Performance of Trading Strategies for Hedging Options

R Project 2025

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Abstract

This project studies and compares three hedging strategies for a six-month call on a European stock *BrightFuture* using a lognormal price model. We use a Monte Carlo engine (100,000 paths, daily rebalancing) to simulate the three strategies: no hedge, *stop-loss* — purchase 100 shares only when the underlying exceeds the strike, and a Black–Scholes delta hedge. We compute the option premium P_0 for each strategy, to find the 99% profit premiums in part b), then analyse the resulting *P&L* (profit and loss) distribution. ß Indifference prices are derived based on a utility function of Constant Absolute Risk Aversion (*CARA*) ußtility. Per our results, the unhedged trader's bank should charge ≈ €3400, with the stop-loss trader being quoted ≈ €1600, and the delta-hedger ≈ €360.

From the perspective of a risk-averse trader, the indifference prices we solve for in section d) push the strategies further apart; while the unhedged bank raises the quote considerably, the delta-hedger's premium rises only marginally. The stop-loss trader once again ends up in the middle, but still presents with a long tail of losses – rare, but significant – once the price slips back under the strike price. In the end, our simulations turn formulae into ready to interpret numbers, that show exactly how much protection each hedge will buy.

Contents

1 Introduction

1.1 Motivation

Generally, selling options is very appealing for an experienced trader because the seller receives an immediate premium. However, like with all financial derivatives, risk can be substantial — if the underlying stock's price rises sharply, so do the losses. Traders manage this risk by employing hedging methods, of which there are three in our project: a *stop-loss hedge*, where the trader buys/sells the stock only if its price crosses the strike; the *delta hedge*, which continuously adjusts a hedge based on the price movements of that stock; or *no hedge* — the trader accepts full exposure.

At this point, probabilistic modelling plays a very important role. It's challenging to evaluate how effective these hedges are, since future stock/asset prices behave randomly and under uncertainty. Therefore, we implement a Monte Carlo simulation, simulating a very large number (in our case 100,000) of possible future stock-price scenarios, letting us analyze and compare the risks of every hedging strategy.

1.2 Objectives and Methodology

The main goal of the project is to use R and especially the Monte Carlo method to meticulously analyze each strategy. Specifically, we implemented the following methods:

We generate 100,000 daily price paths and assume a lognormal model for stock prices, with an expected yearly return of 10% and a 20% standard deviation. To ensure we get realistic daily price movements, we generate random shocks from a normal distribution, then calculate the stock price at each step (through cumulative exponentiation) — this allows us to track how each of the strategies performs over the full six months.

We calculate P_0 — the premium that each trader's bank should charge to guarantee a 99% profit (1% loss probability), to build a clear safety measure for each strategy. We accomplish this by running all the simulations, calculating the final profit or loss for each strategy, and then taking the 1% quantile of the resulting distributions.

Next, we analyse the resulting *P&L* (profit and loss) distributions using histograms to visualize the full range of outcomes. This way we can specifically notice the tail-risk, the aforementioned rare but significant losses that each strategy carries, to gain an intuitive understanding of the frequency of extreme outcomes. All the simulated distributions are then shifted by the future value of the payoff and plotted so that the histogram correctly represents profits and losses.

For the next point, we define a *CARA* (Constant Absolute Risk Aversion) utility function that captures different levels of risk aversion. We supplement this with numerical root-finding (the `uniroot` function in R) to solve for the *indifference premium*, at which the trader is equally well-off (in terms of expected utility) if they buy or sell the option. We can clearly extract how sensitive option prices are to risk preferences by repeating this calculation over an arbitrary range of risk aversion, specifically

$$a \in \{0.001,\ 0.002,\ 0.005,\ 0.01,\ 0.03,\ 0.05\}.$$

We initially experimented with more and higher levels, but for values like 0.05, the expected utility function becomes nearly flat, making the root difficult to bracket and leading to unrealistic results. Our range covers the industry standard for analyses of low to even moderate risk aversion for a decision-maker.

