Optimal Routing and Wavelength Assignment in All-Optical Networks *

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Abstract

This paper considers the problem of routing connections in an optical network using wavelength division multiplexing, where each connection between a pair of nodes in the network is assigned a path through the network and a wavelength on that path, such that connections whose paths share a common link in the network are assigned different wavelengths.

We derive an upper bound on the carried traffic of connections (or equivalently, a lower bound on the blocking probability) for any routing and wavelength assignment (RWA) algorithm in such a network. The bound scales with the number of wavelengths and is achieved asymptotically (when a large number of wavelengths is available) by a fixed RWA algorithm. Although computationally intensive, our bound can be used as a metric against which the performance of different RWA algorithms can be compared for networks of moderate size. We illustrate this by comparing the performance of a simple RWA algorithm via simulation with our bound.

We also derive a similar bound for optical networks using dynamic wavelength converters, which are equivalent to circuit-switched telephone networks, and compare the two cases for different examples.

1 Introduction

Wavelength-division-multiplexing (WDM) technology offers the capability of building very large wide-area networks consisting of thousands of nodes with per-node throughputs of the order of a gigabit-per-second [1, 2, 3]. We study networks consisting of optical nodes interconnected by optical links (Figure 1). We assume each link is bidirectional and actually consists of a pair of unidirectional links. The individual

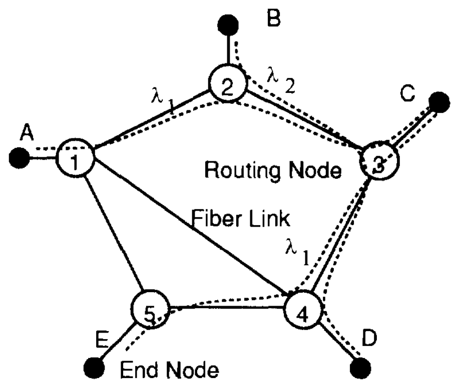

Figure 1: A WDM network consisting of routing nodes interconnected by point-to-point fiber-optic links. The routing nodes have end-nodes attached to them that form the sources and destinations for network traffic.

nodes are capable of routing each wavelength on an incoming link to any outgoing link. However, the same wavelength on two incoming links cannot be routed simultaneously onto a single outgoing link. If there are Λ wavelengths on each link, the optical node may be viewed as consisting of Λ independent switches, one for each wavelength (Figure 2). There is no optical to electronic conversion and vice versa, and hence no buffering, at the intermediate nodes, in these *all-optical networks.* Such networks have been studied in [4, 5], and are currently being explored at the testbed level by several groups [6].

In our network model, connection requests and terminations arrive at random. Each connection must be assigned a specific wavelength and path in the network. The wavelength assigned must be such that no other connection on the same path, or on any path that shares an edge with the assigned path, is assigned the same wavelength. For example, in Figure 1, a

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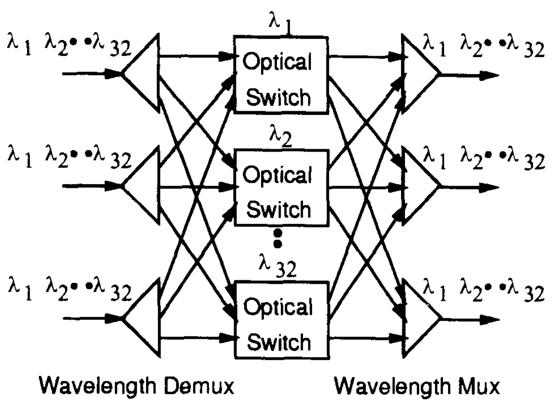

Figure 2: Structure of a reconfigurable routing node. The node can switch each wavelength at its input ports independent of the other wavelengths.

connection between node A and node C is carried on wavelength λ_1, a connection between node C and node E also on the same wavelength λ_1, but a connection between node B and node D must be carried on a different wavelength λ_2.

This routing and wavelength assignment (RWA) problem, or variants of it, have been considered earlier in [7, 8]. Several heuristic RWA algorithms have been proposed and their performance has been quantified via simulation.

A similar routing problem arises in circuit-switched telephone networks. Here, we must route connections by selecting a path for each connection such that there is a circuit available to accommodate the call in every link on the path. In our optical network model, we must not only satisfy the constraint in circuit-switched telephone networks, but also satisfy an additional constraint that the *same* circuit (wavelength) must be assigned to the connection on *every* link in the path. Note that if we are allowed to use dynamic wavelength converters inside the optical network, then it becomes equivalent to the circuit-switched telephone network. Henceforth, we will used the term "circuit-switched" to refer to a circuit-switched telephone network or to an optical network using dynamic wavelength converters. We will use the term "optical network" to refer to a network that does not use wavelength converters.

Related models are considered in [9, 10] where a "virtual topology" is embedded on top of the underlying physical topology. The virtual topology is a graph consisting of the nodes in the network with an edge between two nodes if a connection is set up between the two nodes using some wavelength and path in the physical topology.

