Lecturer : Ayesha

Applied Physics

Topic : Kirchhoff's Law

- Krichhoff's laws are very helpful tool for circuit analysis.
- These laws are used to find current and voltage in complex electrical circuits.

Types of Kirchhoff's Law:

There are two types of Kirchhoff's law:

1. Kirchhoff's Current Law (KCL)
2. Krichhoff's Voltage law (KVL)

Kirchhoff's Current Law (KCL)

KCL States that

"Algebraic sum of current at node or junction is zero"

$$\Sigma I = 0$$

OR

"Current flowing into a node must be equal to current flowing out of it."

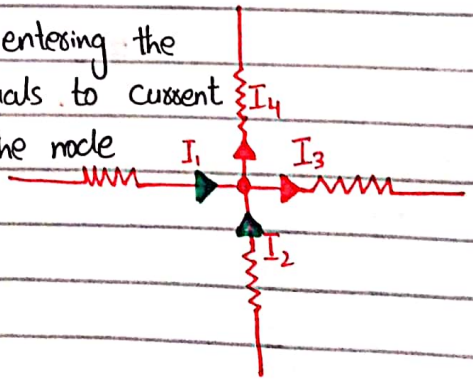
$$I(enter) + I(exit) = 0$$

- This law is based on Conservation of charges.
- KCL is also known as Kirchhoff's 1st law, junction or current law.

- From given diagram

$$I_1 + I_2 = I_3 + I_4$$

- Current entering the node equals to current leaving the node



- we consider, current directed towards a point as Positive and directed away from point as Negative.
- So, according to diagram current directed towards is I_1 and I_2 and current directed away from point is I_3 and I_4

Thus,

$$I_1 + I_2 + (-I_3) + (-I_4) = 0$$

which is same as;

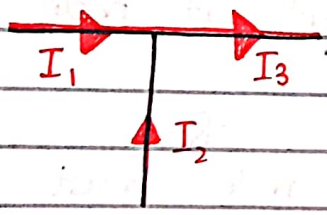
$$I_1 + I_2 = I_3 + I_4$$

$$\downarrow \qquad \downarrow$$

$$\Sigma I_{in} = \Sigma I_{out}$$

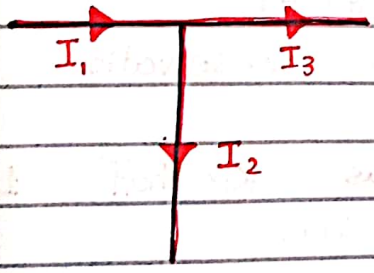
- From Kirchoff's law,

1)



$$I_1 + I_2 = I_3$$

2)



$$I_1 = I_2 + I_3$$

Applications of Kirchoff's Current Law.

- KCL is useful tool for solving complex circuits.
- It is possible to determine unknown value of resistance, current and voltage.
- KCL is useful in understanding the transfer of energy in electric circuit.
- KCL is applicable to almost every circuits, but it can not used for high-frequency circuits.

Kirchhoff's Voltage Law (KVL)

KVL states that;

"Total voltage around any closed loop in an electrical circuit is equal to zero at any instant of time."

- KVL is based on principle of conservation of energy, which states that energy cannot be created or destroyed, only converted from one form to another.
- KVL is applicable to both AC and DC circuits.
- This law is also called voltage rule or loop theorem

That is, $\sum V = 0$

KVL States that;

"Algebric sum of all the voltage and emf in a closed loop is equal to zero."

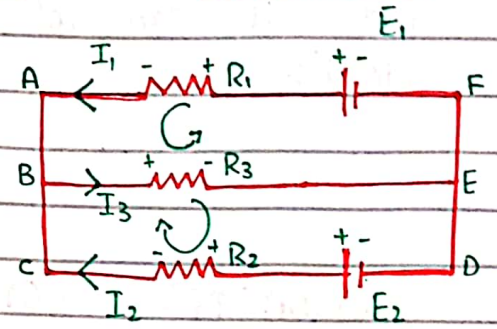
$$\sum E + \sum V = 0$$

As we know

$$V = IR$$

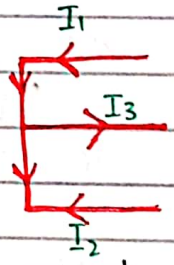
$$\boxed{\sum E + \sum IR = 0} \quad \text{or} \quad \boxed{\sum E = \sum IR}$$