Finally, we evaluate the *conditional risk probability* from the stop-loss strategy, more specifically examining the probability of loss if the stock price temporarily falls below a certain barrier before expiry (which we set at €90, a 10% reduction relative to the current price). We check all the simulated paths to see how often the price falls below this at any given point, and then check which of these paths lead to a loss at the time of expiry. A high conditional probability means that the stop-loss frequently misses the moment — the trader sells the stock near the bottom, cashing out right before the market rallies past the strike. The trader is forced to deliver 100 shares at the time of expiry with no long position, freezing the payoff for the holder — in this case we notice that the stop-loss hedge is effective for steady movements but easily prone to heavy losses under high volatility. In contrast, if only a handful of paths go below the strike, the stop-loss rule does its job correctly. A low result suggests that the barrier is placed well and the trader can effectively "shed" risk during down-trends without missing the rally. We anticipate and eventually show that this strategy introduces *path dependence* into the outcomes of the Profit / Loss computations.

1.3 Structure of the Report

We have organized the report to move logically from modelling to insights and discussion. In the next section we *"set the stage"*, formulating the lognormal process we are using for the share price of BrightFuture and declaring all the parameters (see Table 1). Then, in Section 3 we present our findings from the simulation as follows:

- Quantify the simple probability that the call finishes ITM (in-the-money)
- Derive the P_0 profit premium for each of the three banks
- Visualize and comment on the resulting profit-and-loss-distributions
- Compute and report CARA indifference prices across the interval we defined earlier
- Evaluate the conditional failure probability once the barrier has been reached for the stop-loss strategy.

Section 4 follows, where we discuss and pull all the results together. We compare the efficiency of the hedges, the quotes themselves (or capital requirements) + tail risk, highlight the trade-offs between each. We conclude the report by discussing the main points for a trading desk and contouring points where future work could sharpen our analysis even further. Appendix A contains additional code we use for our that is not included in the original code for compatibility (we call `hrbrthemes` and `patchwork`, the former requiring install through `remotes`, essentially it would be too much clutter in the main code).

1.4 Log-Normal Stock-Price Dynamics

We model the evolution of the share price according to the solution S_t of a geometric Brownian motion (GBM) from the Black–Scholes option pricing model. So over one trading day step $\Delta t = \frac{1}{365}$, the price changes as follows:

$$S_{t+\Delta t} = S_t \, \exp\Big[(\mu - \tfrac{1}{2}\sigma^2)\,\Delta t + \sigma\sqrt{\Delta t}\, Z_{t+\Delta t}\Big]\,, \qquad Z_{t+\Delta t} \sim \mathcal{N}(0,1) \text{ i.i.d.}$$

μ is the mean continuously-compounded return; which is given as $\mu = 10\%$ p.a.
σ is the volatility, which is given at 20% p.a.
The shocks $Z_{t+\Delta t}$ are independent standard normal variables, which verify $\mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B)$.

The lognormal distribution arises from the distribution of the logarithm of S_t. To keep the error low/negligible, we generate 100,000 price paths for the Monte Carlo simulation, each of them with 183 daily computed steps to match the time frame of the option (half a year). Moreover, by the Law of Large Numbers, the averages from the simulation converge quickly; with the N we choose, the sampling error is going to be under 1%.

1.5 Contract Parameters

All the other values are fixed and declared in the table that follows. Risk-free rate: all cash balances grow at this rate $\Rightarrow$ €1 in t years becomes e^{rt}. Barrier for the stop-loss hedge: when the price falls to €90, we essentially "flag" it to isolate the weaknesses in this strategy later on. These parameters are constant throughout the analysis and we declare them together clearly to keep the environment and code clean and free of clutter.

