Q4) Inverse Matrix

Sol:
$$A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \; ; \; A^{-1} = ?$$

$$A^{-1} = \frac{Adj\, A^{-1}}{|A^{-1}|}$$

$$\because |A^{-1}| = \begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} = 2$$

$$\because Adj\, A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}$$

Q7)
$$a = \begin{bmatrix} 4 \\ 5 \\ 2 \\ 2 \end{bmatrix}, \; b = \begin{bmatrix} 4 \\ 0 \\ 2 \\ 0 \end{bmatrix}, \; c = \begin{bmatrix} 2 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

1) Eucildean Distance:

$d_1(a,b) = ?$; $d_2(b,c) = ?$; $d_3(a,c) = ?$

Sol:
$$d(a,b) = \sqrt{(a_1-b_1)^2 + (a_2-b_2)^2 + (a_3-b_3)^2 + (a_4-b_4)^2}$$

$$d_1(a,b) = \sqrt{(4-4)^2 + (0-5)^2 + (2-2)^2 + (0-2)^2}$$

$$= \sqrt{0 + 25 + 0 + 4}$$

$$= \sqrt{29}$$

$$d_1(a,b) = 5.382 \longrightarrow (1)$$

$$d_2(b,c) = \sqrt{(2-4)^2 + (2-0)^2 + (0-2)^2 + (1-0)^2}$$

$$= \sqrt{4 + 4 + 4 + 1}$$

$$= \sqrt{13}$$

$$d_2(b,c) = 3.605 \longrightarrow (2)$$

$$d_3(a,c) = \sqrt{(2-4)^2 + (2-5)^2 + (0-2)^2 + (1-2)^2}$$

$$= \sqrt{4 + 9 + 4 + 1}$$

$$d_3(a,c) = \sqrt{18} \Rightarrow 4.243 \quad \text{—— (3)}$$

from eq. (1), (2) and (3)

$$d_1 > d_3 > d_2$$

(1) b and c users are the closest.

(2) users a and b are farthest apart.

Q7)

② Cosine Distance:-

$$(d)\cos(\theta) = 1 - \frac{a.b}{||a||.||b||}$$

$$||a|| = \sqrt{a_1^2 + a_2^2 + a_3^2 + a_4^2} \Rightarrow \sqrt{16 + 25 + 4 + 4} \Rightarrow \sqrt{49} \Rightarrow 7$$

$$||b|| = \sqrt{b_1^2 + b_2^2 + b_3^2 + b_4^2} \Rightarrow \sqrt{16 + 0 + 4 + 0} \Rightarrow \sqrt{20}$$

$$||c|| = \sqrt{c_1^2 + c_2^2 + c_3^2 + c_4^2} \Rightarrow \sqrt{4 + 4 + 0 + 1} \Rightarrow \sqrt{9} \Rightarrow 3$$

Q8 Find Eigenvalues:

$$D = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Sol:

$\Rightarrow |D - \lambda I| = 0$

$\Rightarrow \begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = 0$

$\Rightarrow (2-\lambda)(2-\lambda) - (1)(1) = 0$

$\Rightarrow 4 + \lambda^2 - 4\lambda - 1 = 0$

$\Rightarrow \lambda^2 - 4\lambda + 3 = 0$

$\Rightarrow \lambda^2 - 3\lambda - 1\lambda + 3 = 0$

$\Rightarrow \lambda(\lambda - 3) - 1(\lambda - 3) = 0$

$\Rightarrow (\lambda - 1)(\lambda - 3) = 0$

$\Rightarrow \lambda_1 = 1$

$\Rightarrow \lambda_2 = 3$

Q9) $\log_2 = 0.69$; $\log_3 = 1.10$

$\log_{12} = ?$

Sol:

$\log_{12} = \log(2 \times 2 \times 3)$

$= \log_2 + \log_2 + \log_3$

$= 0.69 + 0.69 + 1.10$

$\log_{12} = 2.48$

Q10) Find inverse function.

$y = f(x) = \frac{1}{1 + e^{-x}}$

Sol:

$y = 1/1+e^{-x}$

$y - ye^{-x} = 1$

$-ye^{-x} = 1-y$

$e^{-x} = \frac{y-1}{y}$

Taking ln on both sides

$\ln e^{-x} = \ln\left(\frac{y-1}{y}\right)$

$-x(\ln e) = \ln\left(\frac{y-1}{y}\right)$

$x = -\ln\left(\frac{y-1}{y}\right)$

Q11) (A) $f(x) = x^2 - 1$

$f'(x) = \frac{d}{dx}(x^3 - 1) \Rightarrow 3x^2$

(b) $f(x) = \log(x^2 - 3x)$

$f'(x) = \frac{d}{dx}(\log(x^2-3x))$

$\Rightarrow \frac{1}{x^2-3x} \cdot \frac{d}{dx}(x^2-3x)$

$\Rightarrow \frac{1}{x^2-3x}(2x)$

$f'(x) \Rightarrow 2x / x^2 - 3x$

(c) $f(x) = \exp(ax^b)$

$f'(x) = \exp(ax^b) \cdot \frac{d}{dx}(ax^b)$

$\Rightarrow \exp(ax^b) \cdot abx^{b-1}$

$\Rightarrow abx^{b-1} \cdot \exp(ax^b)$

Q12) $f(x,y) = \exp(x^2 + 2y^2)$

$f_x = \frac{d}{dx}(\exp(x^2+2y^2))$

$f_x = 2x \cdot (\exp(x^2+2y^2))$ — (1)

$f_y = \frac{d}{dy}(\exp(x^2+2y^2))$

$f_y = 4y(\exp(x^2+2y^2))$ — (2)

using eq. (1) and (2) we can find f_{xx} and f_{yy} and f_{xy}

$$f_{xx} = \frac{d}{dx}\left(2x\exp(x^2 + 2y^2)\right)$$

$$\boxed{f_{xx} = (2 + 4x^2)\exp(x^2 + 2y^2)}$$

$$f_{yy} = \frac{d}{dy}\left(4y\exp(x^2 + 2y^2)\right)$$

$$\boxed{f_{yy} \Rightarrow (4 + 16y^2)\exp(x^2 + 2y^2)}$$

$$\boxed{f_{xy} = 8xy.\exp(x^2 + 2y^2)}$$