Statistics Kingdom

Kruskal Wallis Test Calculator

Followed by post-hoc Dunn's test

Kruskal Wallis calculator with multiple comparisons, effect size, test power, outliers, and R syntax.

Significance level (α): 0.05

Outliers: Included

Effect size (offsets): 0.3

Correction Method: Bonferroni

Multiple comparisons method: Dunn's

Digits: 4

Step by step

- (selected) Enter raw data directly
- Enter raw data from excel

Compare	How to	List	Elaborate	How to design
3	4	3	5	5
4	4	2	2	5
4	4	2	5	5
5	4	3	4	4
5	5	2	2	5
5	3	2	3	5
4	4	3	2	4
4	4	3	4	3
3	4	4	2	2
2	4	2	3	3
3	4	4	4	3
4	3	4	4	2
3	3	2	2	4
3	3	2	2	4
4	2	4	3	5

Header: you may rename 'Group1', 'Group2', etc.
Data: use Enter or , (comma) or space as delimiters.
The tool ignores empty cells, non-numeric cells, or empty columns.

Calculate | Insert column | Delete column | Clear | Load last run | Load example

Reporting results in APA style

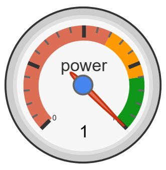
The Kruskal-Wallis H test indicated that there is a significant difference in the *dependent variable* between the different *groups*, $\chi^2(4) = 15.68$, $p = .003$, with a mean rank score of 202.88 for Compare solutions, 164.77 for How to implement, 169.83 for List, 143.16 for Elaborate, 164.64 for How to design.
The Post-Hoc Dunn's test using a Bonferroni corrected alpha of 0.005 indicated that the mean rank of the following pair is significantly different: $\mathbf{x_1}$**-**$\mathbf{x_4}$

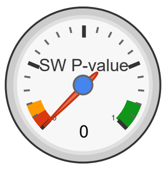
How to do with R?

Kruskal Wallis

Calculators

One-way ANOVA
Mann Whitney U
Levene's test
Two Way ANOVA
Two Sample T-Test
Two Sample Z-Test





Groups:	Compare solutions	How to implement	List	Elaborate	How to design
Skewness:	-0.5444	-0.606	-0.05406	0.1723	-0.09068
Skewness Shape:					
Excess kurtosis:	-0.3506	-0.2751	-1.0726	-1.0476	-1.0635
Tails Shape:	Potentially	Potentially			
Normality	6.573e-8	6.949e-10	0.00006975	0.00001495	0.00001554
Outliers:					
Median:	4	4	4	3	4
Sample size (n):	86	88	52	57	59
Rank sum (R):	17448	14500	8831	8160	9714
R^2/n:	3539915.163	2389204.545	1499741.558	1168168.421	1599352.475

Kruskal-Wallis-test, using Chi-Square(df:4) distribution (right-tailed)

[Validation]

1. H_0 hypothesis

Since the p-value < α, H_0 is rejected.
Some of the groups' mean ranks consider to be not equal.
In other words, the difference between the mean ranks of some groups is big enough to be statistically significant.
When selecting a value from each of the groups, there are some groups with a higher probability of containing the highest value than others.

2. P-value

The p-value equals **0.003483**, ($P(x\leq 15.6781) = 0.9965$). It means that the chance of type I error (rejecting a correct H_0) is small: 0.003483 (0.35%). The smaller the p-value the more it supports H_1.

3. Test statistic

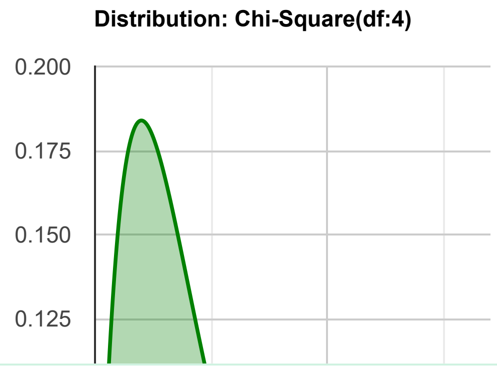
The test statistic **H** equals **15.6781**, which is not in the 95% region of acceptance: [0, 9.4877].

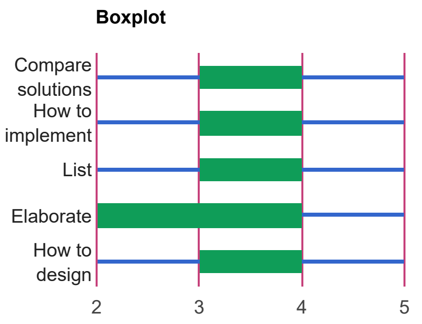
4. Effect size

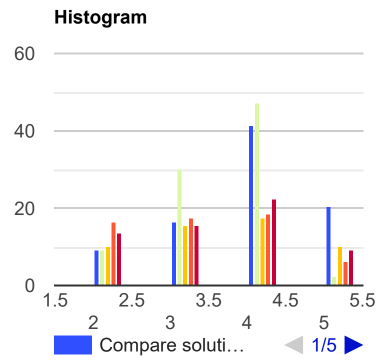
The observed effect size η^2 is **small**, **0.035**. This indicates that the magnitude of the difference between the average is small.

