Inequality, Business Cycles, and Monetary-Fiscal Policy

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Intro

Introduction

Aiyagari based model with aggregate shocks is the standard framework in Macro to study inequality and policy implications.

Notoriously hard to compute, particularly solving for optimal policies (need entire transition dynamics to obtain welfare change).

This paper:

- Provides a general Aiyagari framework with both monetary and fiscal instruments.
- Develops a perturbation method to efficiently solve the transition paths.
- Solve for the optimal monetary and monetary-fiscal policies with various impulse responses.

Overview of the model

Heterogeneous agents with idiosyncratic labor productivity shocks.

Monopolistic competitive firms with heterogeneous productivity shocks and costly adjustments of prices (*Marco won't like this*).

Aggregate markup shocks and aggregate productivity shocks.

Limited insurance due to incomplete financial market.

Fiscal policy instruments: taxes on labor earnings, dividends and interest income (and associated uniform lump-sum transfers).

Monetary policy instruments: interest rates (and associated uniform lump-sum transfers).

Key insights

With costly adjustments of prices under a representative agent framework (**RANK**), the Ramsey planner balances the trade-off between:

- **Price stability**;
- **Output gap**.

Under heterogeneous agent framework (**HANK**), the planner in addition cares about:

- **Ex-ante redistribution** between households;
- Providing better **ex-post insurance** against consumption fluctuations.

Preview on result: **insurance** channel dominates.

Model

Environment:

Continuum of infinitely lived households indexed by i that solves:

$$\max_{c_{i,t},n_{i,t}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_{i,t}, n_{i,t}\right)$$

$$\begin{aligned}\text{sub. to.} \quad & c_{i,t} + Q_t b_{i,t} = \left(1 - \Upsilon_t^n\right) W_t \epsilon_{i,t} n_{i,t} \\ & + T_t + \left(1 - \Upsilon_t^d\right) d_{i,t} + \left(1 - \Upsilon_t^b\right) \frac{b_{i,t-1}}{1 + \Pi_t}\end{aligned}$$

Agent consumes $c_{i,t}$ and supplies $\epsilon_{i,t} n_{i,t}$ units of effective labor.

Q_t equals the inverse of the gross nominal rate between periods t and $t+1$. $b_{i,t}$ denotes the real value of nominal bonds owned by agent i at end of period t, and $d_{i,t}$ to denote real dividends received from intermediate goods producers at t. $d_{i,t} = s_i D_t$, s_i is fixed over time.

$\Pi_t = \frac{P_t}{P_{t-1}} - 1$ denote the net inflation rate.

Households receive a uniform lump-sum transfer T_t and face a linear tax Υ_t^n on their labor earnings, a tax Υ_t^d on their dividends, and a tax Υ_t^b on their interest income.

Environment:

The government's budget constraint at time t is

$$\bar{G} + T_t + \frac{B_{t-1}}{1+\Pi_t} = \int \left[\Upsilon_t^n W_t \epsilon_{i,t} n_{i,t} + \Upsilon_t^d d_{i,t} + \frac{\Upsilon_t^b b_{i,t-1}}{1+\Pi_t} \right] di + Q_t B_t$$

where $\bar{G}$ is a time-invariant level of non-transfer government expenditures.

Denote $\Upsilon_t \equiv \left(\Upsilon_t^n, \Upsilon_t^d, \Upsilon_t^b\right)$.

Monetary policy instruments is a stochastic process: $\{Q_t, T_t\}_t$, given constant-tax-rate $\Upsilon_t = \hat{\Upsilon}$.

Monetary-fiscal policy is a stochastic process: $\{Q_t, \Upsilon_t, T_t\}_t$.

