Area Lights: A Brief Discussion

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Remember the rendering equation:

$$L_o = \int_{\Omega} L_i f(w_i, w_o) (n \cdot w_i) dw_i \tag{1}$$

where L_i represent the incoming light, n is the surface normal at the point being shaded, f accounts for the reflection, and dw_i is the differential solid angle from which the light is received.

We will learn in later weeks that the product $L_i dw_i$ actually represents a radiometric term known as *irradiance*. So the rendering equation computes how much radiance is reflected along w_o due to sum of all irradiances from w_i s.

For point lights, we know that due to the inverse square law the irradiance at a distance of d from a point source of intensity I is $\frac{I}{d^2}$. That is why we were attenuating the point light sources' intensities by the squared distance from them.

However, for area lights, we cannot really use the term intensity as *intensity* is a term reserved for points with zero area (for point lights, that is). Area lights are more suitably defined by the radiance that they emit in different directions. If an area light is emitting the same radiance from all of its points and in all possible directions, such a light is called an ideal diffuse light.

How does this relate to the above equation? Now imagine that you have an ideal diffuse area light, which is emitting a radiance of L in all directions (and from all points on its surface). How much irradiance does this produce at the point being shaded? This is not an easy question to answer as it requires integrating how much contribution each point on the light makes to the point being shaded. But we can approximate this by computing the *solid angle* spanned by the light at the point of intersection. This is given by:

$$dw_i = A \frac{n_l \cdot w_i}{d^2},\tag{2}$$

where A is the area of the light source and n_l is its surface normal. This means we must multiply the radiance of the light source with the above result to compute the irradiance it yields at a distance of d from the light point used in the computation.

As a side note, the computation of dw_i in the equation above is indeed an approximation of the actual solid angle spanned by the surface from the point

being shaded. If we sample a large number of points on the area light and compute their w_i s, some of these will make a shallower angle with the surface normal while others make a steeper one. The mean of solid angles due to each such w_i will yield the actual solid angle of the light as seen from the shading point. Computing integrals by averaging a finite number of points is known as Monte Carlo integration and we will learn more about this topic later in this class.