Supplementary Appendix for "Location Sorting and Endogenous Amenities: Evidence from Amsterdam"

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S.1 Institutional background

S.1.1 The housing market in Amsterdam

The Netherlands is well known for having the largest social housing program in Europe. As of 2017, 42% of the housing stock in Amsterdam correspond to social housing, with the remaining 58% split evenly between traditional private rentals and owner-occupied units. Social housing units are subject to a maximum allowable rent, commonly known as the "liberalization line", which stands at 710.68 euros for 2015-2018. In the private market, the initial rent a landlord charges is not regulated. Eligibility requirements for social housing are generous, as the income cutoff is set at household size-adjusted median income (van Dijk, 2019). Eligible households apply through a centralized city-wide waiting list, with wait times in the range of 7-12 years. A small number of units are allocated by lottery, so that a few lucky households may avoid the long waiting times.

The determination of rents in social housing units

Classification of a unit as social or private is determined by a national point system, based primarily on physical characteristics (size, number of bedrooms and bathrooms, among others) (Fitzsimons, 2013). As of 2013, total points for a unit range between 40 and 250. A unit is classified as social housing if its total score is below an annually updated threshold, 143 for 2013, making it subject to a rent ceiling that is proportional to its score. Both private owners and housing corporations have to follow this system. Units don't have a rent floor, so the actual agreed upon rent could be below the "liberalization line", in which case they are classified as social housing.

There are rent subsidies that are only available for tenants of social housing units. To qualify for these subsidies, the total income in 2018 of the household should be below 30,400 euros (22,400 for a single household) as compared to the 36,798 maximum income for social housing. Second, rent has to be between 225,08 and 710,68 euros for 2018 with different cut-offs depending on the household composition.

Social housing associations

Social housing units are generally built and managed by housing associations, which are non-for-profit organizations. These organizations originated in the mid-1800s with the goal of providing housing and social services for urban workers. After the Housing Act of 1901, the associations were assigned the sole objective of promoting public housing, in return for favorable loans and subsidies for construction and management from the government. In the mid 1990s the housing associations were privatized as part of a

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nationwide strategy to encourage home ownership. This privatization meant that financial support from the state ended but housing associations still remained subject to the statutory obligation of providing good and affordable houses for lower income groups (Regout, 2016). Government policy has actively encouraged housing associations to sell off units to owner occupants. In Amsterdam, the home ownership share of the housing stock increased from 11 to 30% between 1995 and 2015, while the social housing share declined from 58 to 44% (van Duijne and Ronald, 2018). As of recently, two thirds of social housing is owned by housing associations, while one third is owned by private individuals or real estate management companies (recall that the "social housing" label is based on the physical features of the house, not who owns it).

Details on the private rental market

Private housing does not face price restrictions, but rent increases cannot take place more than once a year (Fitzsimons, 2013). Landlords may terminate contracts with their tenants on the following grounds: i) the tenant not behaving in a responsible manner, ii) in the case of temporary tenancy, the landlord can officially end the contract, iii) urgent use by the landlord himself, with the landlord's interest in living in the house being greater than that of the tenant, iv) the tenant turning down a reasonable offer to enter into a new tenancy agreement referring to the same apartment, or v) realization of a zoning plan.

S.1.2 The tourism industry in Amsterdam

Between 2008 and 2017, the number of overnight stays in Amsterdam grew from 8.3 to 15.9 million. This rapid growth in tourist volume has been accompanied by an expansion of the hotel industry, with more high-end hotels being constructed on average. The number of hotels, rooms, and beds have increased by 34%, 65%, and 66% respectively.

The average room price has followed an increasing trend, going from EUR 105 in 2009 to EUR 138 in 2017, with average annual price growth of 3.3% and a peak of 8.8% in 2015. Furthermore, occupancy rates have been steadily increasing from 71% to 84% across hotels of all quality ranges. Overall, average annual hotel revenue has had a total growth of 57% from 2008-2017.[1]

Airbnb

At the same time, this explosion in tourism has been accompanied by the entry of short-term rental platforms such as Airbnb. Airbnb hosts can rent their property in three ways: as an entire home rental, a private room rental, or a shared room rental. Entire home rentals for extended periods of time are typically associated with commercial operators, while live-in hosts are more likely to offer short, private or shared rentals.

Even though this platform originated in 2008, it actually took off in Amsterdam starting in 2011. As of 2017, 10% of the rental stock in Amsterdam corresponds to commercially operated Airbnb listing. However, this number can reach up to 30% in central areas.

Tourism and the labor market

Figure 1 shows sector-level employment and wage trends for the Netherlands. The largest employer is finance/real estate, followed by healthcare and the public sector/education. Hospitality and recreation/culture combined employ less than half of public sector employment, and about a third of the employment in finance/real estate. Furthermore, although employment in hospitality and recreation/culture increases, it does so at a slower rate than other sectors. As of 2017, half of the jobs in the tourism sector correspond to food-catering (bars, restaurants), 18% to hotels, 15% to culture and recreation, and 7% to transportation. Wages are also fairly stagnant during our sample period.

[1] All statistics are from tourism reports commissioned by Onderzoek, Informatie en Statistiek (Research, Information, and Statistics) of the Amsterdam City Data project. Source: ois.amsterdam.nl/toerisme

Figure 1: Sector-level employment and wage trends in the Netherlands.

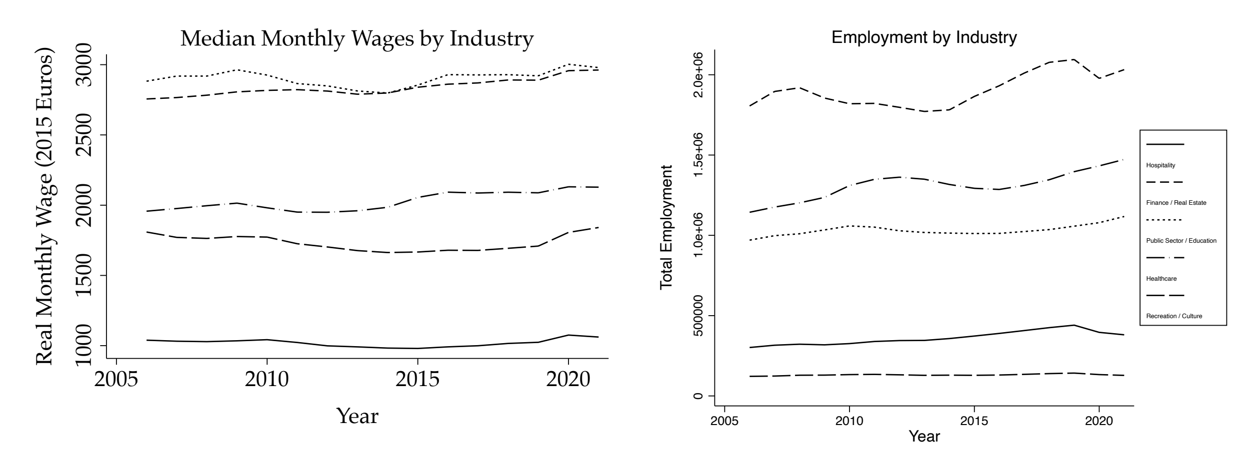
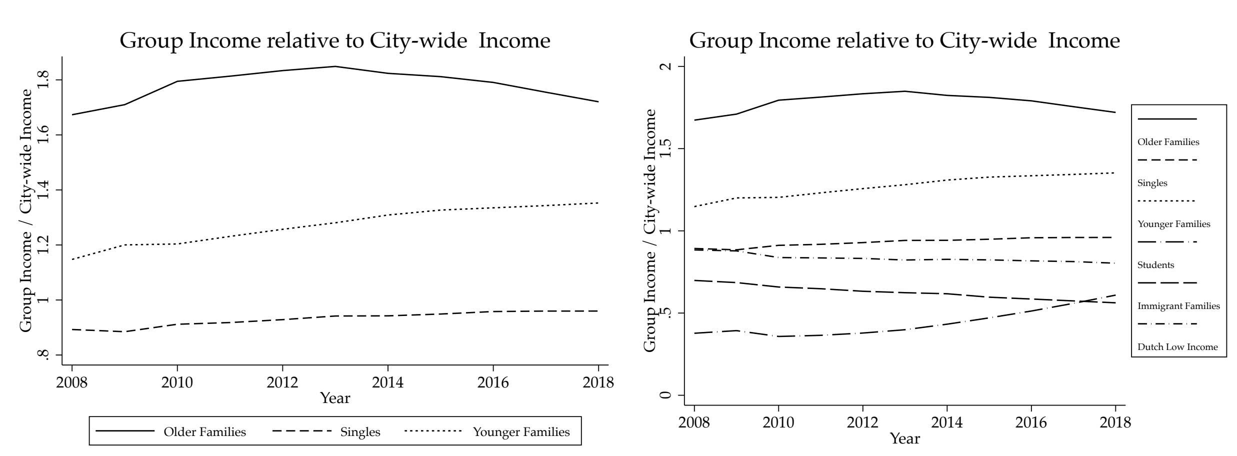


Figure 2: Group-level wage trends in Amsterdam.



We do not have the sector-level data to replicate Figure 1 for Amsterdam. However, according to industry reports, we do know tourism's employment share in Amsterdam is around 11% (Fedorova, de Graaf and Sleutjes, 2019), with the bulk of employment being in financial services and the public sector. Furthermore, we do have the data to compute the wage profiles of the household groups from our k-means classification. We plot each group's wage relative to the city-wide average wage, in order to understand if some groups may be gaining more than others as a result of tourism. Figure 2 shows wages are growing uniformly across the groups we use in our welfare analysis.

S.1.3 Policy changes in the Amsterdam real estate market

Amsterdam saw three policy changes in its rental market during the 2010's, which we describe below.

2011: Change in Housing Point System

Until October 2011, the number of points a housing unit received was based solely on the unit's physical characteristics. A unit in rural Nieuwkoop would receive the same number of points as a unit in the center of Amsterdam, as long as they were comparable in terms of floor area, number of bedrooms, and neighborhood amenities (e.g. access to public transit, nearby green space).[2] In response to growing rental scarcity in metropolitan areas, the Dutch government designated 140 areas across the country as having a 'housing shortage,' and implemented a blanket 25-point increase for all rental units within these areas. Most of these areas were around Amsterdam, Rotterdam, The Hague, and Utrecht, with the entirety of Amsterdam being designated as having a housing shortage (Koninkrijksrelaties, 2011).

In 2011, each point was equivalent to a rent of €4.80, meaning that a 25-point increase resulted in a rent increase of around €120 per month. At this time, the price cap for rental units was set at €650 per month, and this point increase caused a number of units to leave the social market and enter the private sector, where they were no longer bound by any sort of rent ceiling. In Amsterdam, it was estimated that about 28,000 out of a total of 200,000 social housing units would enter the private market following the new rule (van Perlo, 2011). The policy was not applied on incumbent social housing renters. As a direct consequence, it reduced the supply of social housing units that became part of the traditional private rental market.