The routing problem in circuit-switched networks has been extensively studied [11, 12]. It is well known that the routing problem can be formulated as an integer linear program (ILP) [13, chapter 6, section 2]. In [11], it was shown that an upper bound on the carried traffic can be obtained by relaxing the ILP to a linear program (LP), and moreover, that the bound holds for random offered traffic as well. Since the RWA problem in our model is a more constrained version of the routing problem in circuit-switched networks, it is clear that this bound will also be an upper bound for the carried traffic in our network model. However, our objective in this paper is to derive a better upper bound.

For clarity of exposition, we first consider the case when we are given a fixed set of connections to be routed. We formulate the RWA problem as an integer linear program (ILP) where the objective is to maximize the number of connections that are successfully routed. If we relax the integrality constraints in this ILP, we get an LP whose value represents an upper bound on the number of connections that can be successfully routed. We show that a straight-forward formulation, when relaxed into a linear program, yields an upper bound that is the same as the upper bound for the circuit-switched case. We then show how to formulate the ILP suitably so as to get a better upper bound when it is relaxed into an LP.

We then consider the more general case where connections arise at random between some source-destination pair in the network and have a random holding time. (The deterministic case is a special case of this more general model.) For this case, using the results in [11], we show the following. By suitably normalizing the LP for the deterministic case by the number of available wavelengths, we obtain an upper bound on the carried traffic (expected number of connections in progress) per available wavelength that is achievable by *any* RWA algorithm for this network. Moreover, for a large class of traffic models (including the standard Poisson arrivals), there exist RWA algorithms whose carried traffic approaches this upper bound arbitrarily closely when the number of available wavelengths is sufficiently large.

The usefulness of our bound lies in that it can be used as a benchmark against which the performance of various heuristic RWA algorithms can be compared. We illustrate this by comparing the simulated performance of a simple heuristic RWA algorithm against the bound. Moreover the solution to the LP that gives the bound can also indicate how to modify the network to improve its performance.

The paper is organized as follows. The next section formulates the ILPs and corresponding LPs for the RWA problem. Section 3 presents two examples that show that this bound is indeed better than the bound for circuit-switched networks, and also compares the performance of the heuristic and a fixed routing algorithm against our bound in one of the examples. Section 4 discusses the implications of our work and concludes the paper.

2 The Bound

We represent the network by an undirected graph, G. Each node in the graph corresponds to a node in the network and each edge to a link. We assume that all connections to be routed are full duplex, that all links are bidirectional and the two halves of a duplex connection are to be routed over the same path using the same wavelength.

Let N denote the number of source-destination (s-d) pairs in the network, M the number of links and Λ the number of wavelengths available on each link (assumed to be the same for all links). For any RWA algorithm, let $m_i, i = 1, \ldots, N$, denote the number of connections carried between source-destination pair i, and m, the N-vector (m_i). Let ρ denote the total offered load, $p_i\rho, i = 1, \ldots, N$, the offered load between source-destination pair i, and p, the N-vector (p_i). The offered load for the deterministic case (sections 2.1 and 2.2) is the number of connections that are available to be routed. In the random case (section 2.3), it is the expected number of connections that would be in progress if one could successfully route all call arrivals.

Let P denote the total number of available paths on which connections can be routed. The set of paths could either be given or can be computed given the graph G and the set of source-destination pairs. (Note that the number of paths between a source-destination pair in an arbitrary graph/network can be exponential in the number of nodes or links).

Let $A = (a_{ij})$ be the $P \times N$ path–s-d-pair incidence matrix, i.e.,

$$a_{ij} = \begin{cases} 1, & \text{if path } i \text{ is between} \\ & \text{source-destination pair } j, \\ 0, & \text{otherwise.} \end{cases}$$

Let $B = (b_{ij})$ be the $P \times M$ path–edge incidence matrix, i.e.,

$$b_{ij} = \begin{cases} 1, & \text{if link } j \text{ is on path } i, \\ 0, & \text{otherwise.} \end{cases}$$

We first give a straightforward ILP formulation of the RWA problem as a multicommodity flow problem [13, chapter 6, section 2].

2.1 A Simple Formulation

The operation of every RWA algorithm in an optical network can be represented by a $P \times \Lambda$ path–wavelength assignment matrix which we denote by $C = (c_{ij})$ where,

$$c_{ij} = \begin{cases} 1, & \text{if the RWA algorithm assigns} \\ & \text{wavelength } j \text{ to path } i, \\ 0, & \text{otherwise.} \end{cases}$$

Then, the optimal RWA algorithm for the deterministic case is found by solving the following ILP whose value we denote by $C_o(\rho, p)$.

