Spud Launch Solutions, LLC
3990 Fifth Avenue
Pittsburgh, PA 15213

15 April 2016

Dr. Jack Patzer
Undergraduate Coordinator, Bioengineering
University of Pittsburgh
1600 O'Hara Avenue
Pittsburgh, PA 15213

Re: Spud Launcher Design

Dear Dr. Patzer:

As per your request, our team has identified the optimal physical dimensions for maximizing launch distance of the potato wedge. The documents are enclosed.

To simplify the problem, we found the maximum possible exit velocity of the potato, which corresponds directly to the maximum possible distance traveled. We first solved for the exit velocity in terms of our engineering parameters, the dimensions of the chamber and barrel components. For practicality and safety, we implemented constraints on the allowable dimensions of our components. The team then created a MATLAB program that allowed us to analyze the exit velocity under every allowable arrangement of dimensions, and find the maximum value.

Our analysis determined that, under the given constraints and assumptions, the maximum attainable velocity is 105.47 ms^{-1}. This velocity is possible when the radius of the chamber is 7.62 cm, length of the chamber is 40 cm, radius of the barrel is 3 cm, and length of the barrel is 1.5 m.

While we functioned effectively as team throughout the project, each member specialized in specific areas. Nate took the lead on the background information, David focused on the analytical model and results, Ben handled the discussion, and Michael took charge on the abstract and formatting. All group members contributed where necessary.

We appreciate the opportunity to work with you. Please reach out to us for any further inquiries.

Yours truly,

Michael Adams

Nate Myers David Denberg Ben Roadarmel Michael Adams

ABSTRACT

We set out to design a spud launcher that can launch a potato a far as possible. We chose to find maximum exit velocity of the potato because it simplifies the problem while achieving the same result. The spud launcher is composed of three simple parts, a pneumatic chamber, a valve, and a barrel. The dimensions of the chamber and barrel affect exit velocity, so we focused on these engineering parameters when optimizing the system. Taken to be cylindrical, we examined how the radius (r_c and r_b) and the length (l_c and L) of both components affected the exit velocity, and therefore distance, of the potato. Before we could calculate the optimal dimensions, we constrained the acceptable range of dimensions for each component to avoid any unrealistic solutions. For practicality, we also chose to set the radius of the chamber, r_c, to 7.62 cm, which is consistent with commonly available PVC pipe. With these constraints and several simplifying assumptions, we then solved for the exit velocity of the potato in terms of our engineering parameters. To optimize this equation and determine the maximum possible exit velocity, we created a MATLAB program to test each value in the allowable range of our parameters. Through our analysis, we determined the maximum possible velocity to be $v = 105.47\, ms^{-1}$, and the optimal dimensions to be $r_b = 3\ cm$, $l_c = 40\ cm$, and $L = 1.5\ m$. Based on our findings, it is clear that our final result was heavily influenced by our constraints. The relationship between cross sectional area, volume, and exit velocity of the potato predict an increase in exit velocity proportional to an increase in area or volume. However, in reality the velocity would be lower due to retarding forces that we excluded in our analysis.

BACKGROUND

The goal of this project is to design a pneumatic spud launcher for a competition at the Cameron County Fair. The gold medal is awarded to the design team with the launcher that can fire a spud furthest using only 20 psi gauge of compression for the pneumatic chamber. Physical and thermodynamic relationships govern each design output and therefore are considered in each step of development. Proper implementation of thermodynamic concepts such as the first law of thermodynamics, irreversible adiabatic expansion, and conservation of energy will be needed to maximize the launch distance of the spud.

The spud launcher is comprised of a pressurized chamber of air connected to a barrel with a value serving to separate the two inner volumes. When the valve is quickly released, the spud will be forced down the length of the barrel and fire out of the end. Although the competition limits the initial pressure of the air chamber, the other components of the design are left to freely manipulate. Thus, our design objective is to model how changes to each of the design parameters affects the spud's launch distance.

