Let's assume

$$p_n = \frac{2^{a_n} + 3 \times 5 \times \cdots \times p_{n-1}}{k_n}$$

then

$$\begin{aligned} p_{n+1} &= \frac{2^{a_{n+1}} + 3 \times 5 \times \cdots \times p_n}{k_{n+1}} \\ &= \frac{2^{a_{n+1}} k_n + 3 \times 5 \times \cdots \times p_n k_n}{k_n k_{n+1}} \\ &= \frac{2^{a_{n+1}} k_n + 3 \times 5 \times \cdots \times p_{n-1} \times (2^{a_n} + 3 \times 5 \times \cdots \times p_{n-1})}{k_n k_{n+1}} \\ &= \frac{2^{a_{n+1}} k_n + 2^{a_n} \times 3 \times 5 \times \cdots \times p_{n-1} + 3^2 \times 5^2 \times \cdots \times p_{n-1}^2}{k_n k_{n+1}} \end{aligned}$$

hence

$$\begin{aligned} & p_{n+1} - p_n \\ &= \frac{2^{a_{n+1}} k_n + 2^{a_n} \times 3 \times 5 \times \cdots \times p_{n-1} + 3^2 \times 5^2 \times \cdots \times p_{n-1}^2}{k_n k_{n+1}} \\ &- \frac{2^{a_n} + 3 \times 5 \times \cdots \times p_{n-1}}{k_n} \\ &= \frac{2^{a_{n+1}} k_n + 2^{a_n} \times 3 \times 5 \times \cdots \times p_{n-1} + 3^2 \times 5^2 \times \cdots \times p_{n-1}^2}{k_n k_{n+1}} \\ &- \frac{2^{a_n} k_{n+1} + 3 \times 5 \times \cdots \times p_{n-1} k_{n+1}}{k_n k_{n+1}} \\ &= \frac{1}{k_n k_{n+1}} [2^{a_{n+1}} k_n + 2^{a_n} \times 3 \times 5 \times \cdots \times p_{n-1} + 3^2 \times 5^2 \times \cdots \times p_{n-1}^2 \\ &- 2^{a_n} k_{n+1} - 3 \times 5 \times \cdots \times p_{n-1} k_{n+1}] \end{aligned}$$

hence

$$2 | 2^{a_{n+1}} k_n + 2^{a_n} \times 3 \times 5 \times \cdots \times p_{n-1} - 2^{a_n} k_{n+1}$$