Lab 1
Number Systems

Objectives

- Students will come to know about various number systems.
- They will be able to perform conversions between number systems.
- They will be able to perform basic arithmetic operations on binary number systems.
- They will be able to perform addition in hexadecimal and octal number systems.
- They will be able to represent singed numbers.
- They will be able to represent BCD numbers and perform their addition.

NUMBER SYSTEM

Number systems are the technique to represent numbers in the computer system architecture, every value that you are saving or getting into/from computer memory has a defined number system.

Computer architecture supports the following number systems.

- **Binary number system**
- **Octal number system**
- **Decimal number system**
- **Hexadecimal (hex) number system**

BINARY NUMBER SYSTEM

A Binary number system has only two digits that are **0 and 1**. Every number (value) represents with 0 and 1 in this number system. The base of the binary number system is 2, because it has only two digits.

OCTAL NUMBER SYSTEM

The octal number system has only eight (8) digits from **0 to 7**. Every number (value) represents with 0,1,2,3,4,5,6 and 7 in this number system. The base of the octal number system is 8, because it has only 8 digits.

DECIMAL NUMBER SYSTEM

The decimal number system has only ten (10) digits from **0 to 9**. Every number (value) represents with 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 in this number system. The base of the decimal number system is 10 because it has only 10 digits.

HEXADECIMAL NUMBER SYSTEM

A Hexadecimal number system has sixteen (16) alphanumeric values from **0 to 9** and **A to F**. Every number (value) represents with 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, and F in this number system. The base of the hexadecimal number system is 16, because it has 16 alphanumeric values. Here A is 10, B is 11, C is 12, D is 14, E is 15 and F is 16.

Number system	Base(Radix)	Used digits	Example
Binary	2	0, 1	$(11110000)_2$
Octal	8	0, 1, 2, 3, 4, 5, 6, 7	$(360)_8$
Decimal	10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9	$(240)_{10}$
Hexadecimal	16	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F	$(F0)_{16}$

CONVERSIONSDECIMAL TO OTHER

Decimal Number System to Other Base

To convert Number system from a **Decimal Number System** to **Any Other Base** is quite easy; you have to follow just two steps:

A) Divide the Number (Decimal Number) by the base of the target base system (in which you want to convert the number: Binary (2), octal (8), and Hexadecimal (16)).

B) Write the remainder from Step 1 as a Least Signification Bit (LSB) to Step Last as a Most Significant Bit (MSB).

1. DECIMAL TO BINARY

Decimal to Binary Conversion	Result
Decimal Number is: **(12345)$_{10}$**	Binary Number is **(11000000111001)$_2$**

2	12345	1	**LSB**
2	6172	0	
2	3086	0	
2	1543	1	
2	771	1	
2	385	1	
2	192	0	
2	96	0	
2	48	0	
2	24	0	
2	12	0	
2	6	0	
2	3	1	
	1	1	**MSB**

2. DECIMAL TO OCTAL

Decimal to Octal Conversion	Result
Decimal Number is: **(12345)$_{10}$**	Octal Number is **(30071)$_8$**

8	12345	1	LSB
8	1543	7	
8	192	0	
8	24	0	
	3	3	MSB

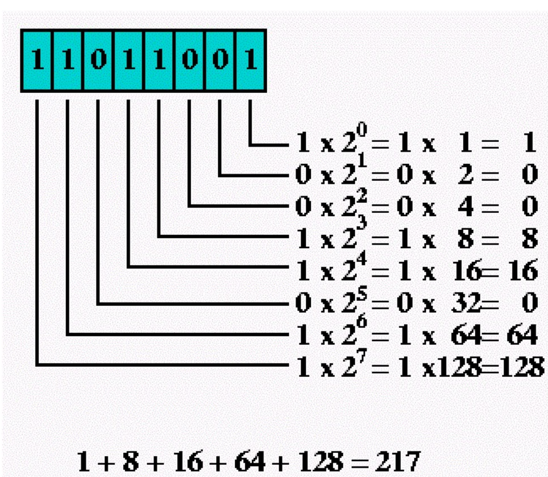
3. DECIMAL TO HEXADECIMAL

Decimal to Hexadecimal Conversion	Result
Example 1 Decimal Number is: **(12345)$_{10}$** 16 12345 9 LSB 16 771 3 16 48 0 8 3 3 MSB	Hexadecimal Number is **(3039)$_{16}$**
Example 2 Decimal Number is: **(725)$_{10}$** 16 725 5 5 LSB 16 45 13 D 2 2 2 MSB	Hexadecimal Number is **(2D5)$_{16}$** **Convert** **10, 11, 12, 13, 14, 15** **to its equivalent...** **A, B, C, D, E, F**

BINARY TO OTHER

Multiply the digit with 2(with place value exponent). Eventually, the sum of all the multiplication becomes the Decimal number.

