

Kruskal Wallis Test Calculator

Followed by post-hoc Dunn's test

Kruskal Wallis calculator with multiple comparisons, effect size, test power, outliers, and R syntax.

Significance level (α):

0.05

Outliers:

Included

Effect size (offsets):

0.3

Correction Method:

Bonferroni

Multiple comparisons method

Dunn's

Digits:

4

☐ Step by step

- ☒ Enter raw data directly
- ☐ Enter raw data from excel

components	Achieve quality attributes	Implement technologies and components design"	analyze decision factors	Compare and evaluate technologies	Design using multiple technologies
5	4	4	2	2	5
4	5	4	3	3	5
2	5	4	3	3	5
2	4	3	2	2	5
3	3	4	2	3	3
4	3	5	3	2	2
4	3	3	4	3	4
3	4	5	2	4	2
3	4	4	2	4	4
3	4	4	2	3	3
4	3	2	2	5	4
3	4	3	2	3	2
2	5	4	4	3	3
3	4	2	3	3	4
3	4	5	5	3	4

Header: you may rename 'Group1', 'Group2', etc.
Data: use or (comma) or as delimiters.
The tool ignores empty cells, non-numeric cells, or empty columns.

Calculate

Insert column

Delete column

Clear

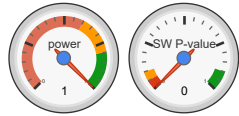
Load last run

Load example

Reporting results in APA style

The Kruskal-Wallis H test indicated that there is a significant difference in the *dependent variable* between the different *groups*, $\chi^2(5) = 12.66$, $p = .027$, with a mean rank score of 157.66 for Design using patterns and components, 185.67 for Achieve quality attributes using components design", 176.64 for Implement technologies and components design, 131.28 for Analyze decision factors, 134.82 for Compare and evaluate technologies, 188.26 for Design using multiple technologies.

[How to do with R?](#)



Groups:	Design using patterns and components	Achieve quality attributes using components design"	Implement technologies and components design	Analyze decision factors	Compare and evaluate technologies	Design using multiple technologies
Skewness:	4.01e-16	-0.331	-0.3147	0.491	0.3531	-0.4579
Skewness Shape:	<div>▲ Potentially</div>	<div>▲ Potentially</div>	<div>▲ Potentially</div>	<div>▲ Potentially</div>	<div>▲ Potentially</div>	<div>▲ Potentially</div>
Excess kurtosis:	-0.8518	-0.6474	-0.5555	-1.0816	0.1952	-0.4587
Tails Shape:	<div>▲ Potentially</div>	<div>▲ Potentially</div>	<div>▲ Potentially</div>	<div>▲ Potentially</div>	<div>▲ Potentially</div>	<div>▲ Potentially</div>
Normality	0.007317	0.000009375	1.37e-11	0.00008215	0.009882	0.0009398
Outliers:						
Median:	3	4	4	3	3	4
Sample size (n):	25	61	173	34	19	29
Rank sum (R):	3941.5	11326	30559	4463.5	2561.5	5459.5
R ² /n:	621416.89	2102922.557	5397991.22	585965.6544	345330.6447	1027797.94

Kruskal-Wallis-test, using Chi-Square(df:5) distribution (right-tailed)[\[Validation\]](#)

- Kruskal Wallis
- Calculators
- [One-way ANOVA](#)
- [Mann Whitney U](#)
- [Levene's test](#)
- [Two Way ANOVA](#)
- [Two Sample T-Test](#)
- [Two Sample Z-Test](#)

1. H₀ hypothesis

Since the p-value < α, H₀ is rejected.
Some of the groups' mean ranks consider to be not equal.

In other words, the difference between the mean ranks of some groups is big enough to be statistically significant.
When selecting a value from each of the groups, there are some groups with a higher probability of containing the highest value than others.

2. P-value

The p-value equals **0.02681**, (P(x≤12.6576) = 0.9732). It means that the chance of type I error (rejecting a correct H₀) is small: 0.02681 (2.68%). The smaller the p-value the more it supports H₁.

