Online Appendix for "Location Sorting and Endogenous Amenities: Evidence from Amsterdam"

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A.1 Institutional background

A.1.1 The housing market in Amsterdam

The Netherlands has the largest social housing program in Europe. As of 2017, 42% of the housing stock in Amsterdam corresponds to social housing, with the remaining 50% evenly split between private rentals and owner-occupied units. Social housing is subject to a maximum allowable rent, commonly known as the "liberalization line" — 710.68 euros for 2015-2018. In the private market, rents are not regulated. Any household below size-adjusted median income is eligible for social housing and can apply through a centralized city-wide system. Wait times range from 7 to 12 years. Some few units are allocated via a lottery (van Dijk, 2019).

The determination of rents in social housing units

Classification of a unit as social is determined by a annually updated national point system, based primarily on physical characteristics (size, number of bedrooms and bathrooms, among others) (Fitzsimons, 2013). A unit is classified as social housing if its score is below 143, where the total range is between 40 and 250 for 2013. Social units are subject to a rent ceiling proportional to its score. All landlords, private and social, have to follow this system. Units don't have a rent floor.

There are rent subsidies only available for tenants of social housing units. To qualify for these subsidies, the total income in 2018 of the household should be

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below 30,400 euros (22,400 for a single household) as compared to the 36,798 maximum income for social housing. Second, rent has to be between 225,08 and 710,68 euros for 2018 with different cut-offs depending on the household composition.

Social housing associations

Social housing units are generally built and managed by housing associations, which are non-for-profit organizations. These organizations originated in the mid-1800s and after the Housing Act of 1901, the associations were assigned the sole objective of promoting public housing, in return for favorable loans and subsidies for construction and management from the government. In the mid 1990s the housing associations were privatized as part of a nationwide strategy to encourage home ownership. While financial support from the state ended, housing associations still remained obliged to provide good and affordable houses for lower income groups (Regout, 2016). Government policy has actively encouraged housing associations to sell off units to owner occupants. In Amsterdam, the home ownership share of the housing stock increased from 11 to 30% between 1995 and 2015, while the social housing share declined from 58 to 44% (van Duijne and Ronald, 2018). As of recently, two thirds of social housing is owned by housing associations, while one third is owned by private individuals or real estate management companies.

Details on the private rental market

Private housing does not face price restrictions, but rent increases cannot take place more than once a year (Fitzsimons, 2013). Landlords may terminate contracts with their tenants on the following grounds: i) the tenant not behaving in a responsible manner, ii) in the case of a temporary tenancy, iii) urgent use by the landlord himself, with the landlord's interest in living in the house being greater than that of the tenant, iv) the tenant turning down a reasonable offer to enter into a new tenancy agreement of the same apartment, or v) realization of a zoning plan.

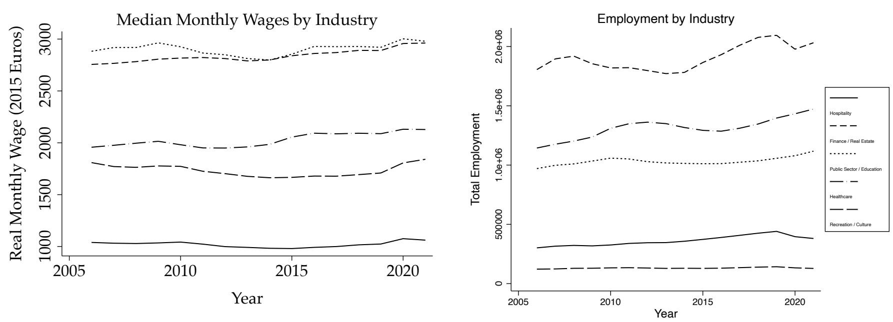
A.1.2 The tourism industry in Amsterdam

Hotels Between 2008 and 2017, the number of overnight stays in Amsterdam grew from 8.3 to 15.9 million. The hotel industry has simultaneously grown: The number of hotels, rooms, and beds have increased by 34%, 65%, and 66% respectively. The average room price has followed an increasing trend, going from EUR 105 in 2009 to EUR 138 in 2017. Furthermore, occupancy rates have been steadily increasing from 71% to 84% across hotels of all quality ranges.[1]

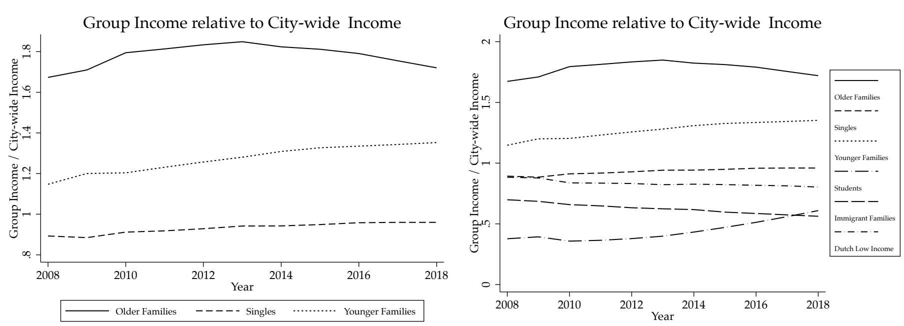
Airbnb This explosion in tourism has been accompanied by the entry of short-term rental platforms such as Airbnb. Hosts can rent their property in three ways: as an entire home rental, a private room rental, or a shared room rental. Entire home rentals for extended periods of time are typically associated with commercial operators, while live-in hosts are more likely to offer short, private or shared rentals. Even though this platform originated in 2008, it only took off starting in 2011. As of 2017, 10% of the rental stock in Amsterdam corresponds to commercially operated Airbnb listing, reaching up to 30% in central areas.

Figure 1: Tourism and the labor market

(a) Sector-level employment



(b) Group-level wage trends in Amsterdam



Tourism and the labor market Figure 1 shows sector-level employment and wage trends for the Netherlands. The largest employer is finance/real estate, followed by healthcare and the public sector/education. Hospitality and recreation/culture combined employ less than half of public sector employment, and about a third

[1] All statistics are from tourism reports by Onderzoek, Informatie en Statistiek (Research, Information, and Statistics) of the Amsterdam City Data project. Source: ois.amsterdam.nl/toerisme

of the employment in finance/real estate. Furthermore, although employment in hospitality and recreation/culture increases, it does so at a slower rate than other sectors. As of 2017, half of the jobs in the tourism sector correspond to food-catering (bars, restaurants), 18% to hotels, 15% to culture and recreation, and 7% to transportation. Wages are also fairly stagnant during our sample period.

We do not have the sector-level data to replicate Figure 1 for Amsterdam. However, tourism's employment share in Amsterdam is around 11% (Fedorova, de Graaf and Sleutjes, 2019), with the bulk of employment being in financial services and the public sector. Furthermore, we can compute the wage profiles of the household groups from our k-means classification. We plot each group's wage relative to the city-wide average wage, in order to understand if some groups may be gaining more than others as a result of tourism. Panel (b) of Figure 1 shows wages are growing uniformly across the groups we use in our welfare analysis.

A.1.3 Policy changes in the Amsterdam real estate market

We describe three policy changes in Amterdam's rental market during the 2010's.

2011: Change in Housing Point System

Until October 2011, the number of points a housing unit received was based solely on the unit's physical characteristics. A unit in rural Nieuwkoop would receive the same number of points as a unit in the center of Amsterdam, as long as they were comparable in terms of floor area, number of bedrooms, and neighborhood amenities (e.g. access to public transit, nearby green space).[2] In response to growing rental scarcity in metropolitan areas, the Dutch government designated 140 areas across the country as having a 'housing shortage,' and implemented a 25-point increase for all rental units within these areas. Most of these areas were around Amsterdam, Rotterdam, The Hague, and Utrecht, with the entirety of Amsterdam being designated as having a housing shortage (Koninkrijksrelaties, 2011).

