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## Homework 3

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**Exercise: IPs with deterministic verifiers** Prove that  $\text{dIP} = \text{NP}$ .<sup>1</sup>

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<sup>1</sup>Let  $\text{dIP}$  be defined like  $\text{IP}$  except that the verifier is deterministic instead of probabilistic: A language  $L$  is in  $\text{dIP}$  if and only if there exists a polynomial-time computable function  $V : \Sigma^* \times \Sigma^* \rightarrow \Sigma^* \cup \{0, 1\}$  and a polynomial  $p$  such that for every  $x \in \Sigma^*$  and for  $t = p(|x|)$  the following hold:

1. if  $x \in L$ , then there exists a  $p$ -bounded prover  $P$  such that  $(V \leftrightarrow_t P)(x) = 1$ ,
2. if  $x \notin L$ , then for every  $p$ -bounded prover  $P$  we have  $(V \leftrightarrow_t P)(x) = 0$ .

Recall that a  $p$ -bounded prover is a function  $P : \Sigma^* \times \Sigma^* \rightarrow \Sigma^*$  satisfying  $|P(x, \langle m_1, \dots, m_r \rangle)| \leq p(|x|)$  for every  $x \in \Sigma^*$  and  $m_1, \dots, m_r \in \Sigma^*$ , and that  $(V \leftrightarrow_t P)(x)$  denotes the output of the  $2t$ -round interaction between  $V$  and  $P$  on input  $x$ ; i.e.,  $(V \leftrightarrow_t P)(x) = V(x, \langle m_1, \dots, m_{2t} \rangle)$  where  $m_{2i-1} = V(x, \langle m_1, \dots, m_{2i-2} \rangle)$  and  $m_{2i} = P(x, \langle m_1, \dots, m_{2i-1} \rangle)$  for  $i = 1, \dots, t$ .