2015: Decrease in Default Lease Durations

A private rental-related policy change was enacted in 2015, which served to drive up rents throughout the Netherlands. First, the default length of a rental lease was decreased from 'indefinite' to two years. In the Netherlands, a landlord can increase rents in one of two ways. The first is to offer a new lease to either an existing renter or a new renter—in the private market, these new leases are not bound by any price caps. The second is to agree with the tenant to index rents to price levels (typically some measure of inflation) upon the initial signing of the lease. In a system where most rental contracts were indefinite, this left little room for landlords to increase rents in excess of inflation. The new law, named "Wet Doorstroming Huurmarkt 2015," changes the standard duration of rental contracts from indefinite to two years, with ample options for contracts of even shorter duration (Koninkrijksrelaties, 2015). After these two years, the landlord can offer the current tenant a new lease, which has to be of indefinite duration. This allowed the landlord to increase rents by any degree after the initial contract expired, and incentivized landlords to continually find new tenants rather than renew an existing tenant's contract using the more restrictive indefinite lease (Koninkrijksrelaties, 2021).

2017: Mandatory Registration of Vacation Rental Properties

A new overnight stay policy was enacted in 2017 in Amsterdam to combat the overcrowding brought to the city by tourists. The policy limited the number of new hotels that could be constructed within city limits, and restricted construction in certain areas to more evenly distribute hotel accommodations throughout the city (Botman, 2021). While already-approved projects could continue construction, 32 out of the 34 petitions for new hotels sent in after the policy came into effect were denied (Couzy, 2019). In conjunction with this policy, the city also enacted a requirement for landlords to report all units they rented out as vacation rentals to the municipal government. The city also announced an intention to limit the maximum number of days a property can be rented out to just thirty days per year, starting in 2019. Together these laws were supposed to, at least in theory, reduce the upward pressure on rents and home prices in Amsterdam by lowering the influx of tourism and increase the supply of rentable properties by lowering the appeal of renting out one's property as a vacation rental. This policy caused an apparent immediate decrease in the number of Airbnb listings in Amsterdam.

[2] huurcommissie.nl

S.2 Data

S.2.1 Residential histories and household characteristics

Information about demographic characteristics come from different data sources and at different frequencies. In this section we describe how we harmonize these different sources. First, we construct a yearly panel of location choices starting in 1995 using the registry (cadaster) data. We observe all individuals in the Netherlands from 1995 to 2020. The cadaster data gives us a history of addresses with registration dates. For every year and individual, we pick the modal address in terms of number of days within that year. In terms of demographics, we keep individuals older than 18 years old and younger than 75 years old. We also observe country of origin of the household head, which we classify into four broad categories: Dutch, Dutch Indies, Western (OECD), and Non-Western. With regards to skill, we observe the graduation date and degree type for everyone who completes a high school degree and beyond in the Netherlands from 1999 until 2020. We classify households according to the highest achieved level of education into low, medium, and high skill for those with high school (VMBO) or less, vocation or selective secundary education (HAVO, VWO, MBO), or college and more (HBO, WO) respectively.[3] Finally, we observe observe tax returns at the household level from 2008 to 2020. In this dataset we observe the total gross and after-tax income, the number of people in the household, an imputed measure of income per person, and categories about household composition. Information about the household composition allows us to infer whether children are part of the household.

For our dynamic location choice estimation sample, we focus on heads of household as identified by the tax data rather than all individuals in the country. We keep those households who have lived at least one year in Amsterdam since 1995, have a household head between 18 and 70 years old, and have at least one year of information about income.

S.2.2 Housing unit characteristics

First, for every housing unit we observe the year it was built, the floor area in square meters, the life stage of the property, and the usage category for 2011-2020. There are 11 usage facility types: residential, sport, events, incarceration, healthcare, industrial, office, education, retail, and other. There are six types of life-stage categories: constructed, not constructed, in process of construction, in use, demolished, not in use. We also observe any changes to these characteristics. For example, we can see if a residential unit that was in use is demolished. With these transitions, we see that there are virtually no residential units that convert to another usage type such as commercial and vice-versa. Given this segmentation of the market, we only keep housing units that are classified as residential.

Second, we observe a panel of housing values and characteristics for all the properties in the Netherlands from 2006 to 2019. We observe the annual tax appraisal value (WOZ) and the geo-coordinates of the property. These data also contain information about the occupant's tenancy status: homeowner, private renter, or social housing renter. The reason these data exist is because each year, the local government assesses every property and issues its resulting WOZ value. According to the Amsterdam city government, WOZ values are mostly based on market values.[4]

S.2.3 Linking households to housing units

We merge the housing unit panel to the household location panel through the property identifier. This merge allows us to infer the occupancy status of the household and the number of occupants in the unit. We only keep housing units with six occupants or less.[5] We do so to remove residential facilities that are not typical households units, such as university student halls.

[3]The education data does not cover people who graduated before 1999. We impute the highest level of achieved education on the rest of the population using data on demographics.

[4]Source: amsterdam.nl/en/taxes/property-valuation/

[5]Six occupants corresponds to the 99th percentile of the number of occupants distribution.

Table 1: Imputation results

	Hedonic Model		Random Forest	
	Rental Prices	Price/m^2	Rental Prices	Price/m^2
β	0.63	0.58	0.89	0.89
	(0.01)	(0.01)	(0.01)	(0.01)
R^2	0.63	0.58	0.94	0.94
N	12674	12674	12674	12674

Note: This table shows regression coefficients and model fit of imputed rental prices on observed rental prices at the property level. We do so for two imputations: a linear hedonic regression and a random forest. Standard errors reported in parenthesis.

S.2.4 Rent imputation

We link microdata from the universe of housing units to a national rent survey which contains approximately 13,000 observations of units in the rental market between 2006 and 2019. We use the matched subset in the rental survey with their tax valuation information to predict rents for housing units that do not appear in the survey but do appear in the property value data as renter-occupied. We keep only properties that are rented in the private rental market and not in social housing rented through housing associations. We predict total rental prices and rental prices by square meter on the properties that are classified as private rental units from the tax appraisal data. We use two methods to predict rental prices: linear regression and random forest. In the two methods, we use tax-appraisal values, official categories for measures of quality, total floor area, number of rooms, latitude and longitude coordinates, time and wijk-code fixed effects. We train our algorithms in 90% of the sample and test out-of-sample predictive power in 10% of the sample. For the hedonic linear regression, the in-sample R^2 for total rental prices is 0.637 while the out-of-sample R^2 is 0.6292. Similarly, the random forest delivers an in-sample R^2 of 0.813 and out-of-sample R^2 of 0.782. The random forest model has a substantially better performance in terms of predictive power, both in-sample and out-of-sample. When regressing the imputed rental prices on true rental prices on the entire data, we also see how the random forest model outperforms classic linear regression in Table 1.

S.2.5 Constructing Airbnb supply and prices

A challenge in working with the web scraped Inside Airbnb data is that some of the listings may be inactive, and thus would overstate Airbnb supply. For example, a listing that was created for a single hosting experience in 2015 and left idle on the site would show up in our raw scrapes after 2015 even though it never had any further reservations. To deal with this we need to define what it means for a listing to be considered "active". Using calendar availability data, we say that a listing is "active" in month t if it has been reviewed by a guest or its calendar has been updated by its host in month t.

Furthermore, we want to separately identify listings in which the host lives in the unit and shares it with guests, from those in which there is no sharing. The former does not reduced housing stock for locals, while the latter does. Hence, we define a listing as "commercially operated" if it is an entire-home listing, it has received new reviews over the past year, and it has "sufficient booking activity" such that it is implausible a local is living in the unit permanently. We consider a listing to have "sufficient booking activity" if it satisfies any of the following three conditions:

1. In the past year it has had over 60 nights booked: this is equivalent to having over 10 new reviews if we assume a review rate of 67% and an average length of 3.9 nights per booking.[6]

2. It shows intent to be booked for many nights over the upcoming year: the listing is available for more than 90 nights over the upcoming year and the "instant book" feature is turned on.

[6]The global average review rate by guests is 67% (Fradkin, Grewal and Holtz, 2018), and the average booking in Amsterdam is for 3.9 nights (source: press.airbnb.com/instant-book-updates/).

3. It has had frequent updates, reflecting intent to be booked even though it may not have the "instant book" feature turned on: the listing has been actively available for more than 90 nights over the upcoming year, and this has happened at least twice within the past year.

A limitation of the listings data is that since our webscrapes begin in 2015 we need to construct Airbnb supply before 2015 using the calendar and review data, but we can only do this for listings that survived up to 2015. For example, a listing that was active in 2011 would only be detected by our methodology if it remained on the site in 2015. Thus, our measure of listings is biased downwards.

S.3 Estimation

S.3.1 Reduced form IV estimation results

The main endogeneity concern from regressing Airbnb listings on housing market outcomes is that any time-varying neighborhood-level unobservable variation that correlates with both variables will lead to biased OLS estimates, with the sign of the bias depending on the sign of such correlations. For example, if the neighborhoods that are becoming unobservably more attractive to locals are also becoming more attractive to tourists, then such areas will have higher housing prices and a higher number of Airbnb listings, leading to upward-biased OLS estimates. By contrast, if the neighborhoods that are becoming less attractive to locals are becoming more attractive to tourists, then those ares will have lower housing prices and a higher number of Airbnb listings, leading to downward-biased OLS estimates. To address such concerns, we complement our analysis in the main text with a shift-share identification strategy, a frequently used research design in the literature measuring the effect of Airbnb on the housing market (Barron, Kung and Proserpio, 2021; Garcia-López, Jofre-Monseny, Martínez-Mazza and Segú, 2020). The "shift" part of the instrument exploits time variation in worldwide demand for Airbnb as proxied by online search volume. The "share" part constructs neighborhood-level exposure to tourism by using the spatial distribution of historic monuments. Our exclusion restriction requires both factors to be orthogonal to time-varying and neighborhood-level unobservables, conditional on the rest of the covariates. First, we do not expect worldwide Airbnb popularity to be informative of neighborhood specific unobservable trends. Second, we assume that the determinants of the spatial distribution of monuments from centuries ago are uninformative of current trends that may affect housing prices.

Table 2: Relationship between rent and Airbnb listings

	Ln (rent/m2)					
	OLS	IV	OLS	IV	OLS	IV
Ln (commercial Airbnb listings)	**0.066*****	**0.090*****	**0.052*****	**0.114*****	**0.115*****	**0.190****
	(0.008)	(0.020)	(0.006)	(0.021)	(0.018)	(0.086)
Ln (housing stock)			-0.056**	-0.095***	-0.111***	-0.163***
			(0.027)	(0.028)	(0.028)	(0.060)
Ln (average income)			-0.492***	-0.490***	-0.353***	-0.313***
			(0.075)	(0.071)	(0.072)	(0.084)
Ln (high-skill population share)			0.330***	0.213***	-0.014	-0.143
			(0.053)	(0.061)	(0.100)	(0.186)
District-year FE					X	X
First stage F-stat		617.51		397.57		86.21
Observations	780	780	773	773	773	773
R2	0.154	0.133	0.422	0.330	0.579	0.546

Notes: Standard errors clustered at the wijk level in parenthesis. We construct commercial Airbnb listings from the Inside Airbnb data, with the exact procedure described in Appendix S.2.5. Rents and house sale values are from a combination of CBS surveys and transaction data, described in section 2. All other variables are from ACD BBGA.