Table 1: Baseline parameter set

Symbol	Meaning	Value
S_0	Initial stock price	€ 100
K	Strike price	€ 110
T	Time to maturity	183 days
r	Risk-free rate	3 % p a
μ	Expected return	10 % p a
σ	Volatility	20 % p a

2 Results and Discussion

2.1 Probability Call Ends In-the-Money

Based on our Monte Carlo simulation, we find the following estimate for the probability that the call option ends in-the-money:

$$\hat{p}_{\mathrm{ITM}} = 0.34778$$

Therefore we can infer that in $\approx$ 34,8% of the cases, the stock exceelds the strike - the option will expire with a positive payoff, giving it intrinsic value at time of maturity.

2.2 Premiums Ensuring 99 % Profit

To make sure the previous assumption of 99% profit holds, we compute the amount that the trader's bank must charge upfront as premium under each of the hedging strategies. This is the 1% quantile of the Profit and Loss distribution. We have the following results:

Table 2: Required premium P_0 to ensure 99% profitability

Strategy	Premium (€)
No-hedge (Alice)	3421.26
Stop-loss (Bradley)	1592.08
Delta-hedge (Claire)	363.98

Due to the high probability of substantial losses, the no-hedge strategy needs to be quoted the highest premium. The other two lower the risk and therefore the premium, with the delta-hedge strategy approaching closely the fair price of the call as it seems to correctly predict losses and safely ”withdraw”.

2.3 Profit & Loss Distributions

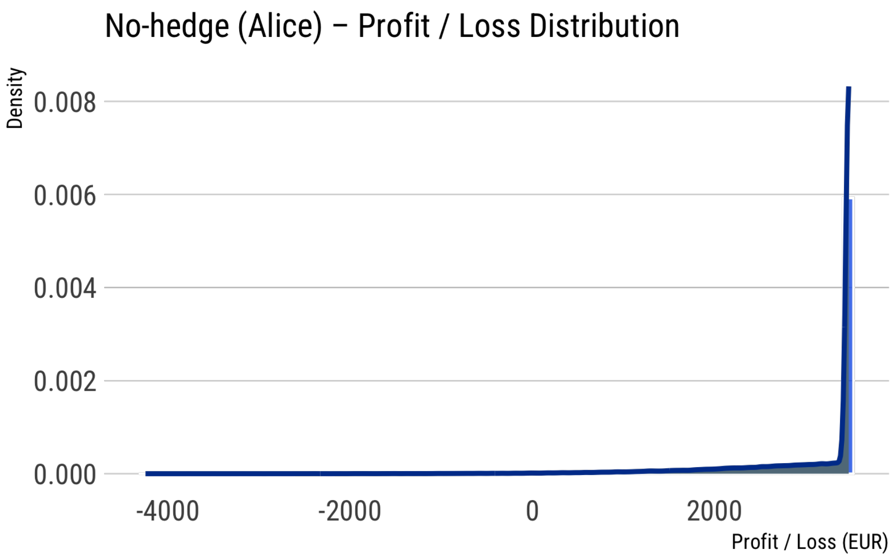


Figure 1: Profit/Loss distribution — No-hedge (Alice)

We start with Alice — who does not hedge at all. Her P&L distribution [Figure 1] is skewed to the right, indicating that the bulk of the outcomes is close to the quoted premium, which is quite intuitive; the lack of any hedging means that the option ends out-of-the-money and is therefore worthless. The main “issue” with this strategy is that in the rare cases the option finishes well in-the-money, the losses can become massive as she must pay the entire intrinsic value. In a real trade scenario, such an event on the tail end would invalidate many small gains on the other end.