(Maximize carried traffic)

$$C_o(\rho, p) = \max \sum_{i=1}^{N} m_i$$

subject to

$$m_i \geq 0, \quad \text{integer}, \qquad i = 1, \ldots, N,$$

$$c_{ij} \geq 0, \text{ integer}, \ i = 1, \ldots, P, \ j = 1, \ldots, \Lambda,$$

(capacity constraint: the same wavelength is used at most once on a given link)

$$C^T B \leq 1_{\Lambda \times M},$$

(traffic demands)

$$m \leq 1_\Lambda C^T A,$$

$$m_i \leq p_i \rho, \qquad i = 1, \ldots, N.$$

($1_{X \times Y}$ represents an $X \times Y$ matrix all of whose elements are unity, and 1_X represents a $1 \times X$ matrix all of whose elements are unity.)

Now consider the corresponding circuit-switched network. The operation of every routing algorithm in such a network can be represented by a vector of path-flows $f = (f_i)$, where f_i denotes the flow on path i.

Then, the optimal routing algorithm for the deterministic case is found by solving the following ILP whose value we denote by $C_c(\rho, p)$.

(Maximize carried traffic)

$$C_c(\rho, p) = \max \sum_{i=1}^{N} m_i$$

subject to

$$m_i \geq 0, \quad \text{integer}, \qquad i = 1, \ldots, N,$$

$$f_i \geq 0, \quad \text{integer}, \qquad i = 1, \ldots, P,$$

(capacity constraint: not more than Λ units of flow on any link)

$$fB \leq 1_M \Lambda,$$

(traffic demands)

$$m \leq fA,$$

$$m_i \leq p_i \rho, \qquad i = 1, \ldots, N.$$

Lemma 1

$$C_o(\rho, p) \leq C_c(\rho, p).$$

Proof: Let c_{ij}^s and m_i^s constitute a feasible solution to the $C_o(\rho, p)$ ILP. Then set $f_i = \sum_{j=1}^{\Lambda} c_{ij}^s$ and $m_i = m_i^s$. Since $f = 1_\Lambda C^T$,

$$fA = 1_\Lambda C^T A \geq m,$$

and

$$fB = 1_\Lambda C^T B \leq 1_\Lambda 1_{\Lambda \times M} = 1_M \Lambda.$$

Therefore, f_i and m_i constitute a feasible solution to the $C_c(\rho, p)$ ILP. Hence,

$$C_o(\rho, p) \leq C_c(\rho, p).$$

□

In order to get upper bounds on $C_o(\rho, p)$ and $C_c(\rho, p)$, we can drop the integrality constraints on m_i, c_{ij} and f_i and solve the corresponding LPs. Let $U_o(\rho, p)$ and $U_c(\rho, p)$ denote the (maximum) values of the LPs for the optical and circuit-switched case respectively. Then, we can show (with a proof virtually identical to that of Lemma 1), that

$$U_o(\rho, p) \leq U_c(\rho, p). \tag{1}$$

Interestingly however, suppose f_i' and m_i' constitute a feasible solution to the $U_c(\rho, p)$ LP. Then set $c_{ij} = f_i'/\Lambda$ and $m_i = m_i'$. Since $C = f^T 1_\Lambda / \Lambda$,

$$1_\Lambda C^T A = 1_\Lambda 1_\Lambda^T f A / \Lambda = fA \geq m,$$

and

$$C^T B = 1_\Lambda^T l B / \Lambda \leq 1_\Lambda^T \Lambda 1_M / \Lambda = 1_{\Lambda \times M}.$$

Therefore, c_{ij} and m_i constitute a feasible solution to the $U_o(\rho, p)$ LP. Thus we have the following lemma.

Lemma 2

$$U_o(\rho, p) \geq U_c(\rho, p).$$

Using (1) and Lemma 2, we have the following proposition.

Proposition 1

$$U_o(\rho, p) = U_c(\rho, p).$$

Thus this formulation of the RWA problem yields an upper bound on carried traffic that is identical to the upper bound for the corresponding circuit-switched network. However, a different formulation of the problem given below can be used to obtain a better upper bound.

2.2 A Better Formulation

We create a new graph G_p where each node corresponds to a path in G and two nodes in G_p are adjacent iff the corresponding two paths in G share a common link. Now our RWA problem is transformed into assigning wavelengths to nodes in G_p so that no two adjacent nodes are assigned the same wavelength. In other words, a set of paths in G can be assigned a common wavelength only if the corresponding nodes in G_p form an independent set. Let L denote the number of maximal independent sets in G_p, let w_i denote the number of wavelengths that are assigned to the nodes in independent set i by a RWA algorithm, and w the L-vector (w_i). (Note that the number of maximal independent sets could be exponential in the number of nodes in G_p.) Let $D = (d_{ij})$ be the $P \times L$ path-independent-set incidence matrix, i.e.,

$$d_{ij} = \begin{cases} 1, & \text{if independent set } j \text{ contains} \\ & \text{path } i, \\ 0, & \text{otherwise.} \end{cases}$$

The ILP, whose value we denote as $T_o^\Lambda(\rho, p)$ can then be formulated as follows:

