

Data-Driven Nests

FDIC Seminar

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Demand Estimation

Discrete Choice Models

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 - Generates unrealistic substitution patterns (IIA) Blue-bus/Red-bus

Demand Estimation

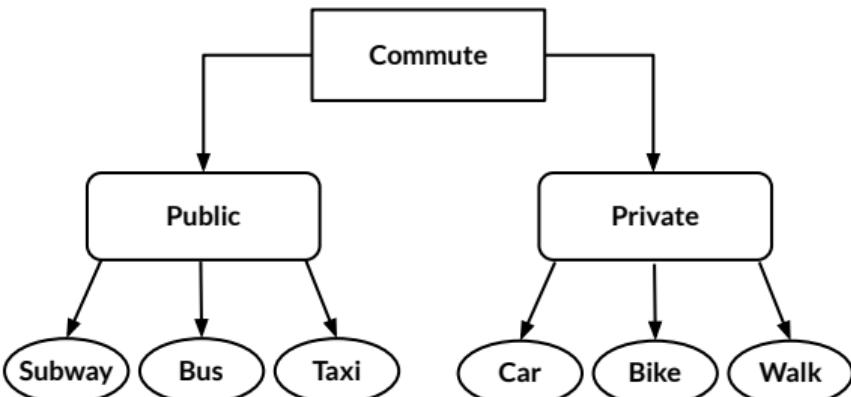
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- When shocks \sim Type I EV \implies Multinomial logit
 - Closed form solutions of choice probability + low number of parameters
 - Generates unrealistic substitution patterns (IIA) Blue-bus/Red-bus
- A common alternative is Nested Logit (NL)
 - Natural extension of Multinomial Logit

Main features of Nested Logit models

- Nests lead to different degrees of substitution:
→ More substitution **within** than **across** nests
- Within-nest correlation of idiosyncratic shocks
- Interpretable model as “sequential” choice
- Low number of parameters
- Closed form solutions of choice probabilities
→ Easy to implement

McFadden (1978)



Nested demand models are ubiquitous across many fields

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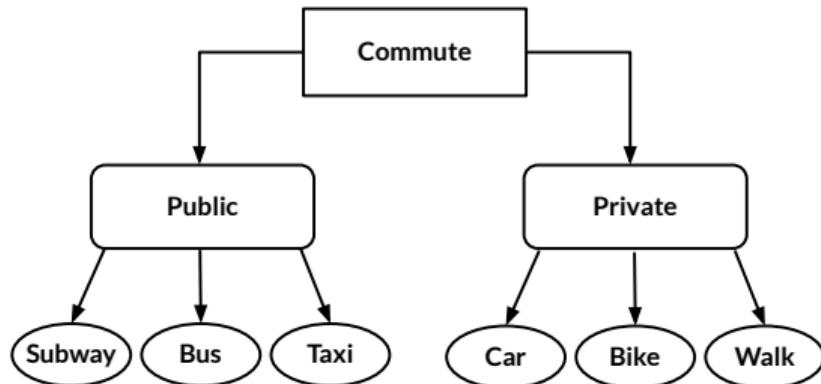
⇒ In many applications, demand can be appropriately described by a nesting structure

What's the right nesting structure?

In general, the practitioner faces the challenge of specifying the nesting structure *ex-ante*.

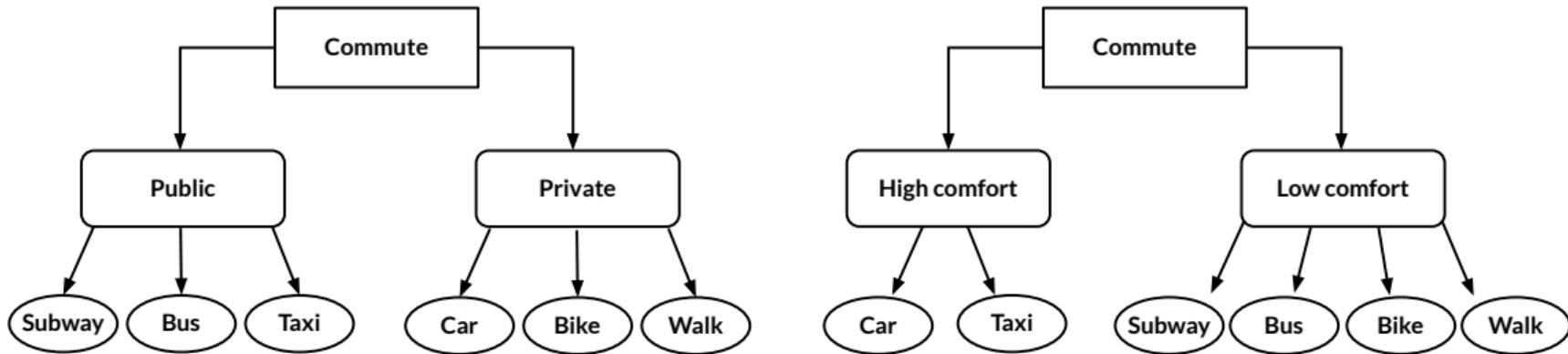
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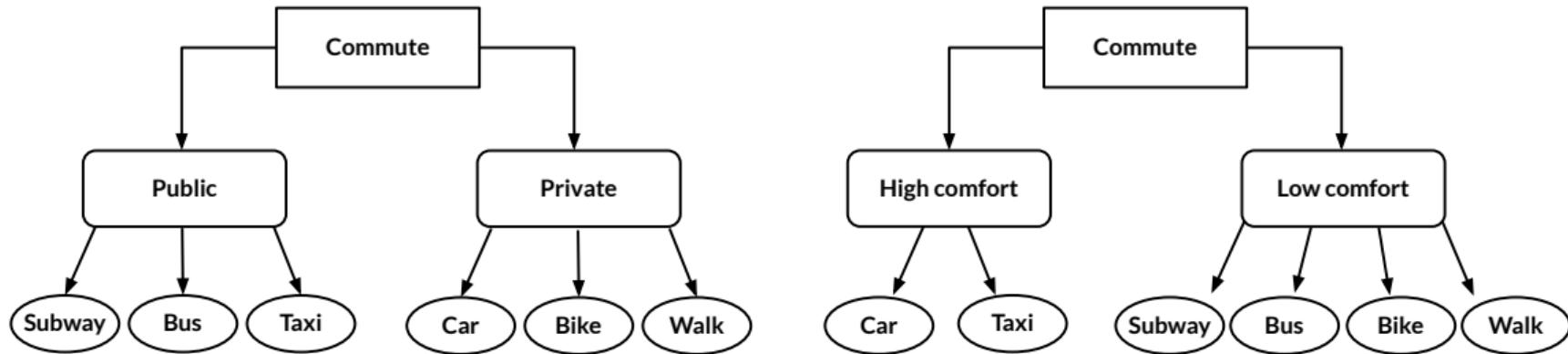
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⇒ Mis-specification of nests can lead to biased estimates (Fosgerau et al., 2024)

This paper: Estimating nests in discrete choice models

- Methodology to estimate the nesting structure as well as preference parameters
- Nesting structure is recovered from **aggregate market share data + product characteristics**
- Two-step estimation procedure:
 1. Estimate nests with k-means clustering on demand curves (Bonhomme and Manresa, 2015):
 - » Nested Logit is a correlated shock that leads to group-specific substitution patterns and elasticities
 - » Demand curves with group-specific coefficients **common within nests** but that **vary across nests**
 2. Estimate model parameters as if the groups were known (Berry, 1994)
- We exploit the structure of the model, the availability of many markets and of many products
- Empirical application: Demand for beer
 - In this setting, data-driven model outperforms Naive Nested Logit and Mixed Logit out of sample

Related literature

- **Discrete Choice Models of Random Utility with Nests:** McFadden (1978, 1981), Berry (1994), Verboven (1996), Cardell (1997), McFadden and Train (2000), Grigolon and Verboven (2014)
- **Group Fixed Effect Estimator:** Han & Moon (2010), Bonhomme & Manresa (2015), Choi and Okui (2024)
- **Alternative Grouping Structure:** Fosgerau, Monardo & De Palma (2024), Hortacsu, Lieber, Monardo & de Paula (ongoing)

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Empirical model

Logit

- Consider the random utility model for agent i when choosing j in market m :

$$V_{ijm} = \delta_{jm} + \varepsilon_{ijm}$$

- Choice of j based on the maximization of the utility:

$$\mathbb{P}_{jm} = \mathbb{P}(V_{ijm} > V_{ij'm} \quad \forall j' \neq j)$$

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- If $\varepsilon_{im} \stackrel{\text{i.i.d.}}{\sim} \exp(-\sum_j e^{-\varepsilon_{ijm}})$, then:

$$\mathbb{P}_{jm} = \frac{e^{\delta_{jm}}}{\sum_{j'=1}^J e^{\delta_{j'm}}}$$

Nested Logit

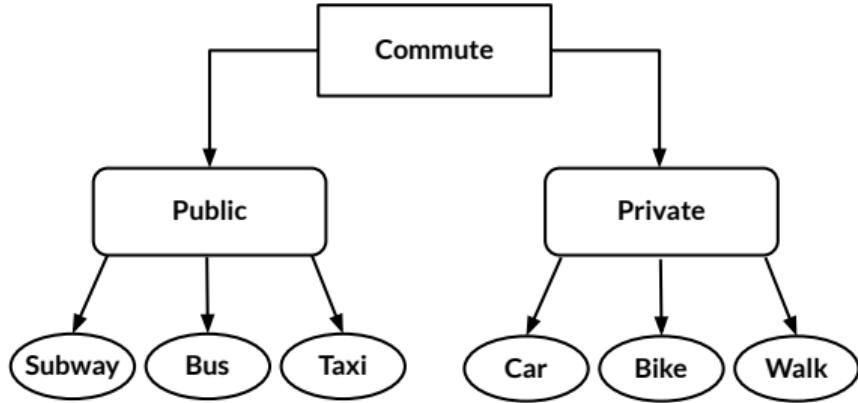
If $\varepsilon_{im} \stackrel{\text{i.i.d.}}{\sim} \exp\left(-\sum_{k=1}^K \left(\sum_{j \in k} e^{-\varepsilon_{ijm}/\sigma_k}\right)^{\sigma_k}\right)$, then

$$\mathbb{P}_{jm} = \frac{e^{\frac{\delta_{jm}}{\sigma^{k(j)}}} \left(\sum_{d \in B_{k(j)}} e^{\frac{\delta_{dm}}{\sigma^{k(j)}}} \right)^{\sigma^{k(j)} - 1}}{\sum_{l=1}^K \left(\sum_{d \in B_l} e^{\frac{\delta_{dm}}{\sigma^l}} \right)^{\sigma^l}}$$