In order to ensure a gold medal is awarded, our design team accessed two other pneumatic spud launcher design concepts. The first model was developed by Mark Denny out of British Columbia and is based on the assumption that the expansion of gas is isothermal. However, by the end of his testing, Denny concluded that his modeling errors were due to his assumption that the gas expansion is isothermal. Denny states that the isothermal assumption would require a method of heat exchanged within the spud launcher that has the capability to function at unrealistic speeds [1]. Therefore, to more properly illustrate the thermodynamic factors at play, our design team will assume the expansion process is irreversibly adiabatic instead.

Additionally, our implementation of the first law of thermodynamics, ideal gas law, and conservation of energy were all inspired by the work of Z.J. Rohrback, T. R. Buresh, and M. J. Madsen from Wabash College [2]. Their work served as an integral starting point for our design, allowing us to blend in the ideas of Denny to receive a properly functioning model.

METHODS

PHYSICAL MODEL

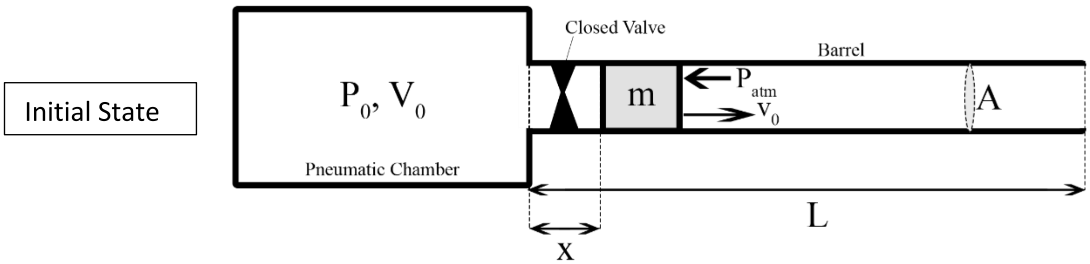

Figure 1. Shows the initial state of spud launcher. The launcher has a pneumatic chamber with an initial volume, V_o containing air at an initial pressure, P_o. When the valve is opened, an adiabatic, irreversible expansion occurs, and the potato with mass m is launched out of the end of the barrel defined with length L and cross-sectional area A. The potato has an exit velocity v_1. The atmosphere exerts a constant pressure on the potato, P_{atm}.

The potato launcher modeled in figure 1 uses a large pneumatic chamber with an attached valve to release pressure. The pneumatic chamber with volume $V_{g,0}$ (m^3) starts with an initial pressure, $P_{g,0}$. A potato with mass m (kg) is situated at position x (m) in the barrel defined with length L (m) and cross sectional area A (m^2). When the valve is opened, the gas in the chamber expands adiabatically to volume $V_{g,1}$ (m^3) and pressure $P_{g,1}$ (Pa). Expansion of the gas within the spud launcher does work on the potato and causes it to move toward the end of the barrel with an eventual exit velocity v (m s^{-1}). Additionally, as the potato moves down the barrel, the external pressure of the atmosphere P_{ext} (Pa) works on the potato.

ANALYTICAL MODEL

The two-part, pneumatic spud launcher system can be broken up into the components: spud and compressed gas. Each part of the spud launcher can be modeled independently using the first law of thermodynamics. Due to the speed of the expansion within the system, the process can be considered adiabatic. The process must also be irreversible as the system is expanding against a constant external pressure. From appendix section (A.1) the following expression for internal energy is derived:

$$\Delta U_g = -P_{ext}(V_{g,1} - V_{g,0}) \tag{1}$$

where U_g(J) is the internal energy of the gas, P_{ext} (Pa) is the external pressure, $V_{g,1}$ (m^3) is the volume of the gas at an expanded state, and $V_{g,0}$ (m^3) is the volume of the gas at a condensed state. ΔU_g, for an ideal gas, is also given as:

