Manipulation

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- A voting rule F is strategy-proof if for every preference profile $\succ' = (\succ_{-i}, \succ'_i)$, it is not the case that $F(\succ') \succ_i F(\succ)$

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- Yes, but not very satisfactory!

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- Onto: Every alternative can win under some preference profile.
- Non-dictatorial: There is no voter i such that $F(\succ)$ is always the top alternative for voter i.

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In ☺words, for $m \geq 3$, any deterministic social choice function must be at least one of the following:

- *dictatorial: there exists a single fixed voter whose most-preferred alternative is chosen for every profile;*
- *imposing: there is at least one alternative that does not win under any profile;*
- *manipulable (i.e., not strategyproof).*

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The first two properties are not acceptable in most voting settings.
So, we need to assume that the voters have an incentive to misreport true preferences.

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- Maybe the rule is manipulable, but it is NP-hard to find a successful manipulation.
  For once NP-hardness can be good!!

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- For plurality, this problem is computationally trivial:
- The only sensible manipulation is to put a as your most preferred candidate!

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- for every preference order P and every alternative a, a score $S(P, a)$ can be defined so that it is,
  - Responsive: the candidate with the largest score wins (in the voting under the joint profile)
  - Monotone: for any two preference orders P and P' and for any candidate a, if for each voter i, $\{b \mid a \ P \ b\} \subseteq \{b \mid a \ P' \ b\}$, then $S(P, a) \leq S(P', a)$.

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  If so, place b in the next position, otherwise terminate claiming that order does not exists.

Manipulable rules by Greedy

Theorem

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- If Greedy-Manipulation succeeds, it constructs a preference order that guarantees that under the joint profile c wins.
- Assume that such an order exists and that Greedy-Manipulation terminates without providing an ordering. Let us reach a contradiction.

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- Consider any completion P of the preference order started by G-Man that places u in the first unassigned place.

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- By initialization $S(P, c) \geq S(P', c)$.
- So, $S(P, c) \geq S(P, u)$.
- But G-Man did not assign u, so $S(P, c) < S(P, u)$ and we get the contradiction.

Manipulable rules by Greedy

Corollary

For any voting rule F satisfying the BTT conditions, and for which the scoring rule can be computed in polynomial time G-Man solves the F-MANIPULATION problem in polynomial time.

By monotonicity, it should be possible to computer the score to of the alternative ranked "first" among a set of unranked alternatives

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- Both, Copeland vote and the score can be computed in polynomial time.
- Copeland is polynomial time manipulable.

Maximin

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  - $Score(x) = min_y n_{x \succ y}$
  - elect x^* with the maximum score
- Working in a similar way, Maximin is polynomial time manipulable.

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- STV-Manipulation is NP-hard (Bartholdi III, Social Choice and Welfare, 1991)