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Computational Complexity
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Homework 1

1 Exercise: Verifiers in L

2 Exercise: NP = NL?

3 Exercise: Comparison in logspace

As it is said in the statement, we know that n is the length of x, and the input tape is as following:

$$\square\ x_1, x_2, ..., x_n \# y_1, y_2, ..., y_n\ \square$$

Starting from this, I describe the TM that outputs 0 if $m < n$ and 1 if $m \geq n$, with the input tape defined above. For a more understandable explanation, I have made a visual representation of both tapes (first input, second work tape) at the end of each general step. Note that the position of the head of each tape is represented as a square surrounding the symbol which is in this position. Let us start with the description.

First, we have to check if $n = 0$. If it happens, we assume that both m and n are 0, so $m = n$.

1. Scan the actual position of the head of the input tape, which is the first position starting from the left. If it is a # symbol, output 1 and halt.

$$\square\ \boxed{x_1}, x_2, ..., x_n \# y_1, y_2, ..., y_n\ \square$$

We copy y into the work tape. Because y is the binary encoding of n, we know that it is $O(\log n)$ in space.

2. Move the input tape to the right until finding the # symbol.
3. Move the input tape to the right until finding a 1.
4. Copy into the work tape from the first 1 (current position) to the $\square$ symbol (end of the input tape), from left to right.

$$\square\ x_1, x_2, ..., x_n \# y_1, y_2, ..., y_n\ \boxed{\square}$$

$$\square\ y_1, y_2, ..., y_n\ \boxed{\square}$$

Before starting to check whether $m < n$ or $m \geq n$, we have to place the head of each tape in the proper position.

5. Move the input tape to the left until finding the # symbol.
6. Move the input and the work tape simultaneously[1] to the left until finding the $\square$ symbol in the work tape.

$$\square\ x_1, x_2, ..., \boxed{x_i}, ..., x_n \# y_1, y_2, ..., y_n\ \square$$

$$\boxed{\square}\ y_1, y_2, ..., y_n\ \square$$

At this point, if we find a 1 between x_1 and x_i, with $i \in \{1, 2, ..., n\}$ or $x_i = \square$, then $m > n$.

[1] First move one, then the other.

7. Move the input tape to the left until finding a 1 or the $\square$ symbol. If it finds a 1, then it outputs 1 and halts.

$$\boxed{\square}\, x_1, x_2, ..., x_i, ..., x_n \# y_1, y_2, ..., y_n\ \square$$

$$\boxed{\square}\, y_1, y_2, ..., y_n\ \square$$

After that, if the TM haven't halted yet, we have to move the heads to the positions like at the end of step 6.

8. Move the input tape to the right until finding the # symbol.
9. Move the working tape to the right until finding the $\square$ symbol.
10. Repeat step 6.

$$\square\ x_1, x_2, ..., \boxed{x_i}, ..., x_n \# y_1, y_2, ..., y_n\ \square$$

$$\boxed{\square}\, y_1, y_2, ..., y_n\ \square$$

Now, we continue checking whether $m < n$ or $m \geq n$, but this time comparing x with y.

11. Scan both tapes and, if the TM does not halt, move the head one position to the right.
    (a) If the value of the header of the input tape is 1, and the one of the work tape is 0, then output 1 and halt.
    (b) Else if the value of the header of the input tape is 0, and the one of the work tape is 1, output 0 and halt.
    (c) Else if the value of both heads are the $\square$ symbol (end of both tapes) output 1 and halt.
    (d) Otherwise repeat step 11.

$$\square\ x_1, x_2, ..., \boxed{x_i}, ..., x_n \# y_1, y_2, ..., y_n\ \square$$

$$\square\ y_1, y_2, ..., \boxed{y_i}, ..., y_n\ \square$$

4 Exercise: Composition of logspace computable functions