Apply KVL, In loop ABEFA



$$+I_1R_1 + I_3R_3 - E_1 = 0 \longrightarrow (1)$$

By applying KCL at junction B



$$I_1 + I_3 - I_3 = 0$$
$$I_1 + I_2 = I_3$$

Now, by putting value of I_3 in equation (1)

$$+I_1R_1 + (I_1 + I_2)R_3 - E_1 = 0 \quad -(2)$$

Apply KVL In loop CBEDC

$$+I_2R_2 + I_3R_3 - E_2 = 0 \longrightarrow (3)$$

put value of I_3 in equation (3)

$$+I_2R_2 + (I_1 + I_2)R_3 - E_2 = 0 \quad -(4)$$

Subtracting equation 2 and 4

$$I_1R_1 + (I_1 + I_2)R_3 - E_1 = 0$$
$$I_2R_2 + (I_1 + I_2)R_3 - E_2 = 0$$

$$I_1R_1 - I_2R_2 - E_1 + E_2 = 0$$

$$(I_1R_1 - I_2R_2) + (E_2 - E_1) = 0$$

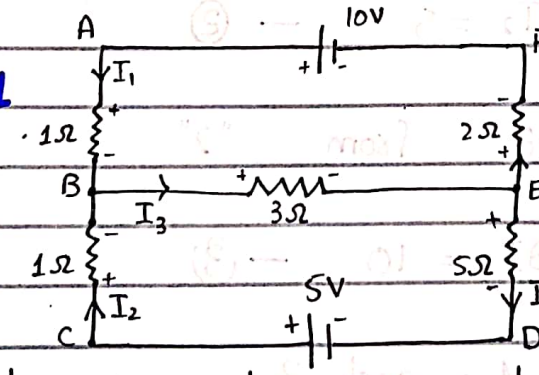
$$\Sigma IR + \Sigma E = 0$$

$$\Sigma V + \Sigma E = 0$$

- KVL is based on principle of conservation of energy, which states that energy cannot be created or destroyed, only converted from one from to another.
- KVL is applicable to both AC and Dc circuits.
- This law is also called voltage rule or loop theorem.

Numericals:

Problem: 01



Given:

$I_1 = ?$ $R_3 = 3\Omega$

$I_2 = ?$ $E_1 = 10V$

$I_3 = ?$ $E_2 = 5V$

$R_1 = 1\Omega$

$R_2 = 1\Omega$

Calculate current in each branch.

Sol:

Apply KVL in closed loop ABEFA

$$I_1R_1 + I_3R_3 + I_1R_2 - E_1 = 0$$

$1I_1 + 3I_3 + 2I_1 - 10 = 0$

$1I_1 + 3(I_1 + I_2) + 2I_1 = 10$

$I_1 + 3I_1 + 3I_2 + 2I_1 = 10$

$6I_1 + 3I_2 = 10 \quad \longrightarrow (1)$

Apply KVL in closed loop CBEDC

$I_2R_2 + I_3R_3 + I_2R_2 - E_2 = 0$

$I_2 + 3I_3 + 5I_2 - 5 = 0$

$I_2 + 3(I_1 + I_2) + 5I_2 = 5$

$I_2 + 3I_1 + 3I_2 + 5I_2 = 5$

$3I_1 + 9I_2 = 5 \quad - (2)$

Multiply eq 2 from "2"

$6I_1 + 18I_2 = 10 \quad - (3)$

Subtract eq 1 and 3

$$\begin{array}{r} 6I_1 + 3I_2 = 10 \\ -\quad 6I_1 + 18I_2 = 10 \\ \hline -15I_2 = 0 \end{array}$$

$I_2 = 0/-15 = 0$

$\boxed{I_2 = 0}$

Put value of I_2 in eq (1)

$$6I_1 + 3I_2 = 10$$

$$6I_1 + 3 \times 0 = 10$$

$$6I_1 = 10 \Rightarrow I_1 = 10/6 = 5/3$$

$$\boxed{I_1 = 1.66}$$

Now we have to find I_3

As we know

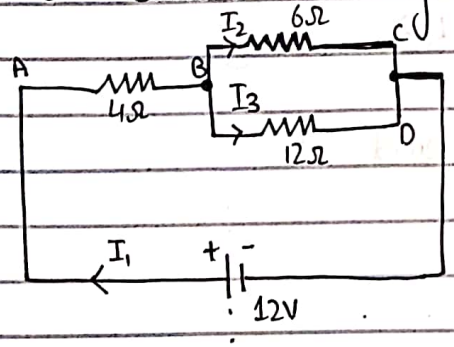
$$I_1 + I_2 = I_3$$

$$1.66 + 0 = I_3$$

$$\boxed{I_3 = 1.66}$$

Problem : 02

Find current flowing in circuit using kirchoff's



Given :