In 2011, each point was equivalent to a rent of €4.80, meaning that a 25-point increase resulted in a rent increase of around €120 per month. At this time, the price cap for rental units was set at €650 per month, and this point increase caused a number of units to leave the social market and enter the private sector, where they were no longer bound by any sort of rent ceiling. In Amsterdam, it was estimated that about 28,000 out of a total of 200,000 social housing units would enter the private market following the new rule (van Perlo, 2011). The policy was not

[2] huurcommissie.nl

applied on incumbent social housing renters. As a direct consequence, it reduced the supply of social housing units and increase the supply of the private market.

2015: Decrease in Default Lease Durations

A private rental-related policy change was enacted in 2015, which served to drive up rents throughout the Netherlands. First, the default length of a rental lease was decreased from 'indefinite' to two years. In the Netherlands, a landlord can increase rents in one of two ways. The first is to offer a new lease to either an existing renter or a new renter—in the private market, these new leases are not bound by any price caps. The second is to agree with the tenant to index rents to price levels (typically some measure of inflation) upon the initial signing of the lease. In a system where most rental contracts were indefinite, this left little room for landlords to increase rents in excess of inflation. The new law, named "Wet Doorstroming Huurmarkt 2015," changes the standard duration of rental contracts from indefinite to two years, with ample options for contracts of even shorter duration (Koninkrijksrelaties, 2015). After these two years, the landlord can offer the current tenant a new lease, which has to be of indefinite duration. This allowed the landlord to increase rents by any degree after the initial contract expired, and incentivized landlords to continually find new tenants rather than renew an existing tenant's contract using the more restrictive indefinite lease (Koninkrijksrelaties, 2021).

2017: Mandatory Registration of Vacation Rental Properties

A new overnight stay policy was enacted in 2017 in Amsterdam to combat the overcrowding brought to the city by tourists. The policy limited the number of new hotels that could be constructed within city limits, and restricted construction in certain areas to more evenly distribute hotel accommodations throughout the city (Botman, 2021). While already-approved projects could continue construction, 32 out of the 34 petitions for new hotels sent in after the policy came into effect were denied (Couzy, 2019). In conjunction with this policy, the city also enacted a requirement for landlords to report all units they rented out as vacation rentals to the municipal government. The city also announced an intention to limit the maximum number of days a property can be rented out to just thirty days per year, starting in 2019. Together these laws were supposed to, at least in theory, reduce the upward pressure on rents and home prices in Amsterdam by lowering the influx of tourism and increase the supply of rentable properties by lowering the appeal of renting out one's property as a vacation rental. This policy caused an apparent immediate decrease in the number of Airbnb listings in Amsterdam.

A.2 Data

A.2.1 Residential histories and household characteristics

Information about demographic characteristics come from different data sources and at different frequencies. In this section we describe how we harmonize these different sources. First, we construct a yearly panel of location choices starting in 1995 using the registry (cadaster) data. We observe all individuals in the Netherlands from 1995 to 2020. The cadaster data gives us a history of addresses with registration dates. For every year and individual, we pick the modal address in terms of number of days within that year. In terms of demographics, we keep individuals older than 18 years old and younger than 75 years old. We also observe country of origin of the household head, which we classify into four broad categories: Dutch, Dutch Indies, Western (OECD), and Non-Western. With regards to skill, we observe the graduation date and degree type for everyone who completes a high school degree and beyond in the Netherlands from 1999 until 2020. We classify households according to the highest achieved level of education into low, medium, and high skill for those with high school (VMBO) or less, vocation or selective secundary education (HAVO, VWO, MBO), or college and more (HBO, WO) respectively.[3] Finally, we observe observe tax returns at the household level from 2008 to 2020. In this dataset we observe the total gross and after-tax income, the number of people in the household, an imputed measure of income per person, and categories about household composition. Information about the household composition allows us to infer whether children are part of the household.

For our dynamic location choice estimation sample, we focus on heads of household as identified by the tax data. We keep those households who have lived at least one year in Amsterdam since 1995, household head's age is between 18 and 70 years, and have at least one year of information about income.

A.2.2 Housing unit characteristics

First, for every housing unit we observe the year it was built, the floor area in square meters, the life stage of the property, and the usage category for 2011-2020. There are 11 usage facility types: residential, sport, events, incarceration, healthcare, industrial, office, education, retail, and other. There are six types of life-stage

[3]The education data does not cover people who graduated before 1999. We impute the highest level of achieved education on the rest of the population using data on demographics.

categories: constructed, not constructed, in process of construction, in use, demolished, not in use. We also observe any changes to these characteristics. For example, we can see if a residential unit that was in use is demolished. With these transitions, we see that there are virtually no residential units that convert to another usage type such as commercial and vice-versa. Given this segmentation of the market, we only keep housing units that are classified as residential.

Second, we observe a panel of housing values and characteristics for all the properties in the Netherlands from 2006 to 2019. We observe the annual tax appraisal value (WOZ) and the geo-coordinates of the property. These data also contain information about the occupant's tenancy status: homeowner, private renter, or social housing renter. These data is annually collected by the local government to assess every property WOZ value and tax accordingly. According to the Amsterdam city government, WOZ values are mostly based on market values.[4]

A.2.3 Linking households to housing units

We merge the housing unit panel to the household location panel through the property identifier. Next, we infer the occupancy status of the household and the number of occupants in the unit. We keep housing units with less than six occupants to remove residential facilities that are not typical households units, such as university student halls.[5]

A.2.4 Rent imputation

We link microdata from the universe of housing units to a national rent survey which contains approximately 13,000 observations of units in the rental market between 2006 and 2019. We use the matched subset in the rental survey with their tax valuation information to predict rents for housing units that do not appear in the survey but do appear in the property value data as renter-occupied. We keep only properties that are rented in the private rental market and not in social housing rented through housing associations. We predict total rental prices and rental prices by square meter on the properties that are classified as private rental units from the tax appraisal data. We use two methods to predict rental prices: linear regression and random forest. In the two methods, we use tax-appraisal values, official categories for measures of quality, total floor area, number of rooms, latitude and longitude coordinates, time and wijk-code fixed effects. We train our

[4]Source: amsterdam.nl/en/taxes/property-valuation/

[5]Six occupants corresponds to the 99th percentile of the number of occupants distribution.

Table 1: Imputation results

	Hedonic Model		Random Forest	
	Rental Prices	Price/m^2	Rental Prices	Price/m^2
β	0.63	0.58	0.89	0.89
	(0.01)	(0.01)	(0.01)	(0.01)
R^2	0.63	0.58	0.94	0.94
N	12674	12674	12674	12674

Note: This table shows regression coefficients and model fit of imputed rental prices on observed rental prices at the property level. We do so for two imputations: a linear hedonic regression and a random forest. Standard errors reported in parenthesis.

algorithms in 90% of the sample and test out-of-sample predictive power in 10% of the sample. For the hedonic linear regression, the in-sample R^2 for total rental prices is 0.637 while the out-of-sample R^2 is 0.6292. Similarly, the random forest delivers an in-sample R^2 of 0.813 and out-of-sample R^2 of 0.782. The random forest model has a substantially better performance in terms of predictive power, both in-sample and out-of-sample. Table 1 shows that when regressing imputed on observed rental prices, the random forest also outperforms classic linear regression.

A.2.5 Constructing Airbnb supply and prices

A challenge in working with the web scraped Inside Airbnb data is that some of the listings may be inactive, and thus would overstate Airbnb supply. To deal with this issue we focus on listings that are sufficiently "active". Using calendar availability data, we say that a listing is "active" in month t if it has been reviewed by a guest or its calendar has been updated by its host in month t.