(Maximize carried traffic)

$$T_o^{\Lambda}(\rho, p) = \max \sum_{i=1}^{N} m_i$$

subject to

$$w_i \geq 0, \quad \text{integer}, \qquad i = 1, \ldots, L,$$

(capacity constraint-not more than Λ units of flow on any link)

$$\sum_{i=1}^{L} w_i \leq \Lambda,$$

(traffic demands)

$$f \leq wD^T,$$

$$m \leq fA,$$

$$m_i \leq p_i\rho, i = 1, \ldots, N.$$

2.3 Random Traffic Demands

Henceforth, we let ρ_i denote the offered traffic to s-d pair i in Erlangs. If calls arrive at random and have random holding times, under the operation of any RWA algorithm, the network is in a random state which we denote by $m = (m_1, m_2, \ldots, m_N)$ where m_i is the number of calls in progress between s-d-pair i. The set of feasible states for this network when the number of available wavelengths is Λ, which we denote by S_Λ, is the set of all non-negative integer vectors m in R^N for which the $T_o^{\Lambda}(\rho, p)$ ILP is feasible, i.e., $m \in R^N$ if there exist non-negative integer vectors $w \in R^L$ and $f \in R^P$ such that,

$$m\mathcal{A} + (\, w \quad f\,)\mathcal{B} \leq \Lambda\mathcal{C},$$

where,

$$\mathcal{A} = (\, I_{N\times N} \quad 0_{N\times P} \quad 0_N\,),$$

$$\mathcal{B} = \begin{pmatrix} 0_{L\times N} & -D^T & 1_L \\ -A & I_{P\times P} & 0_P \end{pmatrix},$$

and

$$\mathcal{C} = (\, 0_N \quad 0_P \quad 1\,).$$

This set of feasible states for the network is of the form described in [11, Section 6].

Using Theorem 3.1 in [11], the carried traffic for any RWA algorithm, R, for this network, satisfies,

$$\frac{1}{\Lambda} T_R(\rho, p) \leq T_o(r, p),$$

where $r = \rho/\Lambda$ and $T_o(r, p)$ is the value of the following linear program.

$$T_o(r, p) = \max \sum_{i=1}^{N} s_i$$

subject to

$$s\mathcal{A} + t\mathcal{B} \leq \mathcal{C},$$

$$s \leq pr.$$

(This is just the $T_o^{\Lambda}(\rho, p)$ program normalized by dividing the objective function and each of the constraints by Λ and dropping the integrality constraints.)

The corresponding circuit-switching LP, obtained by normalizing the objective function and each of the constraints in the $U_c(\rho, p)$ program by Λ and dropping the integrality constraints, is the following.

$$T_c(r, p) = \max \sum_{i=1}^{N} s_i$$

subject to

$$s \leq tA,$$

$$tB \leq 1_M,$$

$$s \leq pr.$$

From Theorem 2.1 in [11], $T_o(r, p)$ and $T_c(r, p)$ are continuous, nondecreasing, convex $\cap$ functions of r. Moreover, the $T_o(r, p)$ and $T_c(r, p)$ programs are parametric linear programs with parameter r in the sense of [14] (section 1.9) and hence, $T_o(r, p)$ and $T_c(r, p)$ are piecewise-linear functions of r ([14], p. 70).

The blocking probability, $B_R(\rho, p)$ for any RWA algorithm, R, is related to its carried traffic by,

$$T_R(\rho, p) = \rho\,(1 - B_R(\rho, p))\,.$$

Therefore,

$$B_o(r, p) = 1 - \frac{T_o(r, p)}{r}$$

and

$$B_c(r, p) = 1 - \frac{T_c(r, p)}{r}$$

are lower bounds for the blocking probability of any RWA algorithm in the wavelength-routing and circuit-switching cases, respectively.

We now show that T_o and T_c (or equivalently B_o and B_c) are achievable when the number of wavelengths (circuits) is large.

2.4 Asymptotic Optimality of $T_o(r,p)$ and $T_c(r,p)$

If $\{s', t'\}$ yield an optimal solution to the $T_o(r,p)$ program, consider a fixed RWA algorithm, F, that assigns the fixed set of $\lfloor t'_i \Lambda \rfloor$, $i = 1, \ldots, L$, wavelengths to each of the paths in independent set i. Let $T_F^{\Lambda}(\rho, p)$ denote the carried traffic for this algorithm.

