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1 Značení

$\Omega \subset \mathbb{R}^n$ omezená oblast

$\partial\Omega$ Liepschitzovská oblast - lze pokrýt konečně mnoha grafy Liepschitzovských zobrazení $(|\phi(x) - \phi(y)| < L||x - y||)$

$\bar{\Omega}$ - uzávěr oblasti Ω

$\mathcal{C}^{(k)}(\Omega)$ spojité funkce s derivacemi do k-tého řádu spojité na Ω

$\mathcal{C}^{(\infty)}(\Omega)$ hladké funkce na Ω

$f \in k, \frac{\partial^{|\alpha|}}{\partial^{\alpha_1} x_1 \partial^{\alpha_2} x_2 ... \partial^{\alpha_n} x_n} f = \mathcal{D}^{\alpha}, \alpha = [\alpha_1, \alpha_2, ..., \alpha_n], |\alpha| = \sum_i \alpha_i$

Lebesgueovy prostory $L_p(\Omega) = \{f : \Omega \mapsto \mathbb{R} | \text{měřitelné}, \int_\Omega |f(x)|^p dx < +\infty\}$ $p \in < 1, +\infty)$

$L_\infty(\Omega) = \{f : \Omega \mapsto \mathbb{R} | \text{měřitelné}, (\exists K > 0)(s.v.x \in \Omega)(|f(x)| \leq K)\}$

2 Vztahy

Minkowskeho nerovnost

$$\left(\int_\Omega |f(x) + g(x)|^p dx\right)^{\frac{1}{p}} \leq \left(\int_\Omega |f(x)|^p dx\right)^{\frac{1}{p}} + \left(\int_\Omega |g(x)|^p dx\right)^{\frac{1}{p}} \tag{1}$$

Holderova nerovnost $\left(\frac{1}{p} + \frac{1}{q} = 1\right), f \in L_p(\Omega), g \in L_q(\Omega)$

$$\int_\Omega |f(x)g(x)|\, dx \leq \left(\int_\Omega |f(x)|^p dx\right)^{\frac{1}{p}} * \left(\int_\Omega |g(x)|^q dx\right)^{\frac{1}{q}} \tag{2}$$

$p_2 > p_1 \geq 1 \implies L_{p_2}(\Omega) \subset L_{p_1}(\Omega)$

$$\int_\Omega |f(x)|^{p_1} dx = \int_\Omega |f(x)|^{p_1} * 1 dx \leq \text{Holder } (\textbf{??}) \leq \left(\int_\Omega (|f(x)|^{p_1})^{\frac{p_1}{p_2}}\right)^{\frac{p_1}{p_2}} * \left(\int_\Omega 1^q dx\right)^{\frac{1}{q}} = \left(\int_\Omega |f(x)|^{p_2}\right)^{\frac{p_1}{p_2}} * |\Omega \tag{3}$$