1. BINARY TO DECIMAL



2. BINARY TO OCTAL

An easy way to convert from binary to octal is to group binary digits into sets of three, starting with the least significant (rightmost) digits.

Binary: 11100101 =	11 100 101	
	011 100 101	Pad the most significant digits with zeros if necessary to complete a group of three.

Then, look up each group in a table:

Binary:	000	001	010	011	100	101	110	111
Octal:	0	1	2	3	4	5	6	7

Binary =	011	100	101	
Octal =	3	4	5	= **$(345)_8$**

3. BINARY TO HEXADECIMAL

An equally easy way to convert from binary to hexadecimal is to group binary digits into sets of four, starting with the least significant (rightmost) digits.

Binary: 11100101 = 1110   0101

Then, look up each group in a table:

Binary:	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001
Hexadecimal:	0	1	2	3	4	5	6	7	8	9

Binary:	1010	1011	1100	1101	1110	1111
Hexadecimal:	A	B	C	D	E	F

Binary =	1110	0101	
Hexadecimal =	E	5	= **$(E5)_{16}$**

OCTAL TO OTHER

1. OCTAL TO BINARY

Converting from octal to binary is as easy as converting from binary to octal. Simply look up each octal digit to obtain the equivalent group of three binary digits.

Octal:	0	1	2	3	4	5	6	7
Binary:	000	001	010	011	100	101	110	111

Octal =	3	4	5	
Binary =	011	100	101	= **(011100101)$_2$**

2. OCTAL TO HEXADECIMAL

When converting from octal to hexadecimal, it is often easier to first convert the octal number into binary and then from binary into hexadecimal. For example, to convert 345 octal into hex:

(from the previous example)

Octal =	3	4	5	
Binary =	011	100	101	= **(011100101)$_2$**

Drop any leading zeros or pad with leading zeros to get groups of four binary digits (bits):
Binary 011100101 = 1110 0101

Then, look up the groups in a table to convert to hexadecimal digits.

Binary:	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001
Hexadecimal:	0	1	2	3	4	5	6	7	8	9

Binary:	1010	1011	1100	1101	1110	1111
Hexadecimal:	A	B	C	D	E	F

Binary =	1110	0101	
Hexadecimal =	E	5	= **(E5)$_{16}$**

Therefore, through a two-step conversion process, octal 345 equals binary 011100101 equals hexadecimal E5.

3. OCTAL TO DECIMAL

The conversion can also be performed in the conventional mathematical way, by showing each digit place as an increasing power of 8.

345 octal = $(3 * 8^2) + (4 * 8^1) + (5 * 8^0) = (3 * 64) + (4 * 8) + (5 * 1)$ = 229 decimal

HEXADECIMAL TO OTHER

1. HEXADECIMAL TO BINARY

Converting from hexadecimal to binary is as easy as converting from binary to hexadecimal. Simply look up each hexadecimal digit to obtain the equivalent group of four binary digits.

Binary:	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001
Hexadecimal:	0	1	2	3	4	5	6	7	8	9

Binary:	1010	1011	1100	1101	1110	1111
Hexadecimal:	A	B	C	D	E	F

Hexadecimal =	A	2	D	E	
Binary =	1010	0010	1101	1110	= 1010001011011110 binary

2. HEXADECIMAL TO OCTAL

To Convert a hexadecimal number to octal octal number, we need to first convert the hexadecimal number into binary number and then from binary to Octal. Simply look up each hexadecimal digit to obtain the equivalent group of four binary digits.

Binary:	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001
Hexadecimal:	0	1	2	3	4	5	6	7	8	9

Binary:	1010	1011	1100	1101	1110	1111
Hexadecimal:	A	B	C	D	E	F

Hexadecimal =	A	2	D	E			
Binary =	1010	0010	1101	1110	= 1010001011011110 binary		
Re-grouping	001	010	001	011	011	110	
Octal	1	2	1	3	3	6	=121336 Octal

Binary Arithmetic

Rules of Binary Addition

- `0 + 0 = 0`
- `0 + 1 = 1`
- `1 + 0 = 1`
- `1 + 1 = 0`, and carry 1 to the next more significant bit

For example,

`00011010 + 00001100 = 00100110`

			1	*1*						*Carries*
	0	0	0	1	1	0	1	0	=	$26_{(\text{base } 10)}$
+	0	0	0	0	1	1	0	0	=	$12_{(\text{base } 10)}$
	0	0	1	0	0	1	1	0	=	$38_{(\text{base } 10)}$

`00010011 + 00111110 = 01010001`

		1	*1*	*1*	*1*	*1*				*carries*
	0	0	0	1	0	0	1	1	=	$19_{(\text{base } 10)}$
+	0	0	1	1	1	1	1	0	=	$62_{(\text{base } 10)}$
	0	1	0	1	0	0	0	1	=	$81_{(\text{base } 10)}$