Our results for rent and house values are presented in table 2. Note that OLS estimates are downward biased, a result that is also found in Barron et al. (2021) in the context in the US. This downward bias can arise for several reasons. One is measurement error or the landlord decision of renting on Airbnb when rental prices are low. Alternatively, this downward bias can also suggest that the unobservables correlating with Airbnb presence are negatively correlated with prices, i.e., they are disamenities for local residents.[7] The purpose of our structural model is precisely to quantify the welfare effects of Airbnb entry beyond housing price effects, in particular the heterogeneous welfare effects that arise due to changes in amenities.

S.3.2 Classification by k-means clustering

Our classification procedure proceeds in two steps. First, given the high persistence in tenancy status, we classify households into three groups: homeowners, private renters, and renters in social housing. This classification is done by identifying their modal tenancy status during the years they live in Amsterdam.

After the first classification step, we construct an invariant vector of demographics as follows. For data that varies over time—age, disposable income (gross income net of tax), disposable income per person, presence of children—we take the average across the years we observe the household in the data. We standardize the complete vector of demographics—skill, region of origin, age, disposable income, disposable income per person, children—because k-means is not invariant to scale and mechanically puts more weight on variables that have larger absolute values. We assign weight equal to $1/\sqrt{C-1}$ to the categorical variables with number of categories equal to C, so that each dimension has a weight of 1.[8] We finally run k-means on the transformed vector of demographics.

To choose the number of groups, we employ two heuristic indexes, the elbow method and the Calinski-Harabasz index, using a cross-validation method. The optimal number of clusters as suggested by the elbow method is pinned down by the largest change of slope in the sum of squared errors curve. The Calinski-Harabasz index suggests that the optimal number of cluster is achieved when the ratio of the sum of between-clusters dispersion and of inter-cluster dispersion is maximized. Panel (a)-(c) of Figure 3 show the results of these heuristics for the three tenancy groups. For homeowners and private renters both methods suggest an optimal number of two clusters. For social housing renters, the first method suggests two clusters and the second method either two or six clusters. Putting both results together, we choose two as the final number of groups for social housing renters.

S.3.3 Housing expenditure shares

With our rental imputations procedure described in Section S.2.4, we can predict rental prices on all residential units of the city. Once we have a predicted annual price, we can compute the share of income that is spent on housing for those household in the private market by dividing the annual rental price by after-tax income. For households in social housing, we take the yearly maximum social hosing rent and similarly divide by income. Finally, we estimate expenditures shares on housing across groups by taking the median observation conditional on demographic type and year. These housing expenditure shares correspond to the term $1-\phi^k$ in Section A.1.

S.3.4 Demand estimation

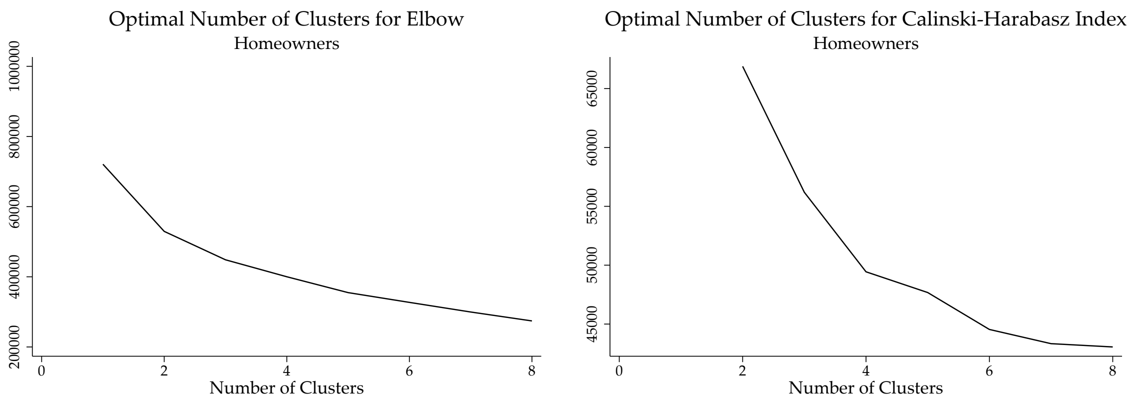
In this section we explain how to deal with the discretization of the state variable location tenure τ, derive the ECCP equation, and provide evidence of validity of our approach using Monte-Carlo simulations. In practice, due to separability of our groups, our demand estimation can be done separately for each group. Given that the same reasoning applies to all groups, in what follows we drop the super-index k to simplify notation.

[7] See Garcia, Miller and Morehouse (2020) for a detailed discussion of how Airbnb externalities lead to heterogeneous effects of short-term rentals on rental prices.

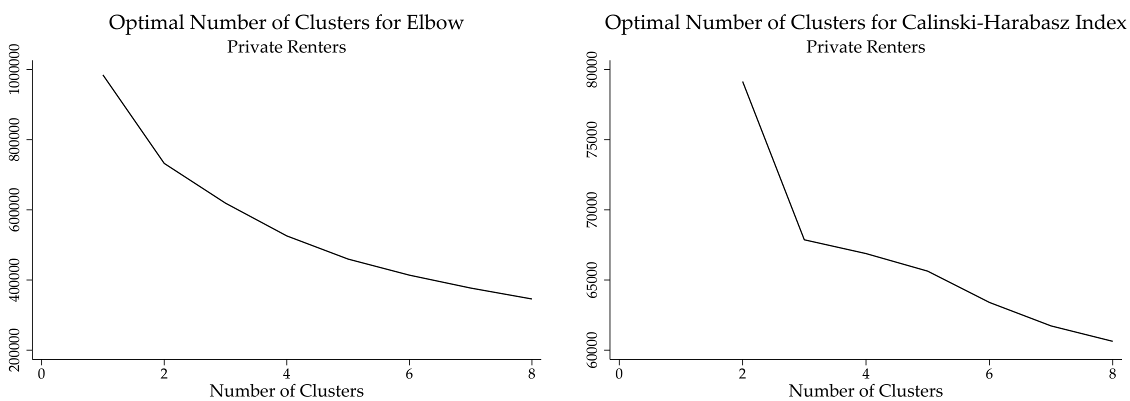
[8] That is, for skill, we retain two categories, one that belongs to low skill and one to medium skill. We divide the standardize dummies by $\frac{1}{\sqrt{2}}$. Four country of origin, we set dutch as the baseline category and divide standardize dummies by $\frac{1}{\sqrt{3}}$.

Figure 3: Heuristics for k-means classification

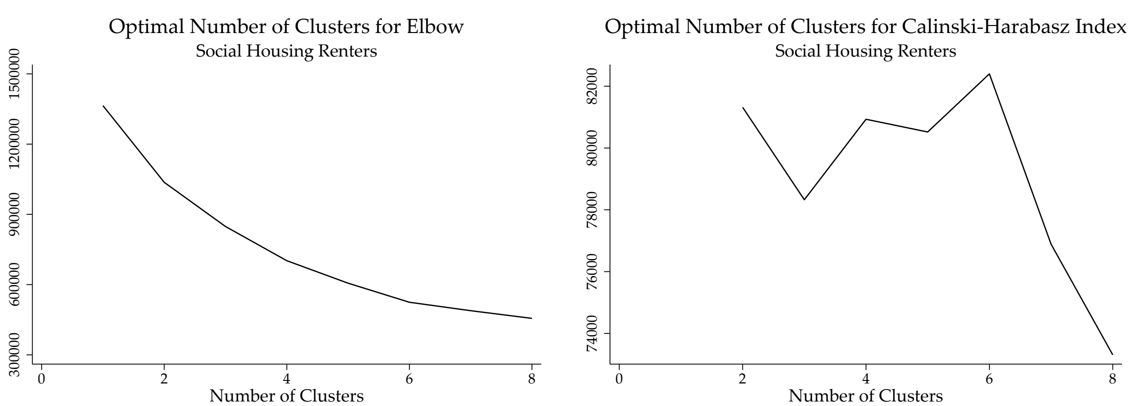
(a) Homeowners



(b) Renters



(c) Social Housing



S.3.4.1 Discretization of a continuous state variable

We closely follow Rust (1987). To keep the number of states low, we discretize location tenure in two buckets as follows:

$$\bar{\tau} = \begin{cases} 1 & \text{if } \tau \leqslant 3 \\ 2 & \text{otherwise} \end{cases}$$

We assume that the discretized location tenure evolves stochastically. Recall that individual state variables x summarize information about location and location tenure in the previous period: $x_t = (j_{t-1}, \tau_{t-1})$. With a slight abuse of notation, we assume that location tenure evolves using transition probabilities $\mathbb{P}_t(x'_{t+1}|j_t, x_t)$. In practice, we assume $\mathbb{P}_t(\tau_t = 1|j_t, x_t) = 1$ if $j_t \neq j_{t-1}$ and

$$\mathbb{P}_t(\tau_t = 2|j_t, x_t) = \begin{cases} 1 & \text{, if } j_t = j_{t-1} \text{ and } \tau_{t-1} = 2 \\ p & \text{, if } j_t = j_{t-1} \text{ and } \tau_{t-1} = 1 \end{cases}$$

where p is estimated using a frequency-based estimator.

S.4 Technical details of the derivation of the ECCP equation

Constructing Expected Value Function. Proceeding similarly as in the main text, the value function is defined as follows.

$$V_t(x, \epsilon) = \max_j \left\{ \mathbb{E}_{x'|j,x}\left[u_t(x', x)\right] + \epsilon_j + \beta \mathbb{E}_t\left[V_{t+1}(x', \epsilon')|j, x, \epsilon\right] \right\}.$$

Under the conditional independence assumption and the assumption that agents are atomistic, we can integrate over future ϵ, defining the ex-ante value function as follows:

$$\begin{aligned} \mathbb{E}_t\left[V_{t+1}(x', \epsilon')|j, x, \epsilon\right] &= \int V_{t+1}(x', \epsilon') dF_t(x', \omega_{t+1}, \epsilon'|j, x, \epsilon) && (1) \\ &= \int \Big(\int V_{t+1}(x', \epsilon') dF(\epsilon') \Big) dF_t(x', \omega_{t+1}|j, x) && (2) \\ &= \int V_{t+1}(x') dF_t(x', \omega_{t+1}|j, x) \equiv EV_t(j, x). && (3) \end{aligned}$$

We next define the conditional value function

$$v_t(j, x) = \sum_{x'} \mathbb{P}_t(x'|j, x)\big(u_t(x', x) + \beta \bar{V}_t(x')\big) \equiv \bar{u}_t(j, x) + \beta EV_t(j, x).$$

If idiosyncratic shocks are distributed according to i.i.d. Type I EV errors, choice probabilities and value functions can be written as:

$$\mathbb{P}_t(j|x) = \frac{\exp(v_t(j, x))}{\sum_{j'} \exp(v_t(j', x))}, \quad \text{and} \quad V_t(x) = \log\left(\sum_j \exp v_t(j, x)\right) + \gamma, \tag{4}$$

where γ is Euler's constant. Combining the two previous equations,

$$V_t(x) = v_t(j, x) - \ln(\mathbb{P}_t(j|x)) + \gamma. \tag{5}$$

Observe that the previous equation holds for any state x, and, more importantly, for any action j. This will be key to exploit renewal actions.

