In the name of beauty

The 5th problem set solution of Optical Networks course

Question 1)

a. The RRC filter bandwidth is given by $(1+\beta)R_s$ where β is the roll-off factor and R_s is symbol rate. A simple substitution yields 36Gbaud.

b. To sample a RRC filter with bandwidth 36G, we need to sample every 1/36 nsec (f_{Sampling}=36G) to avoid aliasing.

c. Denote the uncoded bit stream by b_n. When passing through the **Bit-to symbol converter**, the stream turns into s_n which is a sequence of symbols, e.g. each of which chosen from the alphabet $\{-1-j, -1+j, 1-j, 1+j\}$, hence the output becomes

$$\sum_{n=-\infty}^{\infty} b_n \delta(t-nT_s)$$

which has an inphase component of

$$\sum_{n=-\infty}^{\infty} \mathfrak{R}\{b_n\}\delta(t-nT_s)$$

and a quadrature component of

$$\sum_{n=-\infty}^{\infty} \mathfrak{I}\{b_n\}\delta(t-nT_s)$$

The upsampler is a system block for discretization considerations, and can be ignored since the discussion is fully taken place in analog regime. This leads to

$$S(t) = \sum_{n=-\infty}^{\infty} \mathfrak{R}\{b_n\} s_{\text{RRC}}(t-nT_s)$$

where $s_{\text{RRC}}(t)$ is the RRC pulse shape in time domain.

The signal $S(t)$ is once multiplied in cos carrier and once in sin carrier to give the Tx. midband output signal as

$$\begin{aligned} x(t) = {} & 2\sum_{n=-\infty}^{\infty} \mathfrak{R}\{b_n\} s_{\text{RRC}}(t-nT_s)\cos 2\pi f_c t \\ & -2\sum_{n=-\infty}^{\infty} \mathfrak{I}\{b_n\} s_{\text{RRC}}(t-nT_s)\sin 2\pi f_c t \end{aligned}$$

which is directly input to Rx. block diagram based on back-to-back connection. The upper and lower branch signals after multiplying in cos and $-\sin$ become

$$
\begin{aligned}
x(t)\cos 2\pi f_c t &= 2\sum_{n=-\infty}^{\infty} \Re\{b_n\} s_{\mathrm{RRC}}(t-nT_s)\cos^2 2\pi f_c t \\
&\quad - 2\sum_{n=-\infty}^{\infty} \Im\{b_n\} s_{\mathrm{RRC}}(t-nT_s)\sin 2\pi f_c t\cos 2\pi f_c t \\
&= \sum_{n=-\infty}^{\infty} \Re\{b_n\} s_{\mathrm{RRC}}(t-nT_s)(\cos 4\pi f_c t - 1) \\
&\quad - \sum_{n=-\infty}^{\infty} \Im\{b_n\} s_{\mathrm{RRC}}(t-nT_s)\sin 4\pi f_c t
\end{aligned}
$$

and

$$
\begin{aligned}
x(t)\sin 2\pi f_c t &= \sum_{n=-\infty}^{\infty} \Re\{b_n\} s_{\mathrm{RRC}}(t-nT_s)\sin 4\pi f_c t \\
&\quad - \sum_{n=-\infty}^{\infty} \Im\{b_n\} s_{\mathrm{RRC}}(t-nT_s)(1-\cos 4\pi f_c t)
\end{aligned}
$$

The $\cos 4\pi f_c t$ and $\sin 4\pi f_c t$ are modulated by a baseband signal to midband frequency $2f_c$ and are not passed through the ideal low-pass filter. Hence the output of the filter is

$$
\text{Output of the LPF} = \sum_{n=-\infty}^{\infty} \Re\{b_n\} s_{\mathrm{RRC}}(t-nT_s)
$$

which turns into

$$
Y(t) = \sum_{n=-\infty}^{\infty} \Re\{b_n\} s_{\mathrm{RC}}(t-nT_s)
$$

after passing through the RRC matched filter (RRC pulse shape) block since

$$
s_{\mathrm{RRC}}(t) \star s_{\mathrm{RRC}}(t) = s_{\mathrm{RC}}(t)
$$

d. The above reasoning makes it clear as to why LPF is needed. We need LPF to extract the baseband signal out from a total received, midband signal since our pure information lays there.