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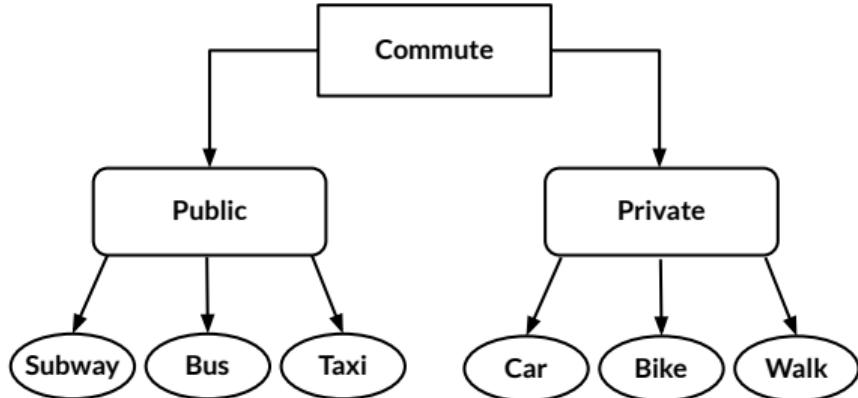
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- Nest structure ~ common within-nest shock

$\rightarrow \varepsilon_{ijm} = \eta_{k(j),m} + \tilde{\varepsilon}_{ijm}$ (Cardell, 1997)

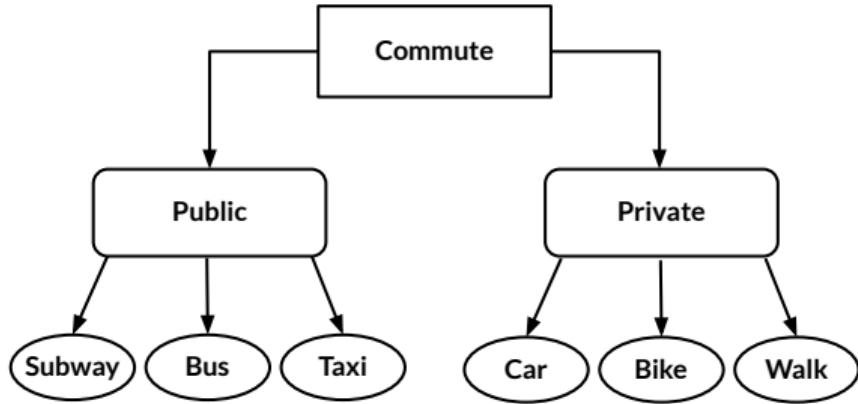


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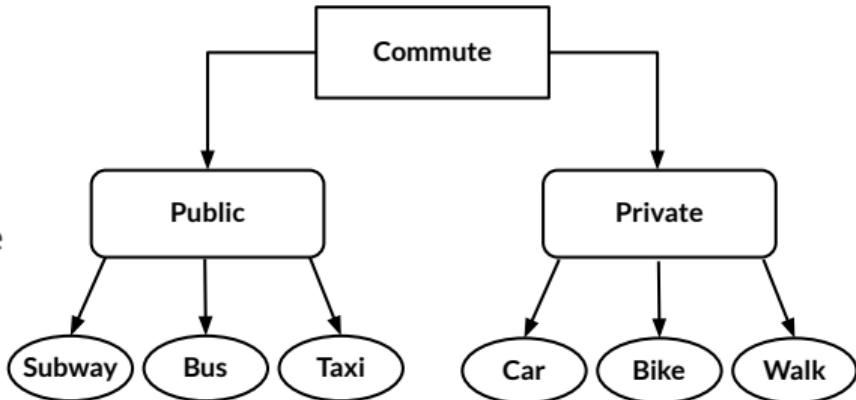


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- $\sigma^{k(j)} \in (0, 1]$ captures within-nest independence
 \rightarrow if $\sigma^{k(j)} = 1 \implies$ back to logit



Towards an empirical equation

$$\mathbb{P}_{jm} = \frac{e^{\frac{\delta_{jm}}{\sigma^{k(j)}}} \left(\sum_{d \in B_{k(j)}} e^{\frac{\delta_{dm}}{\sigma^{k(j)}}} \right)^{\sigma^{k(j)} - 1}}{\sum_{l=1}^K \left(\sum_{d \in B_l} e^{\frac{\delta_{dm}}{\sigma^l}} \right)^{\sigma^l}}$$

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Key observations: $(\beta_x^{k(j)}, \beta_p^{k(j)}, \lambda_m^{k(j)})$ common for products in the same group!

- Common $(\beta_x^{k(j)}, \beta_p^{k(j)})$ implies marginal effects are **common within nests** and **differ across nests**
→ Captures different elasticities of substitution across nests
- Conditional on covariates, $\lambda_m^{k(j)}$ is a **correlated shock within nest** that can vary across markets
→ Example: craft beer has become more popular in the Midwest and coasts relative to the South

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Intuition on nest identification

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$$\overline{\log \mathbb{P}_j} = \beta_x^{k(j)} \bar{x}_j + \beta_p^{k(j)} \bar{p}_j + \bar{\lambda}^{k(j)}, \text{ where } \bar{\xi}_j \rightarrow 0$$

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Key observation: There are K lines with coefficients $(\bar{\lambda}^{k(j)}, \beta_x^{k(j)}, \beta_p^{k(j)})$ that perfectly fit the data

Intuition on nest identification

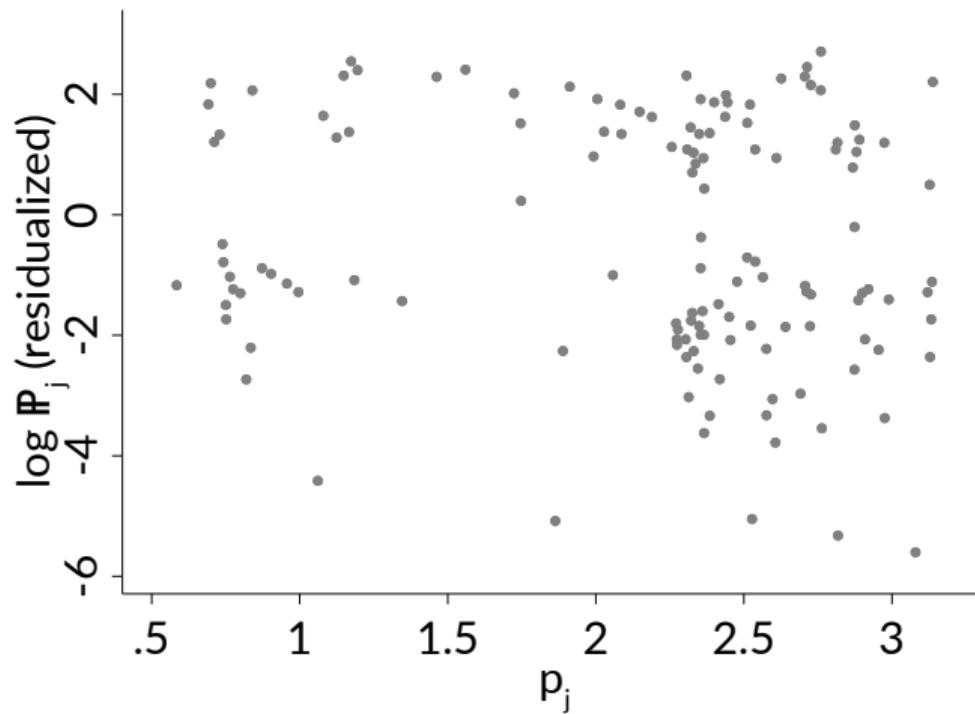
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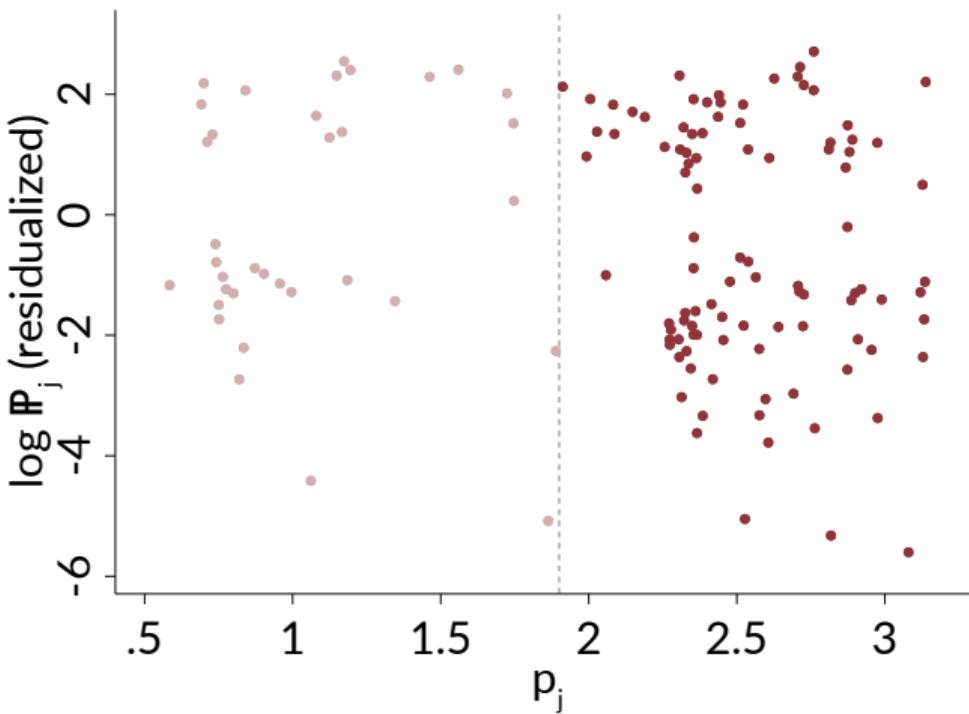
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Key observation: There are K lines with coefficients $(\bar{\lambda}^{k(j)}, \beta_x^{k(j)}, \beta_p^{k(j)})$ that perfectly fit the data
→ Not only search on coefficients, $(\beta^1, \dots, \beta^K, \lambda^1, \dots, \lambda^K)$ but also on how to partition data