S.4.0.1 Toward a demand regression equation.

Our demand regression equation's starting point follows Hotz and Miller (1993), by taking differences on equation 4:

$$\ln\left(\frac{\mathbb{P}_t(j|x_t)}{\mathbb{P}_t(j'|x_t)}\right) = v_t(j,x_t) - v_t(j',x_t). \tag{6}$$

Substituting for the choice specific value function,

$$\ln\left(\frac{\mathbb{P}_t(j|x_t)}{\mathbb{P}_t(j'|x_t)}\right) = \bar{u}_t(j,x_t) - \bar{u}_t(j',x_t) + \beta\big(EV_t(j,x_t) - EV_t(j',x_t)\big). \tag{7}$$

The realized expected value $V_t(x')$ can be decomposed between its expectation at time t and its expectational error, where uncertainty is on the aggregate state variable ω_{t+1}:

$$V_{t+1}(x') = \bar{V}_t(x') + \nu_t(x').$$

Plugging in everything in equation 7 gives us

$$\begin{aligned}
\ln\left(\frac{\mathbb{P}_t(j|x_t)}{\mathbb{P}_t(j'|x_t)}\right) &= \sum_x \mathbb{P}(x|j,x_t)u_t(x,x_t) - \sum_{x'} \mathbb{P}(x'|j',x_t)u_t(x',x_t) \\
&+\beta\left[\sum_x \mathbb{P}(x|j,x_t)\bar{V}_t(x) - \sum_{x'} \mathbb{P}(x'|j',x_t)\bar{V}_t(x')\right] \\
&= \sum_x \mathbb{P}(x|j,x_t)u_t(x,x_t) - \sum_{x'} \mathbb{P}(x'|j',x_t)u_t(x',x_t) \\
&+\beta\left[\sum_x \mathbb{P}(x|j,x_t)\big(V_{t+1}(x) - \nu_{t+1}(x)\big) - \sum_{x'} \mathbb{P}(x'|j',x_t)\big(V_{t+1}(x') - \nu_{t+1}(x')\big)\right]
\end{aligned}$$

Using again equation 5 to replace the continuation values V_{t+1} for choice $\tilde{j}$ gives us

$$\begin{aligned}
\ln\left(\frac{\mathbb{P}_t(j,x_t)}{P_t(j',x_t)}\right) &= \sum_x \mathbb{P}(x|j,x_t)u_t(x,x_t) - \sum_{x'} \mathbb{P}(x'|j',x_t)u_t(x',x_t) \\
&-\beta\Bigg[\sum_x \mathbb{P}(x|j,x_t)\big(v_{t+1}(\tilde{j},x) - \ln \mathbb{P}_{t+1}(\tilde{j}|x) - \nu_{t+1}(x)\big) \\
&- \sum_{x'} \mathbb{P}(x'|j',x_t)\big(v_{t+1}(\tilde{j},x') - \ln \mathbb{P}_{t+1}(\tilde{j}|x') - \nu_{t+1}(x')\big)\Bigg] \\
&= \sum_x \mathbb{P}(x|j,x_t)u_t(x,x_t) - \sum_{x'} \mathbb{P}(x'|j',x_t)u_t(x',x_t) \\
&-\beta\Bigg[\sum_x \mathbb{P}(x|j,x_t)\big(v_{t+1}(\tilde{j},x) - \ln \mathbb{P}_{t+1}(\tilde{j}|x)\big) \\
&- \sum_{x'} \mathbb{P}(x'|j',x_t)\big(v_{t+1}(\tilde{j},x') - \ln \mathbb{P}_{t+1}(\tilde{j}|x')\big)\Bigg] + \tilde{v}_{j,j',x_t},
\end{aligned}$$

where $\tilde{v}_{j,j',x_t}$ is a sum of expectational errors

$$\tilde{v}_{j,j',x_t} = \beta\Big(\sum_x \mathbb{P}(x|j,x_t)\nu_{t+1}(x) - \sum_{x'} \mathbb{P}(x'|j',x_t)\nu_{t+1}(x')\Big).$$

Observe that if $\tilde{j}$ is a renewal action then:

$$v_{t+1}(\tilde{j},x) = \bar{u}_{t+1}(\tilde{j},x) + EV_t(\tilde{j},1) = u_{\tilde{j},x,t+1} + \delta_\tau \cdot 1 + MC(\tilde{j},j) + EV_t(\tilde{j},1)$$

for all $x = (j,\tau)$, regardless of τ, where we have decomposed the per-period utility function, $\bar{u}_{t+1}(\tilde{j},x)$, into a location specific component, $u_{\tilde{j},x_{t+1}}$, a location-tenure component δ_τ, and a moving cost component $MC(\tilde{j},j)$. Substituting and re-arranging,

$$\begin{aligned}
&\ln\Big(\frac{\mathbb{P}_t(j|x_t)}{\mathbb{P}_t(j'|x_t)}\Big) + \beta\Bigg[\sum_x \mathbb{P}(x|j,x_t)\ln\mathbb{P}_{t+1}(\tilde{j}|x) - \sum_{x'}\mathbb{P}(x'|j',x_t)\ln\mathbb{P}_{t+1}(\tilde{j}|x')\Bigg] \\
&= u_{j,x_t} - u_{j',x_t} + \delta_\tau\Big(\sum_x \mathbb{P}(x|j,x_t)\tau(x) - \sum_{x'}\mathbb{P}(x'|j',x_t)\tau(x')\Big) \\
&+ MC(j,j_{t-1}) - MC(j',j_{t-1}) + \beta\Big(MC(\tilde{j},j) - MC(\tilde{j},j')\Big) + \tilde{v}_{j,j',x_t},
\end{aligned}$$

which is the generalized version of equation 9 under stochastic transitions of individual state variables.

S.4.0.2 First-stage estimation of Conditional Choice Probabilities

We follow a similar procedure as in Traiberman (2019) and Humlum (2021), estimating CCPs in a first stage projecting empirical frequencies on a linear model of state characteristics. We depart from their approach and use a multinomial logit on individual decisions for three reasons. First, this approach allows us to directly estimate the CCPs probability using individual data. Second, our data reveal that the likelihood of not moving is approximately 85%, while the probability of moving to any other location remains close to zero. This bimodal nature of our data suggests that an exponential relationship should be better suited to fit individual decisions compared to a linear model. Third, we find that many predicted probabilities of the linear model lie below zero or above one, a feature that requires an ad-hoc extra censoring step.

Observe that our microdata allows us to observe individual moving decisions as well as individual states. That is, for individual i we observe her state at time t $x_{it} = (j_{t-1}, \tau_{t-1})$, where j_{t-1} is the previous location, τ_{t-1} and type $k(i)$, as well as the moving decision variables for all j: $j_{it} = \mathbb{1}\{d(i)_t = j\}$. Therefore, defining some base outcome 0, we estimate the following multinomial logit model across all groups k:

$$\mathbb{P}(j_{it} = j) = \frac{\exp(\beta_j^{k(i)} x_{it})}{1 + \sum_{j'=1}^{J}\exp(\beta_{j'}^{k(i)} x_{it})}.$$

Concretely, we define the model as follows:[9]

$$\beta_j^k s_t = \lambda_j^k + \lambda_t^k + \alpha_1^k \tau_{t-1} + \alpha_2^k \tau_{t-1}^2.$$

In what follows, we show that Monte Carlo simulations reveal that our multinomial predicted CCPs lead to a lower finite sample bias in second-stage estimates compared to the the standard approach where CCPs are estimated using a empirical frequencies.

S.4.0.3 Computation

Let θ be the set of coefficients to be estimated excluding fixed effects and let λ be the set of location fixed effects, with $dim(\theta) = 13$ and $dim(\lambda) = 21$. Also note that we have $J = 22$ locations (gebieds), $T = 10$ time

[9]For computational simplicity, we estimate the following model group by group. In principle, it would be preferred to estimate all groups jointly. Unfortunately this was impractical due to the large nature of our individual data and the limited computational power in the remote environment provided by the Dutch Microcensus facilities.

periods and the maximum location tenure is fixed to $\bar{\tau} = 2$. In total, this yields $(J+1) \times \bar{\tau} \times J \times T \times (J-1) = L = 212,520$ equations.

In matrix notation, if we define by $X_{t,j,\tilde{j},x}$ a vector the right-hand-side variables in Equation 9 excluding $\tilde{\tilde{\xi}}_{t,j,x_t}$ and the fixed effects, and by $Z_{t,j,\tilde{j},x}$ a vector of their corresponding instruments, we stack the variables as follows. We define

$$y \equiv [Y_{t,j,\tilde{j},x}]_{\{t,j',\tilde{j},x\}} \qquad X \equiv [X_{t,j,\tilde{j},x}]_{\{t,j',\tilde{j},x\}} \qquad Z \equiv [Z_{t,j,\tilde{j},x}]_{\{t,j',\tilde{j},x\}} \tag{8}$$

Further, define $g(\theta, \lambda) = Z'(y - X\theta - H\lambda)$, where by H we denote the corresponding matrix of location fixed-effect dummies. For a suitable weighting matrix W, GMM solves the following problem

$$\min_{(\theta,\lambda)\in\mathbb{R}^{34}} g(\theta,\lambda)' W g(\theta,\lambda) \tag{9}$$

Optimization over such a large space is computationally demanding. We use the procedure of Somaini and Wolak (2016) and X and y on the orthogonal complement of the image of the projection matrix of H. However, to define the annihilator matrix of H, we need to store a matrix with more than 45 billion entries, which is a demanding task for most computers. We instead exploit the block structure of the algebraic objects as follows. If we define by D the $((J-1)\cdot T) \times dim(\lambda)$ matrix of dummies for a single location and period, the projection matrix of H is given by

$$H(H'H)^{-1}H' = \frac{1}{(J+1)\cdot\bar{\tau}\cdot(J-1)} \begin{bmatrix} D(D'D)^{-1}D' & \dots & D(D'D)^{-1}D' \\ \vdots & & \vdots \\ D(D'D)^{-1}D' & \dots & D(D'D)^{-1}D' \end{bmatrix} \tag{10}$$

Hence, we obtain

$$\left(I - H(H'H)^{-1}H'\right)Y = Y - \begin{bmatrix} \frac{1}{(J+1)\cdot\bar{\tau}\cdot(J-1)} \sum_{i=1}^{(J+1)\cdot\bar{\tau}\cdot(J-1)} D(D'D)^{-1}D'Y_i \\ \vdots \\ \frac{1}{(J+1)\cdot\bar{\tau}\cdot(J-1)} \sum_{i=1}^{(J+1)\cdot\bar{\tau}\cdot(J-1)} D(D'D)^{-1}D'Y_i \end{bmatrix} \tag{11}$$

Thus, we only need to store the projection matrix of D.

