CS 7649

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PROBLEM 1

Prove: AC-3's Time complexity is $O(ek^3)$ where k is the max. domain size and e is the number of constraints

For every $C_{ij} \in C$
  $Q \leftarrow Q \cup \{\langle x_i, x_j\rangle, \langle x_i, x_j\rangle\}$
END FOR

→ $O(e)$ since e is the no. of constraints.

① While $Q \neq 0$
② Select and delet arc $\langle x_i, x_j\rangle$ from Q
③ Revise (x_i, x_j) ~~from Q~~
④ If Revise (x_i, x_j) caused a change in D_i
⑤ $Q \leftarrow Q \cup \{\langle x_k, x_i\rangle \mid k \neq i, k \neq j\}$
⑥ END IF
END WHILE

→ The highest order term inside the while loop is Revise (x_i, x_j)

x_i $\begin{pmatrix} d_1 \\ d_2 \\ \vdots \\ d_k \end{pmatrix}$ — x_j (k times) — $\begin{pmatrix} d_1 \\ d_2 \\ \vdots \\ d_k \end{pmatrix}$ (k times)

∴ $O(\text{line } ③) = k^2$

Now, while loop runs until Q is not empty. Need to find out when Q is empty.

Consider initially, $len(Q) = \lambda$.

→ When $len(Q) = \lambda$, consider $\langle x_1, x_2 \rangle$

we assume that causes a change to x_1's domain. This will reduce x_1's domain size by 1.

∴ Initially, $len(D_1) = k$

Now, $len(D_1) = k - 1$

∴ Similarly, for the next iteration, it will pop the next arc, $\langle x_2, x_1 \rangle$ and reduce the domain size of x_2.

In the worst case, no assignment will be available, i.e. domain of each variable will be empty.

∴ $\forall x_1$ to x_e.

e times.