$$\Delta U_g = nC_V\Delta T \tag{2}$$

where n (mol) is the number of moles of gas, C_V $(J \cdot mol^{-1} \cdot K^{-1})$ is the heat capacity at constant volume and T (K) is the temperature. Equation (1) and (2) can be equated and then substituted into the ideal gas law to derive an expression for internal pressure:

$$P(x) = \frac{P_{g,0}V_{g,0} - P_{ext}Ax(\gamma - 1)}{Ax + V_{g,0}} \tag{3}$$

where $P(x)$ (Pa) is the internal pressure as a function of spud location, $P_{g,0}$ (Pa) is the initial internal pressure of the gas, x (m) is the position of the spud in the barrel, γ is the adiabatic constant $\frac{C_P}{C_V}$, and A (m^2) is the cross sectional area of the barrel. The full derivation can be found in appendix section (A.3). If we assume a very long barrel, the pressure difference between the external pressure and the internal pressure will reach zero at a certain point after the spud has fired. Such a barrel length is not always infeasible and thus it places a constraint on the design parameters. We must choose a barrel length L that is less than L_{max} (m) where L_{max} is defined as:

$$L_{max} = \frac{V_{g,0}\left(P_{g,0} - P_{ext}\right)}{P_{ext}A\gamma} \tag{4}$$

for which the derivation can be found in appendix section (A.4).

Using the kinetic energy work theorem, we can relate the velocity of the potato to the total work done on the potato. Kinetic energy of the spud equals the total work done on the spud:

$$\frac{1}{2}mv^2 = \int_0^L AP(x) - AP_{ext}dx \tag{5}$$

where v (m s^{-1}) is the velocity of the spud and m (kg) is the mass. We also assume the friction is zero as the spud is slippery. After integrating and solving for v, we get:

$$v = \sqrt{\frac{2}{m}\left(\left(P_{g,0} + P_{ext}(\gamma - 1)\right)V_{g,0}\ln\left(\frac{AL}{V_{g,0}} + 1\right) - \gamma P_{ext}AL\right)} \tag{6}$$

for which the derivation and solution can be found in appendix section A.5.

In order to incorporate more engineering parameters, we will define $V_{g,0}$ as the volume of the pneumatic chamber before the valve is opened to allow gas expansion. We will also assume that the pneumatic chamber will be a cylinder. $V_{g,0}$ can be expressed in terms of the dimensions of the pneumatic chamber as:

$$V_{g,0} = l_c \pi r_c^2 \tag{7}$$

where r_c (m) is the radius of the cylindrical pneumatic chamber, and l_c (m) is the length of the pneumatic chamber. We also define the cross sectional area of the barrel as:

$$A = \pi r_b^2 \tag{8}$$

where r_b(m) is the radius of the barrel.

We know from kinematics that a larger initial velocity will result in a farther distance traveled by the projectile. Thus, by maximizing the muzzle velocity of the spud, we can conclude that the distance traveled by the spud will also be maximized.

IMPLEMENTATION

In order to maximize the distance traveled by the spud, we must maximize the muzzle velocity. From (6) we can see that v is proportional to the square root of $P_{g,0}$. Therefore, a larger value of $P_{g,0}$ will result in a higher muzzle velocity. As we are constrained to a max internal gauge pressure of 20 psi, we set $P_{g,0} = P_{gauge} + P_{atm} = 20\,psi + P_{atm} = 239220\,Pa$. P_{ext} is the atmospheric pressure at sea level which is given as 101325 Pa, and $\gamma = 1.4$, the adiabatic constant for diatomic gases. We define L' as the minimum between L_{max} (4) and a design constraint L_{phys} so that L' does not exceed L_{phys}. We set $L_{phys} = 1.5\,m$.

$$L' := \min\{L_{max}, L_{phys}\} \tag{9}$$

The remaining variables that we choose to manipulate are $V_{g,0}$ and A which are defined as (7) and (8) respectively. Due to the relatively small size of the potato, we decided to constrain the radius of the cross-sectional area, r_b, to some value between 2 and 3 cm. Additionally, the radius of the pneumatic chamber, r_c, is fixed to 7.62 cm as the maximum diameter PVC pipe that can be purchased from most hardware stores is 6 in. Finally, we limit the length of the pneumatic chamber, l_c, to a maximum of 40 cm.