The Deterministic Case: This asymptotic optimality of this fixed RWA algorithm in the deterministic case follows essentially from the fact that appropriately rounding any optimal solution to the $T_o(r,p)$ program yields a feasible solution to the $T_o^{\Lambda}(\rho,p)$ ILP whose value is asymptotically optimal (if Λ is large and $r = \rho/\Lambda$ is fixed). More precisely, if s' and t' constitute an optimal solution to the $T_o(r,p)$ program, $w_j = \lfloor t'_j \Lambda \rfloor$, $f_i = \sum_j s_{ij} w_j$ and $m_i = \min(\sum_j a_{ji} f_j, \lfloor s'_i \Lambda \rfloor)$ yield a feasible solution to the $T_o^{\Lambda}(\rho,p)$ whose value satisfies,

$$\left[\sum_i s'_i - \frac{(L)(P)}{\Lambda}\right] \le \left[\frac{T_F^{\Lambda}(\rho,p)}{\Lambda} = \frac{1}{\Lambda}\sum_i m_i\right] \le \left[\sum_i s'_i = T_o(r,p)\right]$$

and hence is asymptotically optimal for large Λ.

The Random Case: The fixed RWA algorithm is a greedy algorithm in the following sense. It accepts a connection request between an s-d pair if there is a free wavelength on any of the paths between that s-d pair and rejects the connection request otherwise. Using Theorem 4.1 in [11], if the offered traffics to different s-d-pairs are independent of one another, and if the traffic model satisfies the asymptotic traffic property (ATP) [11], this fixed RWA algorithm, F, satisfies,

$$\lim_{\Lambda \to \infty} \frac{1}{\Lambda} T_F^{\Lambda}(\rho, p) = T_o(r,p),$$

i.e., this fixed RWA algorithm is asymptotically optimal.

The ATP can be stated by considering a network with a single s-d pair with an offered traffic of ρ Erlangs and a fixed RWA algorithm that assigns Λ channels to this s-d pair and blocks a connection request if and only if all the Λ channels are occupied. Let $T_F(\rho, 1, \Lambda)$ denote the carried traffic for this fixed RWA algorithm. Then, the statistics of the offered traffic are said to satisfy the ATP if,

$$\lim_{\Lambda \to \infty} \frac{T_F(r\Lambda, 1, \Lambda)}{\Lambda} = \min(r, 1).$$

Informally, the ATP is a kind of law of large numbers and simply states that, for a single s-d pair, if the offered traffic exceeds the number of available wavelengths, all the available wavelengths are asymptotically occupied, and if the offered traffic is less than the number of wavelengths, all connection requests are asymptotically honored. The ATP is satisfied by many common traffic models including Poisson arrivals [11]. It was shown in [11] that a similar fixed routing algorithm is asymptotically optimal for the circuit-switched case.

2.5 Some Simple Bounds on $T_o(r,p)$ and $T_c(r,p)$

The following observations are useful in deriving the values of $T_o(r,p)$ and $T_c(r,p)$ for simple networks.

Proposition 2 *Let K be the maximum number of edge disjoint paths in a graph G, i.e., K is the cardinality of the maximum independent set in the path graph G_p. Then $T_o(r,p) \le K$.*

Proof: Since paths in G that use the same wavelength must be edge-disjoint, the maximum number of times a wavelength can be used is the maximum number of edge-disjoint paths in G; hence $T_o(r,p) \le K$. □

Proposition 3 *Let E be the total number of links in a circuit-switched network G. Let H denote the minimum number of edges in a shortest-path between all desired source-destination pairs between which there is non-zero traffic. Then $T_c(r,p) \le E/H$.*

Proof: Since a connection takes up at least H circuits, and there are a total of $E\Lambda$ circuits, the total number of connections that can be supported is $\le E\Lambda/H$. Thus the maximum number of connections that can be supported per wavelength, $T_c(r,p) \le E/H$. □

2.6 A Heuristic RWA Algorithm

Consider the following RWA algorithm. The set of paths between a source-destination pair is ordered in some manner. The set of wavelengths is ordered in some manner. A new connection is routed on the first path on which a wavelength is available. Among the set of available wavelengths on that path, the first one is selected. If no path can be found, the connection is considered blocked.

We simulated the performance of this simple algorithm for traffic where connection requests are assumed to arrive according to a Poisson process and last for a duration that is exponentially distributed.

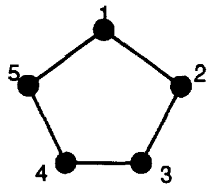

Figure 3: A five node network.

3 Examples

To illustrate the use of our bound, we provide two examples. The first is a simple 5-node network that brings out the difference between the circuit-switched network and the optical network. The second is a more complicated 20-node network and is intended to demonstrate the application of our results to more realistic networks.