3. Test statistic

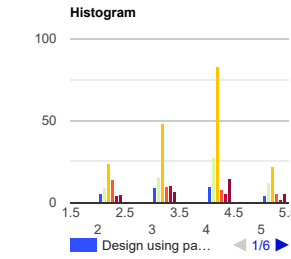
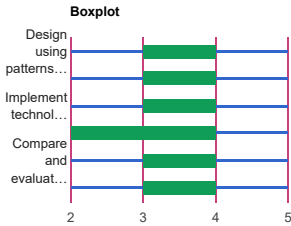
The test statistic **H** equals **12.6576**, which is not in the 95% region of acceptance: [0, 11.0705].

4. Effect size

The observed effect size η² is **small, 0.023**. This indicates that the magnitude of the difference between the average is small.

5. Multiple comparisons

The KW test identified that the mean ranks are not statistically equal but the multiple comparisons do not show enough evidence for a statistical difference between any pair of groups.This seems to be a contradiction, but it is not!. [more](#)
.Besides, the Bonferroni correction usually increasing the probability of false-positive (not rejecting incorrect null assumption).



Validation

Test power

The test priori power is strong **0.9996**

Normality

The normality is **not an assumption for the Kruskal-Wallis Test!** We only check the normality to know if you could use a better test.

The normality was checked based on the [Shapiro-Wilk Test](#). (α=0.05)

When running the SW test on the residuals, the p-value is 2.5e-10.

The ANOVA test is more powerful than the KW test and considered robust for moderate violation of the normality assumption. When checking the grrroups with a small sample size, less than 30, **3** groups don't have normally distributed data (the smaller p-value is 0.00732).

The KW test is probably the correct test.

Multiple comparisons

Compares any pair of groups using the Kruskal Wallis test. In this case, the test is identical to the Mann-Whitney U test with normal approximation.
If you won't correct the significance level (α) and all the comparisons are independent, then the [type I error](#) may get to 0.5367. 1 - (1- 0.05)¹⁵ = 0.5367.
This is the worse case, since usually some aspects of the multiple tests are in common, and the type I error will be lower.

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[illegible]

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$$R_1 = 28.5+28.5+28.5+...+318.5+318.5+318.5= 3941.5.$$
$$R_2 = 28.5+28.5+28.5+...+318.5+318.5+318.5= 11326.$$
$$R_3 = 28.5+28.5+28.5+...+318.5+318.5+318.5= 30559.$$
$$R_4 = 28.5+28.5+28.5+...+318.5+318.5+318.5= 4463.5.$$
$$R_5 = 28.5+28.5+28.5+...+223.5+223.5+318.5= 2561.5.$$
$$R_6 = 28.5+28.5+28.5+...+318.5+318.5+318.5= 5459.5.$$
$$n = n_1 + n_2 +...+ n_k = 341$$
$$H' = \frac{12}{n(n+1)} \left(\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} +...+ \frac{R_k^2}{n_k} \right) - 3(n+1)$$
$$H' = \frac{12}{341(341+1)} \left(\frac{3941.5^2}{25} + \frac{11326^2}{61} + \frac{30559^2}{173} + \frac{4463.5^2}{34} + \frac{2561.5^2}{19} + \frac{5459.5^2}{29} \right) - 3(341+1) = 11.3437$$
$$H = \frac{H'}{1-0.1038} = 12.6576$$
$$\text{MeanRank}_1 = 3941.5 / 25 = 157.66.$$
$$\text{MeanRank}_2 = 11326 / 61 = 185.6721.$$
$$\text{MeanRank}_3 = 30559 / 173 = 176.6416.$$
$$\text{MeanRank}_4 = 4463.5 / 34 = 131.2794.$$
$$\text{MeanRank}_5 = 2561.5 / 19 = 134.8158.$$
$$\text{MeanRank}_6 = 5459.5 / 29 = 188.2586.$$

Kruskal Wallis Test

The Kruskal-Wallis test also called one-way ANOVA on ranks is a non-parametric test.
Use the Kruskal-Wallis test calculator when your data doesn't meet the assumptions of the [one-way ANOVA test calculator](#).

The kruskal wallis test online checks the null assumption that when selecting a value from each of 'n' groups, each of these groups will have an equal probability of containing the highest value.

Target: To check if the difference between the ranks of two or more groups is significant, using a sample data
When the groups have a similar distribution shape, the null assumption is stronger and states that the medians of the groups are equal. When performing the Kruskal Wallis test, we try to determine, if the difference between the ranks reflects a significant difference between the groups, or is due to the random noise inside each group. The Chi-square statistic is an approximation for the exact calculation.

