A glimpse on mechanism design: Auctions

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Fall 2019

1 Auctions

2 Truth telling

3 VCG mechanism

4 Sponsored search

Context

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- What is the right price for objects? groups of objects?
- Objectives:
  - Truth-telling
  - Efficiency: social welfare
  - Revenue: maximize profit
  - Envy-freeness ⋮

Not all of them can be achieved at the same time.

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- Goal: Design an auction so that in equilibrium we get the results we want.
- As in Game theory we rely on rationality.

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      The number (or portion) of goods of each type including legal or other restrictions on how the goods may be allocated.
    - Rules for bidding and clearing.
    - A procedure to determine who wins what (allocation) and how much pays (payment) on the basis of the received information.

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- When?
- What?
- To whom?

Single item auctions

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- We analyze three mechanisms

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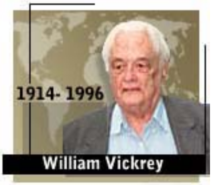
- Second price auctions are also known as Vickrey auctions. defined by William Vickrey in 1961. Vickrey won the Nobel prize in Economics in 1996.



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- Everyone pays the amount of their bid regardless of whether or not they get the good.

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- Bidders have to decide on a strategy to bid, a function applied to their valuation.

SP-Auctions: Equilibrium behaviour

Theorem

In SP-price auctions truth-telling is a dominant strategy.

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- That is, in equilibrium, the auctioneer allocates the item to the bidder with the highest value.
    - With the actual highest value, not just the highest bid.
    - Without assuming anything on the values.
- However the seller does not get maximum revenue.

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  Hard to select an strategy without some information about the others. We continue the analysis on a Bayesian setting.

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- They have beliefs on the valuations of the other players!
- As usual beliefs are modeled with probability distributions.
- Bidders do not know their opponent's values, i.e., there is incomplete information.
  Each bidder's strategy must maximize her expected payoff accounting for the uncertainty about opponent values.

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- What is the equilibrium in this game of incomplete information?

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- maximizing for b_1 we have: $[2b_1\,(v_1 - b_1)]' = 2v_1 - 4b_1 = 0$ which gives $b_1 = v_1/2$

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Theorem

In a FP auction with n bidders under the uniform values model, the strategy $b_i = \frac{n-1}{n} v_i$, for $1 \leq i \leq n$, is a Bayesian Nash equilibrium.

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- Thus, in the uniform value model FP is efficient.

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- When analyzing revenue take into account that when the item is not sold the seller gets a benefit of u.

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- So, with a single bidder,the optimal reserve price is halfway between the value of the object to the seller and the maximum possible bidder value.

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- So, with a single bidder,the optimal reserve price is halfway between the value of the object to the seller and the maximum possible bidder value.
- With more intricate analyses, you can determine the optimal reserve price for a second-price auction with multiple bidders

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  - In a single item auction when i wins the object this payment is 2nd highest bid minus 0

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  Here, again, truthfulness is a dominant strategy.

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  - Without player 1: welfare is 100.
  - $p_1 = 100 - 80$

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  - But, total payment is $20 + 30 < 100$!
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  This is a real problem!

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1. Auctions
2. Truth telling
3. VCG mechanism
4. Sponsored search

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  - Multiple positions, but advertisers submit only a single bid.
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- Late 1990s: Yahoo! and MSN adopt Overture, but mechanism proves unstable. Advertisers constantly change bids to avoid paying more than necessary.
- 2002: Google modifies keyword auction to have advertisers pay minimum amount necessary to maintain their position (GSP) - followed by Yahoo! and MSN.

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- Advertiser i submits a bid b_i, the amount he is willing to pay for a click.

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  The benefit per click is assumed to be independent of the slot.

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  In the simplest model $\gamma_i = 1$.

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- If advertiser i is assigned to slot j at a price of p_i per click then her utility is

$$u_i = \alpha_j \gamma_i (v_i - p_i),$$

which is the number of clicks received times profit per click.

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The social welfare is independent of the payments and the bids!

$$SW(\pi, v, \gamma)$$

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  Exercise: what would be the prices in the VCG auction?

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- The auctioneer sets $\pi(k)$ to be the advertiser with the kth highest effective bid (breaking ties arbitrarily).
- That is, the GSP mechanism assigns slots with higher click-through-rate to advertisers with higher effective bids.

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- In the case $\gamma_i = 1$, for each i,

$$p_i = b_{\pi(k+1)}.$$

GSP:utility

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- $u_i(b,\gamma)$ is the utility derived by advertiser i from the GSP mechanism when advertisers bid according to b:

$$\begin{aligned} u_i(b,\gamma) =& \alpha_{\pi^{-1}(i)}\gamma_i(v_i - p_i) \\ &= \alpha_{\pi^{-1}(i)}[\gamma_i v_i - \gamma_{\pi(\pi^{-1}(i)+1)} b_{\pi(\pi^{-1}(i)+1)}]. \end{aligned}$$

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- It is not a dominant strategy to bid "truthfully"

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GSP: Envy-free equilibrium

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Definition

Given a GSP with n players defined by click-through-rates $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_n$, quality scores $\gamma_1 \geq \gamma_2 \geq \cdots \geq \gamma_n$ and valuations $v_1, \ldots, v_n$.
A bid vector b is an *envy-free* equilibrium if, for any pair j, k of players, player j would not prefer player k's allocation and payments rather than their own.
Formally

$$\alpha_j(\gamma_{\pi(j)} v_{\pi(j)} - \gamma_{\pi(j+1)} b_{\pi(j+1)}) \geq \alpha_k(\gamma_{\pi(j)} v_{\pi(j)} - \gamma_{\pi(k+1)} b_{\pi(k+1)})$$

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This is a envy-free equilibrium!

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- Incentive compatibility (a.k.a. truthfulness): It is a dominant strategy for each player to participate in the auction and report their true value.
- Pareto-optimality: An allocation π and payments p is Pareto-optimal if and only if there is no alternative allocation and payments where all players' utilities and the revenue of the auctioneer do not decrease, and at least one of them increases.