$R_1 = 4\Omega$

$R_2 = 6\Omega$

$R_3 = 12\Omega$

$E = 12V$

$I_1R_1 + I_2R_2 - E = 0$

$4I_1 + 6I_2 - 12 = 0$

As we know:

$4(I_2 + I_3) + 6I_2 = 12$

$4I_2 + 4I_3 + 6I_2 = 12$

$10I_2 + 4I_3 = 12 \quad -①$

Consider loop ABD

$I_1R_1 + I_3R_3 - E = 0$

$4I_1 + I_3 12 - 12 = 0$

$4(I_2+I_3) + I_3 12 = 12$

$4I_2 + 4I_3 + 12I_3 = 12$

$4I_2 + 16I_3 = 12 \quad -②$

Multiply equation 1 by "4"

$40I_2 + 16I_3 = 48 \quad -③$

Subtract equation 2 and 3

$48 = 40I_2 + 16I_3$

$12 = 4I2 + 16I3$

$36 = 36I_2$

$I_2 = 36/36 \Rightarrow$ **$1.0\ A = I_2$**

Put value of I_2 in eq ②

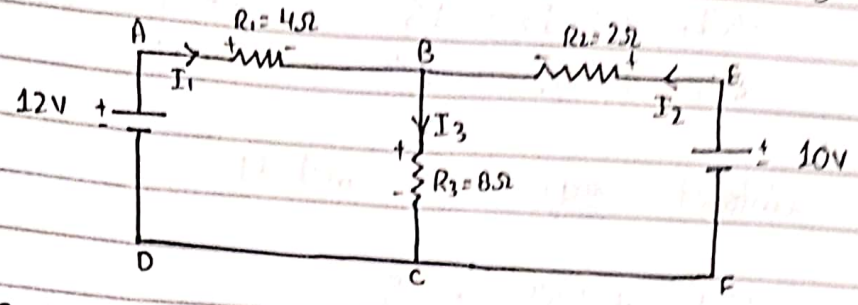
$4(1) + 16I_3 = 12 \Rightarrow 4 + 16I_3 = 12 \Rightarrow 16I_3 = 8$

$I_3 = 1/2 \Rightarrow$ **$0.5 = I_3$**

$I_1 = I_2 + I_3 \Rightarrow 1.0 + 0.5 \Rightarrow$ **$1.5 = I_1$**

Problem 03

Using Kirchhoff's law calculate current:



Solution:

Given data:

$R_1 = 4\Omega$ $R_2 = 2\Omega$ $R_3 = 8\Omega$

$E_1 = 12$ $E_2 = 10V$

To Find:

$I_1 = ?$ $I_2 = ?$ $I_3 = ?$

Consider loop ABCDA then by using KVL

$$I_1R_1 + I_3R_3 - E_1 = 0$$
$$4I_1 + I_3 8 - 12 = 0$$

where $I_3 = I_1 + I_2$

$$4I_1 + (I_1 + I_2)8 - 12 = 0$$
$$4I_1 + 8I_1 + 8I_2 = 12$$
$$12I_1 + 8I_2 = 12$$
$$3I_1 + 2I_2 = 3 \quad -(1)$$

Consider loop EBCFE

$$I_2R_2 + I_3R_3 - E_2 = 0$$
$$I_2 2 + (I_1 + I_2)8 - 10 = 0$$
$$2I_2 + 8I_1 + 8I_2 - 10 = 0$$
$$10I_2 + 8I_1 = 10$$
$$5I_2 + 4I_1 = 5$$
$$4I_1 + 5I_2 = 5 \quad -(2)$$

Multiply eq 1 by "5" and eq 2 by "2"

$15I_1 + 10I_2 = 15 \quad -3$

$8I_1 + 10I_2 = 10 \quad -4$

Subtract eq 3 and 4

$$\begin{array}{r} 15I_1 + 10I_2 = 15 \\ 8I_1 + 10I_2 = 10 \\ \hline 7I_1 = 5 \end{array}$$