Furthermore, we want to separately identify listings in which the host lives in the unit and shares it with guests, from those in which there is no sharing. The former does not reduced housing stock for locals, while the latter does. Hence, we define a listing as "commercially operated" if it is an entire-home listing, has received new reviews over the past year, and has "sufficient booking activity" such that it is implausible a local is living in the unit permanently. A listing has "sufficient booking activity" if it satisfies any of the following three conditions:

1. The listing has been booked over 60 nights in the past year: this is equivalent to having over 10 new reviews assuming a review rate of 67% and an average length of 3.9 nights per booking.[6]

[6] The global average review rate by guests is 67% (Fradkin, Grewal and Holtz, 2018), and the average booking in Amsterdam is for 3.9 nights (source: press.airbnb.com/instant-book-updates/).

2. It shows intent to be booked for many nights over the upcoming year: the listing is available for more than 90 nights over the upcoming year and the "instant book" feature is turned on.

3. It has had frequent updates, reflecting intent to be booked even though it may not have the "instant book" feature turned on: the listing has been actively available for more than 90 nights over the upcoming year, and this has happened at least twice within the past year.

A limitation of the listings data is that since our webscrapes begin in 2015 we need to impute Airbnb supply before 2015 using the calendar and review data. We can only do this for listings that survived up to 2015 and not for those with activity only prior to 2015. Our measure of listings before 2015 is thus downward biased.

A.3 Theory

A.4 Micro-foundation of the utility function

This section derives the amenity demand equation from section 4.1. In our model, households sequentially choose where to live followed by how much quantity of housing and amenities to consume. We solve the household problem backwards. We suppress time subscripts when unnecessary to simplify notation.

Housing and overall amenities expenditure. First, conditional on living in location j, a type k household chooses how much of its wage w^k to spend on housing H_j and on a bundle of locally available consumption amenities C_j,

$$\max_{\{H_j, C_j\}} \quad A_j^k H_j^{1-\phi^k} C_j^{\phi^k} \quad \text{s.t.} \quad r_j H_j + P_{Cj} C_j = w^k.$$

where r_j is the rental price, P_{Cj} is the price of the consumption bundle, and A_j^k is the household's valuation of location attributes. The optimal choice of housing is $H_j^* = (1-\phi^k)\frac{w^k}{r_j}$, so the income left over for amenity consumption is $I^k = \phi^k w^k$.

Individual varieties of amenities. The term C_j aggregates varieties of consumption amenities,

$$C_j \equiv \prod_s \left[\left(\sum_{i=1}^{N_{sj}} q_{isj}^{\frac{\sigma_s - 1}{\sigma_s}} \right)^{\frac{\sigma_s}{\sigma_s - 1}} \right]^{\alpha_s^k},$$

where q_{isj} is the quantity demanded for variety i in sector s and location j, and N_{sj} is the number of firms in the sector-location. The aggregator implies Cobb-Douglas preferences over amenity sectors (with weights α_s^k) and CES preferences over varieties within an amenity sector (with substitution elasticity $\sigma_s > 1$). Given H_j^*, we can redefine the consumer's problem as choosing individual varieties subject to its after-rent income,

$$\max_{\{q_{isj}^k\}_{is}} \quad A_j^k H_j^{*1-\phi^k} \prod_s \left[\left(\sum_{i=1}^{N_{sj}} q_{isj}^{k \frac{\sigma_s - 1}{\sigma_s}} \right)^{\frac{\sigma_s}{\sigma_s - 1}} \right]^{\alpha_s^k \phi^k} \qquad \text{s.t.} \quad \sum_{is} p_{isj} q_{isj}^k = I^k. \tag{1}$$

The solution to the variety choice problem above is identical whether we include the $A_j^k H_j^{*1-\phi^k}$ term or not. In the main text we therefore omit it. First order conditions with respect to q_{isj}^k give,

$$A_j^k H_j^{*1-\phi^k} \alpha_s^k \phi^k \left[\left(\sum_{i=1}^{N_{sj}} q_{isj}^{k \frac{\sigma_s - 1}{\sigma_s}} \right)^{\frac{\sigma_s}{\sigma_s - 1}} \right]^{\alpha_s^k \phi^k - 1} \left(\sum_{i=1}^{N_{sj}} q_{isj}^{k \frac{\sigma_s - 1}{\sigma_s}} \right)^{\frac{1}{\sigma_s - 1}} q_{isj}^{k - \frac{1}{\sigma_s}} \ldots$$

$$\prod_{x' \neq s} \left[\left(\sum_{i=1}^{N_{x'j}} q_{ix'j}^{k \frac{\sigma_{x'} - 1}{\sigma_{x'}}} \right)^{\frac{\sigma_{x'}}{\sigma_{x'} - 1}} \right]^{\alpha_{x'}^k \phi^k} = \lambda^k p_{isj}.$$

By combining the above for two varieties i and i' in the same sector s we obtain,

$$\frac{q_{isj}^k}{q_{i'sj}^k} = \left(\frac{p_{isj}}{p_{i'sj}} \right)^{-\sigma_s}.$$

Furthermore, the total expenditure on sector s is $\alpha_s^k I^k$ is given by $\sum_{i \in s} p_{isj} q_{isj}^k$. Therefore, type-$k$'s demand for variety i in sector-location sj is,

$$q_{isj}^k = \frac{\alpha_s^k I^k}{p_{isj}} \left(\frac{p_{isj}}{P_{sj}} \right)^{1-\sigma_s}, \quad \text{with } P_{sj} \equiv \left(\sum_{i=1}^{N_s} p_{isj}^{1-\sigma_s} \right)^{\frac{1}{1-\sigma_s}},$$

where P_{sj} is the sector's price index. In a symmetric equilibrium, where every firm (variety) within a sector-location faces the same marginal costs we have $p_{isj} =$

p_{sj} $\forall i \in sj$. Demand for each individual variety is therefore,

$$q^k_{isj} = q^k_{sj} = \frac{\alpha^k_s I^k}{p_{sj} N_{sj}} \; \forall i \in sj. \tag{2}$$

Plugging equation 2 into the utility function from 1 gives us,

$$A^k_j H^{*\,1-\phi^k}_j \prod_s \left[N_{sj}^{\frac{1}{\sigma_s - 1}} \frac{\alpha^k_s I^k}{p_{sj}} \right]^{\alpha^k_s \phi^k} = A^k_j w^k r_j{}^{-(1-\phi^k)} \varphi^k \prod_s \left[N_{sj}^{\frac{1}{\sigma_s - 1}} \frac{\alpha^k_s}{p_{sj}} \right]^{\alpha^k_s, \phi^k},$$

where $\varphi^k \equiv (1-\phi^k)^{1-\phi^k} (\phi^k)^{\phi^k}$. We now add time subscripts, allow for location tenure τ_t to affect utility with an elasticity of ν^k, and let attributes variable A to have time-invariant and time-varying components. The indirect utility function is now,

$$\tau_t^{\nu^k} A^k_j A^k_t A^k_{jt} w^k_t r_{jt}{}^{-(1-\phi^k)} \varphi^k \prod_s \left[N_{sjt}^{\frac{1}{\sigma_s - 1}} \frac{\alpha^k_s}{p_{sjt}} \right]^{\alpha^k_s \phi^k}.$$

Taking logs and adding a type I EV error ε_{ijt}, we obtain

$$\mu^k_j + \mu^k_t + \nu^k \log \tau_t - (1-\phi^k) \log r_{jt} + \sum_s \frac{\alpha^k_s \phi^k}{\sigma_s - 1} \log N_{sjt} + \log A^k_{jt} + \psi^k_{jt} + \varepsilon_{ijt}, \tag{3}$$

where $\mu^k_j \equiv \log A^k_j + \log \varphi^k + \phi^k \sum_s \alpha^k_s \log \alpha^k_s$, $\mu^k_t \equiv \log A^k_t + \log w^k_t$, and $\psi^k_{jt} \equiv -\phi^k \sum_s \alpha^k_s \log p_{sjt}$. Furthermore, we decompose A^k_{jt} as follows,

$$\log A^k_{jt} = \log \tilde{A}_{jt} + \sum_l \gamma^k_l \log B_{ljt} + \sum_s \gamma^k_s \log N_{sjt},$$

where $\tilde{A}_{jt}$ is an unobserved location attribute, B_{ljt} is an observed location attribute (other than consumption amenities), and $\gamma^k_s \log N_{sjt}$ represents spillovers from observed consumption amenities (which may be positive or negative) that go beyond the value of consumption itself. Finally, we divide log-utility by the standard deviation of ε_{ijt}, σ^k_ε, in order to normalize the variance of the shock to 1,

$$\delta^k_j + \delta^k_t + \delta^k_\tau \log \tau_t + \delta^k_r \log r_{jt} + \sum_s \delta^k_s \log N_{sjt} + \sum_l \delta^k_l \log B_{ljt} + \xi^k_{jt} + \epsilon_{ijt}, \tag{4}$$

where δ are the normalized parameters in equation 3 after dividing by σ^k_ε.