The mass of the potato is estimated to be 60 g. We assume the potato, once inserted into the barrel, will be cylindrical with a length, 5 cm. We average the volumes of the potato using the minimum and maximum radii of r_b. The density of a raw potato is 590 kg m^{-3} [3] which we use to calculate the approximate mass of our projectile.

In order to maximize the velocity, we employed MATLAB to automate the process. The code is found in appendix section (A.6). We can iteratively calculate the velocity for a range of A and $V_{g,0}$ and determine the maximum from the resulting array. Additionally, L' is used in place of L as the length should not exceed L_{phys}.

RESULTS

Using the code in appendix section (A.6) we were able to optimize the design parameters A and $V_{g,0}$ to achieve the highest possible velocity within our constraints. The maximum velocity was found to be 105.47 m s^{-1}, with $A = 0.0028274\,m^2$, $L = 1.5\,m$, and $V_{g,0} = 0.0072966\,m^3$. Figure 2 shows two different orientations in space of the plot of velocity vs Area and Initial volume.

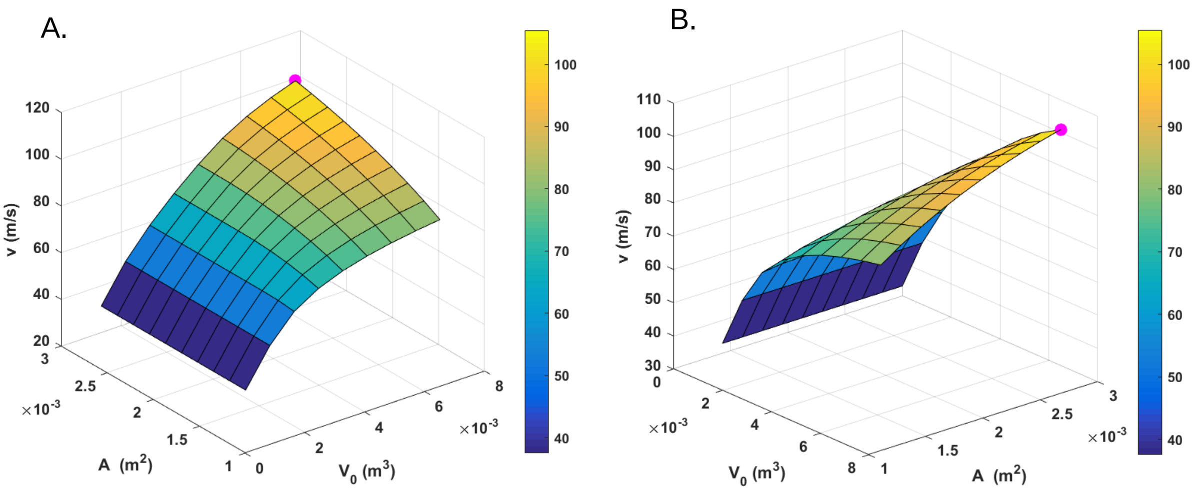

Figure 2. (A) Shows the graph of exit velocity vs. cross-sectional area of the barrel and initial volume. (B) Shows the same graph as in (A) rotated 90˚ to the right. The magenta point is the maximum exit velocity

The graphs in figure 2 show that within the domain $A \in [0.0013\, m^2, 0.0028\, m^2]$ and $V_{g,0} \in [0.0\, m^3, 0.0073\, m^3]$, v approaches a maximum as A and V increase, indicated by the magenta points in Figure 2 (A) and (B). Because of the adjusted length L', as defined in (9), the velocity slopes down as A decreases for a constant $V_{g,0}$. Figures 3 and 4 show v vs A where $V = 0.0073\, m^3$ and v vs $V_{g,0}$ where $A = 0.0028\, m^2$ respectively. The data that corresponds to figures 3 and 4 can be found in appendix section (A.7).