$I_1 = 5/7 \Rightarrow \boxed{I_1 = 0.714 \text{ A}}$

Now put value of I_1 in eq ②

$4I_1 + 5I_2 = 5 \Rightarrow 4(0.714) + 5I_2 = 5$

$2.856 + 5I_2 = 5$

$5I_2 = 2.144$

$I_2 = 2.144/5 \Rightarrow \boxed{I_2 = 0.428 \text{ A}}$

$I_3 = I_1 + I_2 \Rightarrow \boxed{1.143 = I_3}$

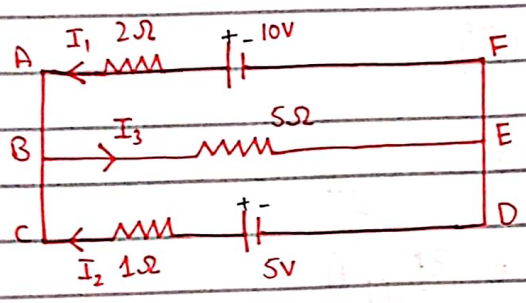
$V_1 = I_1R_1 \Rightarrow 0.714 \times 4 \Rightarrow V_1 = 2.856 \text{ V}$

$V_2 = I_2R_2 \Rightarrow 0.428 \times 2 \Rightarrow V_2 = 0.858 \text{ V}$

$V_3 = I_3R_3 \Rightarrow 1.143 \times 8 \Rightarrow V_3 = 9.144 \text{V}$

Problem 04.

Two batteries having emf 10V and 5V internal resistance are 2Ω and 1Ω respectively are connected in parallel in such as way that they send current in external resistance of 5Ω in the same direction.
Calculate current in each branch and potential difference across 5Ω resistance.



Apply KVL in closed loop ABEFA

$$I_1R_1 + I_3R_3 - E_1 = 0$$
$$2I_1 + (I_1+I_2)5 - 10 = 0$$
$$2I_1 + 5I_1 + 5I_2 = 10$$
$$7I_1 + 5I_2 = 10 \quad -(1)$$

Apply KVL in closed loop CBEDC

$$I_2R_2 + I_3R_3 - E_2 = 0$$
$$1I_2 + (I_1+I_2)5 - 5 = 0$$
$$1I_2 + 5I_1 + 5I_2 - 5 = 0$$
$$6I_2 + 5I_1 = 5 \quad -(2)$$

Multiply eq 1 by "5" and eq 2 by "7" we get;

$$35I_1 + 25I_2 = 50 \quad -(3)$$
$$35I_1 + 42I_2 = 35 \quad -(4)$$

Now subtract eq 3 and 4

$$35I_1 + 25I_2 = 50$$
$$35I_1 + 42I_2 = 35$$
$$-17I_2 = 15$$

$$\boxed{I_2 = -15/17\ A}$$

Put I_2 in eq ①

$$7I_1 + 5(-15/17) = 10$$

$$7I_1 - \frac{75}{17} = 10$$

$$7I_1 = 10 + \frac{75}{17}$$

$$7I_1 = \frac{170 + 75}{17} = \frac{245}{17}$$

$$I_1 = \frac{245}{17 \times 7} = 35/17\ A$$

$$\boxed{I_1 = 35/17\ A}$$

$$I_3 = I_1 + I_2 \Rightarrow \frac{35}{17} + \left(\frac{-15}{17}\right) \Rightarrow \frac{35}{17} - \frac{15}{17}$$

$$\frac{35-15}{17} = \boxed{\frac{20}{17}\ A = I_3}$$

Potential across 5Ω resistance

$$V = IR$$
$$V_3 = I_3 R_3$$
$$= \frac{20}{17} \times 5 \Rightarrow \frac{100}{17}\ V$$

$$\boxed{V_3 = 5.88\ V}$$

If we have to find only potential across 5Ω resistance.

$$V = \frac{\Sigma V/R}{\Sigma 1/R}$$

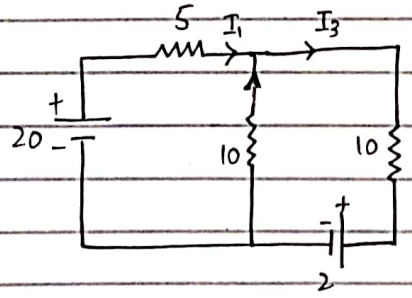
$$= \frac{10/2 + 5/1}{1/2 + 1/5 + 1/1} = \frac{10}{\frac{5+2+10}{10}}$$

$$= \frac{10}{17/10} = \frac{10 \times 10}{17} \Rightarrow \frac{100\ V}{17}$$

$$\boxed{V = 5.88V}$$

Problem:

In circuit E_1 and E_2, R_1, R_2 and R_3 are given. Find I_1, I_2 and I_3.



Problem: Find I_1, I_2, I_3