3.1 A Simple Example

Consider the 5-node pentagon network G_5 shown in Figure 3. Let the source-destination pairs of interest, indexed from 1 to 5, be $(1,3)$, $(2,4)$, $(3,5)$, $(4,1)$, and $(5,2)$, i.e., each node communicates with two other nodes in the network. The paths of interest are then 123, 1543, 234, 2154, 345, 3215, 451, 4321, 512, 5432 and will be indexed from 1 to 10. The maximal independent sets in the path graph are $\{1,2\}$, $\{3,4\}$, $\{5,6\}$, $\{7,8\}$, $\{9,10\}$, $\{1,7\}$, $\{3,9\}$, $\{5,1\}$, $\{7,3\}$, $\{9,5\}$. For simplicity, consider the uniform traffic case, i.e., $p_i = 1/5$, $i = 1,\ldots,5$. In this case, observe that the maximum cardinality of an independent set is 2 and hence from Proposition 2, $T_o(r,p) \leq 2$. The shortest-path has two links ($H = 2$) and there are $E = 5$ links in the graph; hence from Proposition 3, we get $T_c(r,p) \leq 5/2$. Solving the linear programs yield,

$$T_o(r,p) = \begin{cases} r, & 0 \leq r \leq 2, \\ 2, & r > 2\,, \end{cases}$$

and

$$T_c(r,p) = \begin{cases} r, & 0 \leq r \leq 5/2, \\ 5/2, & r > 5/2\,, \end{cases}$$

and hence the upper bounds of Propositions 2 and 3 are tight for large r in this example. Figure 4 plots $T_o(r,p)$ and $T_c(r,p)$ as a function of r.

An asymptotically optimal fixed RWA algorithm for the optical network assigns the first $\Lambda/5$ wavelengths to both paths between s-d pair 1, the second $\Lambda/5$ wavelengths to both paths between s-d pair 2, etc. For $r \geq 2$, this algorithm realizes a total carried traffic of

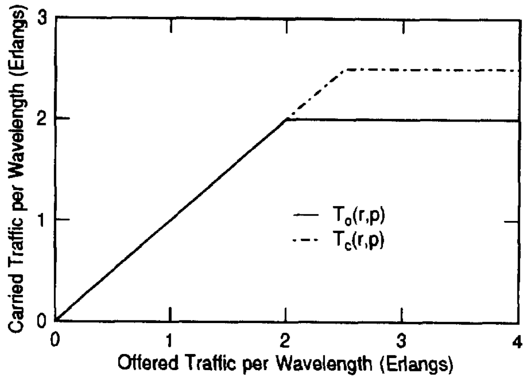

Figure 4: Carried traffic versus offered traffic for the five-node network.

2Λ asymptotically, thus achieving the bound of Proposition 2.

An asymptotically optimal fixed RWA algorithm for the circuit-switched network assigns $\Lambda/2$ circuits for connections between each s-d pair. All connections are routed on the unique shortest paths. For $r \geq 5/2$, this algorithm realizes a total carried traffic of $5\Lambda/2$ asymptotically, thus achieving the bound of Proposition 3.

3.2 A Larger Example

Consider the 20-node skeleton of the arpanet shown in Figure 5 (from [15], pp. 138). For this network we consider the class of algorithms that use only shortest paths. Let the s-d pairs of interest be $\{1,13\}$, $\{2,7\}$, $\{3,15\}$, $\{6,8\}$, $\{11,14\}$, $\{4,20\}$, $\{5,19\}$, $\{9,18\}$, $\{10,17\}$, $\{12,16\}$. In this example there are 14 shortest paths: 3 shortest paths between s-d pairs $\{2,7\}$ and $\{6,8\}$ and a single shortest path between the other 8 s-d pairs. The path graph consists of 14 nodes corresponding to these shortest paths and has 43 maximal independent sets. [1]

Consider first the uniform traffic case. Figure 6 shows the carried traffic and blocking probability as a function of offered traffic. In this case,

$$\begin{aligned} T_c(r,p) &= T_o(r,p) \\ &= \min(r, 7r/10+1, 3r/10+3, 6). \end{aligned}$$

The performance of the fixed RWA algorithm of section 2.4 and the simulated performance of the heuristic

[1] For this example, there are a total of 62 paths between the desired source-destination pairs, and there are 5584 maximal independent sets in the corresponding path graph, illustrating the exponential growth in the number of independent sets as a function of the number of nodes in the path graph.

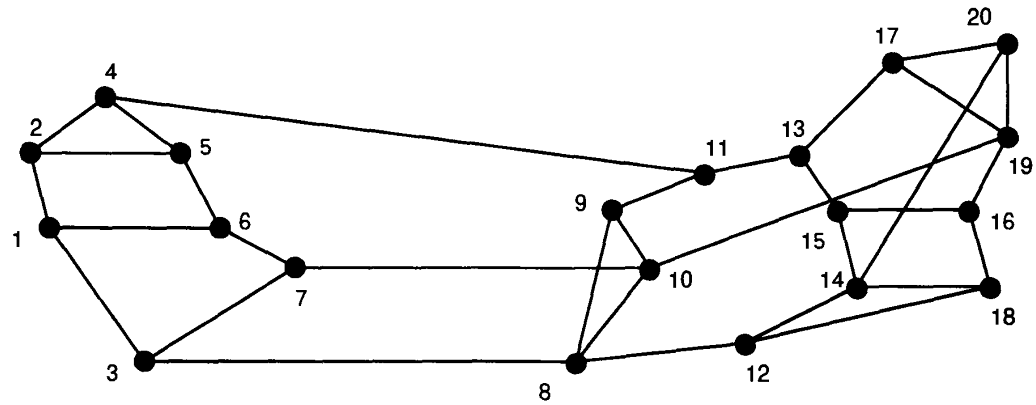

Figure 5: A 20 node network representing a skeleton of the original Arpanet.

algorithm of section 2.6 are also indicated.