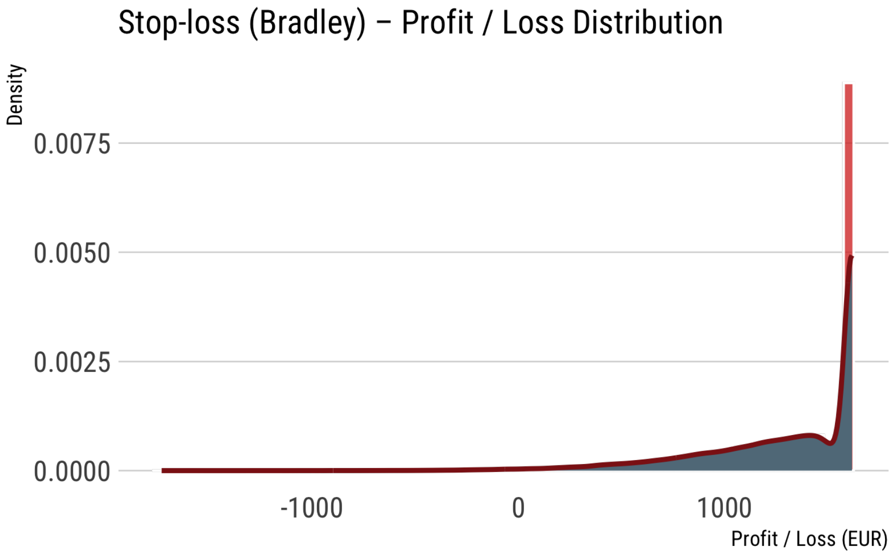


Figure 2: Profit/Loss distribution — Stop-loss (Bradley)

For Bradley, the stop-loss trader, his distribution in [Figure 2] shows that the left tail is smaller but not negligible. Intuitively, from the simulations we understand that when the stock rises steadily, the hedge works — his long position appreciates as the liability of the option does. In this case, the problem is when the price oscillates between gains and losses. A short plunge in price below €90 followed by a rise would force Bradley flat because of his stop-loss before he takes any profits.

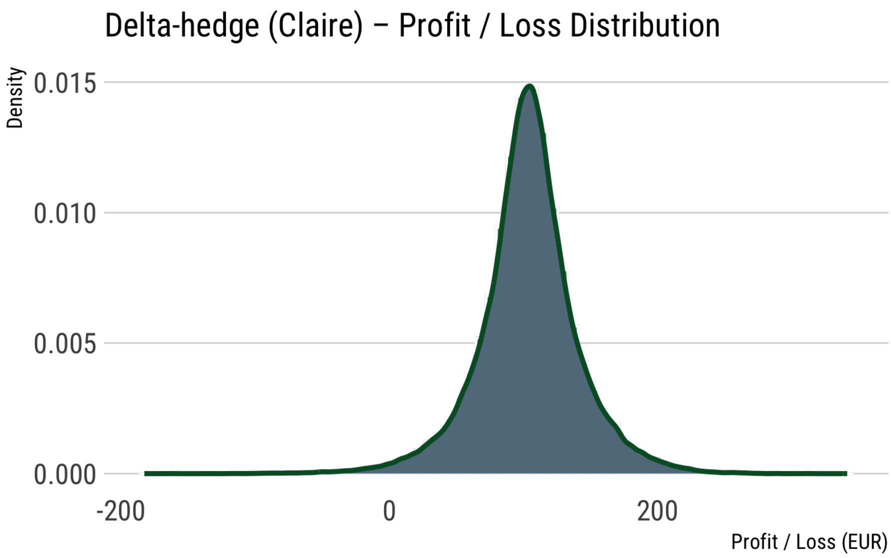
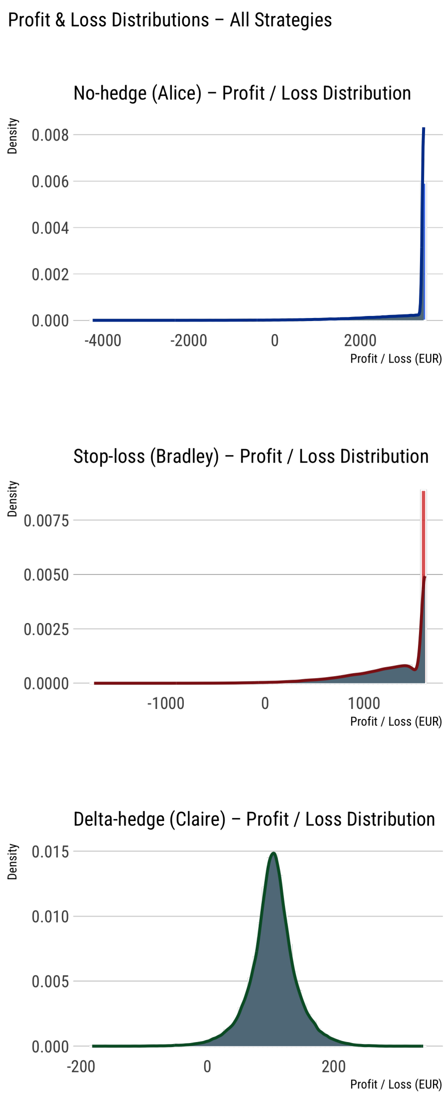


