In the name of beauty

The 3rd problem set solution of Optical Networks course

Question 1)

a. The numerial aperture is given by

$$\mathrm{NA} = \sqrt{n_1^2 - n_2^2}$$

with $n_1 = 1.5$ and $n_2 = 1.4950$, we obtain NA=0.12 .

b.

$$BL < \frac{n_2}{n_1^2}\frac{c}{\Delta}$$

where

$$\Delta = \frac{\mathrm{NA}^2}{2n_1^2}$$

Substituting yields

$$L < 6.23\mathrm{km}$$

Question 2)

Note that

$$D = \frac{\lambda}{c}\frac{d^2n}{d\lambda^2} = -\frac{2\pi c}{\lambda^2}\beta_2$$

hence

$$D = 83 \text{ ps}/(\text{nm}\cdot\text{km})$$

and

$$\beta_2 = -28.29 \text{ ps}^2/\text{km}$$

Question 3)

a. The FT of the partial differential equation is

$$\begin{aligned}\frac{\partial A(z,\omega)}{\partial z} &= -\frac{\alpha}{2}A(z,\omega) + j\frac{\beta_2}{2}\omega^2 A(z,\omega) \\ &= [-\frac{\alpha}{2} + j\frac{\beta_2}{2}\omega^2]A(z,\omega)\end{aligned} \tag{1}$$

which has the following solution

$$A(z,\omega) = C_1 e^{-\frac{\alpha}{2}z + j\frac{\beta_2}{2}\omega^2 z} = A(0,\omega)e^{-\frac{\alpha}{2}z + j\frac{\beta_2}{2}\omega^2 z} \tag{2}$$

The chirped part of the exponent is not considered since it does not influence the amplitude, therefore we must have

$$-\frac{1}{2}(\frac{t}{T_0})^2 = -\frac{1}{2} \implies t = \pm T_0$$

b. The FT of $A(0,t)$ can be written as:

$$A(0,\omega) = \sqrt{\frac{2\pi T_0^2}{1+iC}} \exp\left(-\frac{\omega^2 T_0^2}{2(1+iC)}\right)$$

The $\frac{1}{e}$-intensity condition yields:

$$\left|\exp\left(-\frac{\omega^2 T_0^2}{2(1+iC)}\right)\right| = \frac{1}{\sqrt{e}}$$

since when the spectral intensity increases by a factor of α, the amplitude of the original pulse increases by a factor of $\sqrt{\alpha}$. The latter equality gives us

$$\Delta\omega_0 = \frac{\sqrt{1+C^2}}{T_0}$$

Question 4)

At the end of the fiber we obtain:

$$\begin{aligned}
A(z,\omega) &= A(0,\omega)e^{-\frac{\alpha}{2}z+j\frac{\beta_2}{2}\omega^2 z} \\
&= A_0 e^{-\frac{\omega^2 T_0^2}{2+2jC}} e^{-\frac{\alpha}{2}L+j\frac{\beta_2}{2}\omega^2 L} \\
&= A_0 e^{-\frac{\alpha}{2}L} e^{-\frac{\omega^2 T_0^2}{2+2jC}} e^{j\frac{\beta_2}{2}\omega^2 L} \\
&= A_0 e^{-\frac{\alpha}{2}L} e^{-\frac{\omega^2}{2}(\frac{T_0^2}{1+jC}-j\beta_2 L)} \\
&= A_0 e^{-\frac{\alpha}{2}L} e^{-\frac{\omega^2}{2}\frac{T_0^2-j\beta_2 L+\beta_2 CL}{1+jC}} \\
&= A_0 e^{-\frac{\alpha}{2}L} e^{-\frac{\omega^2}{2}\frac{(T_0^2+\beta_2 CL)^2+(\beta_2 L)^2}{(1+jC)(T_0^2+j\beta_2 L+\beta_2 CL)}} \\
&= A_0 e^{-\frac{\alpha}{2}L} e^{-\frac{\omega^2}{2}\frac{(T_0^2+\beta_2 CL)^2+(\beta_2 L)^2}{T_0^2(1+jC_1)}}
\end{aligned} \tag{3}$$

hence

$$T_1 = T_0\sqrt{\left(1+\frac{\beta_2 CL}{T_0^2}\right)^2+\left(\frac{\beta_2 L}{T_0^2}\right)^2}$$

and

$$C_1 = C + \frac{\beta_2 L}{T_0^2}(1+C^2)$$

Question 5)

The compression factor is

$$\sqrt{\left(1+\frac{\beta_2 CL}{T_0^2}\right)^2+\left(\frac{\beta_2 L}{T_0^2}\right)^2}=\sqrt{1+\frac{2\beta_2 CL}{T_0^2}+\frac{(\beta_2 C)^2L^2+\beta_2^2L^2}{T_0^4}}$$

Since $|\beta_2 C| << 1$ and $\left|\frac{\beta_2 C}{T_0^2}\right| << 1$, the term $\frac{2\beta_2 CL}{T_0^2}$ becomes dominant. Hence

$$\frac{T_1}{T_0}\approx\sqrt{1+\frac{2\beta_2 CL}{T_0^2}}$$

which is greater than 1 for $\beta_2 C > 0$ and less than 1 for $\beta_2 C < 0$.