In the main text, we define the flow utility as 4 net of the type I EV shock, with the addition of the moving cost, and with summations written in vector from. Hence the flow utility of making a location decision j is,

$$\delta_j^k + \delta_t^k + \delta_\tau^k \log \tau_t + \delta_r^k \log r_{jt} + \delta_a^k \log a_{jt} + \delta_b^k \log b_{jt} - MC^k(j, j_{t-1}) + \xi_{jt}^k.$$

where $\delta_a^k \equiv [\delta_1^k, \ldots, \delta_s^k, \ldots, \delta_S^k]$, $\delta_b^k \equiv [\delta_1^k, \ldots, \delta_l^k, \ldots, \delta_L^k]$, $a_{jt} \equiv [N_{1jt}, \ldots, N_{sjt}, \ldots, N_{Sjt}]'$, and $b_{jt} \equiv [B_{1jt}, \ldots, B_{ljt}, \ldots, B_{Ljt}]'$.

A.4.1 Proof of existence of equilibrium

Assumption 1 *Each type of households has a minimum area of housing necessary for subsistence, $\gamma_k, \forall k$. Denote by $\psi_k \equiv w_k/\gamma_k$ the maximum rental price that a type-khousehold can pay to reach its subsistence level of housing. Define $\psi_{min} \equiv \min_k \psi_k$ and $\psi_{max} \equiv \max_k \psi_k$. Further, we assume there is a constant $q \in (0, 1]$ such that $q \times \sum_k M_k \cdot \gamma_k = \sum_j H_j$, so that the city can accommodate at most fraction q of households.*

Proposition 1 *Suppose Assumption 1 holds. Then a stationary equilibrium exists.*

Proof. We perform the change of variables to $\rho_j \equiv \log(r_j)$ and seek a vector of log-prices and a matrix of amenities that clear all markets. To this end, define

$$\mathcal{D} \equiv \bigtimes_j \left[\min \left\{ -H_j, \log \left(\frac{q \cdot \min_k \alpha_h^k w_k}{\sum_{j=1}^J H_j} \right) \right\}, \log \left(\max \left\{ \sum_k \frac{\alpha_h^k w^k M_k}{\psi_{min}}, \psi_{max} \right\} \right) \right] \times \left[0, \sum_{k=1}^K \frac{M_k \alpha_s^k \alpha_c^k}{F_{js} \sigma_s} \right]^{J \cdot S},$$

and observe $\mathcal{D}$ is convex and compact. We let $\mathcal{D}$ be the domain of the value function for each type. Denote by ω a generic concatenation of ρ and $\mathbf{a}$.

That the (expected) value function of households on this domain is well-defined, unique, and continuous follows by a standard argument presented, for example, in Rust (1988) and Stokey, Lucas and Prescott (1989). Further, that it is a strictly decreasing function in $\rho_j, \forall j \in \mathcal{J}$ with $\rho_j < \log(\psi_k)$ and decreasing otherwise follows by an adaptation of Corollary 1 to Theorem 3.2. in Stokey et al. (1989)).

It follows that for any household type k, the one-step transition probabilities, defined in equation 6, are continuous in $(\rho, \mathbf{a}) \in \mathcal{D}$, and since the flow utility function and the expected value functions are both strictly decreasing in ρ_j, such that

$$\rho_j \in \left[\min \left\{ -H_j, \log \left(\frac{q \cdot \min_k \alpha_h^k w_k}{\sum_{j=1}^J H_j} \right) \right\}, \log(\psi_k) \right] \equiv \mathcal{I}_j.$$

It follows that the transition probabilities are strictly decreasing in ρ_j in this set.

We stack up the transition probabilities into a transition matrix $\mathbf{\Pi}^k$. Observe that the resulting Markov chain is regular if $\rho_j \in \mathcal{I}_j, \forall j$. To this end, suppose $\rho_j < \log(\psi_k), \forall j \in \mathcal{J}$. We claim that the entries of $\mathbf{\Pi}^k(\omega)^{\bar{\tau}+2}$ are all strictly positive. Indeed, this follows from (i) $\mathbb{P}^k(d = j'|j, \tau, \omega) > 0, \forall j, \tau, j' \neq j$, (ii) $\mathbb{P}^k(d = s|j, \tau, \omega) > 0, \forall j, \tau$, and (iii) it takes at most two steps to arrive to a state $(j, 1), \forall j \in \mathcal{J}$. Whenever $\exists j \in \mathcal{J}$ such that $\rho_j > \log(\psi_k)$, since such a location is not affordable, the household will never select it. Hence, in this case we may restrict the Markov chain to the rest of the locations, and the restricted chain is again regular by an analogous argument. It follows a stationary distribution exists, is unique, and equals the limiting distribution. Denote the stationary distribution by $\pi^k(\omega) \equiv \pi^k(\mathbf{\Pi}^k(\omega))$.

Next, we claim $\pi^k(\omega)$ is a continuous function of ω. We define the following auxiliary matrices following Schweitzer (1968): (i) The time-averaged transition matrix and (ii) the fundamental matrix of Kemeny and Snell (1983), respectively,

$$\mathbf{\Pi}^{k,\infty}(\omega) = \lim_{m\to\infty} \frac{1}{m} \sum_{j=1}^{m} \mathbf{\Pi}^k(\omega)^j \quad \text{and} \quad \mathbf{Z}^k(\omega) = \Big(\mathbf{I} - \mathbf{\Pi}^k(\omega) + \mathbf{\Pi}^{k,\infty}(\omega)\Big)^{-1}.$$

For any feasible ω, ω', define $\Delta^k(\omega, \omega') \equiv \mathbf{\Pi}^k(\omega') - \mathbf{\Pi}^k(\omega)$, and vector $\mathbf{g}^k(\omega, \omega')$ given by

$$g_l^k(\omega, \omega') \equiv \sum_{s,m=1}^{N} \pi_s^k(\mathbf{\Pi}^k(\omega)) Z_{sl}^k(\omega) \Delta_{sl}^k(\omega, \omega').$$

It follows from expression 15 in Schweitzer (1968)

$$\begin{aligned}
||\pi^k(\omega) - \pi^k(\omega')||_\infty &= ||\pi^k(\mathbf{\Pi}^k(\omega)) - \pi^k(\mathbf{\Pi}^k(\omega'))||_\infty \\
&= ||\pi^k(\mathbf{\Pi}^k(\omega)) - \pi^k(\mathbf{\Pi}^k(\omega) + \Delta^k(\omega, \omega'))||_\infty \\
&= ||\pi^k(\mathbf{\Pi}^k(\omega)) - \big(\pi^k(\mathbf{\Pi}^k(\omega)) + \mathbf{g}^k(\omega, \omega') + \mathcal{O}\big(\Delta^k(\omega, \omega')^2\big)\big)||_\infty \\
&\leqslant ||\mathbf{g}^k(\omega, \omega')||_\infty + ||\mathcal{O}\big(\Delta^k(\omega, \omega')^2\big)||_\infty.
\end{aligned}$$

Further, we have

$$||\mathbf{g}^k(\omega, \omega')||_\infty \leqslant ||\mathbf{Z}^k(\omega)||_\infty ||\Delta^k(\omega, \omega')||_\infty \to \mathbf{0} \text{ as } \omega' \to \omega,$$

since $\mathbf{\Pi}^k(\omega)$ is a continuous function of ω and so $||\Delta^k(\omega, \omega')||_\infty \to \mathbf{0}$ as $\omega' \to \omega$. Similarly, $||\mathcal{O}\big(\Delta^k(\omega, \omega')^2\big)||_\infty \to \mathbf{0}$ as $\omega' \to \omega$. Hence, $||\pi^k(\omega) - \pi^k(\omega')||_\infty \to \mathbf{0}$ as $\omega' \to \omega$. Since ω was arbitrary, it follows that $\pi^k(\omega)$ is a continuous function of

ω. Further, by the definition of a stationary distribution, $\sum_\tau \pi^k_{j\tau}(\omega)$ is a decreasing function of ρ_j and an increasing function of $\rho_{j'}$ for $j \neq j'$.