Figure 3: Profit/Loss distribution — Delta-hedging (Claire)

For Claire and the delta hedge strategy, we have the "best" distribution. Her exposure throughout the option is kept near zero, so the distribution is centered just above the break-even price with an almost symmetrical spread (skewness and kurtosis are close to a normal curve). Claire pays a "running cost" in fees (the residual spread of ±€50) to cover her risk and avoid the loss tail that affects Alice and Bradley.

From these histograms, we clearly see that Alice's strategy (no hedge) is cheap but very risky. Hedging with stop-loss reduces Bradley's expected losses, but an oscillation can still cause problems, while Claire's proactive delta hedging strategy almost eliminates the downside risk but demands very tight discipline. The trade-offs are quantified in the rest of the report in terms of both utility and euros.



2.4 Indifference Prices

In this section we examine the preferences of selling the option for a utility-based indifference price under CARA – Constant Absolute Risk Aversion utility preference. This function is given as

$$u(x) \;=\; \frac{1-\exp(-a\,x)}{a},$$

where $a > 0$ is the absolute risk aversion. This is a concave utility function that captures risk aversion – smaller values of a make $u(x)$ almost linear, denoting risk-neutrality. We consider six different levels of risk aversion ranging from an almost risk-neutral $a = 0,001$ to the risk-averse $a = 0,05$. For each of these risk aversions we compute the indifference selling price – the price that the writer of a risk-averse call would need to be indifferent between writing the option and not trading at all.

To find this price, we solve for the p that equalizes the expected utility function both in case he sells or buys the option. Essentially we equate the utility of the current situation – the "status quo" – to the expected utility of the final wealth including p and the option payoff. Formally, the equation we solve is

$$E[u(\Delta W + p)] \;=\; u(0),$$

In R we solve this equation numerically with the `uniroot` function, which finds the root of $E[u(\Delta W+p)] - u(0)$ as a function of p. For all values we report the indifference prices computed in Table 3.

Table 3: CARA indifference prices P_0^* (in €) for six risk-aversion levels

a	$P^*_{0,A}$	$P^*_{0,B}$	$P^*_{0,C}$
0.001	1093	405	263
0.002	2822	548	263
0.005	5445	1316	265
0.010	6500	2208	269
0.030	7230	2919	292
0.050	7380	3069	344

Initially we tried a higher value for a to see how very risk-averse investors would behave, however the root-finding function becomes difficult to use and led to unpredictable results. This is because $u(x)$ declines exponentially for a large negative x. Small changes of p can change the sign of the equation and `uniroot` will either give an absurd answer or fail to bracket which we have handled in the code as well. The range we chose allows us to model behaviour from risk aversion $(0,03\text{–}0,05)$ to risk tolerance $(0,001\text{–}0,005)$. The key implication of the result is that for the unhedged strategy higher risk aversion substantially increases the required price by 600 % over the interval. In contrast, the delta-hedger's price increases by only $\approx$ 30 % over the range – risk is cheaper for Claire.

2.5 Down-and-Out Conditional Probability

In this part we compute the probability that the option is down-and-out given it finishes in-the-money at the time of expiry. In such a situation, the option would technically end with a positive payoff, with the underlying BrightFuture stock above the strike price, however it would have earlier hit the €90 barrier we set for stop-loss therefore becoming worthless.