S.4.1 Monte Carlo simulations

We validate our demand estimation strategy using Monte Carlo studies. We perform the exercise for a model with agents' beliefs given by (i) rational expectation and (ii) perfect foresight. Since the main estimation procedure is performed independently for each type, we conduct the exercise for a single type.

The period flow utility function used in the simulations is as follows (with rational expectations, we set $\eta_t = 0, \forall t$)

$$u_t((d,\tau), x_t) = \alpha \log(r_{dt}) + \sum_s \beta_s \log N_{dst} + \xi_{dt} + \eta_t + \lambda_d + MC(d, j_{t-1}) + \delta_\tau \tau.$$

We first construct the unobservable term ξ_{dt}, which contains the sum of two i.i.d. terms, u and v, representing unobservable characteristics of locations that vary across time and an expectational error component, respectively. Next, we construct the exogenous parts of household budget terms and amenities for each location-period pair i.i.d. from the Log-Normal distribution. The terms are a weighted sum of the exogenous terms and v, depending on whether we consider a model with endogeneity. We also draw the distance between inner-city locations and location and time fixed effects, constructing the flow utility function for all states. The full set of parameters is in Table 3.

We compute the EV function for each time period as follows. Starting in the last period T, we assume that the economy is in steady-state. We define EV_T as the unique solution to the following fixed-point

Table 3: Parameters used in simulations

Variables		Parameters	
Name	Distribution (i.i.d.) / Value	Name	Value
u	$N(0, 0.05)$	α	- 0.05
v	$N(0, 0.05)$	β_1, β_2	0.1
ξ	$u + v$	γ_0, γ_1	-0.0025
b_{exo}	$LogN(0.5, 0.1)$	γ_2	-0.5
r	$0.75 \cdot b_{\text{exo}} + 0.25 \cdot v$	δ	0.1
a_{exo}	$LogN(1.5, 0.5)$	$N_{\text{households}}$	$\in [5 \cdot 10^5, 10^6]$
a	$0.75 \cdot a_{\text{exo}} + 0.25 \cdot v$	J	24
$\text{dist}(j, j'), j, j' \neq 0; \rho_d$	$LogN(1, 0.5)$	S	2
λ_j	$N(0, 0.1)$	$\bar{\tau}$	2
λ_t (Perfect foresight)	$N(0, 0.1)$	T	10
λ_t (Rational expectations)	0	tol in EV iteration	10^{-10}

equation

$$EV_T(j_T, \tau_T) = \log \left(\sum_d \exp \Big(\sum_{x'} \mathbb{P}_T(x'|d, x_T) \big[u_T(x', x_T) + \beta EV_T(d, x_T) \big] \Big) \right) \tag{12}$$

For periods $t = 1, \ldots, T-1$, we compute EV_t using backward substitution as follows

$$EV_t(j_t, \tau_t) = \log \Big(\sum_d \exp \big(\sum_{\tau'} \mathbb{P}_{t+1}(x'|d, x_t) \big[u_{t+1}(d, x_t) + \beta EV_{t+1}(d, x') \big] \big) \Big) \tag{13}$$

Assuming a uniform initial distribution of individuals across states, we simulate each individual forward for 10 time periods using the the conditional choice probabilities computed using the flow utilities and EV functions presented above. We repeat each procedure 10 times and average the resulting output. The selected population sizes are 50 thousand and 1 million; the former roughly corresponding to the cardinality of each type of households, and the latter simulates large samples and provides insight into convergence properties. We test two models correcting for zero observed conditional choice probabilities: (i) we predict conditional choice probabilities using a multi-nomial logit model and (ii) we use observed frequencies and replace zero probabilities with a small $\epsilon = 10^{-5}$.

The results are presented below for the models with perfect foresight and rational expectations in Tables 4 and 5, respectively. In both cases, models with transition probabilities estimated using the multi-nomial logit model yield a strictly dominant finite sample performance. The gap is most pronounced in small samples where the likelihood of observing zero flows between states in the data is higher, where the frequency-based estimator uses small but arbitrary values imputed by the researcher, which can be far from the true transition probabilities. The multi-nomial logit approximates the true probabilities well, reducing finite-sample bias in the final estimation stage. As we increase the sample size, the number of observed zero flows diminishes, and we observe convergence of both estimators to the performance of the first-best estimator using the true transition probabilities.

Table 4: Monte Carlo simulations with fixed effects and an indicator for high location capital

			Mean of the absolute value of bias						
ξ	Pop (in 10^3)	Prob.	α	β_1	β_2	γ_0	γ_1	γ_2	δ
zero	50	T	7.6E-16	1.6E-16	1.8E-16	1.7E-15	4.8E-19	1.1E-15	1.7E-15
		L	1.7E-02	3.5E-03	1.7E-03	4.5E-02	2.0E-04	4.8E-02	1.0E-01
		F	3.2E-01	2.1E-01	1.8E-01	1.1E+00	2.5E-03	8.0E-01	8.8E-01
	1000	T	9.1E-16	1.3E-16	1.9E-16	1.6E-15	1.1E-18	1.2E-15	1.9E-15
		L	3.4E-03	1.1E-03	5.9E-04	3.8E-02	2.2E-05	3.4E-02	1.0E-01
		F	7.7E-03	2.3E-03	1.6E-03	3.3E-02	6.7E-05	1.6E-02	3.9E-02
exogenous	50	T	3.0E-02	8.5E-03	6.1E-03	1.7E-15	1.0E-18	9.9E-16	1.8E-15
		L	3.3E-02	8.2E-03	7.5E-03	4.9E-02	2.0E-04	3.4E-02	1.0E-01
		F	3.0E-01	1.5E-01	1.3E-01	1.2E+00	2.4E-03	8.9E-01	7.6E-01
	1000	T	3.8E-02	5.5E-03	5.6E-03	1.8E-15	1.4E-18	1.6E-15	2.0E-15
		L	3.7E-02	5.7E-03	5.4E-03	4.5E-02	3.6E-05	3.6E-02	1.0E-01
		F	4.0E-02	5.7E-03	6.6E-03	2.5E-02	1.0E-04	7.3E-03	3.0E-02
endogenous	50	T	4.2E-02	9.8E-03	8.4E-03	1.8E-15	1.3E-18	1.2E-15	1.8E-15
		L	3.9E-02	1.0E-02	1.1E-02	4.7E-02	1.6E-04	3.5E-02	1.0E-01
		F	2.7E-01	1.7E-01	1.2E-01	1.1E+00	2.8E-03	8.6E-01	6.6E-01
	1000	T	5.0E-02	1.0E-02	6.7E-03	1.7E-15	1.6E-18	1.3E-15	1.8E-15
		L	5.4E-02	1.0E-02	6.6E-03	3.7E-02	2.6E-05	3.4E-02	1.0E-01
		F	6.0E-02	1.0E-02	6.8E-03	2.3E-02	8.5E-05	9.3E-03	2.6E-02

Notes: The table presents averaged absolute distance between the estimated parameter and the true parameter, over 10 random draws of datasets. T represents estimation using the true transition probabilities; L represents estimation using predicted probabilities by a multi-nomial logit model; and F represents estimation using transition probabilities computed based on empirical shares.

Table 5: Monte Carlo simulations with location fixed effects only and an indicator for high location capital

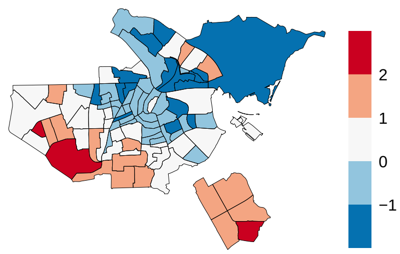
			Mean of the absolute value of bias						
ξ	Pop (in 10^3)	Prob.	α	β_1	β_2	γ_0	γ_1	γ_2	δ
zero	50	T	1.2E-15	1.9E-16	3.2E-16	1.3E-15	8.2E-19	1.3E-15	1.8E-15
		L	2.3E-02	2.3E-03	4.5E-03	5.7E-02	1.4E-04	3.9E-02	1.0E-01
		F	6.1E-01	1.7E-01	1.7E-01	1.3E+00	2.9E-03	7.8E-01	1.3E+00
	1000	T	8.1E-16	2.5E-16	2.6E-16	1.2E-15	6.5E-19	9.4E-16	1.6E-15
		L	5.6E-03	1.2E-03	8.0E-04	3.3E-02	2.8E-05	3.6E-02	1.0E-01
		F	1.4E-02	3.9E-03	2.6E-03	1.3E-02	5.0E-05	1.4E-02	3.8E-02
exogenous	50	T	2.7E-02	3.8E-03	3.5E-03	7.2E-03	5.0E-06	1.4E-15	2.0E-15
		L	2.7E-02	3.5E-03	5.7E-03	5.9E-02	2.2E-04	2.8E-02	1.0E-01
		F	4.6E-01	2.2E-01	2.7E-01	7.7E-01	2.9E-03	5.2E-01	1.5E+00
	1000	T	4.2E-02	1.5E-02	1.4E-02	7.9E-02	3.6E-04	1.3E-15	2.0E-15
		L	4.4E-02	1.5E-02	1.4E-02	1.0E-01	3.7E-04	3.4E-02	1.0E-01
		F	4.8E-02	1.6E-02	1.4E-02	9.7E-02	4.7E-04	1.3E-02	4.3E-02
endogenous	50	T	2.5E-02	9.9E-03	1.1E-02	9.1E-03	1.3E-05	1.2E-15	2.0E-15
		L	2.8E-02	9.5E-03	1.0E-02	5.6E-02	1.6E-04	4.4E-02	1.0E-01
		F	4.3E-01	2.5E-01	2.4E-01	7.9E-01	2.8E-03	6.5E-01	1.0E+00
	1000	T	3.0E-02	6.8E-03	3.2E-03	1.1E-02	1.0E-05	1.1E-15	1.9E-15
		L	2.8E-02	6.9E-03	3.0E-03	2.5E-02	5.5E-05	3.7E-02	1.0E-01
		F	4.2E-02	8.3E-03	5.5E-03	2.8E-02	1.0E-04	9.2E-03	3.3E-02

Notes: The table presents averaged absolute distance between the estimated parameter and the true parameter, over 10 random draws of datasets. T represents estimation using the true transition probabilities; L represents estimation using predicted probabilities by a multi-nomial logit model; and F represents estimation using transition probabilities computed based on empirical shares.