d_1 → Removed in itr = 1

d_2 → Removed in itr = 2

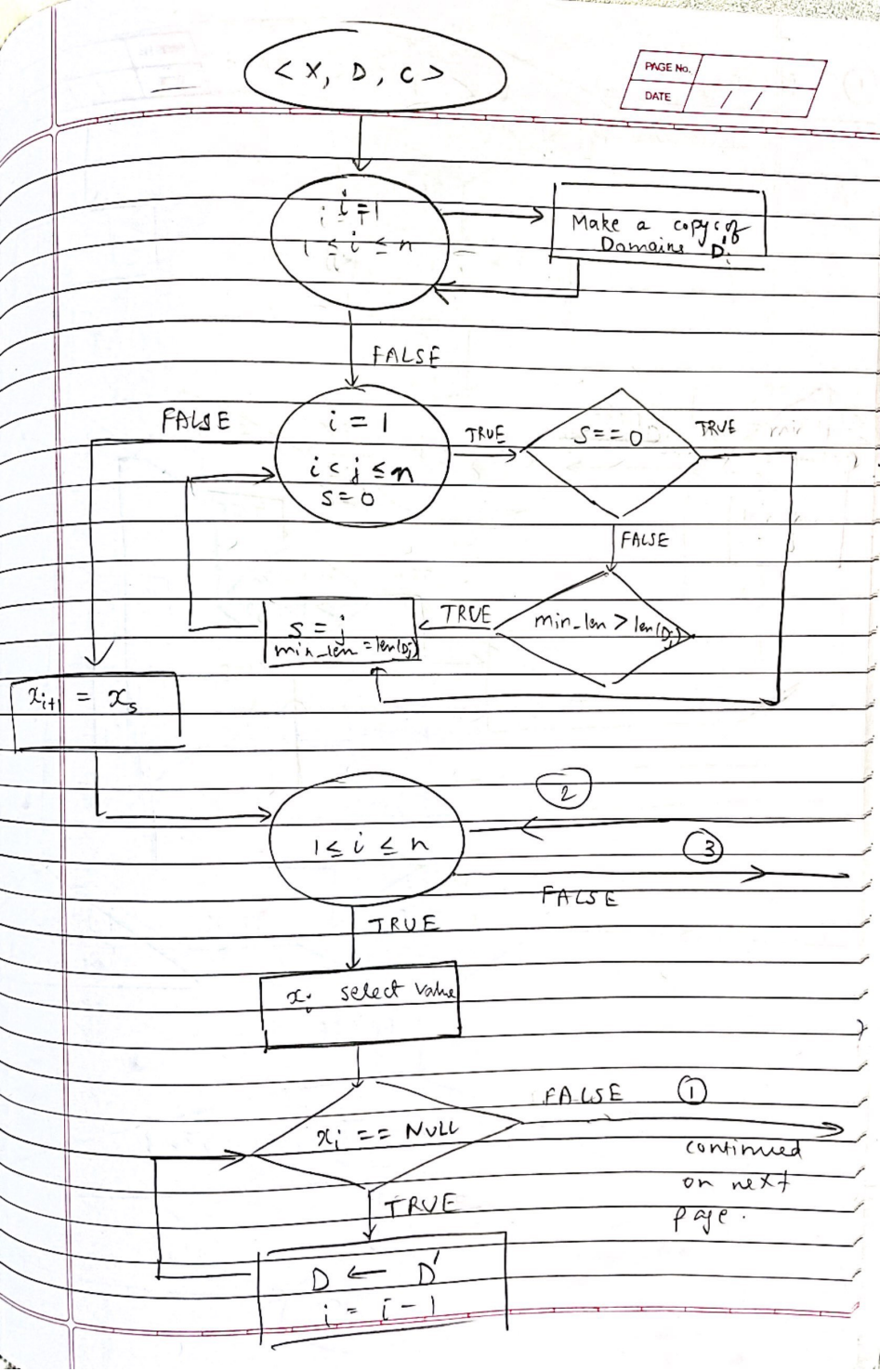
d_k → Removed in itr = k

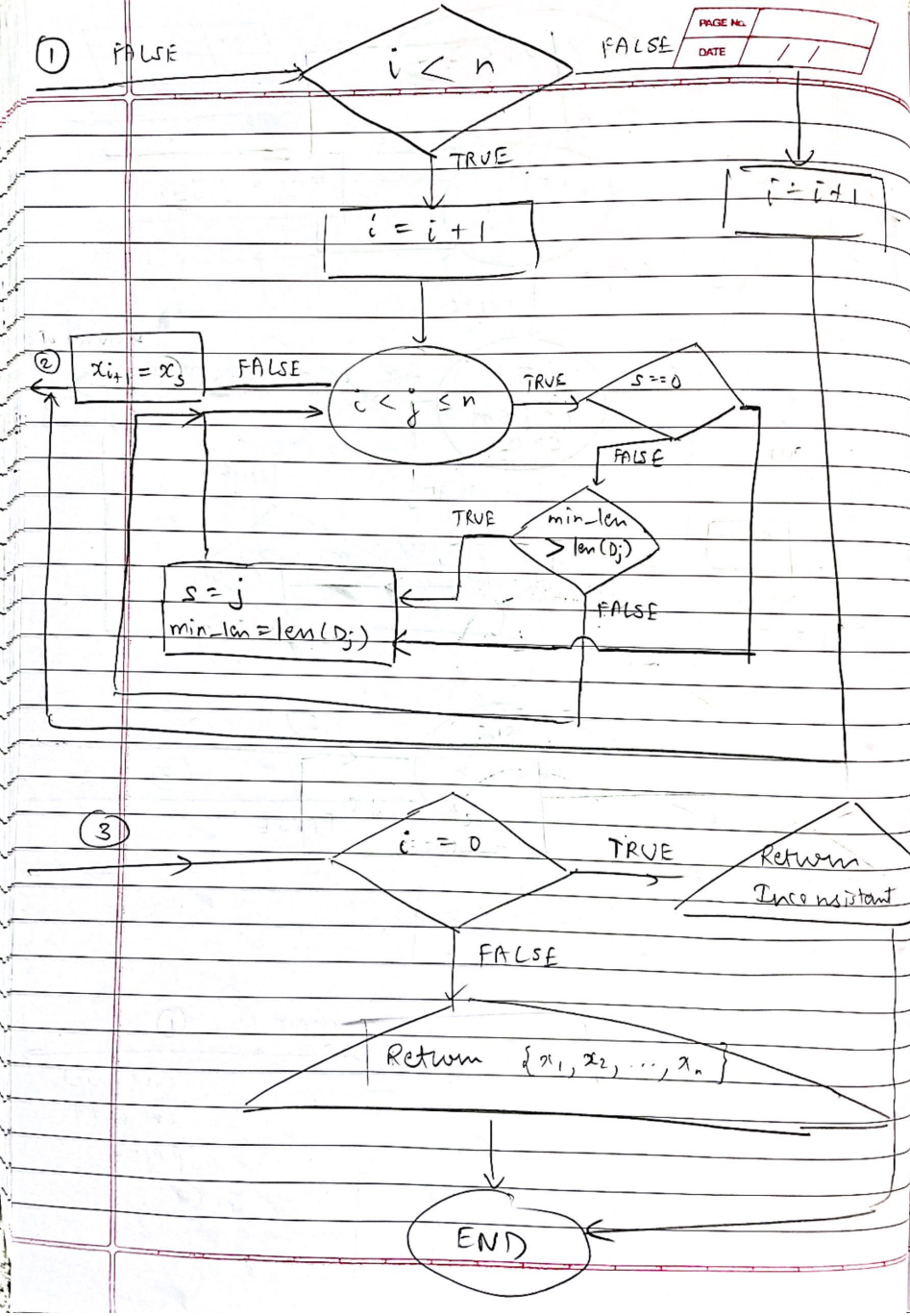
↓

Q will be empty.

∴ While loop runs $O(ek)$ times

∴ $O(AC-3) = O(ek * k^2) = O(ek^3)$





SELECT VALUE