5. Multiple comparisons

The mean rank of the following pair is significantly different: $\mathbf{x_1}$**-**$\mathbf{x_4}$







Validation

Test power

The test priori power is strong **1**

Normality

The normality is **not** an assumption for the Kruskal-Wallis Test! We only check the normality to know if you could use a better test.
The normality was checked based on the Shapiro-Wilk Test. (α=0.05)
When running the SW test on the residuals, the p-value is 1.179e-8.
It is assumed that all the groups distribute normally or have a big sample size, at least 30.
It is valid to use the KW test on a normal data! But in this case you should consider the more powerfull ANOVA test calculator

Multiple comparisons

Compares any pair of groups using the Kruskal Wallis test. In this case, the test is identical to the Mann-Whitney U test with normal approximation.
If you won't correct the significance level (α) and all the comparisons are independent, then the type I error may get to 0.4013. $1 - (1- 0.05)^{10} = 0.4013$.
This is the worse case, since usually some aspects of the multiple tests are in common, and the type I error will be lower.
In this case the Bonferroni correction would be an over correction and would reduce the test power.
The corrected α using **Bonferroni** correction method is **0.005**.
m - the number of tests / pairs.
Corrected α = α / m = 0.05 / 10 = 0.005.

Pair	Mean Rank difference	Z	SE	Critical value	p-value	p-value/2
x_1-x_2	38.111	2.6849	14.1945	39.8444	0.007255	0.003627
x_1-x_3	33.0568	2.0102	16.4446	46.1606	0.04441	0.02221

x_1-x_4	59.7258	3.7355	15.9888	44.8812	0.0001874	0.00009368
x_1-x_5	38.2397	2.4164	15.825	44.4214	0.01567	0.007837
x_2-x_3	-5.0542	0.3087	16.3741	45.9625	0.7576	0.3788
x_2-x_4	21.6148	1.358	15.9162	44.6774	0.1745	0.08723
x_2-x_5	0.1287	0.008168	15.7517	44.2155	0.9935	0.4967
x_3-x_4	26.669	1.4856	17.9519	50.3915	0.1374	0.06869
x_3-x_5	5.1829	0.2911	17.8061	49.9824	0.771	0.3855
x_4-x_5	-21.4862	1.2358	17.3861	48.8032	0.2165	0.1083

Group	**How to implement**	**List**	**Elaborate**	**How to design**
Compare solutions	**38.11**	**33.06**	**59.73**	**38.24**
How to implement	0	**-5.05**	**21.61**	**0.13**
List	**-5.05**	0	**26.67**	**5.18**
Elaborate	**21.61**	**26.67**	0	**-21.49**

Calculation

Ranks

Group	**Value**	**Rank**
Compare solutions	2	**29**
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R_1 = 29+29+29+...+319+319+319= 17448.
R_2 = 29+29+29+...+223+319+319= 14500.
R_3 = 29+29+29+...+319+319+319= 8831.
R_4 = 29+29+29+...+319+319+319= 8160.
R_5 = 29+29+29+...+319+319+319= 9714.
$n = n_1 + n_2 + ... + n_k = 342$

$$H' = \frac{12}{n(n+1)}\left(\frac{R_1^2}{n_1}+\frac{R_2^2}{n_2}+...+\frac{R_k^2}{n_k}\right) - 3(n+1)$$

$$H' = \frac{12}{342(342+1)}\left(\frac{17448^2}{86}+\frac{14500^2}{88}+\frac{8831^2}{52}+\frac{8160^2}{57}+\frac{9714^2}{59}\right) - 3(342+1) = 14.0548$$

$$H = \frac{H'}{1 - 0.1035} = 15.6781$$

$MeanRank_1$ = 17448 / 86 = 202.8837.
$MeanRank_2$ = 14500 / 88 = 164.7727.
$MeanRank_3$ = 8831 / 52 = 169.8269.
$MeanRank_4$ = 8160 / 57 = 143.1579.
$MeanRank_5$ = 9714 / 59 = 164.6441.

Kruskal Wallis Test

The Kruskal-Wallis test also called one-way ANOVA on ranks is a non-parametric test.
Use the Kruskal-Wallis test calculator when your data doesn't meet the assumptions of the one-way ANOVA test calculator.

The kruskal wallis test online checks the null assumption that when selecting a value from each of 'n' groups, each of these groups will have an equal probability of containing the highest value.

Target: To check if the difference between the ranks of two or more groups is significant, using a sample data
When the groups have a similar distribution shape, the null assumption is stronger and states that the medians of the groups are equal. When performing the Kruskal Wallis test, we try to determine, if the difference between the ranks reflects a significant difference between the groups, or is due to the random noise inside each group. The Chi-square statistic is an approximation for the exact calculation.