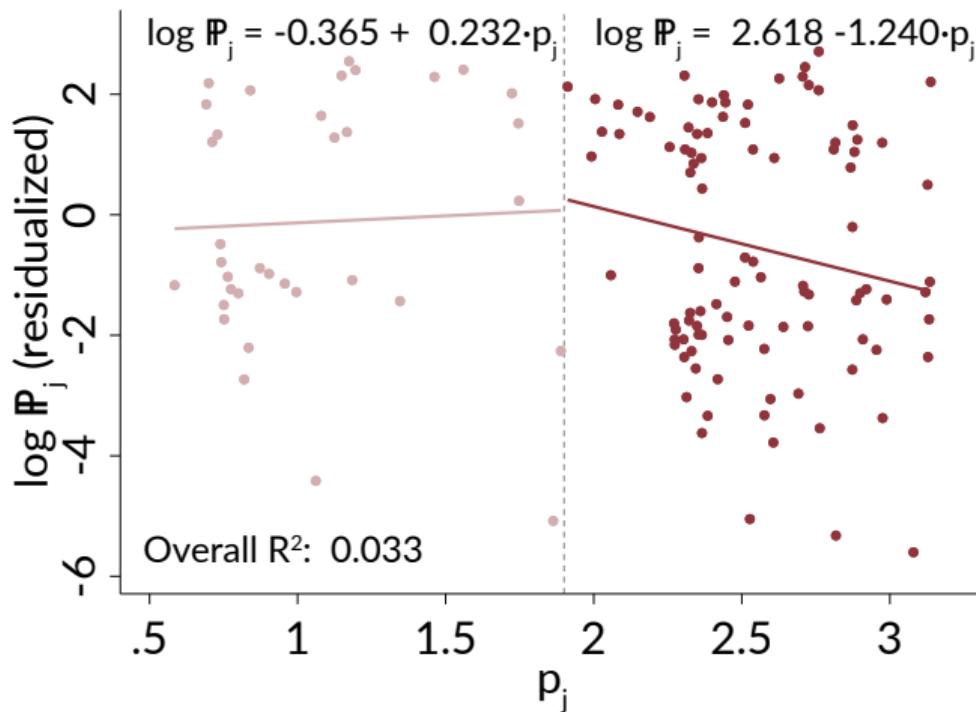
Intuition on nest identification: graphic example with $K = 2$



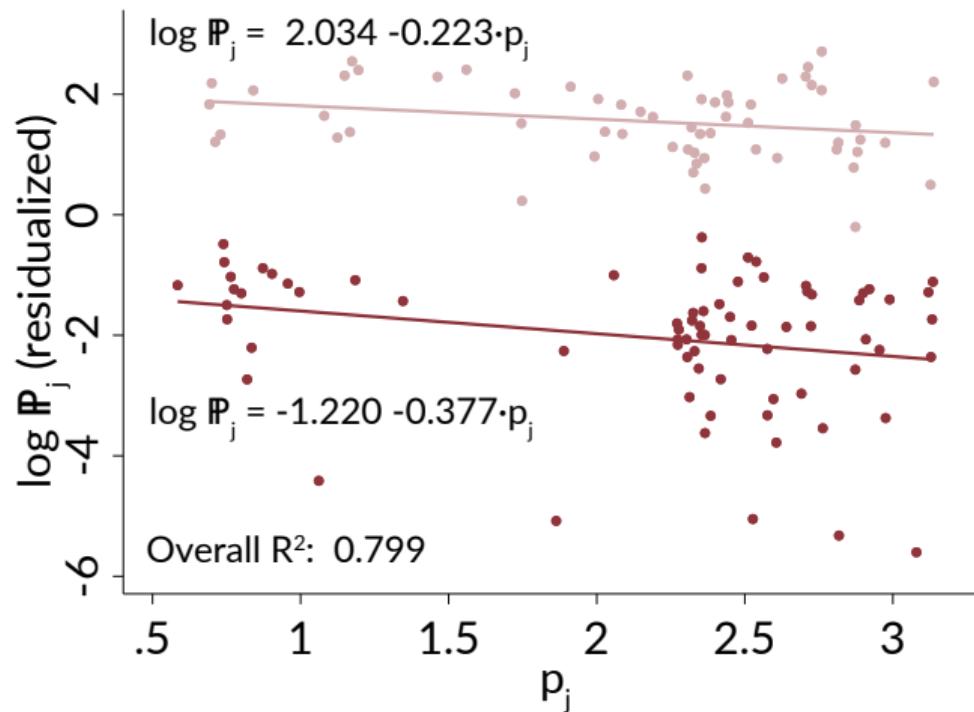
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Estimation of groups

First-step: Classification

Group-fixed effect estimator defined by the following clustering problem:

$$\arg \min_{\substack{k(1), \dots, k(J) \\ \beta^1, \dots, \beta^K, \lambda_1^1, \dots, \lambda_M^K}} \sum_{m=1}^M \sum_{j=1}^J \left(\log \mathbb{P}_{jm} - (\beta_p^k p_{jm} + \beta_x^k x_{jm} + \lambda_m^k) \right)^2$$

→ Equivalent to k-means but using regressions instead of cluster means

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Combinatorial, non-convex problem!

→ Solution: iterative clustering algorithm extending Bonhomme and Manresa (2015)

Two-step strategy

First Step: Classification

1. Let $\theta^0 \equiv (\beta^{1,0}, \dots, \beta^{K,0}, \lambda_1^{K,0}, \dots, \lambda_M^{K,0})$ be a starting value.

Two-step strategy

First Step: Classification

1. Let $\theta^0 \equiv (\beta^{1,0}, \dots, \beta^{K,0}, \lambda_1^{K,0}, \dots, \lambda_M^{K,0})$ be a starting value.
2. For θ^s , compute for all $j \in J$:

$$k(j)^{s+1} = \arg \min_{k \in \{1, \dots, K\}} \sum_{m=1}^M \left(\log \mathbb{P}_{jm} - (\beta_p^{k,s} p_{jm} + \beta_x^{k,s} x_{jm} + \lambda_m^{k,s}) \right)^2,$$

to compute grouping structure \mathcal{B}^{s+1} .

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4. Repeat until convergence of parameters.
5. In practice, to find global optimum, repeat for different starting values and take minimum

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Second step: Estimation of structural parameters

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We follow Berry (1994):

$$\log \frac{\mathbb{P}_{jm}}{\mathbb{P}_{0m}} = \beta_p p_{jm} + \beta_x x_{jm} + (\sigma^{k(j)} - 1) \log \mathbb{P}_{j,m|k(j)} + \xi_{jm}$$

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Two sets of instruments:

- Instrument for prices, z_{jm}
- Instrument for $\log \mathbb{P}_{j,m|k(j)}$ = $\frac{e^{\frac{\beta_p p_{jm} + \beta_x x_{jm}}{\sigma^{k(j)}}}}{\sum_{d \in B_{k(j)}} e^{\frac{\beta_p p_{jm} + \beta_x x_{jm}}{\sigma^{k(j)}}}}$ \implies Functions of $x_{j'm}$ for $j' \in k(j)$

Practical aspects and extensions

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Choosing K: Cross-validation with Elbow method

So far we have assumed the number of groups is known.

In practice, we can also estimate the number of groups using a **N -fold cross-validation** procedure.

For $k \in \{1, \dots, K\}$:

- Divide products into N equal subsets, P_1, \dots, P_N .
- Pick subset P_n and estimate grouping structure and grouping parameters in the other $N - 1$ parts.
- Classify products across estimated groups in part P_n and **compute out-of-sample MSE**

$$MSE_n(k) = \frac{1}{J \cdot M} \sum_{m=1}^M \sum_{j \in P_n} (y_j - \beta_{m,-n}^{k(j)} x_j - \lambda_{m,-n}^{k(j)})^2$$

- Take average across N folds:

$$MSE(k) = \frac{1}{N} \sum_{n=1}^N MSE_n(k)$$

- Choose k according to

$$k^* = \{k(j) | \text{where slope of } MSE(k) \text{ changes}\}$$

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Endogenous prices

Exogeneity of characteristics would imply:

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In the presence of exogenous shifter z_{jm} , consider for now a linear price projection:

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Substitute into main equation:

$$\log \mathbb{P}_{jm} = \beta_x^{k(j)} x_{jm} + \beta_p^{k(j)} p_{jm} + \xi_{jm} + \lambda_m^{k(j)} = \underbrace{\tilde{\beta}_x^{k(j)}}_{\beta_x^{k(j)} + \beta_p^{k(j)} \gamma_x} x_{jm} + \underbrace{\tilde{\beta}_z^{k(j)}}_{\beta_p^{k(j)} \gamma_z} z_{jm} + \underbrace{v_{jm}}_{\xi_{jm} + \beta_p^{k(j)} v_{jm}} + \lambda_m^{k(j)},$$

More general pricing equations

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- Non-linear functions of x_{jm} and z_{jm} with groupings:

$$p_{jm} = f^{g(j)}(x_{jm}, z_{jm}) + v_{jm}$$

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Individual heterogeneity with observed conditional shares

Assume heterogeneity can be described by types t (e.g. income quintiles) and utility given by:

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Two cases:

1. No common nesting structures: classify even type-by-type
2. Common nesting structures: solve joint problem

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Higher-level Nesting Structures

Upstream and downstream nests given by A_1, \dots, A_N and B_1, \dots, B_K , respectively.

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We can run same classification algorithm!

→ Note: needs normalization of some $\lambda_m^k = 0$ to avoid co-linear group-market fixed effects.

Consistency, statistical properties, and Monte Carlo

Regularity conditions with unknown nests

1. K fixed. Let $J \rightarrow \infty$, $M \rightarrow \infty$, and $\frac{\log M}{J} \rightarrow 0$

- Need to observe market shares multiple times so error ξ_{jm} vanishes out Why $M \rightarrow \infty$?