Redefine demand function for squared footage of long-term housing as a function of log-rent and amenities, $\mathcal{D}^L_j(\omega)$. Observe that $\mathcal{D}^L_j(\omega)$ is a continuous function of ω and that $\mathcal{D}^L_j(\omega)$ is decreasing in ρ_j and weakly increasing in $\rho_{j'}$ for $j \neq j'$. We redefine the share of long-term houses as a function of log-rent $s^L_j(\rho)$.

Fixing amenities $\mathbf{a}$, solving for an equilibrium in the market for long-term housing corresponds to solving the following system of equations in $\rho \in \mathbb{R}^J$, $\mathcal{D}^L_j(\rho, \mathbf{a}) = s^L_j(\rho)H_j, \forall j \in \mathcal{J}$. This system has a solution. Indeed, fix amenities $\mathbf{a}$ and define the excess demand function $\mathbf{z} : \mathbb{R}^J \to \mathbb{R}^J$ by $z_j(\rho) = \mathcal{D}^L_j(\rho, \mathbf{a}) - s^L_j(\rho)H_j, \forall j \in \mathcal{J}$. To solve for equilibrium, we seek a root of $\mathbf{z}$. Before we proceed, note that the following holds using the above and by Assumption 1. If we denote $\rho_{min} \equiv \log\left(\frac{q \cdot \min_k \alpha^k_h w_k}{\sum_{j=1}^J H_j}\right)$, we have

$$\begin{aligned}
\lim_{\rho \to \rho_{min}} z_j(\rho) &= \frac{\sum_{k=1}^K \alpha^k_h w^k M_k \sum_\tau \pi^k_{j\tau}(\rho_{min}, \rho_{-j})}{q \cdot \min_k \alpha^k_h w^k} \sum_{j=1}^J H_j - s^L_j(\rho_{min}, \rho_{-j})H_j \\
&\geqslant \frac{\sum_{k=1}^K M_k \sum_\tau \pi^k_{j\tau}(\rho_{min}, \rho_{-j})}{q} \sum_{j=1}^J H_j - s^L_j(\rho_{min}, \rho_{-j})H_j \\
&\geqslant \sum_{k=1}^K M_k \times \sum_{j=1}^J H_j - s^L_j(\rho_{min}, \rho_{-j})H_j > 0,
\end{aligned}$$

and

$$\lim_{\rho_j \to \log(\psi_{max})} z_j(\rho) = -s^L_j(\log(\psi_{max}), \rho_{-j})H_j < 0.$$

We transform the root-finding problem to a fixed-point problem by defining $\mathbf{f} : \mathbb{R}^J \to \mathbb{R}^J$, $\mathbf{f}(\rho) = \mathbf{z}(\rho) + \rho$. Observe that $\mathbf{f}$ is continuous, and since z_j is decreasing in ρ and by the above, we must have

$$\mathbf{f}\Big(\bigtimes_{j\in\mathcal{J}} \left[\min\left\{ -H_j, \log\Big(\frac{q\cdot \min_k \alpha_h^k w_k}{\sum_{j=1}^J H_j}\Big)\right\}, \log\left(\max\Big\{ \sum_k \frac{\alpha_h^k w^k M_k}{\psi_{min}}, \psi_{max}\Big\}\right)\right]\Big) \subseteq$$
$$\bigtimes_{j\in\mathcal{J}} \left[\min\left\{ -H_j, \log\Big(\frac{q\cdot \min_k \alpha_h^k w_k}{\sum_{j=1}^J H_j}\Big)\right\}, \log\left(\max\Big\{ \sum_k \frac{\alpha_h^k w^k M_k}{\psi_{min}}, \psi_{max}\Big\}\right),\right]$$

where the considered set is convex and compact. Applying Brouwer's fixed point theorem, an equilibrium exists. Finally, an equilibrium vector of strictly positive rental prices must exist by the properties of the logarithmic function.

Further, since the aggregate demand function is strictly decreasing in $r_j, \forall j \in \mathcal{J}$ such that $r_j \leqslant \psi_{\max}$ and since all equilibrium prices are at most $\psi_{\max}$, restricting attention to $(0, \psi_{max}]$, the strict gross substitutes property holds. Hence, the equilibrium is unique. This allows us to define the equilibrium price vector as a function of the fixed vector of amenities $\mathbf{r}(\mathbf{a})$. Continuity of this function follows by applying the Implicit Function Theorem to $\mathbf{z}$ at market-clearing prices.

Define $\psi : \mathbb{R}^{J\cdot S} \to \mathbb{R}^{J\cdot S}$ by

$$\psi_{js}(\mathbf{a}) = \sum_{k=1}^K \frac{\mathcal{D}_j^{Lk}(\mathbf{r}(\mathbf{a}), \mathbf{a})\alpha_s^k \alpha_c^k w^k}{F_{js}\sigma_s}, \forall j \in \mathcal{J}, s \in \mathcal{S}.$$

By the above, it follows that ψ is continuous and

$$\psi\left(\Big[0, \sum_{k=1}^K \frac{M_k \alpha_s^k \alpha\ _c^k w^k}{F_{js}\sigma_s}\Big]^{J\cdot S}\right) \subseteq \Big[0, \sum_{k=1}^K \frac{M_k \alpha_s^k \alpha_c^k w^k}{F_{js}\sigma_s}\Big]^{J\cdot S}.$$

Existence of equilibrium hence follows by Brouwer's fixed-point theorem. ■

A.4.2 Outline of the equilibrium solver algorithm

The equilibrium existence argument presented above suggests a natural nested fixed-point algorithm to solve for equilibrium. The algorithm proceeds as follows. Fix parameters $\lambda \in (0, 1)$ and $\delta > 0$.

For step $t = 1, \ldots,$ the outer loop proceeds as follows:
($\mathbf{O_1^t}$) Guess $\mathbf{a}^{(t)}$. For step $g = 1, \ldots,$ the inner loop proceeds as follows:
- ($\mathbf{I}_1^g$) Guess $\mathbf{r}^{(g)}$.
- ($\mathbf{I}_2^g$) Compute excess demand for housing $\mathbf{z}(\mathbf{r}^{(g)}, \mathbf{a}^{(t)})$.

($\mathbf{I}_3^g$) Update $\mathbf{r}^{(g+1)} = \mathbf{r}^{(g)} + \delta \cdot \mathbf{z}(\mathbf{r}^{(g)}, \mathbf{a}^{(t)})$.

($\mathbf{I}_3^g$) Compute $d_r^{(g)} = ||\mathbf{r}^{(g+1)} - \mathbf{r}^{(g)}||_\infty$.

Iterate until step G such that $d_r^{(G)} < \epsilon_r$ for a tolerance level $\epsilon_r > 0$.

Denote $\mathbf{r}^{(et)} \equiv \mathbf{r}^{(G)}$.