Right-tailed the Kruskal Wallis test can use only the right tail. [Why?](#)

Hypotheses

H₀: MR₁ = .. = MR_k
H₁: not(MR₁ = .. = MR_k)

MR - Mean rank.

Test statistic

H' =

12

R_j²

$$\overline{n(n+1)} \sum \left(\overline{n_j} \right) = 3(n+1)$$

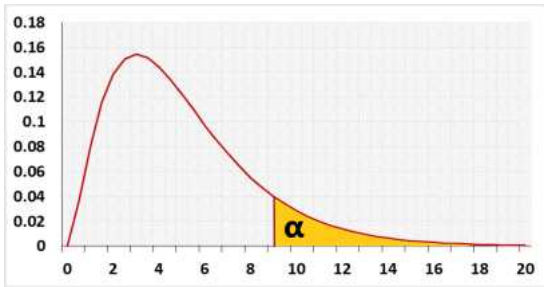
$$H = \frac{H'}{1 - \text{correction}}$$

R_j - the rank sum of group j .

n_j - the sample size of group j .

n - the total sample size across all groups, $n = n_1 + \dots + n_j$.

χ^2 distribution



Assumptions

- Independent samples from independent groups. One subject can't be in more than one group.
- The dependent variable is ordinal variable or continuous variable
- Two or more groups (the independent variable is categorical variable with two or more values)

Required Sample Data

- Sample data from all compared groups.

Multiple comparisons

Even if we know that not all the ranks are equal, we don't know which groups are not equal, hence we run a [Multiple comparisons](#) test to compare all the pairs.

We support two methods for the Multiple comparisons:

- **Dunn's test calculator** - takes into consideration the total number of groups (k) even when comparing only two groups
- **Mann Whitney U test** - the calculator uses the normal approximation of the Mann Whitney U test and supports the same results as the Kruskal Wallis test with two groups.

R Code

The following R code should produce the same results

```
if(!"MultNonParam" %in% installed.packages()){install.packages("MultNonParam")}
library(MultNonParam)
if(!"dunn.test" %in% installed.packages()){install.packages("dunn.test")}
library(dunn.test)
x1 <- c(5,4,2,2,3,4,4,3,3,3,4,3,2,3,3,4,2,5,2,3,4,4,5,4,4)
x2 <- c(4,5,5,4,3,3,3,4,4,4,3,4,5,4,4,3,3,5,4,4,3,3,4,5,2,4,4,2,4,2,5,5,4,5,5,4,5,4,2,4,2,3,4,3,3,2,4,4,4,4,3,4,4,4,4,3,5,3,3,2,2)
x3 <-
c(4,4,4,3,4,5,3,5,4,4,2,3,4,2,5,4,4,4,5,4,4,3,2,2,3,2,4,2,4,3,2,2,3,3,4,4,4,4,4,4,2,4,3,4,2,3,4,5,4,4,4,4,5,5,4,3,3,3,3,4,4,4,5,4,4,4,4,4,3,5,5,3,4,4,2,5,4,5,4,4,4,4,4,4,4,4,3,3,5,3,2,4,2,4,4,4,4,3,2,2,2,3,4,3,3,3,2,5,3,4,3,3,3,5,3,4,4,4,4,5,4,
x4 <- c(2,3,3,2,2,3,4,2,2,2,2,2,4,3,5,3,4,2,3,3,2,3,2,5,5,4,5,4,4,5,4,2,3,2)
x5 <- c(2,3,3,2,3,2,3,4,4,3,5,3,3,3,3,4,4,4,3)
x6 <- c(5,5,5,5,3,2,4,2,4,3,4,2,3,4,4,4,5,4,3,4,3,4,4,2,4,4,3,4)
list1=list(x1,x2,x3,x4,x5,x6)
kruskal.test(list1)
dunn.test(list1)
nreps <- c(25,61,173,34,19,29)
shifts <- c(0,0.3,0.6,0.8999999999999999,1.2,1.5)
kwpower(nreps,shifts,'normal')
```

The dunn.test results show half of the p-values, that you should compare to half alpha.

This calculator results show full p-values.

When using corrections, the `dunn.test` corrects the p-value while this calculator corrects the alpha.