Rules of Binary Multiplication

- `0 x 0 = 0`
- `0 x 1 = 0`
- `1 x 0 = 0`
- `1 x 1 = 1`, and no carry or borrow bits

For example,

`00101001 × 00000110 = 11110110`

			0	0	1	0	1	0	0	1	=	$41_{(\text{base } 10)}$
×			0	0	0	0	0	1	1	0	=	$6_{(\text{base } 10)}$
			0	0	0	0	0	0	0	0		
		0	0	1	0	1	0	0	1			
	0	0	1	0	1	0	0	1				
	0	0	1	1	1	1	0	1	1	0	=	$246_{(\text{base } 10)}$

Binary Division

Binary division is the repeated process of subtraction, just as in decimal division.

For example,

`00101010 ÷ 00000110 =` `1 1 1` $= 7_{(\text{base } 10)}$

```
00000111
                  _______________________________
        1 1 0 ) 0  0  1  ¹0  1  0  1  0   =  42(base 10)
                      -  1   1  0         =   6(base 10)
                      ___________
                         1                   borrows
                         1  0  ¹0  1
                      -  1  1   0
                      ______________
                               1  1  0
                         -     1  1  0
                         _____________
                                     0
```

```
10000111 ÷ 00000101 =                1  1  0  1  1   =  27(base 10)
             00011011
                  _______________________________
        1 0 1 ) 1  0  0  ¹0  0  1  1  1   =  135(base 10)
              - 1  0  1                   =   5(base 10)
              _________
                   1  1  ¹0
                -  1  0   1
                ___________
                         1  1
                      -     0
                      _______
                         1  1  1
                      -  1  0  1
                      __________
                               1  0  1
                         -     1  0  1
                         _____________
                                     0
```

Example − Division

101010 / 000110 = 000111

```
              1 1 1      = 7₁₀
        ____________
000110 ) 1 ¹0 1 0 1 0    = 42₁₀
        -1 1 0           = 6₁₀
         ______
          1 ¹0 0 1
         -1 1 0
         ________
            1 1 0
           -1 1 0
           ______
                0
```

Hexadecimal Addition

Following hexadecimal addition table will help you greatly to handle Hexadecimal addition.

+	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
1	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F	10
2	2	3	4	5	6	7	8	9	A	B	C	D	E	F	10	11
3	3	4	5	6	7	8	9	A	B	C	D	E	F	10	11	12
4	4	5	6	7	8	9	A	B	C	D	E	F	10	11	12	13
5	5	6	7	8	9	A	B	C	D	E	F	10	11	12	13	14
6	6	7	8	9	A	B	C	D	E	F	10	11	12	13	14	15
7	7	8	9	A	B	C	D	E	F	10	11	12	13	14	15	16
8	8	9	A	B	C	D	E	F	10	11	12	13	14	15	16	17
9	9	A	B	C	D	E	F	10	11	12	13	14	15	16	17	18
A	A	B	C	D	E	F	10	11	12	13	14	15	16	17	18	19
B	B	C	D	E	F	10	11	12	13	14	15	16	17	18	19	1A
C	C	D	E	F	10	11	12	13	14	15	16	17	18	19	1A	1B
D	D	E	F	10	11	12	13	14	15	16	17	18	19	1A	1B	1C
E	E	F	10	11	12	13	14	15	16	17	18	19	1A	1B	1C	1D
F	F	10	11	12	13	14	15	16	17	18	19	1A	1B	1C	1D	1E

To use this table, simply follow the directions used in this example – Add A_{16} and 5_{16}. Locate A in the X column then locate the 5 in the Y column. The point in 'sum' area where these two columns intersect is the sum of two numbers.

$A_{16} + 5_{16} = F_{16}$

Example

$4A6_{16} + 1B3_{16} = 659_{16}$

1		carry
	4 A 6	$= 1190_{10}$
+	1 B 3	$= 435_{10}$
	6 5 9	$= 1625_{10}$

Octal Addition

Following the octal addition table will help you to handle octal addition.

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	10
2	2	3	4	5	6	7	10	11
3	3	4	5	6	7	10	11	12
4	4	5	6	7	10	11	12	13
5	5	6	7	10	11	12	13	14
6	6	7	10	11	12	13	14	15
7	7	10	11	12	13	14	15	16

A

Sum

B

To use this table, simply follow the directions used in this example: Add 6_8 and 5_8. Locate 6 in the A column then locate the 5 in the B column. The point in 'sum' area where these two columns intersect is the 'sum' of two numbers.

$6_8 + 5_8 = 13_8$.

Example

$456_8 + 123_8 = 601_8$

$$
\begin{array}{rl}
1\,1 & \text{carry} \\
4\,5\,6 & = 302_{10} \\
+\,1\,2\,3 & = 83_{10} \\
\hline
6\,0\,1 & = 385_{10}
\end{array}
$$