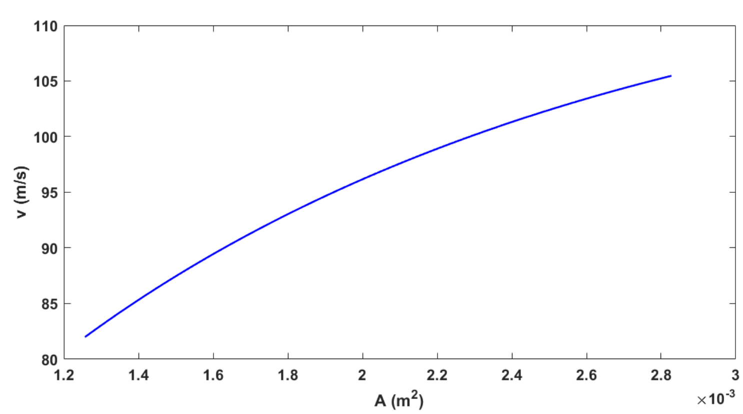

Figure 3. Shows the graph of exit velocity (m/s) vs cross-sectional area (m^2), where $V_{g,0}$ is fixed at .0073 m^3.

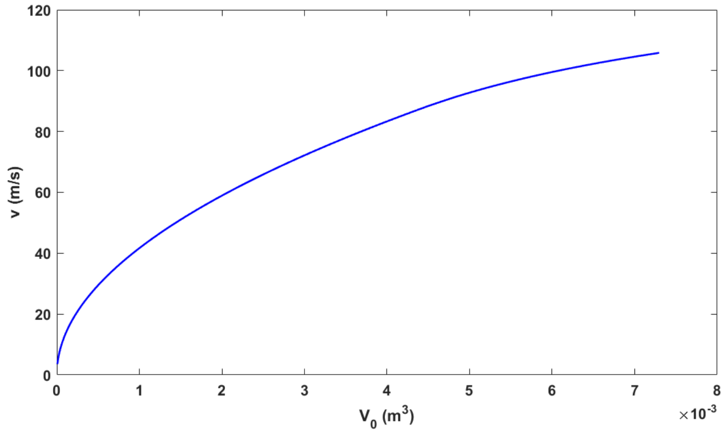

Figure 4. Shows the graph of exit velocity (m/s) vs initial volume (m^3) where A is fixed at 0.0028 m^2

DISCUSSION

Given several design parameters to change we can maximize the initial velocity of the spud from the end of the barrel. Internal pressure, $P_{g,0}$ is set at 20 psi gauge. Assuming the pneumatic chamber to be a cylinder, the max diameter we can use is 6 in, resulting in an area of 9π inches squared. Our value for r_c will be 3 in or .0762 m. By using this maximum area, we can determine the volume of the pneumatic chamber by varying only its length, l_c. The barrel itself will have a variable area and length, giving us two more design parameters, the radius and length of the barrel r_c and L respectively. Because we want the launcher to be practical, we imposed a restriction of 1.5 m maximum on L. We do not have the means to set the external pressure, we will assume that it is atmospheric pressure at sea level, 101 kPa.

By assuming that the gas in inside and outside the launcher to be ideal, an irreversible adiabatic expansion, and no friction we can determine an equation for muzzle velocity in terms of our parameters and given values. This velocity, v, is a function of r_b, L, and l_c. We know that the maximum work produced by the system will be at the point where the external pressure is equal the internal pressure just as the potato leaves the end of the barrel. Using this we can relate the volume of the pneumatic chamber and the barrel to find the optimal length for the barrel given a volume for the pneumatic chamber and radius for the barrel. By then graphing velocity as a function of $V_{g,0}$ area of the barrel, A, and L, we can determine the optimal values for r_b, L, and l_c. If the value of L, is greater than our constraint of 1.5 m we will use the constraint.