For finding the optimum length at which the width is minimized, we must differentiate the compressing factor w.r.t. L as the fiber length. The zero-derivation equation is:

$$C\left(1+\frac{\beta_2 CL}{T_0^2}\right)+\left(\frac{\beta_2 L}{T_0^2}\right)=0$$

which yields

$$L_{\mathrm{opt}}=-\frac{T_0^2 C}{\beta_2(1+C^2)}$$

which is valid for $\beta_2 C < 0$. The minimum width is then derived as

$$T_{1,\min}=T_0\frac{1}{\sqrt{1+C^2}}$$

Question 6)

Based on the equation 3.2.8 of the Agrawal's textbook, we have

$$T_{\mathrm{FWHM}}=2\sqrt{\ln 2}T_0$$

hence

$$\begin{aligned}T_0 &= 30\text{ps}\\ L &= 50\text{km}\\ \beta_2 &= -20.4\text{ps}^2/\text{km}\\ C &= 0\end{aligned}$$

and we obtain

$$T_{1,\mathrm{FWHM}}=73.03\text{ps}$$

Question 7)

$$L = 10 \text{ km}$$
$$\sigma_\lambda = 30 \text{ nm}$$
$$D = -80 \text{ ps}/(\text{nm} \cdot \text{km})$$

therefore

$$B_{\max} = 10.42MHz$$

Question 8)

a.

$$\text{Fiber 1}: \ \beta_2 = -31.87\text{ps}^2/\text{km}$$
$$\text{Fiber 2}: \ \beta_3 = 0.0992\text{ps}^3/\text{km}$$

$$\text{Transmitter 1 , Fiber 1}: \ L_{\text{Dispersion}} = 19.9 \text{ km}$$
$$\text{Transmitter 1 , Fiber 2}: \ L_{\text{Dispersion}} = 17502 \text{ km}$$
$$\text{Transmitter 2 , Fiber 1}: \ L_{\text{Dispersion}} = 3486 \text{ km}$$
$$\text{Transmitter 2 , Fiber 2}: \ L_{\text{Dispersion}} = 812717419 \text{ km}$$

b.

$$\text{Transmitter 1 , Fiber 1}: \ L_{\text{Attenuation}} = 95.24 \text{ km}$$
$$\text{Transmitter 1 , Fiber 2}: \ L_{\text{Attenuation}} = 86.96 \text{ km}$$
$$\text{Transmitter 2 , Fiber 1}: \ L_{\text{Attenuation}} = 95.24 \text{ km}$$
$$\text{Transmitter 2 , Fiber 2}: \ L_{\text{Attenuation}} = 86.96 \text{ km}$$

c.
The combinations

$$\text{Transmitter 1 , Fiber 2}$$
$$\text{Transmitter 2 , Fiber 2}$$

lead to maximum optical reach due to both attenuation and dispersion, i.e, when fiber 2 is used.

Question 9)

a. The transmission length should be 95.24 km, just like part c- of the previous question. Since we have

$$25L = 16L_{\text{DCF}}$$

we obtain

$$L = 37.17 \text{ km}$$
$$L_{\mathrm{DCF}} = 58.07 \text{ km}$$

b.

If signal is dropped 20 dB due to total attenuation, we need an amplifier of gain 20 dB to fully compensate for the fiber loss. Hence:

$$G_{\mathrm{dB}} = 20 \implies G = 100$$
$$\mathrm{NF} = 5.5 \implies F = 3.55$$

By using the following relation for ASE noise power spectral density:

$$\sigma^2_{\mathrm{ASE,PSD}} = \frac{1}{2} h\nu_{\mathrm{opt}} G F$$

where $\nu_{\mathrm{opt}} = \frac{c}{\lambda}$, we finally calculate

$$\sigma^2_{\mathrm{ASE,PSD}} = 22.69 \frac{\mu\mathrm{W}}{\mathrm{THz}}$$

Question 10)

The total splice loss is $1 + 1 + 0.2 = 2.2$ dB. Also, the total fiber loss can be given by $50\mathrm{km} \times 0.5 \frac{\mathrm{dB}}{\mathrm{km}} = 25\mathrm{dB}$. Since there are 9 intermediate splices, the total loss imposed on signal is

$$\mathrm{Loss} = 9 \times 2.2 + 25 = 44.8\mathrm{dB}$$

The minimum sensitivity power is $0.3\mu\mathrm{W} \equiv -5.23\mathrm{dBu}$, giving the least launch power as

$$P = -5.23\mathrm{dBu} + 44.8\mathrm{dB} = 39.57\mathrm{dBu} = 9.57\mathrm{dBm} \equiv 9.06\mathrm{mW}$$