S.4.2 Housing supply estimation

Figure 4 shows the spatial distribution of the estimates for κ_j, the landlord's differential operating costs between short- and long-term markets. The results suggest landlords in central locations face lower costs of renting short-term relative to long-term. This is consistent with the notion that they match with guests easier and face lower vacancy risk.

Figure 4: Spatial distribution of location fixed effects κ_j.



Notes: estimates of κ_j are from the elasticity IV specification with wijk and year fixed effects. The values have been standardized, so positive values are above average, negative are below, and a value of 1 indicates a 1 standard deviation above the mean κ_j.

S.5 Theory

S.5.1 Proof of existence of equilibrium

Assumption 1 *Each type of households has a minimum area of housing necessary for subsistence; let us denote this level by $\gamma_k, \forall k$. Denote by $\psi_k \equiv w_k/\gamma_k$ the maximum rental price that a household of type k is able to pay to reach its subsistence level of housing. We define $\psi_{min} \equiv \min_k \psi_k$ and $\psi_{max} \equiv \max_k \psi_k$. Further, we assume there is a constant $q \in (0,1]$ such that $q \times \sum_k M_k \cdot \gamma_k = \sum_j H_j$. In words, this assumption says that the city can accommodate at most fraction q of households.*

Proposition 1 *Suppose Assumption 1 holds. Then a stationary equilibrium exists in the dynamic model.*

Proof. We perform the change of variables to $\rho_j \equiv \log(r_j)$ and seek a vector of log-prices and a matrix of amenities that clear all markets. To this end, define

$$\mathcal{D} \equiv \bigtimes_j \left[\min\left\{ -H_j, \log\left(\frac{q \cdot \min_k \alpha_h^k w_k}{\sum_{j=1}^J H_j}\right)\right\}, \log\left(\max\left\{\sum_k \frac{\alpha_h^k w^k M_k}{\psi_{min}}, \psi_{max}\right\}\right)\right] \times \left[0, \sum_{k=1}^K \frac{M_k \alpha_s^k \alpha_c^k}{F_{js}\sigma_s}\right]^{J \cdot S}$$

and observe $\mathcal{D}$ is convex and compact. We let $\mathcal{D}$ be the domain of the value function for each type. The choice of domain will become clear later in the proof. Denote by ω a generic concatenation of ρ and $\mathbf{a}$.

That the (expected) value function of households on this domain is well-defined, unique, and continuous follows by a standard argument presented, for example, in Rust (1988) and Stokey, Lucas and Prescott (1989). Further, that it is a strictly decreasing function in $\rho_j, \forall j \in \mathcal{J}$ with $\rho_j < \log(\psi_k)$ and decreasing otherwise follows by an adaptation of Corollary 1 to Theorem 3.2. in Stokey et al. (1989)). We omit these arguments for brevity.

It follows that for any household type k, the one-step transition probabilities, defined in equation 6, are continuous in $(\rho, \mathbf{a}) \in \mathcal{D}$, and since the flow utility function and the expected value functions are both strictly decreasing in ρ_j, such that

$$\rho_j \in \left[\min\left\{ -H_j, \log\left(\frac{q \cdot \min_k \alpha_h^k w_k}{\sum_{j=1}^J H_j}\right)\right\}, \log(\psi_k)\right] \equiv \mathcal{I}_j$$

it follows that the transition probabilities are strictly decreasing in ρ_j on this domain.

We stack up the transition probabilities into a transition matrix $\mathbf{\Pi}^k$. Observe that the resulting Markov chain is regular if $\rho_j \in \mathcal{I}_j, \forall j$. To this end, suppose $\rho_j < \log(\psi_k), \forall j \in \mathcal{J}$. We claim that the entries of $\mathbf{\Pi}^k(\omega)^{\bar{\tau}+2}$ are all strictly positive. Indeed, this follows from (i) $\mathbb{P}^k(d = j'|j, \tau, \omega) > 0, \forall j, \tau, j' \neq j$, (ii)

$\mathbb{P}^k(d = s|j, \tau, \omega) > 0, \forall j, \tau$, and (iii) it takes at most two steps to arrive to a state $(j, 1), \forall j \in \mathcal{J}$. Whenever $\exists j \in \mathcal{J}$ such that $\rho_j > \log(\psi_k)$, since such a location is not affordable, the household will never select it. Hence, in this case we may restrict the Markov chain to the rest of the locations, and the restricted chain is again regular by an analogous argument. It follows a stationary distribution exists, is unique, and equals the limiting distribution. Denote the stationary distribution by $\pi^k(\omega) \equiv \pi^k(\mathbf{\Pi}^k(\omega))$.

Next, we claim $\pi^k(\omega)$ is a continuous function of ω. To this end, we define the following auxiliary matrices following Schweitzer (1968): (i) The time-averaged transition matrix and (ii) the fundamental matrix of Kemeny and Snell (1983), respectively,

$$\mathbf{\Pi}^{k,\infty}(\omega) = \lim_{m\to\infty} \frac{1}{m} \sum_{j=1}^{m} \mathbf{\Pi}^k(\omega)^j \qquad \text{and} \qquad \mathbf{Z}^k(\omega) = \Big(\mathbf{I} - \mathbf{\Pi}^k(\omega) + \mathbf{\Pi}^{k,\infty}(\omega)\Big)^{-1}.$$

For any feasible ω, ω', define $\Delta^k(\omega, \omega') \equiv \mathbf{\Pi}^k(\omega') - \mathbf{\Pi}^k(\omega)$, and vector $\mathbf{g}^k(\omega, \omega')$ given by

$$g_l^k(\omega, \omega') \equiv \sum_{s,m=1}^{N} \pi_s^k(\mathbf{\Pi}^k(\omega)) Z_{sl}^k(\omega) \Delta_{sl}^k(\omega, \omega')$$

It follows from expression 15 in Schweitzer (1968)

$$\begin{aligned}
||\pi^k(\omega) - \pi^k(\omega')||_\infty &= ||\pi^k(\mathbf{\Pi}^k(\omega)) - \pi^k(\mathbf{\Pi}^k(\omega'))||_\infty \\
&= ||\pi^k(\mathbf{\Pi}^k(\omega)) - \pi^k(\mathbf{\Pi}^k(\omega) + \Delta^k(\omega, \omega'))||_\infty \\
&= ||\pi^k(\mathbf{\Pi}^k(\omega)) - \big(\pi^k(\mathbf{\Pi}^k(\omega)) + \mathbf{g}^k(\omega, \omega') + \mathcal{O}\big(\Delta^k(\omega, \omega')^2\big)\big)||_\infty \\
&\leqslant ||\mathbf{g}^k(\omega, \omega')||_\infty + ||\mathcal{O}\big(\Delta^k(\omega, \omega')^2\big)||_\infty
\end{aligned}$$

Further, we have

$$||\mathbf{g}^k(\omega, \omega')||_\infty \leqslant ||\mathbf{Z}^k(\omega)||_\infty ||\Delta^k(\omega, \omega')||_\infty \to \mathbf{0} \text{ as } \omega' \to \omega$$

since $\mathbf{\Pi}^k(\omega)$ is a continuous function of ω and so $||\Delta^k(\omega, \omega')||_\infty \to \mathbf{0}$ as $\omega' \to \omega$. Similarly, $||\mathcal{O}\big(\Delta^k(\omega, \omega')^2\big)||_\infty \to \mathbf{0}$ as $\omega' \to \omega$. Hence, $||\pi^k(\omega) - \pi^k(\omega')||_\infty \to \mathbf{0}$ as $\omega' \to \omega$. Since ω was arbitrary, it follows that $\pi^k(\omega)$ is a continuous function of ω. Further, by the definition of a stationary distribution, $\sum_\tau \pi_{j\tau}^k(\omega)$ is a decreasing function of ρ_j and an increasing function of $\rho_{j'}$ for $j \neq j'$.

Redefine demand function for squared footage of long-term housing as a function of log-rent and amenities, $\mathcal{D}_j^L(\omega)$. Observe that $\mathcal{D}_j^L(\omega)$ is a continuous function of ω and that $\mathcal{D}_j^L(\omega)$ is decreasing in ρ_j and weakly increasing in $\rho_{j'}$ for $j \neq j'$ Further, we correspondingly redefine the share of long-term houses as a function of log-rent $s_j^L(\rho)$.

Fixing amenities $\mathbf{a}$, solving for an equilibrium in the market for long-term housing corresponds to solving the following system of equations in $\rho \in \mathbb{R}^J$, $\mathcal{D}_j^L(\rho, \mathbf{a}) = s_j^L(\rho) H_j, \forall j \in \mathcal{J}$. This system has a solution. Indeed, fix amenities $\mathbf{a}$ and define the excess demand function $\mathbf{z} : \mathbb{R}^J \to \mathbb{R}^J$ by $z_j(\rho) = \mathcal{D}_j^L(\rho, \mathbf{a}) - s_j^L(\rho) H_j, \forall j \in \mathcal{J}$. To solve for equilibrium, we seek a root of $\mathbf{z}$. Before we proceed, note that the following holds using the above and by Assumption 1. If we denote by

$$\rho_{min} \equiv \log\left(\frac{q \cdot \min_k \alpha_h^k w_k}{\sum_{j=1}^J H_j}\right)$$

we have

$$\begin{aligned}\lim_{\rho\to\rho_{min}} z_j(\rho) &= \frac{\sum_{k=1}^K \alpha_h^k w^k M_k \sum_\tau \pi_{j\tau}^k(\rho_{min},\rho_{-j})}{q\cdot \min_k \alpha_h^k w^k}\sum_{j=1}^J H_j - s_j^L(\rho_{min},\rho_{-j})H_j \\ &\geqslant \frac{\sum_{k=1}^K M_k \sum_\tau \pi_{j\tau}^k(\rho_{min},\rho_{-j})}{q}\sum_{j=1}^J H_j - s_j^L(\rho_{min},\rho_{-j})H_j \\ &\geqslant \sum_{k=1}^K M_k \times \sum_{j=1}^J H_j - s_j^L(\rho_{min},\rho_{-j})H_j > 0\end{aligned}$$

and

$$\lim_{\rho_j\to\log(\psi_{max})} z_j(\rho) = -s_j^L(\log(\psi_{max}),\rho_{-j})H_j < 0$$

We transform the root-finding problem to a fixed-point problem by defining $\mathbf{f} : \mathbb{R}^J \to \mathbb{R}^J$, $\mathbf{f}(\rho) = \mathbf{z}(\rho) + \rho$. Observe that $\mathbf{f}$ is continuous, and since z_j is decreasing in ρ and by the above, we must have

$$\begin{aligned}&\mathbf{f}\Big(\bigtimes_{j\in\mathcal{J}}\left[\min\left\{-H_j,\log\left(\frac{q\cdot\min_k \alpha_h^k w_k}{\sum_{j=1}^J H_j}\right)\right\},\log\left(\max\left\{\sum_k \frac{\alpha_h^k w^k M_k}{\psi_{min}},\psi_{max}\right\}\right)\right]\Big) \subseteq \\ &\qquad \bigtimes_{j\in\mathcal{J}}\left[\min\left\{-H_j,\log\left(\frac{q\cdot\min_k \alpha_h^k w_k}{\sum_{j=1}^J H_j}\right)\right\},\log\left(\max\left\{\sum_k \frac{\alpha_h^k w^k M_k}{\psi_{min}},\psi_{max}\right\}\right)\right]\end{aligned}$$

where the considered set is convex and compact. Applying Brouwer's fixed point theorem, an equilibrium exists. Finally, an equilibrium vector of strictly positive rental prices must exist by the properties of the logarithmic function.