Using Matlab to graph our function for v, and then using our answers to determine our parameters, we obtained the following values:

v	105 m s^{-1}
$V_{g,0}$	0.0073 m^3
A	0.0029 m^2

r_b	0.030 m
L	1.5 m
r_c	0.076 m
l_c	0.40 m
P_{ext}	101 kPa
$P_{g,0}$	240 kPa
m	0.06 kg
γ	1.4

Table 1. Table containing all design parameters and given values determined from the equation for v.

Our graph provided us the values for $V_{g,0}$, A, and L, however, the maximum theoretical length, L_{max}, was found to be greater than our design constraint, L_{phys}. which is why we chose a value of 1.5 m to be used for L. By using the equations for volume and area we used our values for $Vg, 0$ and A we determined r_b and l_c given our constraint for r_c. The mass of the potato was estimated using assumptions concerning density and size of the potato in the implementation section. We also assumed the adiabatic constant, γ, to be the diatomic value of 1.4 which is a reasonable assumption given air is mostly composed of diatomic gases. Using these values, our spud launcher will result in the maximum muzzle velocity. We know from kinematics, that our launcher design will also result in the largest distance traveled.

REFERENCES

[1] Mark Denny. (2011. Feb). The Internal Ballistics of an Air Gun. Victoria, British Columbia, Canada. Retrieved from:
http://www.kiledjian.elac.org/phys%20001/Ballistics%20of%20Air%20Gun.pdf

[2] Z.J. Rohrback, T. R. Buresh, M. J. Madsen. (June 2011). The Exit Velocity of a Compressed Air Cannon. Crawfordsville, IN, USA. Retrieved from: http://arxiv.org/pdf/1106.2803.pdf

[3] U. R. Charrondiere, D. Haytowitz, B. Stadlmayr. (2012). FAO/INFOODS Density Database Version 2.0. Retrieved from: http://www.fao.org/3/a-ap815e.pdf

APPENDIX

(A.1)

The first assumption we make is that the gas in the barrel and chamber is ideal and follows the ideal gas law:

$$PV = nRT \tag{A.1.1}$$

where P (Pa) is pressure, V (m^3) is volume, n (mol) is moles of gas, R (J mol^{-1} K^{-1}) is the gas constant, and T (K) is the temperature.

(A.2)

First we state the first law in its entirety:

$$\Delta U_g + \Delta ME_g = W_g + Q_g \tag{A.2.1}$$

where U_g(J) is the internal energy of the gas, ME_g (J) is the mechanical energy of the gas, Q_g(J) is the heat of the gas, and W_g (J) is the work done on the system. The system is at rest so the mechanical energy does not change ($\Delta ME_g = 0$), and from the assumption of an irreversible adiabatic process ($Q_g = 0$), the first law reduces to:

$$\Delta U_g = W_g \tag{A.2.2}$$

Work can be calculated from a pressure volume relationship in (A.2.3).

$$W_g = -\int P_{ext}\, dV_g \tag{A.2.3}$$

where dV_g (m^3) is the volumetric change in gas and P_{ext} (Pa) is the external pressure.

After integrating the final equation is:

$$\Delta U_g = -P_{ext}(V_{g,1} - V_{g,0}) \tag{A.2.4}$$

where $V_{g,1}$ (m^3) is the volume of the gas at an expanded state, and $V_{g,0}$ (m^3) is the volume of the gas at the condensed state.

