PROBLEM 1

$$T = \begin{bmatrix} 0.6 & 0.1 & 0.3 \\ 0.0 & 0.1 & 0.9 \\ 0.7 & 0.1 & 0.2 \end{bmatrix}$$

$$P = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

At steady state,

$$TP = P$$

Using Gauss Jordon elimination, set a 4x4 matrix such that

$$\begin{bmatrix} 1-0.6 & 0.1 & 0.3 & 0 \\ 0 & 1-0.1 & 0.9 & 0 \\ 0.7 & 0.1 & 1-0.2 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

↓ Plugging it in Gauss-Jordon elimination, ~~we get~~ after transposing it,

~~$P_1 = 0.353$~~

~~$P_2 = 0.333$~~

~~$P_4 = 0.333$~~

$P_1 = 0.572$

$P_2 = 0.1$

$P_3 = 0.327$

PROBLEM 2

For state S1

$$V^{\pi}(S1) = (0.9)\Big[(0.1)\big[(0.95)V^{\pi}(S1) - 0.1\big] + (0.2)\big[0.95\,V^{\pi}(S2) - 0.1\big] + (0.6)\big[0.95\,V^{\pi}(S3) - 0.1\big] + (0.1)(0.95\,V^{\pi}(S4) - 0.1)\Big] +$$

$$(0.1)\Big[(0.3)\big[0.95\,V^{\pi}(S1) - 0.1\big] + (0.2)(0.95\,V^{\pi}(S2) - 0.1) + (0.4)(0.95\,V^{\pi}(S3) - 0.1) + (0.1)(0.95\,V^{\pi}(S4) - 0.1)\Big]$$

$$V^{\pi}(S1) = V^{\pi}(S1)\big[(0.9)(0.1)(0.95) + (0.1)(0.3)(0.95)\big] + V^{\pi}(S2)\big[(0.9)(0.2)(0.95) + (0.1)(0.2)(0.95)\big] + V^{\pi}(S3)\big[(0.9)(0.6)(0.95) + (0.1)(0.4)(0.95)\big] + V^{\pi}(S4)\big[(0.9)(0.1)(0.95) + (0.1)(0.1)(0.95)\big] + (0.9)\big[(0.1)(-0.1) + (0.2)(-0.1) + (0.6)(-0.1) + (0.1)(-0.1)\big] + (0.1)\big[(-0.1)(0.3) + (0.2)(-0.1) + (0.4)(-0.1) + (0.1)(-0.1)\big]$$

Similarly, evaluating for all states, we get the following equations in matrix form.

$$\begin{bmatrix} -0.886 & 0.19 & 0.551 & 0.095 \\ 0.285 & -0.81 & 0.285 & 0.19 \\ 0.275 & 0.095 & -0.515 & 0.095 \\ 0.285 & 0.285 & 0.285 & -0.905 \end{bmatrix} \begin{bmatrix} V^{\pi}(s_1) \\ V^{\pi}(s_2) \\ V^{\pi}(s_3) \\ V^{\pi}(s_4) \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.1 \\ -0.67 \\ 0.1 \end{bmatrix}$$

$\therefore V^{\pi}(s) =$ ~~$\begin{bmatrix} -5.6 & -3.129 & -8.92 \\ -4.74 & -4.158 & -8.61 \\ -4.74 & -3.038 & -9.88 \\ -4.75 & -3.25 & -8.63 \end{bmatrix}$~~

$\therefore V^{\pi}(s) = \begin{bmatrix} 4.886 \\ 4.654 \\ 5.629 \\ 4.666 \end{bmatrix}$