The result $\hat{p}_{\mathrm{DO|ITM}} = 0.0476$ means that ≈4,8 % of the time the call would end in-the-money, the payoff would be negated by the breach of the 90 % stop-loss hedge. In the code, this probability is calculated from the outputs of stop-loss stochastic paths we simulate in the prior subquestions.

Generally, a down-and-out option expires worthless if the underlying stock falls below the barrier. Consequently, if the stock experiences a "whipsaw" – a sharp price fluctuation or reversal – this stop-loss trader's option will sell at €90 immediately. If he wants to re-enter at recovery he will incur even extra loss, essentially buying low and selling high around his barrier. In this scenario a strategy with a smaller barrier or even no-hedge would continue to hold this long position and be able to get a payoff. The measure $\hat{p}_{\mathrm{DO|ITM}} = 0.0476$ is evidence of what is called in probability theory a tail risk for the strategy – the chance that the "whipsaw" scenario happens – a result which would not be visible in the initial analysis of only the final payoff distribution. So as we make our model stricter, tail risk starts to emerge in the stop-loss strategy, which intuitively makes this approach far riskier than delta-neutral hedging.

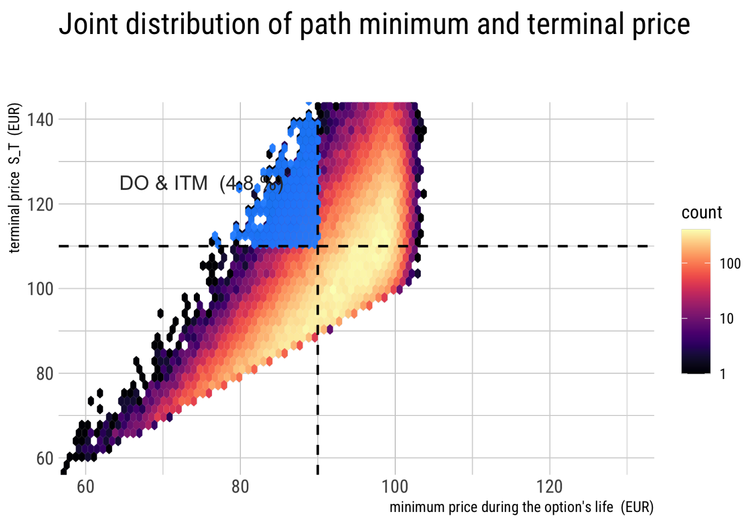


Figure 4: Joint distribution of the path minimum and terminal price. The shaded part $\min S_t \leq 90$ and $S_T > 110$ represents the conditional event DO | ITM

For a visual representation, in Figure 4 we aggregated in each hexagon simulations that end with a similar minimum price (x-axis) and terminal price (y-axis). The lighter the colour of each hexagon, the higher the number of paths that end with those prices. The dashed vertical and horizontal lines are, respectively, the down-and-out barrier and the strike price. The shaded upper-left quadrant indicates the paths that we are looking at – breaching the €90 barrier and ending above €110, clearly showing that this is not a negligible mass.

3 Conclusion

In conclusion, through this study and its objectives we can outline the following conclusions and their real trading implications: $\approx 34\%$ of simulations finish above the strike price, so the option has intrinsic value, but keeping the loss rate below 1% would cost €3,4k ; €1,6k; €364 for the no hedge, stop-loss, and delta-hedge respectively. If we look at the results from the perspective of investor preference through CARA, we notice premiums increasing by up to 600% over the risk-aversion range for no hedge, while barely changing for the delta hedge. This shows that through financial analysis we can derive a type of transaction cost – risk capital. We also outline in point (e) the weakness of the stop-loss strategy – that $\approx 4,8\%$ of outcomes that finish in the money and are therefore "winners" for all the other strategies in fact are rendered worthless and the trader cannot re-enter the market in a favourable position. This situation is seen with cases where the stock "whipsaws" and exhibits a sharp fluctuation in price.