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 - This requires that two groups contain same products and look *identical* across all markets

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\Rightarrow Asymptotic **super-consistency** to the infeasible estimator where the groups are known
 \rightarrow Convergence rate is $O_p(JM^{-\eta})$ for any $\eta > 0$

Monte Carlo design: Data generating process

Fix $K = 3$. Set $\sigma = (0.2, 0.3, 0.6)$. Classify products using uniform draws $k(j) \sim \mathcal{U}\{1, \dots, K\}$

Models for the average utility δ_{jm} and prices p_{jm} are given by:

$$\delta_{jm} = \beta_p p_{jm} + \beta_x x_{jm} + \xi_{jm} \quad \text{and} \quad p_{jm} = mc_{jm} + \rho \cdot \xi_{jm}$$

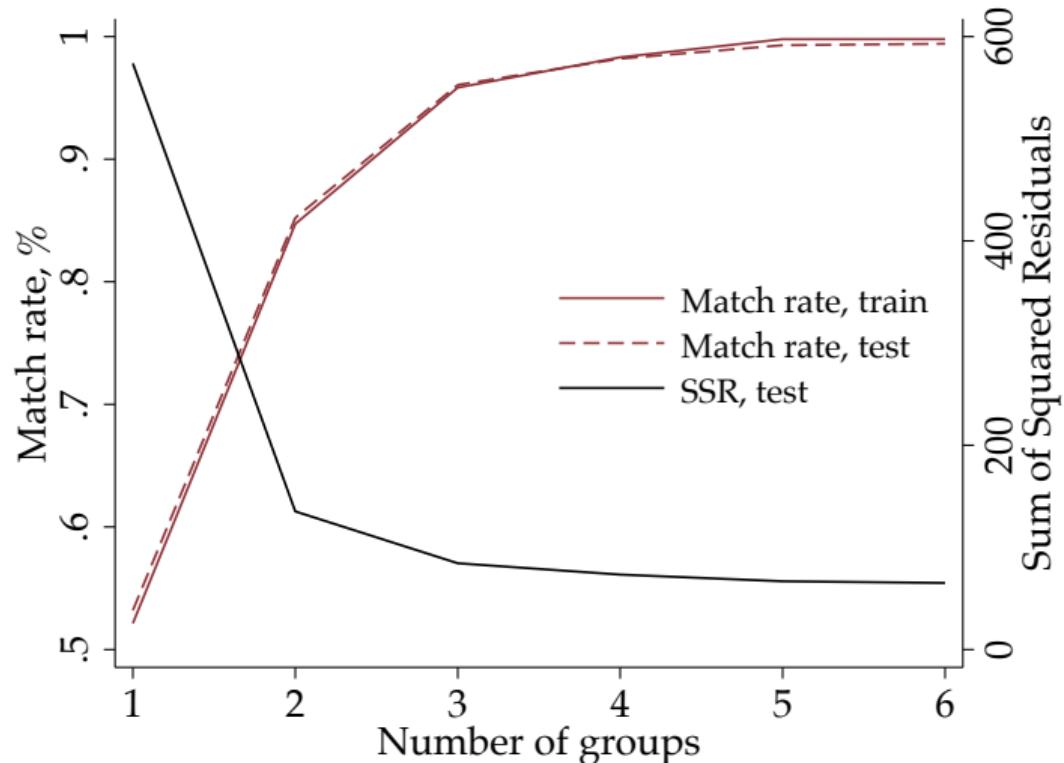
Generate data as follows:

$$\begin{bmatrix} \mu_x^k \\ \mu_{mc}^k \end{bmatrix} \stackrel{i.i.d.}{\sim} \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0.3 \\ 0.3 & 1 \end{bmatrix}\right) \Rightarrow \begin{bmatrix} x_{jm} \\ mc_{jm} \\ \xi_{jm} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_x^k \\ \mu_{mc}^k \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0.3 & 0 \\ 0.3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)$$

Set $\rho = 0.3$

Estimating the number of groups: Monte Carlo

Set $K^0 = 3$, the total number of products $J = 500$, and the total number of markets is $M = 10$.



Results with $K = 3$

50 Bootstrap iterations

J	M	Runtime	Matched	True	β_p	β_c	σ_1	σ_2	σ_3
					-1	1	0.2	0.3	0.6
100	10	00:02	0.996	Mean β	-0.992	0.991	0.189	0.297	0.602
				Std β	0.032	0.033	0.034	0.024	0.007
100	50	00:25	1.0	Mean β	-1.001	0.998	0.2	0.3	0.6
				Std β	0.01	0.011	0.001	0.002	0.003
100	100	01:07	1.0	Mean β	-1.0	1.0	0.2	0.3	0.6
				Std β	0.006	0.007	0.001	0.001	0.003
500	10	00:06	0.995	Mean β	-1.0	0.998	0.199	0.298	0.6
				Std β	0.015	0.02	0.006	0.016	0.003
500	50	07:14	1.0	Mean β	-1.0	0.999	0.2	0.3	0.6
				Std β	0.004	0.004	0.0	0.001	0.001
500	100	29:24	1.0	Mean β	-1.0	0.999	0.2	0.3	0.6
				Std β	0.003	0.003	0.0	0.0	0.001
1000	10	00:12	1.0	Mean β	-1.0	0.999	0.199	0.3	0.6
				Std β	0.007	0.007	0.003	0.001	0.002
1000	50	44:57	1.0	Mean β	-1.0	1.0	0.2	0.3	0.6
				Std β	0.002	0.003	0.0	0.0	0.001
1000	100	2:11:14	1.0	Mean β	-1.0	0.999	0.2	0.3	0.6
				Std β	0.002	0.002	0.0	0.0	0.001

Empirical application: Demand for beer

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Nielsen IQ store panel data

Data Description:

- We define products as Universal Product Codes (UPCs)
- Focus on year 2022 and on UPCs categorized as beer (24,188 unique UPCs)
- Define markets as states (exclude Alaska and Hawaii) $\implies M = 48$
- Each observation contains data on:
 - Total number of sales by UPC and state, aggregating across stores
 - Product characteristics: Prices, unit quantity, total units, type of beer, brand, packaging, domestic

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Keep products that are sold in at least ten states

- Unbalanced panel with $J = 2,806$ unique UPCs (11.6%) and 64,497 observations
- Cover between 61.5% to 99% of volume sales across states
- Average UPC is sold across 23.32 states ~ 50% missing markets

Summary statistics

Large variation in shares and covariates

Panel A: Full Sample Summary Statistics

Variable	Mean	Std. Dev.	Median	Min	Max
Price per ml (\$)	0.0063	0.0053	0.0049	0.0002	0.0519
Price (\$)	10.8193	5.8026	10.7381	1.6338	25.6800
Unit quantity (units)	5.9316	5.8582	6.0000	1.0000	36.0000
Unit size (ml)	607.80	856.33	354.88	207.01	8517.17

Panel B: Median Characteristics of the Top Ten National Brands

Brand	Market share	Unit price/ml	Unit price	Unit quantity	Unit size (ml)
Bud Light	13.79%	0.004	9.54	6.00	354.88
Modelo	8.91%	0.004	9.90	4.00	354.88
Miller	7.92%	0.003	12.31	12.00	354.88
Coors	7.43%	0.003	12.68	12.00	354.88
Michelob	7.03%	0.004	13.26	7.00	354.88
Bud	6.71%	0.003	8.43	6.00	473.18
Corona	6.03%	0.004	11.24	6.00	354.88
Busch	3.62%	0.003	10.87	12.00	354.88
Natural	3.25%	0.002	10.74	12.00	354.88
New Belgium	2.53%	0.005	10.81	6.00	354.88

Empirical Model

We model the average utility δ_{jm} as follows:

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Instrument for prices: Gandhi-Houde instruments

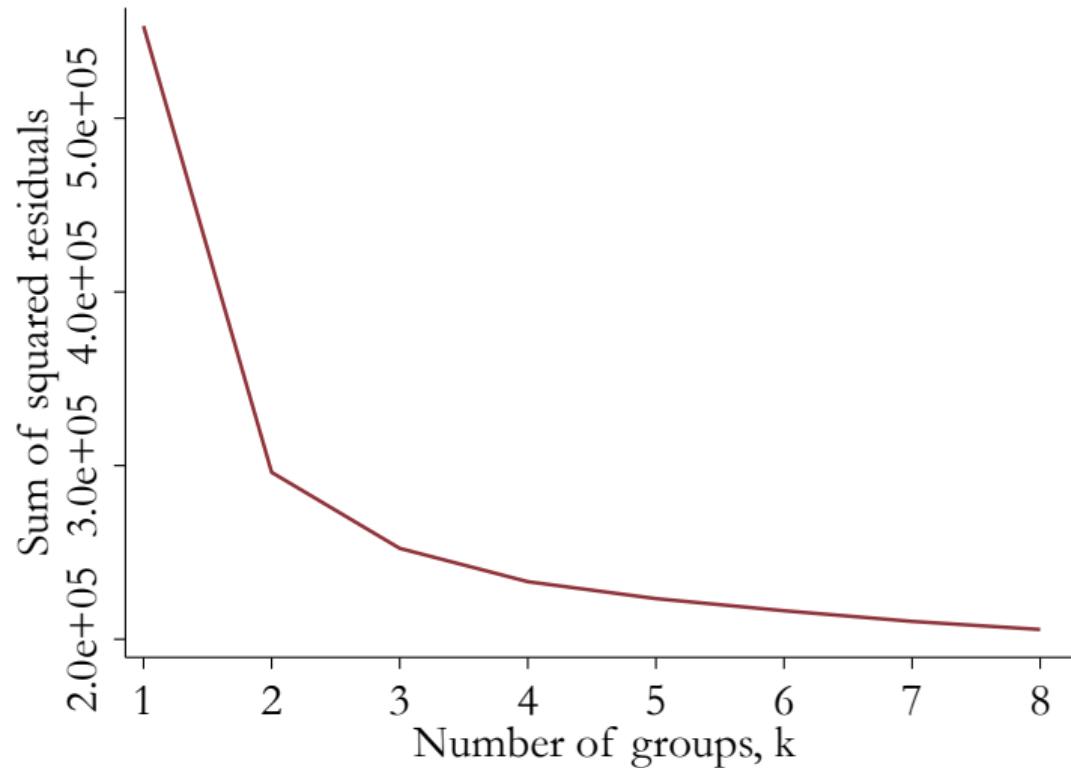
$$gh_{jm}^1 = \sum_{j' \notin \text{Brand}(j,m)} (\log \text{size}_j - \log \text{size}'_{j'})^2 \quad \text{and} \quad gh_{jm}^2 = \sum_{j' \notin \text{Brand}(j,m)} (\log \text{quantity}_j - \log \text{quantity}'_{j'})^2$$

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First-step: Choosing the number of groups

Elbow method suggest $K^* = 4$



First-step: Group characteristics

Characteristics	Unconditional values	Conditional values by group			
		1	2	3	4
Unit size (ml)	354.882	354.882	354.882	354.882	354.882
Unit quantity	6.000	6.000	6.000	6.000	4.000
Unit price	10.738	10.346	10.209	11.205	10.782
Price per ml	0.005	0.005	0.004	0.005	0.006
Share of domestic beer	0.751	0.793	0.802	0.737	0.719
Share of ale	0.421	0.394	0.219	0.449	0.466
Share of regular beer	0.335	0.366	0.380	0.324	0.311
Share of stout and porter	0.079	0.050	0.029	0.077	0.121
Share of light beer	0.123	0.140	0.343	0.103	0.068
Share of other types	0.042	0.050	0.029	0.047	0.034
Share of top ten brands	0.170	0.207	0.579	0.108	0.101
# of products	2806	658	242	1103	803
Label		Regular Lager	Light Beer	Special /Craft	Boutique Craft