($\mathbf{O_2^t}$) Compute

$$\mathbf{a}_{js}^{(et)} = \sum_{k=1}^{K} \frac{\mathcal{D}_j^k(\mathbf{r}^{(et)}, \mathbf{a}^{(t)})\alpha_s^k \alpha_c^k w^k}{F_{js}\sigma_s}.$$

($\mathbf{O_3^t}$) Update $\mathbf{a}^{(t+1)} = (1-\lambda)\mathbf{a}^{(et)} + \lambda \mathbf{a}^{(t)}$.

($\mathbf{O_4^t}$) Compute $d_a^{(t)} = ||\mathbf{a}^{(t+1)} - \mathbf{a}^{(t)}||_\infty$.

Iterate until step T such that $d_a^{(T)} < \epsilon_a$ for a tolerance level $\epsilon_a > 0$.

A.5 Estimation

A.5.1 Reduced form IV estimation results

The main endogeneity concern from regressing Airbnb listings on housing market outcomes is that any time-varying neighborhood-level unobservable variation that correlates with both variables will lead to biased OLS estimates, with the sign of the bias depending on the sign of such correlations. For example, if the neighborhoods that are becoming unobservably more attractive to locals are also becoming more attractive to tourists, then such areas will have higher housing prices and a higher number of Airbnb listings, leading to upward-biased OLS estimates. By contrast, if the neighborhoods that are becoming less attractive to locals are becoming more attractive to tourists, then those ares will have lower housing prices and a higher number of Airbnb listings, leading to downward-biased OLS estimates. To address such concerns, we complement our analysis in the main text with a shift-share identification strategy, a frequently used research design in the literature measuring the effect of Airbnb on the housing market (Barron, Kung and Proserpio, 2021; Garcia-López, Jofre-Monseny, Martínez-Mazza and Segú, 2020). The "shift" part of the instrument exploits time variation in worldwide demand for Airbnb as proxied by online search volume. The "share" part constructs neighborhood-level exposure to tourism by using the spatial distribution of historic monuments. Our exclusion restriction requires both factors to be orthogonal to time-varying and neighborhood-level unobservables, conditional on the rest of the covariates.

Table 2: Relationship between rent and Airbnb listings

	Ln (rent/m2)					
	OLS	IV	OLS	IV	OLS	IV
Ln (commercial Airbnb listings)	**0.066***** (0.008)	**0.090***** (0.020)	**0.052***** (0.006)	**0.114***** (0.021)	**0.115***** (0.018)	**0.190**** (0.086)
Ln (housing stock)			-0.056** (0.027)	-0.095*** (0.028)	-0.111*** (0.028)	-0.163*** (0.060)
Ln (average income)			-0.492*** (0.075)	-0.490*** (0.071)	-0.353*** (0.072)	-0.313*** (0.084)
Ln (high-skill population share)			0.330*** (0.053)	0.213*** (0.061)	-0.014 (0.100)	-0.143 (0.186)
District-year FE					X	X
First stage F-stat		617.51		397.57		86.21
Observations	780	780	773	773	773	773
R2	0.154	0.133	0.422	0.330	0.579	0.546

Notes: Standard errors clustered at the wijk level in parenthesis. We construct commercial Airbnb listings from the Inside Airbnb data, with the exact procedure described in Appendix A.2.5. Rents and house sale values are from a combination of CBS surveys and transaction data, described in section 2. All other variables are from ACD BBGA.

First, Airbnb worldwide popularity is unlikely to be informative of neighborhood-specific trends. Second, the spatial distribution of monuments determined centuries ago are unlikely to be informative of current trends affecting housing prices.

Our results for rent and house values are presented in table 2. Note that OLS estimates are downward biased, a result that is also found in Barron et al. (2021) in the context in the US. This downward bias can arise for several reasons. One is measurement error or the landlord decision of renting on Airbnb when rental prices are low. Alternatively, this downward bias can also suggest that the unobservables correlating with Airbnb presence are negatively correlated with prices, i.e., they are disamenities for local residents.[7] The purpose of our structural model is precisely to quantify the welfare effects of Airbnb entry beyond housing price effects, in particular the welfare effects that arise due to changes in amenities.

A.5.2 Classification by k-means clustering

Our classification procedure proceeds in two steps. First, given the high persistence in tenancy status, we classify households into three groups: homeowners, private renters, and renters in social housing. This classification is done by identifying their modal tenancy status during the years they live in Amsterdam.

Then, we construct an invariant vector of demographics as follows. For data that varies over time—age, disposable income (gross income net of tax), dispos-

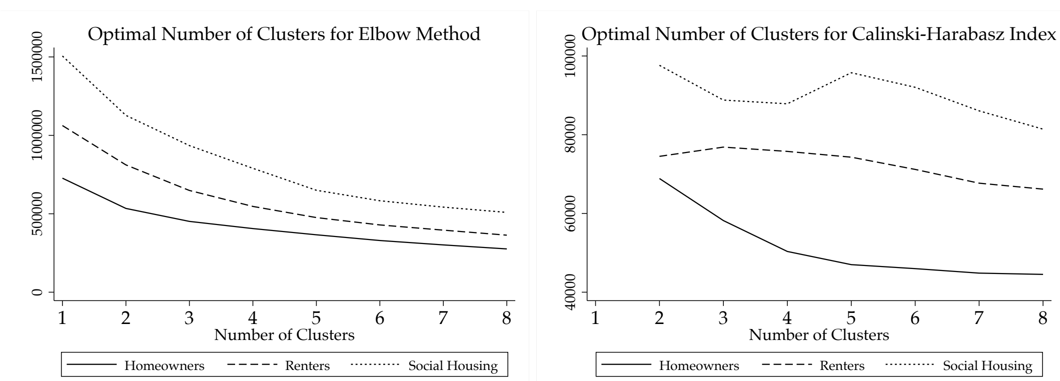
[7]See Garcia, Miller and Morehouse (2020) for a detailed discussion of how Airbnb externalities lead to heterogeneous effects of short-term rentals on rental prices.

able income per person, presence of children—we take the averages across years. We standardize the complete vector of demographics—skill, region of origin, age, disposable income, disposable income per person, children—because k-means is not invariant to scale and mechanically puts more weight on variables that have larger absolute values. We assign weight equal to $1/\sqrt{C-1}$ to the categorical variables with number of categories equal to C, so that each dimension has a weight of 1.[8] We finally run k-means on the transformed vector of demographics.

To choose the number of groups, we use a cross-validation method using two heuristics: the elbow method and the Calinski-Harabasz index. The optimal number of clusters as suggested by the elbow method is pinned down by the largest change of slope in the sum of squared errors curve. The Calinski-Harabasz index suggests that the optimal number of cluster is achieved when the ratio of the sum of between-clusters dispersion and of inter-cluster dispersion is maximized. Figure 2 shows the results of these heuristics for the three tenancy groups. For homeowners and private renters both methods suggest an optimal number of two clusters. For social housing renters, the first method suggests two clusters and the second method either two or six clusters. Putting both results together, we choose two as the final number of groups for social housing renters.

Figure 2: Heuristics for k-means classification

(a) Elbow Method (b) Calinski-Harabasz



[8]That is, for skill, we retain two categories, one that belongs to low skill and one to medium skill. We divide the standardize dummies by $\frac{1}{\sqrt{2}}$. Four country of origin, we set dutch as the baseline category and divide standardize dummies by $\frac{1}{\sqrt{3}}$.

A.5.3 Housing expenditure shares

With our rental imputations procedure described in Section A.2.4, we can predict rental prices on all residential units of the city. With this predicted annual price, we compute the share of income that is spent on housing for those household in the private market by dividing it by household's after-tax income. For households in social housing, we use instead the yearly maximum social hosing rent. Finally, we estimate expenditures shares on housing across groups by taking the median observation conditional on demographic type and year. These housing expenditure shares correspond to the term $1 - \phi^k$ in Section A.4.

