\therefore$ If $G = 2N+2$.

No. of paths examined $= \dfrac{\cancel{2N+2}^{\,1}}{\cancel{2}_{\,1}} + 2 = N+3$

Ans: $\boxed{N+3}$ where $G = 2N+2$

B). Largest no. of paths

1+ no. of paths depending on level.

at ~~node~~ = 2, 6, 10, ..., 2N+2
a path is added.

$\therefore\ 2N+2 = 2 + (x-1)(4)$

$\therefore\ \dfrac{N}{2} + 1 = x$

$\therefore$ Max paths in queue $= 1 + \dfrac{N}{2} + 1$

Ans: $\boxed{2 + \dfrac{N}{2}}$ where $G = 2N+2$

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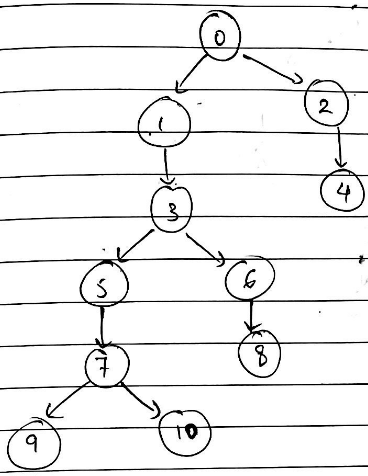
PAGE No.
DATE / /

c) No. ~~to~~ paths that BFS examines:



If $G = 2$, paths examined $= 3$.
$G = 6$ paths " $= 3 + 4 = 7$
$G = 10$ " " $= 7 + 4 = 11$

$\therefore$ let A.P. $= 2, 6, 10, \ldots, 2N+2$
index $= 1, 2, \ldots, x$

$\therefore 2N+2 = 2 + (x-1)(4)$

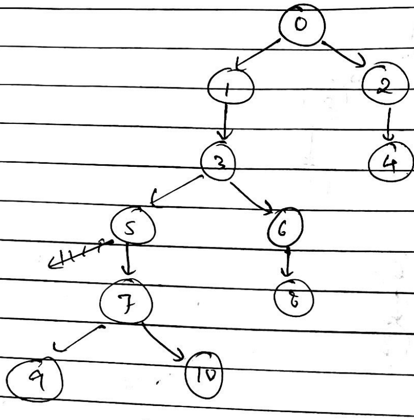
$\therefore \frac{N}{2} + 1 = x.$

Let A.P. $= 3, 7, 11, \ldots, \lambda$
index $= 1, 2, 3, \ldots, \frac{N}{2}+1$

$\therefore \lambda = 3 + \left(\frac{N}{\cancel{2}}\right)(\cancel{4}^{\,2}) = 2N + 3$

Ans: $\boxed{2N+3}$ where $G = 2N+2$

D) Largest queue size.

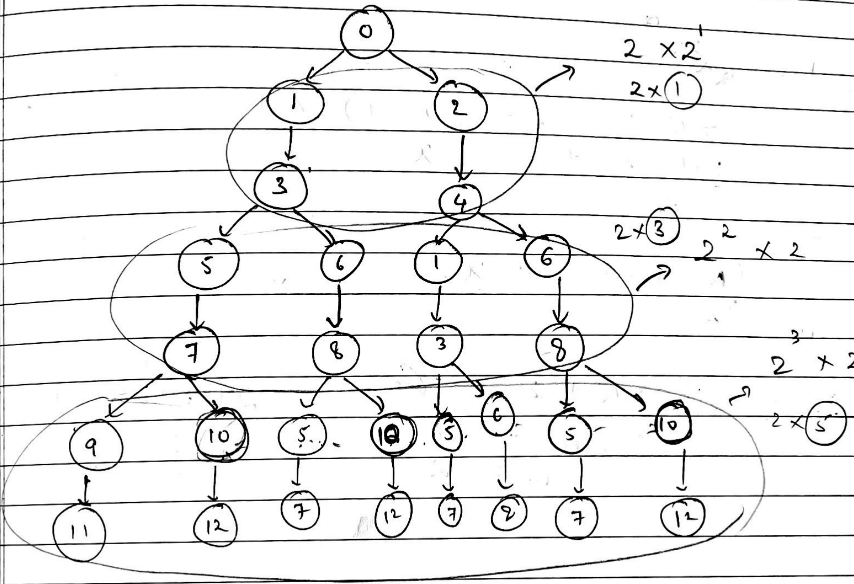


Ans: By inspection, we see that any level, the maximum number of paths is 3. But, if $G = 2$, max is 2.

∴ Ans: Max queue size $= \begin{cases} 3, & \text{if } G > 2 \\ 2, & \text{if } G = 2 \end{cases}$

where $G = 2N + 2$

E). Number of paths BFS examines:



Ans: By inspection, consider the A.P.

A.P. $\quad 1, 5, 10, \ldots, 2N$ ~~+2~~

index $\quad 1, 2, 3, \ldots, x.$

At each index number of paths explored correspond to $2^{index} \times 2$.

∴ $\cancel{2N + 2 = 1 + (x-1)(2)}$

∴ $\cancel{N = (x-1)}$ $\quad N = \cancel{x}$

∴ $\cancel{x = N+1}$

∴ ~~No of paths explored~~ $\cancel{= 2\sum_{i=1}^{N} 2^i + 3}$

PAGE No.
DATE / /

$$2N + \cancel{2} = \cancel{2} + (x-1)(4)$$

$$\frac{N}{2} + 1 = x$$

Consider one less term as the algorithm ends when it sees the first goal node.

$$\therefore x = \frac{N}{2}$$

$\therefore$ No. of paths explored $= 2\sum_{i=1}^{N/2} 2^i + 3$

$$\Rightarrow (2)\cdot\left[\frac{2\left(1-(2)^{N/2}\right)}{(1-2)}\right] + 3$$

$$\Rightarrow (2)^2\left(2^{N/2} - 1\right) + 3$$

$$\Rightarrow 2^{N/2+2} - 4 + 3$$

Ans: $\boxed{2^{(N/2+2)} - 1}$, where $G = 2N + 2$

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F) Using the graph from part E, by inspection, consider

A.P. = 2, 6, 10, ..., 2N + 2

index = 1, 2, 3, ..., $\frac{N}{2}+1$

No. of nodes = 2^{i} where i is the index in the above A.P.

∴ For $G = 2N + 2$, $i = \frac{N}{2} + 1$

∴ Max queue size = $2^{\left(\frac{N}{2}+1\right)}$