Environment ctd.:

A final good Y_t is produced by competitive firms that use a continuum of intermediate goods $\{y_t(j)\}_{j\in[0,1]}$ as inputs into a production function

$$Y_t = \left[\int_0^1 y_t(j)^{\frac{\Phi_t - 1}{\Phi_t}} dj\right]^{\frac{\Phi_t}{\Phi_t - 1}}$$

where the elasticity of substitution Φ_t is stochastic. Final good producers take the final good price P_t and the intermediate goods prices $\{p_t(j)\}_j$ as given and solve

$$\max_{\{y_t(j)\}_{j\in[0,1]}} P_t \left[\int_0^1 y_t(j)^{\frac{\Phi_t - 1}{\Phi_t}} dj\right]^{\frac{\Phi_t}{\Phi_t - 1}} - \int_0^1 p_t(j) y_t(j) dj$$

which yields a standard demand function for intermediate goods and a final goods price

$$y_t(j) = \left(\frac{p_t(j)}{P_t}\right)^{-\Phi_t} Y_t$$

$$P_t = \left(\int_0^1 p_t(j)^{1-\Phi_t}\right)^{\frac{1}{1-\Phi_t}}$$

Environment ctd.:

Intermediate goods $y_t(j)$ are produced by monopolists with production functions

$$y_t(j) = \left[n_t^D(j)\right]^\alpha \left[h_t(j)\right]^{1-\alpha}$$

where $n_t^D(j)$ is effective labor hired by firm j and $h_t(j)$ is an intermediate input measured in units of the final good.

Intermediate goods monopolists face downward sloping demand curves and choose prices $p_t(j)$, while bearing quadratic Rotemberg (1982) price adjustment costs $\frac{\psi}{2}\left(\frac{p_t(j)}{p_{t-1}(j)} - 1\right)^2$ measured in units of the final consumption good. Intermediate goods producing firm j chooses prices $\{p_t(j)\}_t$ and factor inputs $\left\{h_t(j), n_t^D(j)\right\}_t$ that solve

$$\max_{\left\{p_t(j), n_t^D(j), h_t(j)\right\}_t} \mathbb{E}_0 \sum_t S_t \left(1 - \Upsilon_t^d\right)$$
$$\left\{\frac{p_t(j)}{P_t} y_t(j) - W_t n_t^D(j) - h_t(j) - \frac{\psi}{2}\left(\frac{p_t(j)}{p_{t-1}(j)} - 1\right)^2\right\},$$

where W_t is the real wage per unit of effective labor.

Environment ctd.:

S_t is a stochastic discount factor (**SDF**) that follows:

$$S_t = S_{t-1} Q_{t-1} \left(1 + \Pi_t\right) / \left(1 - \Upsilon_t^b\right)$$

with $S_{-1} = 1$. In a symmetric equilibrium, $p_\iota(j) = P_\iota, y_\iota(j) = Y_\iota, h_\iota(j) = H_\iota$, and $n_t^D(j) = N_t$ for all j. Market clearing conditions in labor, goods, and bond markets are

$$C_t = \int c_{i,t} di, \quad N_t = \int \epsilon_{i,t} n_{i,t} di, \quad D_t = Y_t - H_t - W_t N_t - \frac{\psi}{2} \Pi_t^2,$$

$$Y_t = N_t^\alpha H_t^{1-\alpha}, \quad \Pi_t = P_t / P_{t-1} - 1$$

$$C_t + \bar{G} = Y_t - H_t - \frac{\psi}{2} \Pi_t^2,$$

$$\int b_{i,t} di = B_t.$$

Environment ctd. (almost there):