Question 2)

a. We have 4 inner points of power $1^2+1^2=2$, 8 points with power $1^2+3^2=10$ and 4 outer points with equal power $3^2+3^2=18$, all of which equally likely. Hence the average power is

$$
\frac{1}{16}(4\times 2 + 8\times 10 + 4\times 18) = 10
$$

b. The desired average power is assumed 6.8, which is

$$2P_1 + 10P_2 + 10P_3 + 18P_4 = 6.8$$

or equivalently

$$24P_3 + 18P_4 = 6.8$$

also

$$P_1 + P_2 + P_3 + P_4 = 1 \implies 4P_3 + P_4 = 1$$

with the following solution

$$\begin{aligned} P_1 &= 0.47 \\ P_2 &= 0.23 \\ P_3 &= 0.23 \\ P_4 &= 0.07 \end{aligned}$$

Question 3)

a. Each 4 bits of an input stream are mapped to a symbol in 16QAM, leading to a bit rate of 96 Gbps.

b. Since the channel encoder adds redundant bits to the input bit stream, each 3 input bits are mapped to 4 bits, thereby yielding a total bit rate of

$$96 \times \frac{4}{3} Gbps = 128 Gbps$$

c.

$$96 \times \frac{6}{5} Gbps = 115.2 Gbps$$

which is less than that in part b- since the redundancy is decreased.

d. For unshaped 8QAM and 32QAM, the bit-to-symbol conversion ratio is 3 and 5, respectively, hence

$$\begin{aligned} \text{8QAM uncoded bit rate} &= 24 \times 3 = 72 Gbps \\ \text{32QAM uncoded bit rate} &= 24 \times 5 = 120 Gbps \end{aligned}$$

similarly

$$\begin{aligned} \text{8QAM encoded bit rate} &= 24 \times 3 = 86.4 Gbps \\ \text{32QAM encoded bit rate} &= 24 \times 5 = 144 Gbps \end{aligned}$$

e. The bit-to-symbol ratio can be calculated from

$$H = -\sum_{i=1}^{16} \hat{P}_i \log_2 \hat{P}_i$$

which with the probabilisitic shaping parameters calculated in question 2, gives

$$H = 3.76 \text{ bits}$$

f. The bit rate is correspondingly equal to $24G \times 3.76 = 90.24$ Gbps, which when encoded, increases by $\frac{6}{5}$ to 108.29 Gbps, though less than $115.2 Gbps$ since the number of bits per symbol is reduced.

Question 4)

a and b. The ASE power spectral density is

$$\sigma^2_{\text{ASE,PSD}} = \frac{1}{2} N_{\text{Span}} h\nu_{\text{opt}} GF = 16.06\mu\text{W/Gbaud}$$

hence the ASE noise variance becomes

$$\begin{aligned}\sigma^2_{\text{ASE}} &= R_{\text{Receiver}} \sigma^2_{\text{ASE,PSD}} \\ &= (1+\beta) R_s \sigma^2_{\text{ASE,PSD}} \\ &= 0.462\text{mW} \equiv -3.35\text{dBm}\end{aligned}$$

and the SNR is calculated as

$$\text{SNR (dB)} = \text{Power (dBm)} - \sigma^2_{ASE} \text{ (dBm)} = 12.75\text{dB}$$

c. A LDPC code with parameters (16193,9713) is needed to obtain a probability of error as $\sim 2 \times 10^{-3}$. With such probability of error, a RS code with parameters (294,244) can reduce the probability of error by an astounding factor to 10^{-10}.

d. That probabilistic shaping reduces the launch power by 32% and hence the SNR; therefore

$$\text{SNR}_{\text{Shaped}} = 11.08 \text{ dB}$$

with an approximate probability of error of $\sim 5 \times 10^{-3}$. Similarly, the LDPC and RS code parameters are (16195,12595) and (294,244) and the reduced probability of error is $\sim 10^{-6}$, respectively.