Miguel Alcón Doganoc
Combinatorial Problem Solving
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Logic Synthesis

1 Variables

Given the number of input signals n, the depth d, and the specification (truth table) of the logical circuit, I defined the following variables:

- $\mathbf{c_{i,j}}$:= "Code of the node (i,j)", where
  - $i \in \{0, 1, ..., d\}$
  - $j \in \{0, 1, ..., 2^i - 1\}$
- $\mathbf{b^{(t)}_{i,j}}$:= "Boolean value of the node (i, j) for the row t of the truth table", where
  - $i \in \{0, 1, ..., d\}$
  - $j \in \{0, 1, ..., 2^i - 2\}$
  - $t \in \{0, 1, ..., 2^n - 1\}$

For example, for a NOR-circuit that implements the functionality of an AND gate (see figure 1), with $n = d = 2$, one possible solution for the variables $c_{i,j}$ and $b^{(0)}_{i,j}$, with $i \in \{0, 1, ..., 6\}$, is shown in figures 2b and 2c, respectively.

x_1	x_2	y	t
0	0	0	0
0	1	0	1
1	0	0	2
1	1	1	3

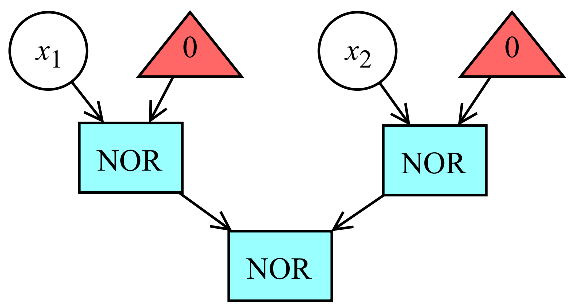
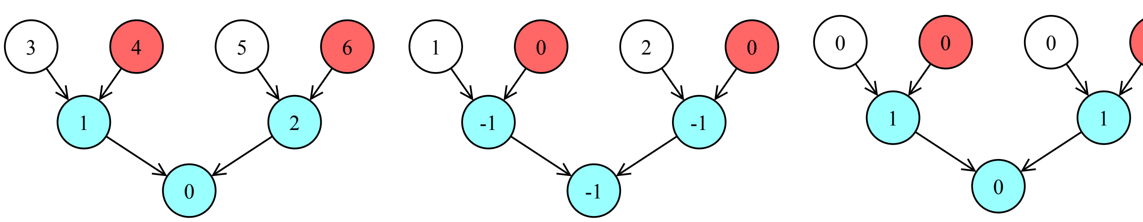


Figure 1: Truth table of $y = \text{AND}(x_1, x_2)$ and NOR-circuit implementing it.



(a) i (b) $c_{i,j}$ (c) $b^{(0)}_{i,j}$

Figure 2: Visual representation of (i, j) and both variables.

2 Constraints

In order to simplify the definition of the constraints, I define three functions. Given the variable $v_{i,j}$, with $v_{i,j} = c_{i,j}$ or $b^{(t)}_{i,j}$,

- **left**$(v_{i,j})$:= "Variable corresponding to the one on the left of $v_{i,j}$"
- **right**$(v_{i,j})$:= "Variable corresponding to the one on the right of $v_{i,j}$"
- **bit-up(k,t)** := "Boolean value of x_k in the t row of the truth table"

So, the constraints are the following:

1. $c_{d,j} \geq 0$
2. $c_{i,j} \geq 0 \Rightarrow (\textbf{left}(c_{i,j}) = 0 \land \textbf{right}(c_{i,j}) = 0)$
3. $c_{i,j} = -1 \Rightarrow (\textbf{left}(c_{i,j}) \geq \textbf{right}(c_{i,j}))$
4. $(c_{i,j} = -1 \land (\textbf{left}(c_{i,j}) > 0 \lor \textbf{right}(c_{i,j}) > 0)) \Rightarrow (\textbf{left}(c_{i,j}) \geq \textbf{right}(c_{i,j}))$
5. $c_{i,j} = -1 \Rightarrow (b^{(t)}_{i,j} = \neg(\textbf{left}(b^{(t)}_{i,j}) \lor \textbf{right}(b^{(t)}_{i,j})))$
6. $c_{i,j} = 0 \Rightarrow \neg b^{(t)}_{i,j}$
7. $c_{i,j} = k \Rightarrow b^{(t)}_{i,j}$
8. $c_{i,j} = k \Rightarrow \neg b^{(t)}_{i,j}$