Further, since the aggregate demand function is strictly decreasing in $r_j, \forall j \in \mathcal{J}$ such that $r_j \leqslant \psi_{\max}$ and since all equilibrium prices are at most $\psi_{\max}$, restricting attention to $(0, \psi_{max}]$, the strict gross substitutes property holds. Hence, the equilibrium is unique. This allows us to define the equilibrium price vector as a function of the fixed vector of amenities $\mathbf{r}(\mathbf{a})$. Continuity of this function follows by applying the Implicit Function Theorem to $\mathbf{z}$ at market-clearing prices.

Define $\psi : \mathbb{R}^{J\cdot S} \to \mathbb{R}^{J\cdot S}$ by

$$\psi_{js}(\mathbf{a}) = \sum_{k=1}^K \frac{\mathcal{D}_j^{Lk}(\mathbf{r}(\mathbf{a}),\mathbf{a})\alpha_s^k\alpha_c^k w^k}{F_{js}\sigma_s}, \forall j \in \mathcal{J}, s \in \mathcal{S}$$

By the above, it follows that ψ is continuous and

$$\psi\left(\left[0,\sum_{k=1}^K \frac{M_k\alpha_s^k\alpha_c^k w^k}{F_{js}\sigma_s}\right]^{J\cdot S}\right) \subseteq \left[0,\sum_{k=1}^K \frac{M_k\alpha_s^k\alpha_c^k w^k}{F_{js}\sigma_s}\right]^{J\cdot S}$$

Existence of equilibrium hence follows by Brouwer's fixed-point theorem. ■

S.5.2 Outline of the equilibrium solver algorithm

The equilibrium existence argument presented above suggests a natural nested fixed-point algorithm to solve for equilibrium. The algorithm proceeds as follows. Fix parameters $\lambda \in (0, 1)$ and $\delta > 0$.

The outer loop proceeds as follows for step $t = 1, \ldots$

($\mathbf{O_1^t}$) Guess $\mathbf{a}^{(t)}$. The inner loop proceeds as follows for step $g = 1, \ldots$

($\mathbf{I}_1^g$) Guess $\mathbf{r}^{(g)}$
($\mathbf{I}_2^g$) Compute excess demand for housing $\mathbf{z}(\mathbf{r}^{(g)}, \mathbf{a}^{(t)})$
($\mathbf{I}_3^g$) Update $\mathbf{r}^{(g+1)} = \mathbf{r}^{(g)} + \delta \cdot \mathbf{z}(\mathbf{r}^{(g)}, \mathbf{a}^{(t)})$
($\mathbf{I}_3^g$) Compute $d_r^{(g)} = ||\mathbf{r}^{(g+1)} - \mathbf{r}^{(g)}||_\infty$
Iterate until step G such that $d_r^{(G)} < \epsilon_r$ for a tolerance level $\epsilon_r > 0$. Denote

$$\mathbf{r}^{(et)} \equiv \mathbf{r}^{(G)}$$

($\mathbf{O_2^t}$) Compute

$$\mathbf{a}_{js}^{(et)} = \sum_{k=1}^{K} \frac{\mathcal{D}_j^k(\mathbf{r}^{(et)}, \mathbf{a}^{(t)})\alpha_s^k \alpha_c^k w^k}{F_{js}\sigma_s}$$

($\mathbf{O_3^t}$) Update

$$\mathbf{a}^{(t+1)} = (1-\lambda)\mathbf{a}^{(et)} + \lambda \mathbf{a}^{(t)}$$

($\mathbf{O_4^t}$) Compute $d_a^{(t)} = ||\mathbf{a}^{(t+1)} - \mathbf{a}^{(t)}||_\infty$
Iterate until step T such that $d_a^{(T)} < \epsilon_a$ for a tolerance level $\epsilon_a > 0$.

S.6 Simulations and Counterfactuals

S.6.1 Model fit

We first construct the amenity supply equation using the parameters estimated in section 5.2. We set the unobservable component of entry costs equal to the residuals of equation 8. We treat house owners and tourists in hotels as exogenous consumers of amenities.

For housing demand, we first fix the exogenous characteristics of demand, such as the characteristics of the housing stock at its 2017 level. Recall that the endogenous components of neighborhoods, prices and consumption amenities, are both found as the solution of our equilibrium solver. We set unobservable demand shocks $\xi_{j,t}^k$ equal to zero, their conditional mean.[10] Then, we take the estimated parameters from section 5.3 and sum across groups k to compute aggregate demand for long-term housing.

To rectify potential measurement error and alleviate concerns about the level of aggregation, we calibrate the differential costs of short- versus long-term rentals to match the number of Airbnb tourists in each location in 2017. Airbnb tourists are an endogenous object, as defined in section 4.4

Finally, we define out starting value of our equilibrium solver algorithm equal to the observed equilibrium and set our tolerance level equal to 1E-5, measured against the infinite norm for the excess demand function for the vector of prices and amenities $(\mathbf{r}, \mathbf{a})$.

S.6.2 Multiplicity of equilibria

Given that endogenous amenities act as agglomeration forces, the model may feature multiple equilibria. Computationally, a standard way of detecting multiple equilibria is by initiating the equilibrium algorithm solver in section S.5.2 from many different starting values. When we do so, we do find that multiple equilibria exist. Therefore, we define an equilibrium selection rule as the resulting equilibrium when we set the initial value of our algorithm solver equal to the observed equilibrium. Using this selection rule, we see that our model can reproduce the patterns observed in the data fairly accurately, as shown in section 5.5.

[10] We do not set unobservable demand shocks $\xi_{j,t}^k$ equal to the structural demand residuals because the residuals that we recover from equation 10 contain two components: unobservable demand shocks $\xi_{j,t}^k$ and expectational errors $\tilde{v}_{t,j,x_{it}}$, which are impossible to separate.

For the counterfactual analysis presented in section 6, we initially computed the following equilibria using observed quantities for rent and amenities as starting values: (i) an equilibrium with endogenous amenities and no entry of short-term rentals (NS) and (ii) an equilibrium with endogenous amenities and short-term rentals in full equilibrium (S), as defined in section 4.4. The rest of the counterfactual equilibria analyzed in section 6 were computed using one of these equilibria as starting values. In particular, the homogeneous case in section 6.1 was computed using (S), the exogenous case in section 6.2 was computed using (NS), and all equilibria in section 6.3 were computed using (S).

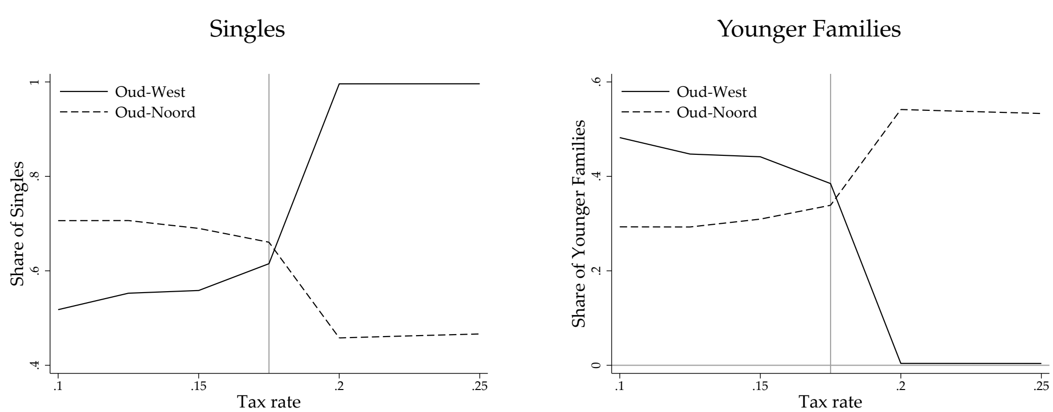
To test the robustness of the equilibria (S) and (NS) to deviations in the initial value supplied to the equilibrium solver, we perform a local search around them. Denote an equilibrium vector of rents by $\mathbf{r} \in \mathbb{R}^J$ and an equilibrium matrix of amenities by $\mathbf{a} \in \mathbb{R}^{J \times S}$, respectively. Denote by $\mathbf{a}^m \in \mathbb{R}^S$ a vector comprised of the minimum quantity of each amenity in $\mathbf{a}$. We proceed by repeatedly taking i.i.d. draws from the uniform distribution on the interval $[a_{js} - \gamma a_s^m, a_{js} + \gamma a_s^m]$, for each location j and service s, and a constant $\gamma \in (0, 1)$. We use each draw as a new initial value for amenities. For rent, we use the observed vector of rents.[11] We let each iteration converge using the tolerance level equal to 1E-4,[12] and we repeat this procedure 80-times using $\gamma = 0.5$ and $\gamma = 0.9$.

For both models (S) and (NS), while we find multiple equilibria for the case of $\gamma = 0.9$, each of the 80 iterations for $\gamma = 0.5$ converges to the same equilibrium, suggesting that our equilibria are locally stable in a sufficiently small neighborhood.

S.6.3 Tipping points in policy counterfactuals

Because the model's endogenous amenities act as agglomeration forces, policy counterfactuals may affect sorting and equilibrium outcomes in a non-linear way. Specifically, the demographic composition of neighborhoods may respond non-linearly to policy counterfactuals. Figure 5 shows that this indeed occurs for a few selected neighborhoods as the Airbnb tax described in Section 6.3 is gradually increased.

Figure 5: Tipping points



Notes: The figure shows how the share of each type of household in selected neighborhoods changes as the Airbnb tax is increased. The selected neighborhoods shown are those experiencing non-linear changes due to tipping. Most neighborhoods do not exhibit tipping behavior.

[11] Recall that for fixed amenities, equilibrium in the rental market is unique. Hence, the choice of an initial value for rent is immaterial.

[12] Note that the tolerance level is slightly less strict than the tolerance level used for computing the equilibrium presented in section S.5.2. We found this level to be a sufficient compromise between decreasing the computational burden of computing the equilibrium many times and keeping the precision of the computed equilibrium high.