A.5.4 Demand estimation

In this section we explain how to deal with the discretization of the state variable location tenure τ, derive the ECCP equation, and provide evidence of validity of our approach using Monte-Carlo simulations. In practice, due to separability of our groups, our demand estimation can be done separately for each group. Therefore, in what follows we drop the super-index k to simplify notation.

Discretization of a continuous state variable We closely follow Rust (1987). To keep the number of states low, we discretize location tenure in two buckets:

$$\bar{\tau} = \begin{cases} 1 & \text{if } \tau \leqslant 3 \\ 2 & \text{otherwise} \end{cases}$$

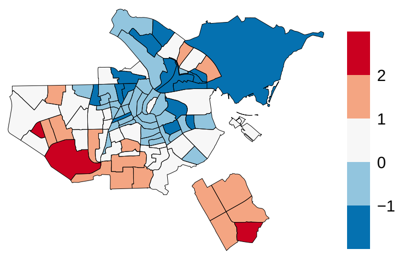
We assume that the discretized location tenure evolves stochastically. Recall that individual state variables x summarize information about location and location tenure in the previous period: $x_t = (j_{t-1}, \tau_{t-1})$. With a slight abuse of notation, we assume that location tenure evolves using transition probabilities $\mathbb{P}_t(x'_{t+1}|j_t, x_t)$. In practice, we assume $\mathbb{P}_t(\tau_t = 1|j_t, x_t) = 1$ if $j_t \neq j_{t-1}$ and

$$\mathbb{P}_t(\tau_t = 2|j_t, x_t) = \begin{cases} 1 & \text{, if } j_t = j_{t-1} \text{ and } \tau_{t-1} = 2 \\ p & \text{, if } j_t = j_{t-1} \text{ and } \tau_{t-1} = 1 \end{cases}$$

where p is estimated using a frequency-based estimator.

A.5.5 Housing supply estimation

Figure 3: Spatial distribution of location fixed effects κ_j.



Notes: estimates of κ_j are from the elasticity IV specification with wijk and year fixed effects. The values have been standardized, so positive values are above average, negative are below, and a value of 1 indicates a 1 standard deviation above the mean κ_j.

Figure 3 shows the spatial distribution of the estimates for κ_j, the landlord's differential operating costs between short- and long-term markets. The results suggest landlords in central locations face lower costs of renting short-term relative to long-term. This is consistent with higher matching rates and lower vacancy risk.

A.6 Simulations and Counterfactuals

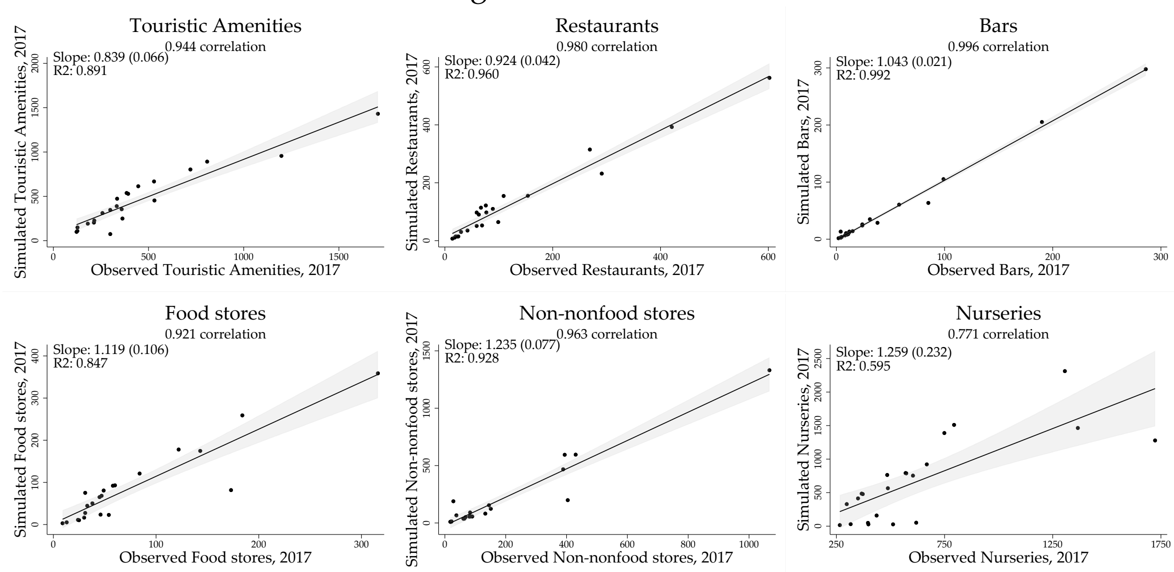
A.6.1 Baseline Equilibrium Details and Model fit

We construct the amenity supply equation using the parameters estimated in section 5.2. We set the unobservable component of entry costs equal to the residuals of equation 8. We treat tourists in hotels as exogenous consumers of amenities.

For housing demand, we first fix the exogenous characteristics of demand at their 2017 level. Recall that the endogenous components of neighborhoods, prices and consumption amenities, are both found as the solution of our equilibrium solver. We set unobservable demand shocks $\xi^k_{j,t}$ equal to zero, their conditional mean.[9] Then, we take the estimated parameters from section 5.3 and sum across groups k to compute aggregate demand for long-term housing. We calibrate the differential costs of short- versus long-term rentals to match the number of Airbnb tourists in each location in 2017.

[9] We do not set unobservable demand shocks $\xi^k_{j,t}$ equal to the structural demand residuals because the residuals that we recover from equation 10 contain two components: unobservable demand shocks $\xi^k_{j,t}$ and expectational errors $\tilde{\nu}_{t,j,x_{it}}$, which are impossible to separate.

Figure 4: Model fit.



Notes: The figure presents scatter plots, linear fit, and 95% confidence intervals of the simulated objects against the observed objects for the equilibrium described in Section 4.4 for 22 districts in Amsterdam in 2017. The model is defined in Section 4. Estimation details are in Section 5. To compute the equilibrium, we used a nested fixed-point algorithm, outlined in Section A.4.2, initiated at the observed prices and amenities.

Finally, our starting value of our equilibrium solver algorithm is equal to the observed equilibrium. We define convergence when the infinite norm of the excess demand function for the vector of prices and amenities $(\mathbf{r}, \mathbf{a})$ is less than 1E-5.

Figure 4 presents plots of observed against simulated endogenous objects.

A.6.2 Multiplicity of equilibria

Given that endogenous amenities act as agglomeration forces, the model may feature multiple equilibria. Computationally, a standard way of detecting multiple equilibria is by initiating the equilibrium algorithm solver in section A.4.2 from many different starting values. When we do so, we do find that multiple equilibria exist. Therefore, we define an equilibrium selection rule as the resulting equilibrium when we set the initial value of our algorithm solver equal to the observed equilibrium. Using this selection rule, we see that our model can reproduce the patterns observed in the data fairly accurately, as shown in section 5.5.

For the counterfactual analysis presented in section 6, we initially computed the following equilibria using observed quantities for rent and amenities as starting values: (i) an equilibrium with endogenous amenities and no entry of short-term rentals (NS) and (ii) an equilibrium with endogenous amenities and short-term rentals in full equilibrium (S), as defined in section 4.4. The rest of the counterfactual equilibria analyzed in section 6 were computed using one of these equilibria as starting values. In particular, the homogeneous case in section 6.1 was computed using (S), the exogenous case in section 6.2 was computed using (NS), and all equilibria in section 6.3 were computed using (S).

To test the robustness of the equilibria (S) and (NS) to deviations in the initial value supplied to the equilibrium solver, we perform a local search around them. Denote an equilibrium vector of rents by $\mathbf{r} \in \mathbb{R}^J$ and an equilibrium matrix of amenities by $\mathbf{a} \in \mathbb{R}^{J \times S}$, respectively. Denote by $\mathbf{a}^m \in \mathbb{R}^S$ a vector comprised of the minimum quantity of each amenity in $\mathbf{a}$. We proceed by repeatedly taking i.i.d. draws from the uniform distribution on the interval $[a_{js} - \gamma a_s^m, a_{js} + \gamma a_s^m]$, for each location j and service s, and a constant $\gamma \in (0, 1)$. We use each draw as a new initial value for amenities. For rent, we use the observed vector of rents.[10] set the tolerance level equal to 1E-4 and repeat this procedure 80-times using $\gamma = 0.5$.