(A.3)

We first state the two equations for internal energy for an ideal gas:

$$\Delta U_g = -P_{ext}(V_{g,1} - V_{g,0}) \tag{A.3.1}$$

$$\Delta U_g = nC_V\Delta T = nC_V\left(T_{g,1} - T_{g,0}\right) \tag{A.3.2}$$

where n (mol) is the number of moles of gas, C_V $(J \cdot mol^{-1} \cdot K^{-1})$ is the heat capacity at constant volume and T (K) is the temperature. We define $V_{g,1}$ as a function of spud location:

$$V_{g,1}(x) = Ax + V_{g,0} \tag{A.3.3}$$

where A (m^2) is the cross-sectional area of the barrel. We then equate (A.3.1) and (A.3.2), substitute (A.3.3), and solve for $T_{g,1}$:

$$T_{g,1} = T_{g,0} - \frac{P_{ext}Ax}{nC_V} \tag{A.3.4}$$

where $T_{g,1}$ and $T_{g,0}$ are the temperatures at some second state and the initial state of the system respectively. The ideal gas law (A.1.1) can be rearranged to (A.3.5) as the number of moles of the gas remains constant.

$$P_{g,1} = \frac{nRT_{g,1}}{V_{g,1}} \tag{A.3.5}$$

We then substitute (A.3.4) and (A.3.2) into (A.3.5) and solve for $P_{g,1}$ as a function of x:

$$P_{g,1} = \frac{nRT_{g,0}}{Ax + V_{g,0}} - \frac{P_{ext}Ax(\gamma - 1)}{Ax + V_{g,0}} \tag{A.3.6}$$

we also use the substitute the adiabatic constant $\gamma = \frac{C_P}{C_V}$ to eliminate C_V. Because moles of gas remain constant, we can substitute (A.1.1) in for $nRT_{g,0}$. The final equation for internal pressure is:

$$P(x) = P_{g,1} = \frac{P_{g,0}V_{g,0} - P_{ext}Ax(\gamma - 1)}{Ax + V_{g,0}} \tag{A.3.7}$$

(A.4)

As the spud moves down the barrel, the internal pressure drops, but the external pressure remains constant. The spud will continue to accelerate until the pressure difference reaches zero. It is at that point in the barrel, which we define as the maximum barrel length. First we set (A.3.7) equal to the external pressure and set x equal to L_{max}:

$$P_{ext} = \frac{P_{g,0}V_{g,0} - P_{ext}AL_{max}(\gamma - 1)}{AL_{max} + V_{g,0}} \tag{A.4.1}$$

Then after rearranging, we get the expression:

$$L_{max} = \frac{V_{g,0}\left(P_{g,0} - P_{ext}\right)}{P_{ext}A\gamma} \tag{A.4.2}$$

(A.5)

First we state Newton's second law in order to find the forces acting on the spud:

$$m\frac{d^2x}{dt^2} = mv\frac{dv}{dx} = AP(x) - AP_{ext} \tag{A.5.1}$$

The internal pressure $P(x)$ acts on the spud in the positive x direction and the external pressure acts on the spud in the negative x direction hence the positive and negative coefficients. We integrate both sides of (A.5.1) to obtain the Kinetic Energy Work relationship:

$$\frac{1}{2}mv^2 = \int_0^L AP(x) - AP_{ext}dx \qquad \text{(A.5.2)}$$

Where v (m s^{-1}) is the muzzle velocity of the spud. Substituting (A.3.7) into (A.5.2) and integrating we get:

$$\frac{1}{2}mv^2 = \left[P_{g,0}V_{g,0}\ln\left(Ax + V_{g,0}\right)\right]_0^L - \left[(\gamma - 1)P_{ext}\left(Ax - V_{g,0}\ln\left(Ax + V_{g,0}\right)\right)\right]_0^L - [AP_{ext}x]_0^L \qquad \text{(A.5.3)}$$

After evaluating at L and 0 and simplifying:

$$\frac{1}{2}mv^2 = \left(P_{g,0} - (\gamma - 1)P_{ext}\right)V_{g,0}\ln\left(\frac{AL}{V_{g,0}} + 1\right) - \gamma P_{ext}AL \qquad \text{(A.5.4)}$$

Solving for v leads us to this equation for muzzle velocity:

$$v = \sqrt{\frac{2}{m}\left(\left(P_{g,0} + P_{ext}(\gamma - 1)\right)V_{g,0}\ln\left(\frac{AL}{V_{g,0}} + 1\right) - \gamma P_{ext}AL\right)} \qquad \text{(A.5.5)}$$