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Share of domestic beer	0.751	0.793	0.802	0.737	0.719
Share of ale	0.421	0.394	0.219	0.449	0.466
Share of regular beer	0.335	0.366	0.380	0.324	0.311
Share of stout and porter	0.079	0.050	0.029	0.077	0.121
Share of light beer	0.123	0.140	0.343	0.103	0.068
Share of other types	0.042	0.050	0.029	0.047	0.034
Share of top ten brands	0.170	0.207	0.579	0.108	0.101
# of products	2806	658	242	1103	803
Label		Regular Lager	Light Beer	Special /Craft	Boutique Craft

First-step: Group characteristics

Characteristics	Unconditional values	Conditional values by group			
		1	2	3	4
Unit size (ml)	354.882	354.882	354.882	354.882	354.882
Unit quantity	6.000	6.000	6.000	6.000	4.000
Unit price	10.738	10.346	10.209	11.205	10.782
Price per ml	0.005	0.005	0.004	0.005	0.006
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Brands by group

Regular Beer



Light Beer



Special/Craft

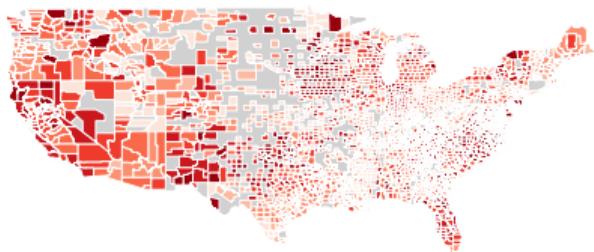


Boutique Craft



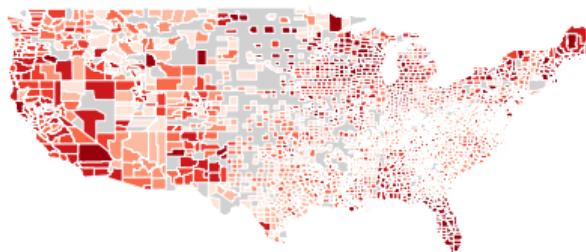
Spatial correlation of county market shares by group

Regular lager



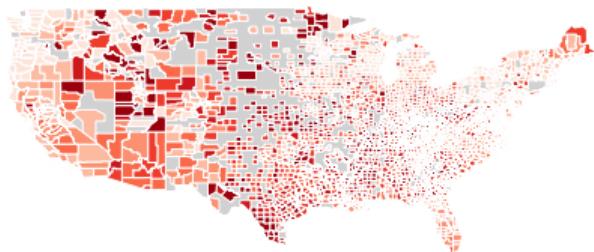
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Special/craft



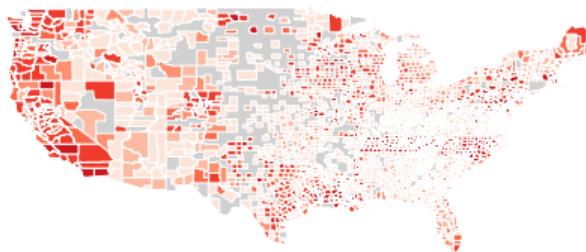
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Light beer



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(.78,.846]
(.725,.78]
(.672,.725]
(.617,.672]
.544,.617]
[0,.544]
No data

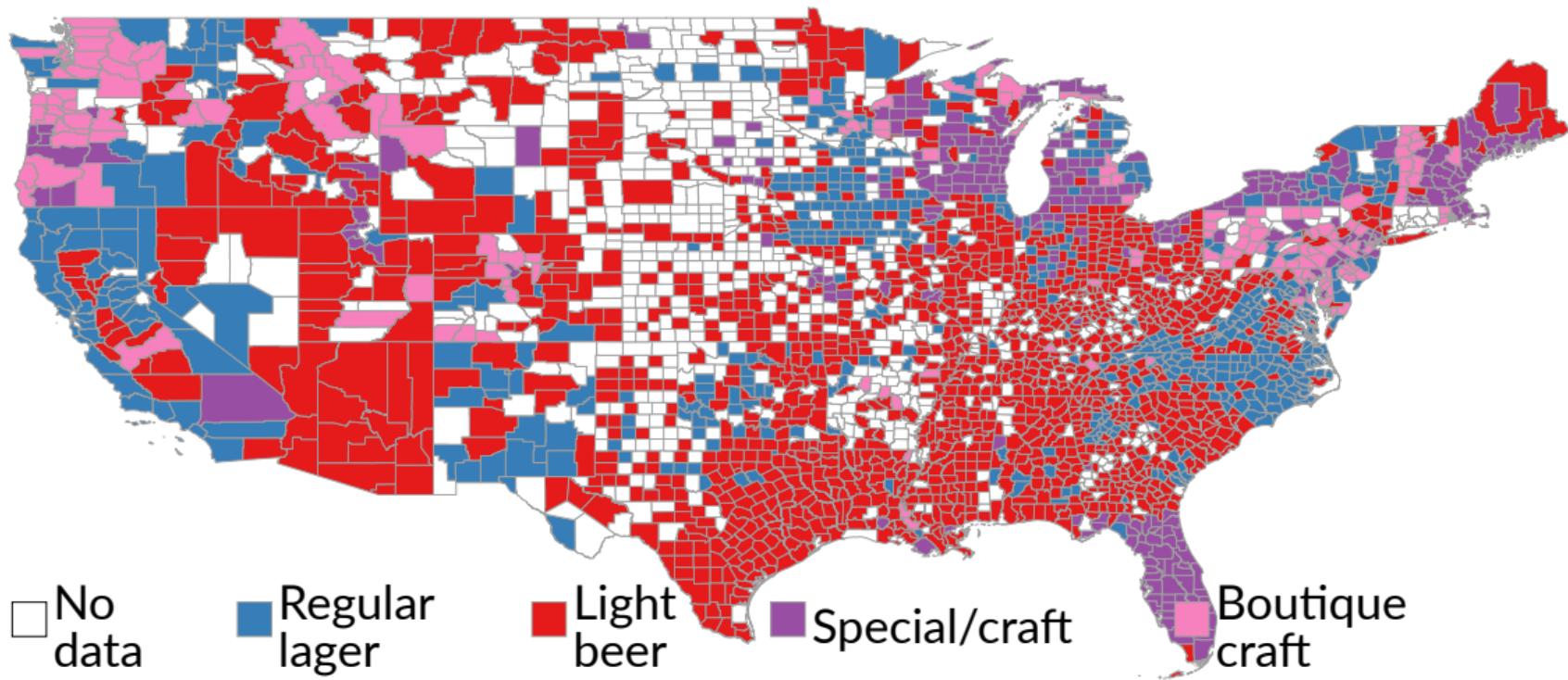
Boutique craft



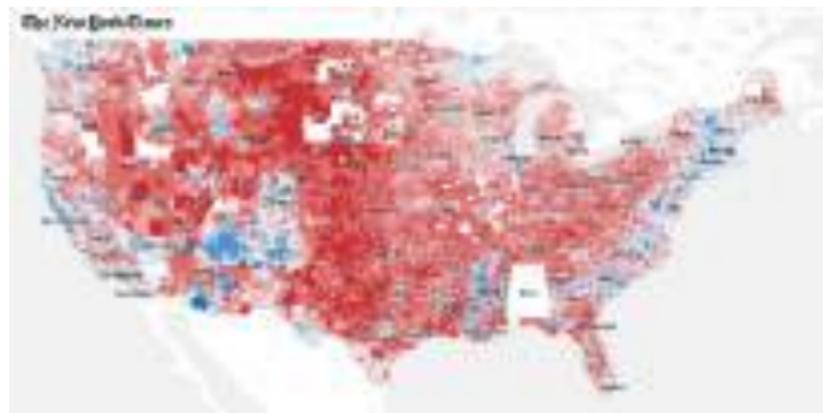
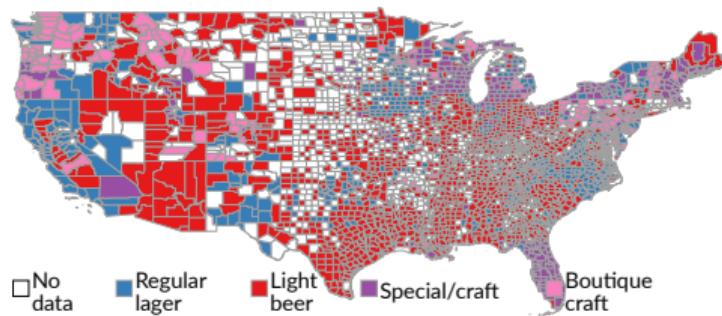
(.007,1]
(.003,.007]
(.002,.003]
(.001,.002]
(.0001]
[0,0]
No data

Most popular group (relative to national trends)

High degree of spatial correlation



Group popularity correlated with political outcomes!



Correlation between Light Beer share and 2024 Republican share is 0.39

Groups are capturing political preferences

This is not a mere coincidence: there are empirical studies on this!

The screenshot shows a journal article from the *Journal of Wine Economics*. The title of the article is "Alcohol Consumption and Political Ideology: What's Party Got to Do with It?" by Pavel A. Vaynshteyn and Walter P. Dornbusch. The article was published online by Cambridge University Press on 19 October 2013. The page includes a "Read" button, citation information (DOI 10.1017/jwe.2013.1), and download links for PDF, EPUB, and Mobi. There are also links for Rights & Permissions and a "Bookmark" option.

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Key takeaway: The Data-driven NL is also able to capture unobserved drivers of demand
(so long as they can be described or approximated by groups)

Group validation: Bud Light boycott

Testing nest substitution

In April 2023, Dylan Mulvaney—an actress, social media influencer, and transgender woman—releases a social media promotion of Bud Light



Group validation: Bud Light boycott

Testing nest substitution

The promotion sparked a backlash and calls for a boycott



Group validation: Bud Light boycott

Testing nest substitution

The boycott appeared to have some bite, and its effects were widely covered in the media



BBC News website article about the Bud Light boycott.

BBC BBC News

Dylan Mulvaney: Bud Light loses top spot in US after boycott

By Mark Hairston

A large Bud Light logo is shown with a thick red diagonal slash through it, set against a background of a cloudy sky.