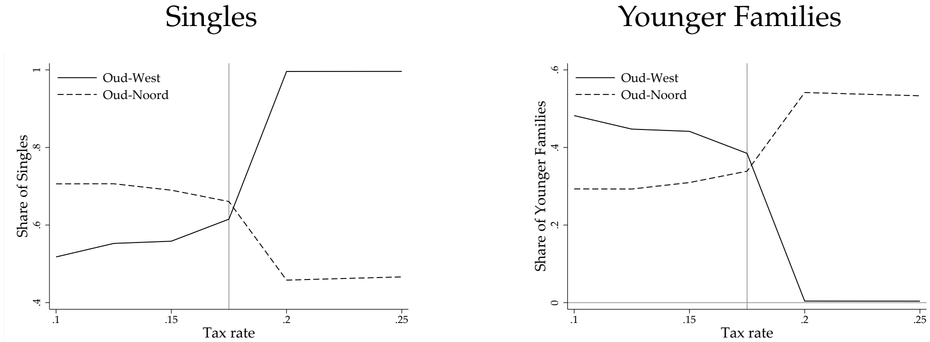
For both models (S) and (NS), each of the 80 iterations for $\gamma = 0.5$ converges to the same equilibrium, suggesting that our equilibria are locally stable in a sufficiently small neighborhood. We take this result as evidence that our selection rule leads to a locally unique equilibrium. Given that the functions in the system of equations that characterizes the equilibrium are continuous, the previous result suggests that the equilibrium is locally unique in a neighborhood of the estimated parameters. Therefore, for sufficiently small deviations in primitives, the economy is unlikely to change to a new equilibrium regime.

A.6.3 Tipping points in policy counterfactuals

Because the model's endogenous amenities act as agglomeration forces, policy counterfactuals may affect sorting and equilibrium outcomes in a non-linear way. Specifically, demographic composition may respond non-linearly to policy counterfactuals. Figure 5 shows that this indeed occurs for a few selected neighborhoods as the Airbnb tax described in Section 6.3 is gradually increased.

[10] Recall that for fixed amenities, equilibrium in the rental market is unique.

Figure 5: Tipping points



Notes: The figure shows how the share of each type of household in selected neighborhoods changes as the Airbnb tax is increased. The selected neighborhoods shown are those experiencing non-linear changes due to tipping. Most neighborhoods do not exhibit tipping behavior.

A.6.4 Measuring Welfare

In measuring welfare we mostly follow Small and Rosen (1981).

First, we exploit our assumption that idiosyncratic shocks are distributed as Type I Extreme Value errors. In such a case, type-k renter's welfare in steady-state for a vector of prices and amenities $(\mathbf{r}, \mathbf{a})$ is given by:

$$W^k(\mathbf{r}, \mathbf{a}) \equiv \sum_{j,\tau} EV^k_{j,\tau}(\mathbf{r}, \mathbf{a})\pi^k_{j,\tau}(\mathbf{r}, \mathbf{a}),$$

where $\pi^k_{j,\tau}(\mathbf{r}, \mathbf{a})$ is the stationary distribution and $EV^k(\mathbf{r}, \mathbf{a})$ is the expected value function of type k, respectively. For a formal definition of the latter, see equation 1.

Second, we can write renter's consumer surplus in log euros by multiplying by the variance of the idiosyncratic shocks: $\tilde{W}^k(\mathbf{r}, \mathbf{a}) \equiv \sigma^k_\epsilon W^k(\mathbf{r}, \mathbf{a})$. We estimate the variance of the idiosyncratic shock, σ^k_ϵ, as $\sigma^k_\epsilon = -(1-\phi^k)/\delta^k_r$, given our estimates of expenditure shares on housing, ϕ^k, and the rent preference parameters, δ^k_r, where we follow our micro-foundation in section A.4.

Second, we define consumer surplus as an implicit function of $\tilde{w}^k$ as follows:

$$\tilde{W}^k(\mathbf{r}, \mathbf{a}; \tilde{w}^k) = \sigma^k_\epsilon \sum_{j,\tau} \tilde{EV}^k_{j,\tau}(\mathbf{r}, \mathbf{a}; \tilde{w}^k)\pi^k_{j,\tau}(\mathbf{r}, \mathbf{a}),$$

where we define $\tilde{EV}^k_{j,\tau}$ as the following re-scaled version of the utility function u^k_{jt}:

$$\tilde{u}^k_j = u^k_j + \log \tilde{w}^k - \log w^k,$$

and w^k are fixed to the baseline wage level. Since re-scaling only changes the utility level but not choice probabilities, we obtain[11]

$$\tilde{W}^k(\mathbf{r}, \mathbf{a}; \tilde{w}^k) = \Big(\frac{1}{1-\beta}\log(\tilde{w}^k) - \frac{1}{1-\beta}\log(w^k) + \tilde{W}^k(\mathbf{r}, \mathbf{a})\Big).$$

Denote by $\tilde{W}_0^k = \tilde{W}^k(\mathbf{r}_0, \mathbf{a}_0)$ and by $\tilde{W}_1^k = \tilde{W}^k(\mathbf{r}_1, \mathbf{a}_1)$ the consumer surplus of group k in log euros in the baseline and in a counterfactual equilibrium, respectively. We define our welfare changes in terms of *consumption equivalent*, CE^k:

$$\tilde{W}_0^k = \tilde{W}^k(\mathbf{r}_1, \mathbf{a}_1; w^k + CE^k).$$

In other words, our consumption equivalent measure gives us how annual wages should change to keep households on the baseline equilibrium's indifference curve. We can write CE^k in closed-form solution as follows

$$CE^k = w^k\Bigg(\exp\Big(\big[\tilde{W}_0^k - \tilde{W}_1^k\big](1-\beta)\Big) - 1\Bigg).$$

To define homeowner's overall welfare, we can add rental income, ρ^k, to define *consumption equivalent with rental income*, CE_r^k:

$$CE_r^k = \exp\Big(\big[\tilde{W}_0^k - \tilde{W}_1^k\big](1-\beta)\Big)(w^k + \rho_0^k) - w^k - \rho_1^k.$$

To compute rental income, we assume that each homeowner owns a representative portfolio of the city. Under that assumption, ρ^k can be defined as follows:

$$\rho^k(\mathbf{r}) = \sum_j s_j^L(\mathbf{r})\omega_j^L(\mathbf{r}) r_j f_j + s_j^S(\mathbf{r})\omega_j^S(\mathbf{r}) p_j,$$

with

$$\omega_j^L(\mathbf{r}) \equiv \frac{s_j^L(\mathbf{r})\mathcal{H}_j^{SL} + \mathcal{H}_j^L}{\sum_{j'} s_{j'}^L(\mathbf{r})\mathcal{H}_{j'}^{SL} + \mathcal{H}_{j'}^L} \quad \text{and} \quad \omega_j^S(\mathbf{r}) \equiv \frac{s_j^S(\mathbf{r})\mathcal{H}_j^{SL}}{\sum_{j'} s_{j'}^S(\mathbf{r})\mathcal{H}_{j'}^{SL}},$$

where $\mathcal{H}_j^{SL}$ is the quantity of housing units that can be supplied both to the market for short- and long-term rentals and enters the landlords' problem, and $\mathcal{H}_j^L$ is the quantity of housing units supplied only to the market for long-term rentals.

[11] Observe that $\tilde{W}^k(\mathbf{r}, \mathbf{a}; w^k) = \tilde{W}^k(\mathbf{r}, \mathbf{a})$.

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