Now consider the case of non-uniform traffic with the (arbitrarily chosen) vector

$$p = (2/23, 6/23, 2/23, 6/23, 2/23, 1/23, \\ 1/23, 1/23, 1/23, 1/23).$$

Figure 7 shows the carried traffic and blocking probability as a function of offered traffic. In this case,

$$T_c(r,p) = \min(r, 10r/23 + 2, 5r/23 + 3, \\ 2r/23 + 4, 6),$$

and

$$T_o(r,p) = \min(r, 10r/23 + 2, 4r/23 + 3, \\ 2r/23 + 4, 6).$$

Figures 6 and 7 also illustrate the asymptotic optimality of the fixed RWA algorithm for large Λ proved in section 2.4. However, this is a different (but fixed) RWA algorithm for each value of the offered traffic. Therefore, the implementation of the fixed RWA algorithm requires that the offered traffic be known and since in practice the offered traffic may be unknown and changing, this is an impractical algorithm.

In this case, an RWA algorithm whose implementation does not require an estimate of the offered traffic is desirable. The heuristic RWA algorithm of section 2.6 has this property and it is gratifying to note, from Figures 6–9, that its performance is close to that achievable by *any* [2] RWA algorithm in this example (for both the uniform and nonuniform cases). Moreover, we would not have been able to draw this conclusion in the nonuniform case, if our tighter upper bound had not been available.

It must also be noted that since a fixed RWA algorithm implies a fixed setting of the switches shown in Figure 2, if the mean offered traffic is known and not changing, the network can be implemented *without optical switches*. Thus the principal advantage of optical switching in an all-optical network is probably the ability to adapt to the unknown traffic and not just the amount of wavelength reuse that can be obtained.

4 Conclusion

Although the bound presented in this paper can be obtained by solving a linear program, the number of variables in the linear program may be an exponential function of the number of source-destination pairs in the case of the optical network without wavelength converters. However, since the bound is on a per-wavelength basis, it need be computed only once and can then be scaled easily with the number of wavelengths. Moreover, the bound is achieved by a fixed-routing algorithm asymptotically (when the number of wavelengths is large). It can be used as a metric against which the performance of different heuristics can be compared. Moreover, solving the LP enables

[2]Note that even though a RWA algorithm has no choice regarding the path on which to route the connection for most of the s-d pairs in this example, it still has the choice of deliberately blocking the connection request and the task of choosing one of the (potentially) many available wavelengths on this path. Even if there were a fixed path between every s-d pair in the network, a number of RWA heuristics can be designed simply based on the way in which they pick one of the available wavelengths on this path—least used, most used, etc.[7].