Group validation: Bud Light boycott

Testing nest substitution

Use Bud Light boycott as an exogenous negative shock to demand for Bud Light

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Hypothesis: if consumers are substituting consumption to other beers

⇒ Demand for beers within the same nest as Bud Light ("Light Beer") should increase more relative to beers in other nests

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Empirical model

$$\log \mathbb{P}_{jmt} = \kappa_j + \eta_t + \sum_{k=1}^K \alpha_k \cdot \mathbb{1}\{\widehat{k}(j) = k\} \mathbb{1}\{t = 2023\} + \epsilon_{jmt}$$

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Because of the inclusion of UPC fixed effects κ_j

⇒ exploit variation (temporal changes) not used in the classification of groups

Group validation: DiD results

$$\log \mathbb{P}_{jmt} = \kappa_j + \eta_t + \sum_{k=1}^K \alpha_k \mathbb{1}\{\widehat{k}(j) = k\} \mathbb{1}\{t = 2023\} + \epsilon_{jmt}$$

	First stage		DiD		Placebo	
t=2023	-0.287***	(0.009)	-0.314***	(0.009)	-0.353***	(0.014)
Treatment (Budlight)	-0.156***	(0.054)				
Treatment same group			0.289***	(0.031)		
Regular beer \times t=2023					0.087***	(0.025)
Light beer \times t=2023					0.328***	(0.033)
Non-lager/craft \times t=2023					0.025	(0.024)
Boutique craft \times t=2023					0.132***	(0.032)
Placebo Regular beer \times t=2023						0.020
Placebo Light beer \times t=2023						0.020
Placebo Non-lager/craft \times t=2023						0.030
Placebo Boutique craft \times t=2023						0.007
Observations	219728		213959		208829	208829
UPC FE	YES		YES		YES	YES
Exclude Bud Light	NO		YES		YES	YES
Baseline group	No Bud Light		Other groups		Outside option	Outside option

Notes: *** p<0.01, ** p<0.05, * p<0.1

Outline

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 - 1.1 Identification
 - 1.2 Estimation
2. Practical aspects and extensions
 - 2.1 Choosing the number of groups
 - 2.2 Endogenous prices
 - 2.3 Introducing consumer heterogeneity
 - 2.4 Higher-level nesting structures
3. Consistency, statistical properties, and Monte Carlo
4. Empirical application: Demand for beer
 - 4.1 Data description and empirical model
 - 4.2 First-step estimation: Classification
 - 4.3 Second-step estimation: Structural parameters**
 - 4.4 Comparison with alternative empirical models

Second-step results: Data-driven Nested Logit structural parameters

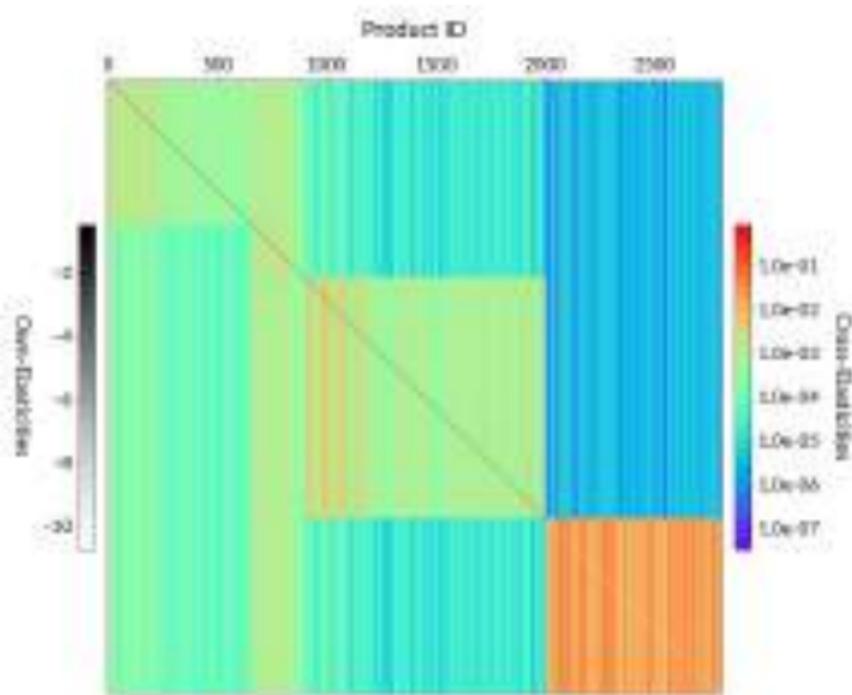
	OLS	IV
log unit size	0.070 (0.006)***	0.331 (0.025)***
log upc quantity	0.080 (0.005)***	0.432 (0.026)***
log price	-0.054 (0.006)***	-0.406 (0.033)***
$\sigma_{\text{regular/lager}}$	0.142 (0.001)***	0.520 (0.014)***
$\sigma_{\text{light beer}}$	0.343 (0.002)***	0.802 (0.016)***
σ_{craft}	-0.015 (0.001)***	0.333 (0.013)***
$\sigma_{\text{boutique craft}}$	-0.320 (0.001)***	0.038 (0.015)***
Mean Own-price Elasticity	1.357 (1.808)	-3.779 (4.414)
Mean Cross-price Elasticity	-0.001 (0.006)	0.004 (0.022)
Market Fixed Effects	✓	✓
IV Type		gh_1, gh_2
Number of Products		2806
Number of Observations		64497

Literature range of own-price elasticities is $[-6.3, -2.4]$

(Pinske and Slade, 2004; Slade, 2004; Romeo, 2016; Miller and Weinberg, 2017)

Data-driven Nested Logit matrix of elasticities

Rich heterogeneity in substitution patterns



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Naive Nested Logit: Using NielsenIQ “Beer Type” as Nests

	OLS	IV
log unit size	-0.085 (0.008)***	1.468 (0.066)***
log upc quantity	0.090 (0.006)***	1.433 (0.066)***
log price	-0.092 (0.007)***	-1.525 (0.082)***
$\sigma_{\text{regular/lager}}$	0.110 (0.001)***	0.611 (0.015)***
$\sigma_{\text{light beer}}$	0.121 (0.001)***	0.653 (0.015)***
σ_{ale}	0.006 (0.001)***	0.568 (0.017)***
$\sigma_{\text{stout/porter}}$	-0.255 (0.002)***	0.483 (0.025)***
$\sigma_{\text{malt liquor}}$	-0.415 (0.004)***	0.260 (0.026)***
$\sigma_{\text{cocktails}}$	-0.616 (0.008)***	0.197 (0.039)***
σ_{flavored}	-1.915 (0.020)***	-0.866 (0.087)***
$\sigma_{\text{consumer packaged}}$	-1.744 (0.021)***	-1.328 (0.085)***
$\sigma_{\text{combo pack}}$	-0.471 (0.004)***	0.189 (0.028)***
Mean Own-price Elasticity	-6.878 (7.354)	-2.778 (0.904)
Mean Cross-price Elasticity	0.004 (0.022)	0.001 (0.011)
Market Fixed Effects	✓	✓
IV Type		gh_1, gh_2
Number of Products	2806	
Number of Observations	64497	

Bud Light boycott: Comparing Data-driven vs. Naive Nested Logit

Data-driven Nested Logit “Beer Light” category predicts better out-of-sample substitution

	Data-driven NL		Naive NL		Combined
t=2023	-0.284*** (0.013)	-0.353*** (0.014)	-0.253*** (0.012)	-0.353*** (0.014)	-0.285*** (0.013)
Treatment same group	0.259*** (0.032)				
Regular beer × t=2023		0.087*** (0.025)			
Light beer × t=2023		0.327*** (0.033)			
Non-lager/craft × t=2023		0.024 (0.024)			
Boutique craft × t=2023		0.131*** (0.032)			
Naive light beer × t=2023			0.076** (0.036)	0.176*** (0.036)	
Naive ale × t=2023				0.047** (0.023)	
Naive regular beer × t=2023				0.198*** (0.024)	
Naive stout × t=2023				-0.028 (0.046)	
Only Data-driven Light Beer					0.252*** (0.037)
Only Naive Light Beer					0.010 (0.044)
Both Light Beer					0.280*** (0.057)
Observations	214353	209209	214353	209209	214353
UPC FE	YES	YES	YES	YES	YES
Exclude Bud Light	YES	YES	YES	YES	YES
Baseline group	Outside option				

*** p<0.01, ** p<0.05, * p<0.1

Mixed logit

We estimate the following mixed-logit model using PyBLP Conlon and Gortmaker (2020):

$$u_{ijm} = \delta_{jm} + \mu_{ijm} + \epsilon_{ijm},$$

where

$$\delta_{jm} = \beta_p \log \text{price}_{jm} + \beta_s \log \text{size}_j + \beta_q \log \text{quantity}_j + \eta_m + \xi_{jm},$$

and μ_{ijm} is drawn from

$$\mu_{ijm} = (\sigma_p \nu_i) \log p_{jm},$$

where σ_p governs the variance of the idiosyncratic component and $\nu_i \sim N(0, 1)$.

Mixed Logit

Structural estimates

Panel A: Linear Coefficients		β
log price		-3.790*** (0.091)
log unit size		3.086*** (0.063)
log unit quantity		3.232*** (0.063)
Panel B: Non-linear Coefficient		σ_p
log price		0.002*** (0.0003)
Own-price elasticity		-4.702 (2.875)
Cross-price elasticity		0.002 (0.005)
Market Fixed Effects		✓
IV Type		Gandhi-Houde
Number of Products		2806
Number of Observations		64497

Bud Light boycott comparison

	Data-driven Nested Logit		Naive Nested Logit		Mixed Logit	
t=2023	-0.284*** (0.013)	-0.353*** (0.014)	-0.253*** (0.012)	-0.353*** (0.014)	-0.255*** (0.032)	-0.232*** (0.015)
Treatment same group	0.259*** (0.032)					
Regular beer \times t=2023		0.087*** (0.025)				
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Naive regular beer \times t=2023				0.198*** (0.024)		
Naive stout \times t=2023				-0.028 (0.046)		
Above median $\mathcal{E}_{\text{Bud Light}}^i \times$ t=2023					0.013 (0.034)	
$\mathcal{E}_{\text{Bud Light}}^i \times$ t=2023						-4.315 (3.240)
Observations	214353	209209	214353	209209	214353	214353
UPC FE	YES	YES	YES	YES	YES	YES
Exclude Bud Light	YES	YES	YES	YES	YES	YES
Baseline group	Outside option	Outside option	Outside option	Outside option	Below median	Outside option

*** p<0.01, ** p<0.05, * p<0.1

Conclusions

Conclusions: Proposed a two-step estimator to estimate nesting structure

Showed conditions for consistency of estimator

Empirical application based on US beer market shows:

- Reasonable results and rich substitution patterns with **minimal data and computational requirements**
- Nests correlated with political preferences
 - Data-driven Nested Logit can capture unobservable factors of demand
 - To the best of our knowledge, no empirical IO paper considers politics as a key demand driver
- For this application, our method outperforms Naive Nested Logit and Mixed Logit in out-of-sample test

Thanks!