(A.6)

```
P_ext = 101325;
gamma = 1.4;
n = 1000;

r_barrel_max = 3/100;
r_barrel_min = 2/100;
A_max = pi * r_barrel_max ^ 2;
A_min = pi * r_barrel_min ^ 2;

r_chamber_max = 7.62/100;
r_chamber_min = 0/100;
depth_chamber_max = .4;
depth_chamber_min = 0;
V_max = depth_chamber_max * pi * r_chamber_max ^ 2;
V_min = depth_chamber_min * pi * r_chamber_min ^ 2;

step_A = (A_max - A_min) / (n-1);
step_V = (V_max - V_min) / (n-1);

m = .06;
P_0 = 137895 + P_ext;
```

```matlab
V_0 = linspace(V_min, V_max, n);
A = linspace(A_min, A_max, n);
[V_0grid, A_grid] = meshgrid(V_min:step_V:V_max, A_min:step_A:A_max);
L_max = 1.5;

v_max = 0;
for i = 1:n
    for j = 1:n
        [v(i,j), L(i,j)] = fun(m, P_0, gamma, P_ext, V_0(i), A(j), L_max);
        if imag(v(i,j)) ~= 0
            v(i,j) = NaN;
        end
        if v(i,j) > v_max
            v_max = v(i,j);
            index = [i, j];
        end
    end
end

disp(['Max v = ', num2str(v_max)])
disp(['V_0 = ', num2str(V_0(index(1))), ' A = ', num2str(A(index(2)))]);
disp(['L = ', num2str(L(index(1), index(2)))]);

figure;
scatter3(V_0(index(1)), A(index(2)), v_max, 'filled', 'm');
hold on;
surf(V_0grid, A_grid, v', 'CDataMapping','scaled', 'LineStyle','none');
colorbar
xlabel('V_0');
ylabel('A');
zlabel('v');



function [v, L] = fun(m, P_0, gamma, P_ext, V_0, A, L)
temp = V_0 .* (P_0 - P_ext) ./ (P_ext .* gamma .* A);
if temp < L
    L = temp;
end
v = sqrt((2 ./ m) .* ((P_0 + (gamma - 1) .* P_ext) .* V_0 .* ...
    log(A .* L ./ V_0 + 1) - gamma .* A .* P_ext .* L));
end
```

(A.7)

Table A.7.1 Corresponding to Figure 3

$A \cdot 10^3\ m^2$	$V_{g,0} \cdot 10^3\ m^3$	$v\ (m\ s^{-1})$
1.26	7.30	81.99
1.34	7.30	83.95
1.42	7.30	85.81
1.50	7.30	87.55
1.59	7.30	89.20
1.67	7.30	90.76
1.75	7.30	92.23
1.84	7.30	93.62
1.92	7.30	94.93
2.00	7.30	96.18
2.08	7.30	97.36
2.17	7.30	98.47
2.25	7.30	99.53
2.33	7.30	100.53
2.41	7.30	101.47
2.50	7.30	102.37
2.58	7.30	103.21
2.66	7.30	104.01
2.74	7.30	104.76
2.83	7.30	105.47

Table A.7.2 Corresponding to Figure 4

$A \cdot 10^3\ m^2$	$V_{g,0} \cdot 10^3\ m^3$	$v\ (m\ s^{-1})$
2.87	0.00	NaN
2.87	0.36	25.17

2.87	0.73	35.59
2.87	1.09	43.59
2.87	1.46	50.33
2.87	1.82	56.27
2.87	2.19	61.65
2.87	2.55	66.58
2.87	2.92	71.18
2.87	3.28	75.50
2.87	3.65	79.58
2.87	4.01	83.47
2.87	4.38	87.18
2.87	4.74	90.61
2.87	5.11	93.61
2.87	5.47	96.24
2.87	5.84	98.58
2.87	6.20	100.68
2.87	6.57	102.57
2.87	6.93	104.29
2.87	7.30	105.85