Algorithmic Methods for Mathematical Models

Assignment MH

Due 3 December 2018

We want to open a new supermarket and we need to decide how many employees to hire and which hours they should be assigned to a checkout station, which is the most critical task they will perform in the supermarket.

The supermarket opens from Monday to Saturday, from 10am to 10pm. We have available a set E of possible employees to hire, and for each hour t, with $1 \leq t \leq 72$ (there are 72 working hours per week), a given parameter c_t estimates the number of customers that the supermarket will receive during the t-th hour. Considering that each customer spends pt minutes at the checkout station, we can assume that floor(60/pt) is the number of customers that a checkout station can process in one hour. If this capacity is exceeded, another checkout station should be opened. Assume that we have as many checkout stations as necessary.

Employees can only work whole hours (i.e. they cannot work fractions of them) and that no employee can work more than two consecutive hours along the same day. Note that it is OK, for example, to work the last two hours on Monday and the first hours on Tuesday. There are pairs of employees that cannot be working in checkout stations simultaneously. This information is stored in a matrix I where, for each pair of employees $e1$ and $e2$, $I[e1][e2]$ is 1 if these employees are incompatible and 0 otherwise.

Because of the complexity of the optimization problem, we want to develop a heuristic algorithm. We are considering greedy plus local search, as well as the GRASP meta-heuristic.

(a) Specify the greedy algorithm, including the greedy function $q(\cdot)$. Specify $q(\cdot)$ using mathematical notation and a short descriptive text.

(b) Specify the algorithm for the GRASP *constructive phase*, including the equation describing the RCL. We can assume that the same greedy function id used here.

(c) Let us assume that the GRASP *constructive phase* is being executed and the following assignments have already been done:

t	E
1	5 6
2	5 6 11 15
3	11 15 16 18

Write down the RCL at t=4 (assume α=0.3) after applying the greedy function you proposed in (a). For each element in the RCL, provide the value of $q(\cdot)$.

(d) Design an algorithm (pseudo-code) for the *local search* phase. Define the neighborhood $N(S)$ that you consider and describe how $N(S)$ is explored.