S.6.4 Segregation measure

We follow White (1986) and use the entropy index for the city as our measure of segregation. We construct the index as follows. First, we define the entropy index for a single location. Let d_j^k be the fraction of households of type k in location j, i.e. if the demand for location j from household type k is D_j^k, we have $d_{jk} \equiv D_j^k / \sum_k D_j^k$. For location j, the entropy index v_j is defined as

$$v_j \equiv -\sum_{k=1}^{K} d_j^k \log(d_j^k)$$

We proceed to defining the entropy index for the whole city. First, we define the following objects: $D_j \equiv \sum_{k=1}^{K} D_j^k$, $D^k \equiv \sum_{j=1}^{J} D_j^k$, and $D \equiv \sum_{j=1}^{J} \sum_{k=1}^{K} D_j^k$. Pooling all locations together and a weighted average of the location-wise indices, weighted by the total demand for a location, respectively, we obtain

$$\widehat{v} \equiv -\sum_{k=1}^{K} \frac{D^k}{D} \log\left(\frac{D^k}{D}\right) \qquad \text{and} \qquad \overline{v} \equiv \sum_{j=1}^{J} v_j \frac{D_j}{D}.$$

Finally, we normalize the above indices to obtain an index in the interval $[0,1]$. Our measure of segregation then is

$$v \equiv \frac{\widehat{v} - \overline{v}}{\widehat{v}},$$

Observe that higher v means higher segregation, v equals 0 if and only if the share of each type in each location is given by its population share in the whole population, and v equals 1 if and only if each location is occupied by exactly one type.

The entropy index is 0.04 in the observed data and 0.7 in the simulated equilibrium. While the difference might seem concerning, we argue that a high level of segregation is natural for a stationary equilibrium, as we take the observed dynamics to the infinite limit. Indeed, such patterns in steady-state equilibrium is consistent with the trends described in section 3. These more extreme segregation patterns are a product of the combination of large moving costs and endogenous amenities. A simple comparative static exercise presented in section S.6.5 reveals that segregation in the steady-state equilibrium is increasing in the moving cost. Finally, focusing on a stationary equilibrium allows us to decrease the computational burden of computing the full equilibrium path while extracting the core economic lessons.[13]

S.6.5 Relationship between segregation in steady-state and moving costs

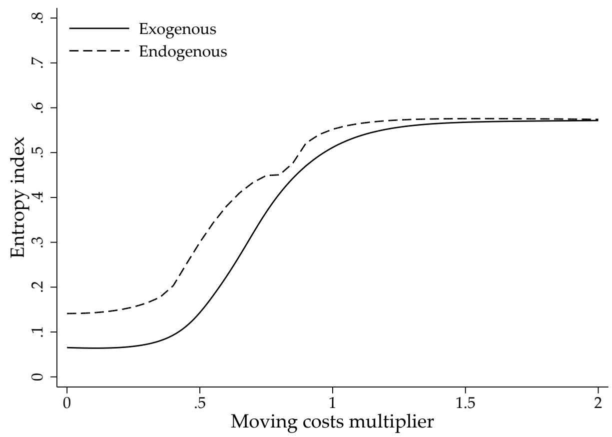
In this section, we shed light on the relationship between the level of moving costs and the level of segregation in equilibrium. Due to the complexity of the non-linear dynamic system, we cannot construct a closed-form solution of this mapping and perform instead a simulation exercise.

We use the same specification outlined in Section S.6.1. We vary moving costs by scaling each parameter of the moving costs and location capital by the same scalar, and compute the entropy index in equilibrium for an equally-spaced grid of scalars on the interval $[0,2]$.[14] We perform this exercise both with exogenous and endogenous amenities. The resulting mappings are presented in Figure 6.

[13] One caveat is that by focusing only on steady-state, we are not able to say anything about welfare along the transition paths.

[14] We also scale the location capital as it serves an analogous role to moving costs. As we increase location capital, the opportunity cost of moving to a different location also increases.

Figure 6: The relationship between moving costs and entropy index for the city.



Notes: The figure presents a plot from a simulation exercise of recomputing equilibrium for a set of moving cost multipliers taken from an equally-spaced grid on the interval $[0, 2]$. For the model with exogenous amenities, we used 201 points, with an increment of 0.01. For the model with endogenous amenities, due to the computational burden of repeatedly solving for equilibrium, we used 41 points, with an increment of 0.05.

Several patterns stand out. First, the function mapping moving costs to the entropy index follows a sigmoidal curve. The curve is essentially flat for moving costs below a third of the estimated level, increases at an increasing rate between one and two thirds of the estimated level of moving costs, which is followed by a concave region on the rest of the domain. As moving costs increase beyond the estimated level, we observe convergence to an upper bound below 0.6. This pattern is robust for both models.

Intuitively, higher moving costs imply both greater persistence and path dependence in the dynamics, decreasing the likelihood of moving to a different location for each state. Agglomeration forces attract peers to move to the same set of locations, increasing homogeneity of the type composition of neighborhoods. The interaction of these two forces leads to a higher degree of segregation in the steady state equilibrium. The shape of the relationship is closely related to the shape of the distribution of the error term in flow utility and the definition of our segregation index. An increase in moving costs for a small base level has a higher impact on dampening the likelihood of moving, generating the convex relationship. The marginal impact of an increase in moving costs is low when moving costs are already high.

Second, for each level of moving costs, the entropy index for the full equilibrium with endogenous amenities is strictly higher than for the model with exogenous amenities. This follows because amenities act as an agglomeration force. Third, the entropy index is strictly positive for each level of moving costs. Intuitively, heterogeneity among households leads to heterogeneous sorting patterns even in the absence of moving costs.

S.6.6 Measuring Welfare

In this section we describe how to measure welfare and welfare changes across scenarios. We mostly follow Small and Rosen (1981) to construct consumer surplus as a function of (log) euros.

First, we exploit our assumption that idiosyncratic shocks are distributed as Type I Extreme Value errors. In such a case, type-k renter's welfare in steady-state for a vector of prices and amenities $(\mathbf{r}, \mathbf{a})$ is given by:

$$W^k(\mathbf{r}, \mathbf{a}) \equiv \sum_{j,\tau} EV^k_{j,\tau}(\mathbf{r}, \mathbf{a}) \pi^k_{j,\tau}(\mathbf{r}, \mathbf{a}),$$

where $\pi^k_{j,\tau}(\mathbf{r}, \mathbf{a})$ is the stationary distribution and $EV^k(\mathbf{r}, \mathbf{a})$ is the expected value function of type k, respectively. For a formal definition of the latter, see equation 1.

Second, we can write renter's consumer surplus in log euros by multiplying by the variance of the idiosyncratic shocks: $\tilde{W}^k(\mathbf{r},\mathbf{a}) \equiv \sigma_\epsilon^k W^k(\mathbf{r},\mathbf{a})$. We estimate the variance of the idiosyncratic shock, σ_ϵ^k, as $\sigma_\epsilon^k = -(1-\phi^k)/\delta_r^k$, given our estimates of expenditure shares on housing, ϕ^k, and the rent preference parameters, δ_r^k, where we follow our micro-foundation in section A.1.

Second, we define consumer surplus as an implicit function of $\tilde{w}^k$ as follows:

$$\tilde{W}^k(\mathbf{r},\mathbf{a};\tilde{w}^k) = \sigma_\epsilon^k \sum_{j,\tau} E\tilde{V}_{j,\tau}^k(\mathbf{r},\mathbf{a};\tilde{w}^k)\pi_{j,\tau}^k(\mathbf{r},\mathbf{a}),$$

where we define $E\tilde{V}_{j,\tau}^k$ as the following re-scaled version of the utility function u_{jt}^k in equation 5:

$$\tilde{u}_j^k = u_j^k + \log\tilde{w}^k - \log w^k,$$

where w^k are fixed to the baseline wage level. Since re-scaling only changes the utility level but not choice probabilities, we obtain

$$\tilde{W}^k(\mathbf{r},\mathbf{a};\tilde{w}^k) = \Big(\frac{1}{1-\beta}\log(\tilde{w}^k) - \frac{1}{1-\beta}\log(w^k) + \tilde{W}^k(\mathbf{r},\mathbf{a})\Big).$$

Observe that $\tilde{W}^k(\mathbf{r},\mathbf{a};w^k) = \tilde{W}^k(\mathbf{r},\mathbf{a})$.

Denote by $\tilde{W}_0^k = \tilde{W}^k(\mathbf{r}_0,\mathbf{a}_0)$ and by $\tilde{W}_1^k = \tilde{W}^k(\mathbf{r}_1,\mathbf{a}_1)$ the consumer surplus of group k in log euros in the baseline and in a counterfactual equilibrium, respectively. We define our welfare changes in terms of *consumption equivalent*, CE^k:

$$\tilde{W}_0^k = \tilde{W}^k(\mathbf{r}_1,\mathbf{a}_1;w^k + CE^k).$$

In other words, our consumption equivalent measure gives us how annual wages should change to fix a type-k household Hicksian demand and keeping households on the baseline equilibrium's indifference curve. We can write CE^k in closed-form solution as follows

$$CE^k = w^k\Bigg(\exp\Big(\big[\tilde{W}_0^k - \tilde{W}_1^k\big](1-\beta)\Big) - 1\Bigg).$$

We can add rental income, ρ^k, accruing to homeowners to define homeowner's overall welfare $\tilde{W}^k(\mathbf{r},\mathbf{a};w^k + \rho^k)$ and the *consumption equivalent with rental income*, CE_r^k:

$$CE_r^k = \exp\Big(\big[\tilde{W}_0^k - \tilde{W}_1^k\big](1-\beta)\Big)(w^k + \rho_0^k) - w^k - \rho_1^k.$$

To compute rental income, we assume that each homeowner owns a representative portfolio of the city. Under that assumption, ρ^k can be defined as follows:

$$\rho^k(\mathbf{r}) = \sum_j s_j^L(\mathbf{r})\omega_j^L(\mathbf{r})r_j f_j + s_j^S(\mathbf{r})\omega_j^S(\mathbf{r})p_j,$$

with

$$\omega_j^L(\mathbf{r}) \equiv \frac{s_j^L(\mathbf{r})\mathcal{H}_j^{SL} + \mathcal{H}_j^L}{\sum_{j'} s_{j'}^L(\mathbf{r})\mathcal{H}_{j'}^{SL} + \mathcal{H}_{j'}^L} \quad \text{and} \quad \omega_j^S(\mathbf{r}) \equiv \frac{s_j^S(\mathbf{r})\mathcal{H}_j^{SL}}{\sum_{j'} s_{j'}^S(\mathbf{r})\mathcal{H}_{j'}^{SL}},$$

where $\mathcal{H}_j^{SL}$ is the quantity of housing units that can be supplied both to the market for short- and long-term rentals and enters the landlords' problem, and $\mathcal{H}_j^L$ is the quantity of housing units supplied only to the market for long-term rentals.

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