There are aggregate and idiosyncratic shocks. Aggregate shocks are a "markup" shock Φ_t and an aggregate productivity shock Θ_t that follow $\mathrm{AR}(1)$ processes

$$\ln \Phi_t = \rho_\Phi \ln \Phi_{t-1} + (1-\rho_\Phi) \ln \bar{\Phi} + \mathcal{E}_{\Phi,t},$$
$$\ln \Theta_t = \rho_\Theta \ln \Theta_{t-1} + (1-\rho_\Theta) \ln \bar{\Theta} + \mathcal{E}_{\Theta,t},$$

where $\mathcal{E}_{\Phi,t}$ and $\mathcal{E}_{\Theta,t}$ are mean-zero random variables that are i.i.d. over time and uncorrelated with each other at all times. Individual productivity $\epsilon_{i,t}$ follows a stochastic process described by

$$\ln \epsilon_{i,t} = \ln \Theta_t + \ln \theta_{i,t} + \varepsilon_{\epsilon,i,t},$$
$$\ln \theta_{i,t} = \rho_\theta \ln \theta_{i,t-1} + \varepsilon_{\theta,i,t},$$

where innovations $\varepsilon_{\epsilon,i,t}$ and $\varepsilon_{\theta,i,t}$ are mean-zero, uncorrelated with each other, and i.i.d. across time.

Competitive equilibrium:

Definition 1: Given initial conditions and a monetary-fiscal policy $\{Q_t, \Upsilon_t, T_t\}_t$, a competitive equilibrium is a stochastic process $\left\{\{c_{i,t}, n_{i,t}, b_{i,t}\}_i, C_t, N_t, B_t, W_t, P_t, Y_t, H_t, D_t, \Pi_t, S_t\right\}_t$ that satisfies:

(i) $\{c_{i,t}, n_{i,t}, b_{i,t}\}_{i,t}$ maximize utility subject to budget constraint and natural debt limits;

(ii) final goods firms choose $\{y_t(j)\}_j$ to maximize its profit;

(iii) intermediate goods producers' prices and factor inputs solve its problem and satisfy $p_t(j) = P_t, y_t(j) = Y_t, h_t(j) = H_t$, and $n_t^D(j) = N_t$ for all j; and

(iv) market clearing conditions are satisfied.

Competitive equilibrium ctd.:

The competitive equilibria can be characterized by feasibility constraints and consumers' and firms' optimality conditions

$$\left(1-\Upsilon_t^n\right) W_t \epsilon_{i,t} u_{c,i,t} = -u_{n,i,t},$$

$$Q_t u_{c,i,t} = \beta \mathbb{E}_t u_{c,i,t+1} \left(1-\Upsilon_{t+1}^b\right) / \left(1+\Pi_{t+1}\right),$$

$$0 = \frac{1}{\psi} Y_t \left[1 - \Phi_t \left(1 - \frac{1}{1-\alpha} \left(\frac{1-\alpha}{\alpha} W_t\right)^{\alpha}\right)\right] - \Pi_t \left(1+\Pi_t\right)$$

$$+\mathbb{E}_t \frac{S_{t+1}}{S_t} \left(\frac{1-\Upsilon_{t+1}^d}{1-\Upsilon_t^d}\right) \Pi_{t+1} \left(1+\Pi_{t+1}\right),$$

$$\frac{1-\alpha}{\alpha} W_t = \frac{H_t}{N_t}$$

and derived agents' budget constraints that,

$$c_{i,t} - T_t - \left(1-\Upsilon_t^d\right) s_i D_t - \frac{\left(1-\Upsilon_t^b\right) b_{i,t-1}}{1+\Pi_t}$$

$$= \left(\frac{u_{n,i,t}}{u_{c,i,t}}\right) n_{i,t} + \mathbb{E}_t \left(\frac{u_{c,i,t+1}}{u_{c,i,t}}\right) \frac{\left(1-\Upsilon_{t+1}^b\right) b_{i,t}}{1+\Pi_{t+1}}$$

Ramsey problem:

A Ramsey planner orders allocations by

$$\mathbb{E}_0 \int \sum_{t=0}^{\infty} \beta^t \vartheta_i u\left(c_{i,t}, n_{i,t}\right) di$$

where ϑ_i are the pareto weights that has an integral equals to one.

Definition 2: Given initial conditions and a time-invariant tax policy, an optimal **monetary policy** chooses Q_t, T_t to bring about a CE allocation that maximizes the planner's objective function.