3.1 Outlook

For future projects/research on this topic, we could expand the movement of the stock (currently a Brownian Motion) to a jump-diffusion model – which would introduce shocks or "flash crashes" to the simulation and test the delta-hedge more strictly. Moreover, for more accurate results we could consider a bid-ask spread when re-hedging, adding a previously not considered cost to Bradley and Claire's strategies.

A Appendix A

```
        # ==================================================================
# Profit Loss Appendix A
        # ==================================================================

        library(ggplot2)
        library(hrbrthemes)    # theme_ipsum_rc()
        library(patchwork)     # for combined panel

        theme_set(theme_ipsum_rc(base_size = 12,
        grid = "Y",
        plot_title_size = 14))

        plot_pnl_hist <- function(pnl_vec, strategy, fill_col, line_col,
        bins = 70, file_out = NULL,
        width = 6, height = 4) {

                g <- ggplot(data.frame(pnl = pnl_vec), aes(x = pnl)) +
                geom_histogram(aes(y = after_stat(density)),
                bins   = bins,
                fill   = fill_col,
                alpha = 0.75,
                colour = "white") +          # thin white bar borders
                geom_density(size = 1, colour = line_col) +
                labs(title = paste(strategy, "- Profit / Loss Distribution"
                   ),
                x = "Profit / Loss (EUR)",
                y = "Density") +
                coord_cartesian(expand = 0.01)

                if (!is.null(file_out)) {
                        ggsave(file_out, g, width = width, height = height,
                            dpi = 300)
                }
                invisible(g)
        }

        p_A <- plot_pnl_hist(profit_A,
        strategy = "No-hedge (Alice)",
        fill_col = "#0057e7", line_col = "#003d99",
        file_out = "pnl_alice.png")
```

```
p_B <- plot_pnl_hist(profit_B,
strategy = "Stop-loss (Bradley)",
fill_col = "#d62d20", line_col = "#8d1b17",
file_out = "pnl_bradley.png")

p_C <- plot_pnl_hist(profit_C,
strategy = "Delta-hedge (Claire)",
fill_col = "#008744", line_col = "#005a2e",
file_out = "pnl_claire.png")

combined <- (p_A / p_B / p_C) +
plot_annotation(title = "Profit & Loss Distributions - All
    Strategies")

ggsave("pnl_all_strategies.png",
combined, width = 6, height = 12, dpi = 300)


df_hex <- data.frame(
min_price = apply(S_paths, 1, min),
S_T       = ST
)
df_hex$wedge_flag <- (df_hex$min_price <= 90) & (df_hex$S_T > 110)

prob_DO_ITM <- cond_prob   #  0.04 with current seed

gg_hex <- ggplot() +
# BACKGROUND
geom_hex(
data = df_hex,                    # full data set
aes(min_price, S_T, fill = after_stat(count)),
bins = 60, colour = NA
) +
scale_fill_viridis(
option = "magma", trans = "log10",
name = "count"
) +
# FOREGROUND
geom_hex(
data = subset(df_hex, wedge_flag),
aes(min_price, S_T),
bins = 60, fill = "dodgerblue", colour = NA, alpha = .95
) +
## barrier & strike
geom_vline(xintercept = 90,  linetype = "dashed", linewidth = .7) +
geom_hline(yintercept = 110, linetype = "dashed", linewidth = .7) +
## label inside the wedge
annotate(
"text", x = 75, y = 125,
label = sprintf("DO & ITM  (%.1f %%)", 100 * prob_DO_ITM),
size = 4.2, colour = "grey20"
) +
coord_cartesian(xlim = c(60, 130), ylim = c(60, 140)) +
labs(
title    = "Joint distribution of path minimum and terminal price",
subtitle = "Blue hexes = paths that breached 90 yet finished above
    110",
x        = "minimum price during the option's life  (EUR)",
```

```
        y         = "terminal price  S_T  (EUR)"
        ) +
        theme_ipsum_rc(base_size = 11)

        ggsave("min_vs_terminal_hex_wedge.png", gg_hex,
        width = 7, height = 5, dpi = 300)
        print(gg_hex)



}
```
Listing 1: R code used for generating Figures 1,2,3,4, and 5