Right-tailed the Kruskal Wallis test can use only the right tail. Why?

Hypotheses

H_0: MR_1 = .. = MR_k

H_1: not(MR_1 = .. = MR_k)

MR - Mean rank.

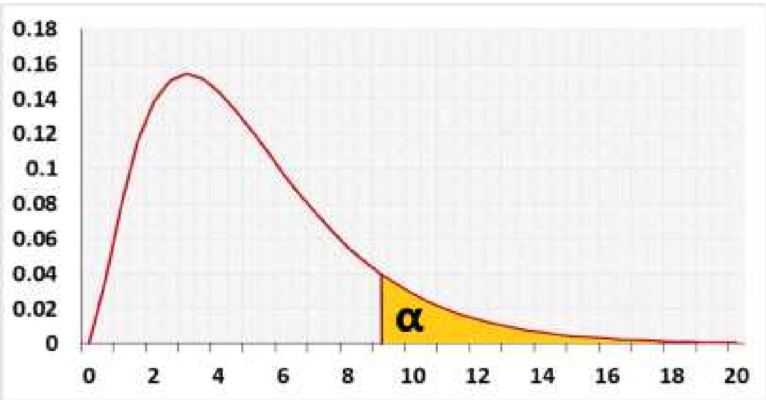
Test statistic

$$H' = \frac{12}{n(n+1)} \sum \left(\frac{R_j^2}{n_j}\right) - 3(n+1)$$

$$H = \frac{H'}{1 - \text{correction}}$$

R_j - the rank sum of group j.
n_j - the sample size of group j.
n - the total sample size across all groups, n = n_1 +...+ n_j.

χ^2 distribution



Assumptions

- Independent samples from independent groups. One subject can't be in more than one group.
- The dependent variable is ordinal variable or continuous variable
- Two or more groups (the independent variable is categorical variable with two or more values)

Required Sample Data

- Sample data from all compared groups.

Multiple comparisons

Even if we know that not all the ranks are equal, we don't know which groups are not equal, hence we run a Multiple comparisons test to compare all the pairs.
We support two methods for the Multiple comparisons:

- **Dunn's test calculator** - takes into consideration the total number of groups (k) even when comparing only two groups
- **Mann Whitney U test** - the calculator uses the normal approximation of the Mann Whitney U test and supports the same results as the Kruskal Wallis test with two groups.

R Code

The following R code should produce the same results

```
if(!"MultNonParam" %in% installed.packages()){install.packages("MultNonParam")}
library(MultNonParam)
if(!"dunn.test" %in% installed.packages()){install.packages("dunn.test")}
library(dunn.test)
x1 <-
c(3,4,4,5,5,5,4,4,3,2,3,4,3,3,4,4,4,4,4,4,3,4,5,4,4,3,4,5,5,4,4,3,5,5,4,4,4,5,5,2,2,4,4,4,2,2,4,2,3,4,3,3,5,3,5,4,4,5,3,4,5,4,5,5,4,3,4,3,5,4,4,4,5,4,3,5,4,4,5,2,4,4,4,4,2,2)
x2 <-
```

```
c(4,4,4,4,5,3,4,4,4,4,4,3,3,3,2,3,3,2,2,4,3,4,4,3,2,3,4,4,4,4,4,4,4,4,4,4,3,5,3,4,4,4,4,4,4,2,4,4,4,4,3,2,2,3,3,3,4,3,3,3,4,4,4,4,3,3,4,4,3,4,4,3,3,3,3,2,3,4,4,4,3,3,2,4,4,4,3,3)
x3 <- c(3,2,2,3,2,2,3,3,4,2,4,4,2,2,4,4,3,2,5,5,5,3,3,5,4,4,3,3,3,4,4,4,5,5,2,3,3,5,5,4,5,4,3,4,4,3,4,4,4,5,3,2)
x4 <- c(5,2,5,4,2,3,2,4,2,3,4,4,2,2,3,3,3,2,3,2,4,4,3,3,2,3,3,5,4,4,3,3,2,2,5,2,2,4,5,3,3,4,4,2,5,4,4,4,4,4,4,4,3,3,3,2,2)
x5 <-
c(5,5,5,4,5,5,4,3,2,3,3,2,4,4,5,3,4,2,3,2,3,4,4,4,4,3,2,4,4,3,2,2,2,2,5,3,5,5,4,4,4,4,4,2,3,3,3,3,4,2,4,4,2,4,2,3,4,3,4)
list1=list(x1,x2,x3,x4,x5)
kruskal.test(list1)
dunn.test(list1)
nreps <- c(86,88,52,57,59)
shifts <- c(0,0.3,0.6,0.8999999999999999,1.2)
kwpower(nreps,shifts,'normal')
```

The dunn.test results show half of the p-values, that you should compare to half alpha.
This calculator results show full p-values.
When using corrections, the dunn.test corrects the p-value while this calculator corrects the alpha.