Appendix

Blue-bus/Red-bus problem

Implications of IIA property

A traveler has a choice of commuting by **car** or taking a **blue bus**

Blue-bus/Red-bus problem

Implications of IIA property

A traveler has a choice of commuting by **car** or taking a **blue bus**

Assume indirect utility from *car* and *blue bus* is the same so

$$P_{car} = P_{blue\ bus} = \frac{1}{2} \implies \frac{P_{car}}{P_{blue\ bus}} = 1$$

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Given IIA, we still have $\frac{P_{car}}{P_{blue\ bus}} = 1$. The only consistent model with both is

$$P_{car} = P_{blue\ bus} = P_{red\ bus} = \frac{1}{3}$$

Connection to IIA

$$\begin{aligned}\log \mathbb{P}_{jm} &= \frac{\beta_x x_{jm} + \beta_p p_{jm} + \xi_{jm}}{\sigma^{k(j)}} + \lambda_m^{k(j)} \\ &= \underbrace{\beta_x^{k(j)}}_{\frac{\beta_x}{\sigma^k}} x_{jm} + \underbrace{\beta_p^{k(j)}}_{\frac{\beta_p}{\sigma^k}} p_{jm} + \lambda_m^{k(j)} + \xi_{jm},\end{aligned}$$

Intuition:

- The marginal effect of variation in covariates varies by nest
- IIA within a nest: only j and j' covariates affect $\log \frac{\mathbb{P}_{jm}}{\mathbb{P}_{j'm}}$
- Not IIA across nests: If $k(j) \neq k(j')$, $\log \frac{\mathbb{P}_{jm}}{\mathbb{P}_{j'm}}$ also function of $\lambda_m^{k(j)}$ and $\lambda_m^{k(j')}$

One-step group estimation

Combine steps 1 + 2 by solving the following constrained problem:

$$\underset{\substack{k(1), \dots, k(J) \\ \beta^1, \dots, \beta^K, \lambda_1^1, \dots, \lambda_M^K \\ \beta, \sigma^1, \dots, \sigma^K}}{\arg \min} \sum_{m=1}^M \sum_{j=1}^J \left(\log \left(\frac{\mathbb{P}_{jm}}{\mathbb{P}_{0m}} \right) - (x_{jm} \beta^k + \lambda_m^k) \right)^2,$$

where

$$\beta^k = \frac{\beta}{\sigma^k} \quad \text{and} \quad \lambda_m^k = (\sigma^k - 1) \log \left(\sum_{d \in B_k} e^{\log \left(\frac{\mathbb{P}_{dm}}{\mathbb{P}_{0m}} \right) - \lambda_m^k} \right),$$

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but

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but

- No theory has been developed for consistency of non-linear models
- In practice it may work, but substantial computational burden due to non-linearities

Regularity conditions with unknown nests

Regularity conditions to ensure consistent classification:

- K fixed. Let $J \rightarrow \infty$, $M \rightarrow \infty$, and $\frac{\log M}{J} \rightarrow 0$ Why $M \rightarrow \infty$?
- Variation in x_{jm}, z_{jm} conditional on groups

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- K fixed. Let $J \rightarrow \infty$, $M \rightarrow \infty$, and $\frac{\log M}{J} \rightarrow 0$ Why $M \rightarrow \infty$?
- Variation in x_{jm}, z_{jm} conditional on groups
- Exogeneity of ξ_{jm} :

$$\mathbb{E}[\xi_{jm}, v_{jm} | x_{jm}, z_{jm}] = 0$$

- Thin tails
- $\lambda_m^{k(j)}$ is a random variable that diverges when $J \rightarrow \infty$. Conditions so they are well behaved:
 - Balanced nests: $|B_k| = O_p(J)$ for $k = 1, \dots, K$
 - Normalization by $\log J$ to stay in compact space + conditions so that $\log J \lambda_m^k(J)$ is well-defined
 - Sequence $(\sigma_{01,J}, \dots, \sigma_{0K,J})_{J=1}^\infty$ such that $\lambda_m^{k(j)}$ is defined when J grows
- Group separation: cannot have $\beta^k = \beta^{k'}$ and $\lambda_m^k = \lambda_m^{k'}$ for all m for some $k \neq k'$

Regularity conditions with unknown nests

Regularity conditions to ensure consistent classification:

- K fixed. Let $J \rightarrow \infty$, $M \rightarrow \infty$, and $\frac{\log M}{J} \rightarrow 0$ Why $M \rightarrow \infty$?
- Variation in x_{jm}, z_{jm} conditional on groups
- Exogeneity of ξ_{jm} :

$$\mathbb{E}[\xi_{jm}, v_{jm} | x_{jm}, z_{jm}] = 0$$

- Thin tails
- $\lambda_m^{k(j)}$ is a random variable that diverges when $J \rightarrow \infty$. Conditions so they are well behaved:
 - Balanced nests: $|B_k| = O_p(J)$ for $k = 1, \dots, K$
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Why asymptotics on M ?

Graphical (literal) toy example: playing in the dark



- You can't see
- Different shapes are placed in different urns
- Every minute m , you can learn by touching an unknown shape from an urn for 5 seconds
- Your goal is to get as many shapes through the box as possible
- The more minutes m , the more you learn!

Why asymptotics on M ?

Simplified example

- Consider the following simplified model with $G = 2$:

$$y_{jm} = \alpha_{k_j^*}^* + \xi_{jm}, \quad k_j \in \{1, 2\}.$$

- We characterize the misclassification probability:

$$\Pr(\widehat{k}_j(\alpha) = 2 | k_j^* = 1) = \Pr((\bar{y}_j - \alpha_2)^2 < (\bar{y}_j - \alpha_1)^2 | k_j^* = 1).$$

- If ξ_{jm} are iid normal $(0, \sigma^2)$ and $\alpha_1 < \alpha_2$ then this is:

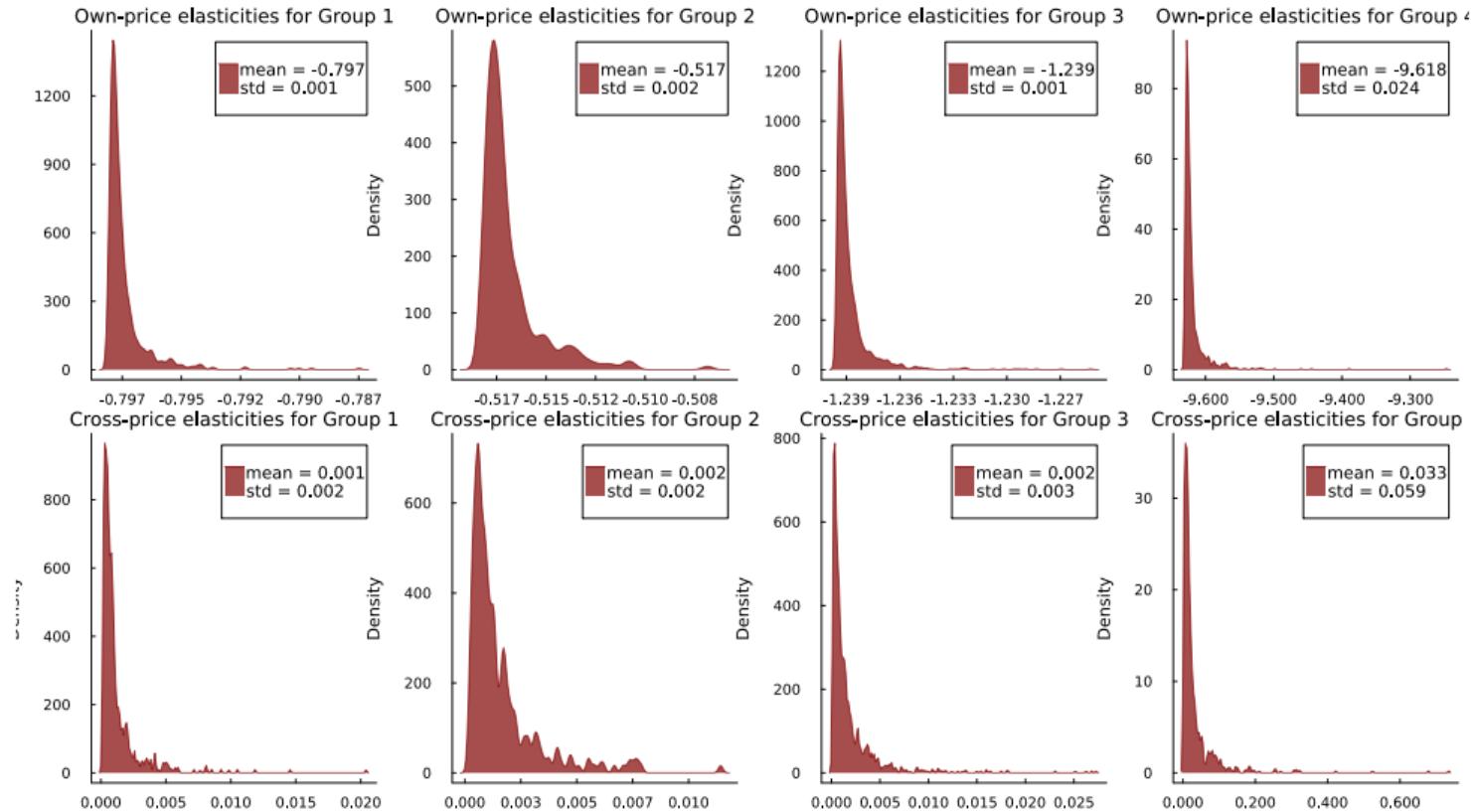
$$\Pr\left(\bar{\xi}_j > \frac{\alpha_1 + \alpha_2}{2} - \alpha_1^*\right) = 1 - \Phi\left(\frac{\sqrt{M}}{\sigma} \left(\frac{\alpha_1 + \alpha_2}{2} - \alpha_1^*\right)\right),$$

which vanishes exponentially fast as M increases.