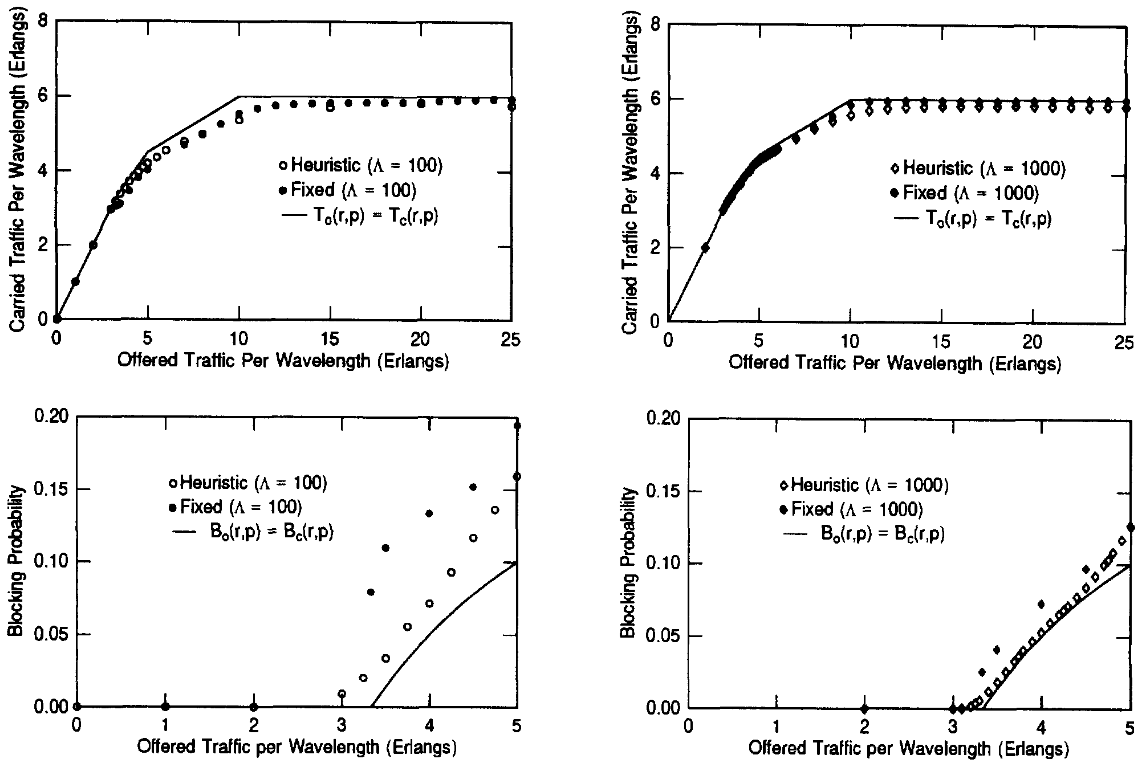

Figure 6: Carried traffic and blocking probability versus offered traffic for the 20-node network, assuming uniform traffic.

us to determine the assignment of paths and wavelengths used by an asymptotically optimal fixed RWA algorithm. This information can be useful in indicating which links are congested and how the network can be modified by the addition of more links/wavelengths to relieve this congestion and improve its performance.

Using two examples, we showed that this bound yields a better bound on the carried traffic than the bound for the corresponding circuit-switched network, or equivalently the corresponding optical network using dynamic wavelength converters. Indeed using this bound, we showed in the second example that the performance of the simple RWA heuristic described here, assuming Poisson arrivals and exponential holding times, is very close to that of an optimal algorithm.

Although the difference between the bounds for the optical network and the corresponding circuit-switched network was not large for the two examples considered here, there are networks and traffic patterns for which the difference is significant. An example given in [16] shows a network and traffic pattern that can be supported without blocking by a small constant number of wavelengths in the circuit-switched case (using dynamic wavelength converters), but requires as many wavelengths as the number of nodes in the optical case.

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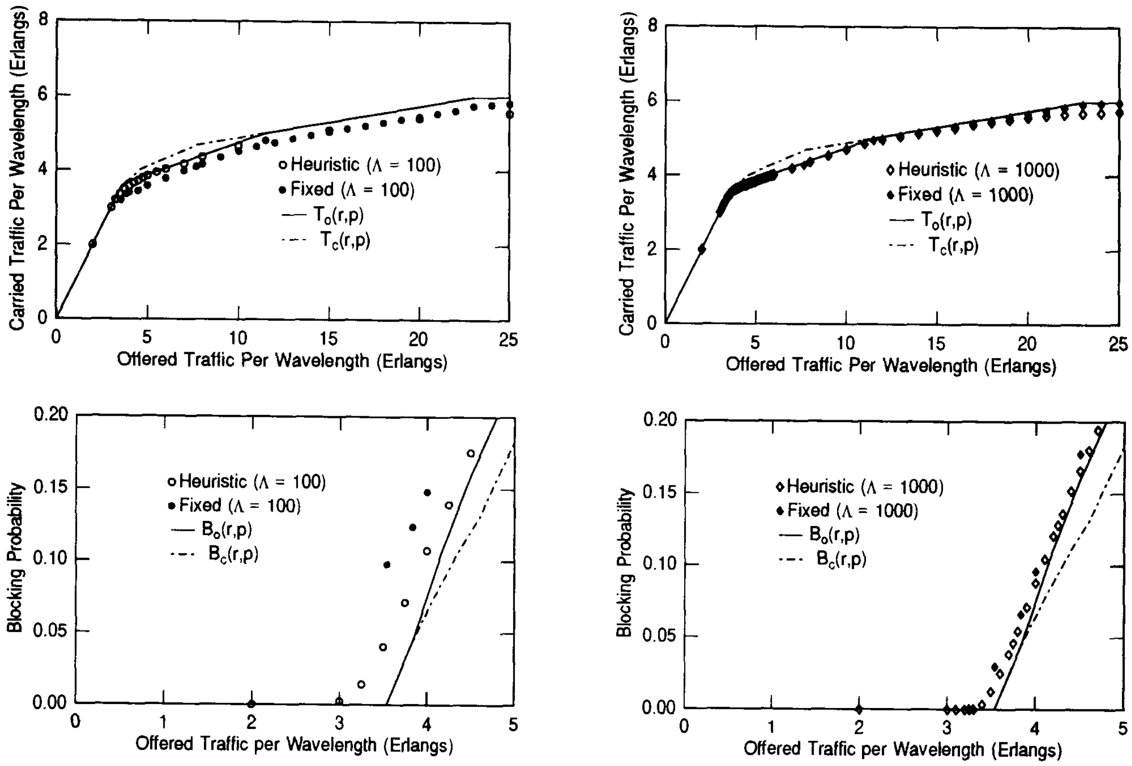

Figure 7: Carried traffic and blocking probability versus offered traffic for the 20-node network, for the non-uniform traffic case.

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