- **Intuition:** When M grows
 - Mean \bar{y}_j converges to α_1
 - If mis-classified, then error $\bar{y}_j - \alpha_2$ eventually should become larger than $\bar{y}_j - \alpha_1$
 - Every m is a chance to learn: With enough opportunities, we'll eventually learn the truth!

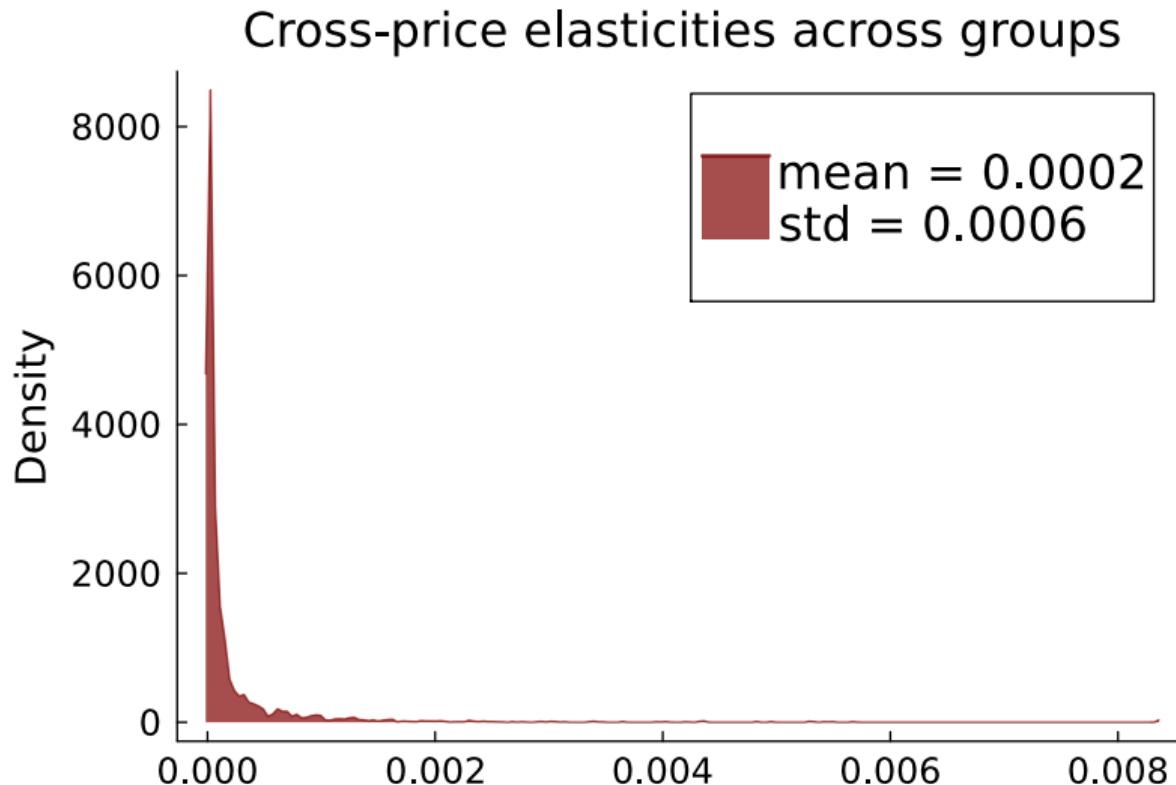
Substitution patterns: Within-group price elasticities

Back



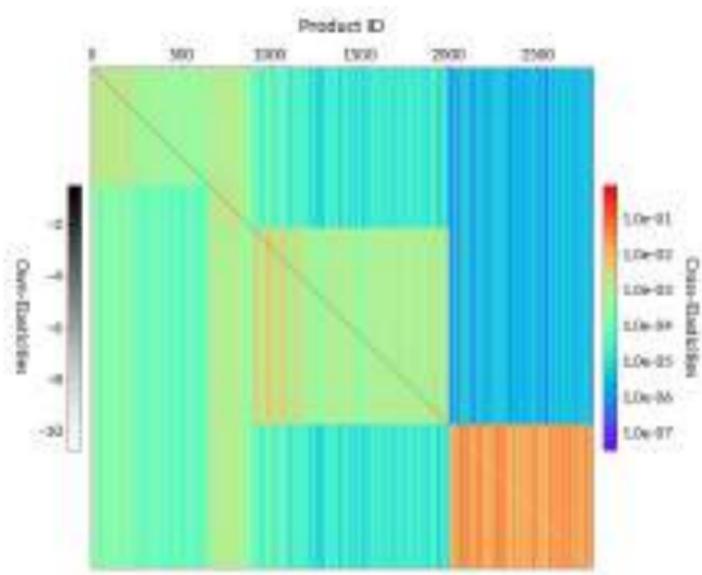
Substitution Patterns: Cross-group Cross-price Elasticities

Back

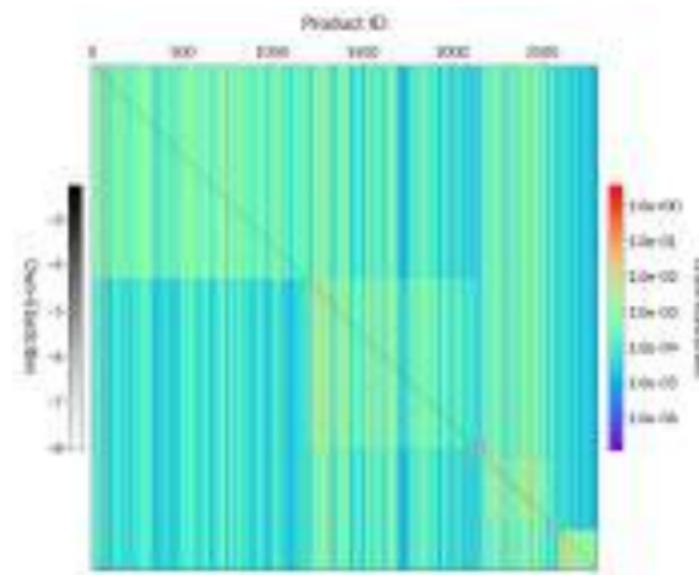


Comparison: Matrix of price elasticities

Data-driven NL captures more heterogeneity, while ML resembles Logit patterns



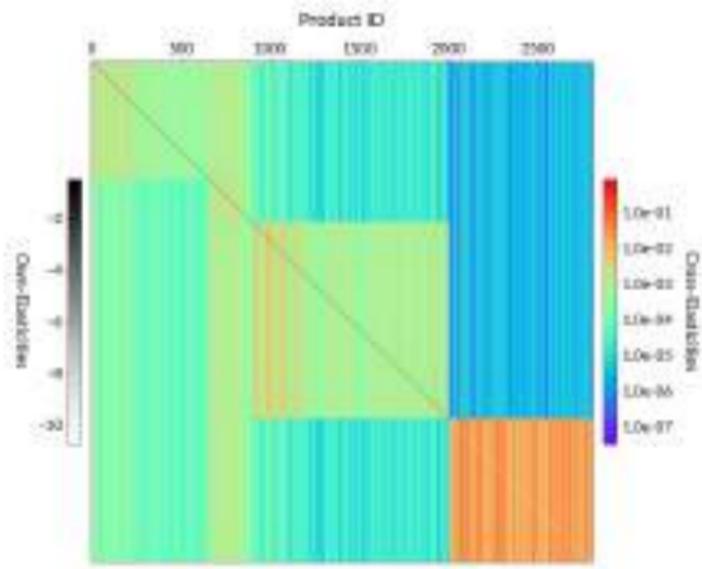
(a) Nested logit ($k^* = 4$)



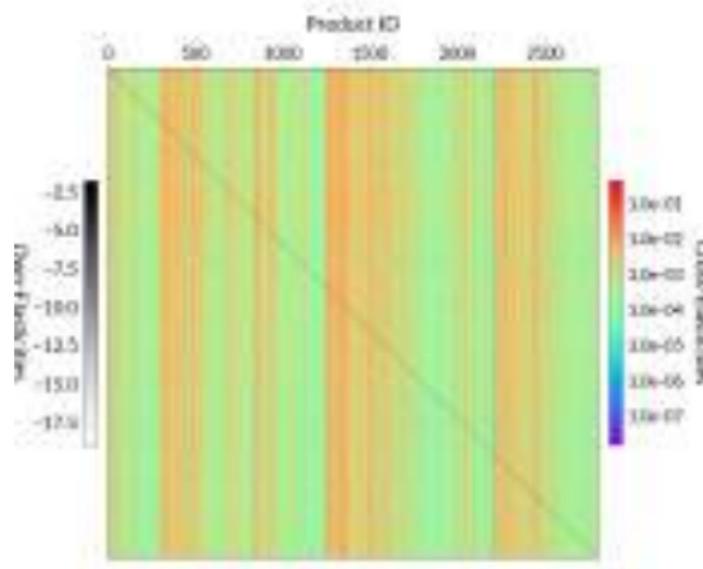
(b) Naive Nested Logit

Comparison: Matrix of price elasticities

Data-driven NL captures more heterogeneity, while ML resembles Logit patterns



(a) Nested logit ($k^* = 4$)



(b) Mixed Logit

Mixed Logit

Bud Light boycott DiD results

$$\log \mathbb{P}_{jmt} = \kappa_j + \eta_t + f(\text{elasticity}_{BudLight}^j) \mathbb{1}\{t = 2023\} + \epsilon_{jmt}$$

	Continuous		DiD			
$\mathbb{1}\{t = 2023\}$	-0.232***	(0.015)	-0.224***	(0.016)	-0.247***	(0.013)
ML elasticity $\mathbb{1}\{t = 2023\}$	-4.315	(3.240)	-0.041*	(0.023)	0.013	(0.027)
ML elasticity above median $\mathbb{1}\{t = 2023\}$						
ML elasticity top quartile $\mathbb{1}\{t = 2023\}$					0.181***	(0.027)
ML elasticity quartile 1 $\mathbb{1}\{t = 2023\}$					0.078***	(0.027)
ML elasticity quartile 2 $\mathbb{1}\{t = 2023\}$					0.058**	(0.027)
ML elasticity quartile 3 $\mathbb{1}\{t = 2023\}$					0.119***	(0.027)
ML elasticity quartile 4 $\mathbb{1}\{t = 2023\}$						
Observations	120265		213959		213959	
UPC FE	YES		YES		YES	
Exclude Bud Light	YES		YES		YES	
Baseline group	Outside option		Below median		Other quartiles	

*** p<0.01, ** p<0.05, * p<0.1