An optimal **monetary-fiscal policy** chooses Q_t, Υ_t, T_t to bring about a CE allocation that maximizes the planner's objective function.

Such policies are **Ramsey policies** and associated allocations are **Ramsey allocations**.

Computation Strategy (not going into details)

Computation Strategy:

Anmol came and taught a class but I didn't really follow. The idea is the following:

They constructs a stochastic sequence of small-noise expansions along a simulated optimal path. A key step uses functional derivative techniques to characterize how government decisions depend on a high-dimensional state vector that changes over time in response to aggregate shocks.

The small noise expansions of policy functions take the form

$$\tilde{\boldsymbol{X}}(\Omega, \sigma\mathcal{E}; \sigma) = \overline{\boldsymbol{X}} + \sigma\left(\overline{\boldsymbol{X}}_{\mathcal{E}}\mathcal{E} + \overline{\boldsymbol{X}}_{\sigma}\right) + \mathcal{O}\left(\sigma^2\right)$$

and

$$\tilde{\boldsymbol{x}}(\boldsymbol{z}, \Omega, \sigma\varepsilon, \sigma\mathcal{E}; \sigma) = \overline{\boldsymbol{x}}(\boldsymbol{z}) + \sigma\left(\overline{\boldsymbol{x}}_{\boldsymbol{\varepsilon}}(\boldsymbol{z})\boldsymbol{\varepsilon} + \overline{\boldsymbol{x}}_{\mathcal{E}}(\boldsymbol{z})\mathcal{E} + \overline{\boldsymbol{x}}_{\sigma}(\boldsymbol{z})\right) + \mathcal{O}\left(\sigma^2\right).$$

where $\boldsymbol{X}$ is the aggregate policy function and $\boldsymbol{x}$ is the individual policy function.

Comparison of results

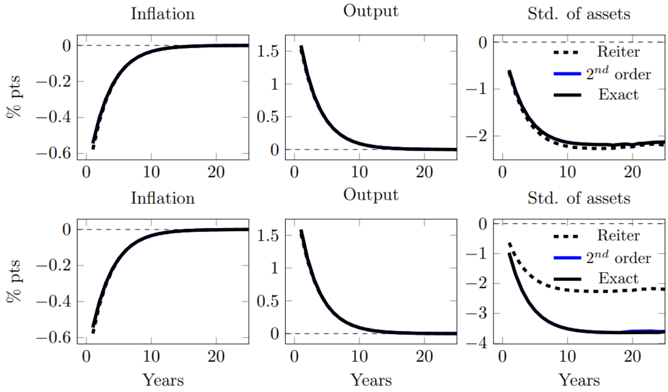

Figure I: Comparisons of impulse responses to a 1% TFP shock at $t = 1$ in the top panel and $t = 250$ in the bottom panel across approximation methods. The bold lines are are the exact solution (black) and our method applied to second-order (blue). The dashed black line are responses under the Reiter method.

Comparison of results, ctd.

Maximum Errors (%)	Ind. Consumption	Agg. Output	Inflation	Interest Rate
2^{nd} Order				
$\gamma = 1, \sigma_\epsilon = 0.50$	0.0039	4.2e-6	3.1e-5	4.3e-5
$\gamma = 1, \sigma_\epsilon = 0.75$	0.0134	2.6e-5	1.5e-4	2.2e-4
$\gamma = 1, \sigma_\epsilon = 1.00$	0.0328	8.2e-5	4.9e-4	6.9e-4
$\gamma = 3, \sigma_\epsilon = 0.5$	0.0453	0.0011	0.0024	0.0034
Reiter-based				
$\gamma = 1, \sigma_\epsilon = 0.50$	0.0374	0.0616	0.0337	0.0505
$\gamma = 1, \sigma_\epsilon = 0.75$	0.0466	0.0610	0.0335	0.0501
$\gamma = 1, \sigma_\epsilon = 1.00$	0.0492	0.0602	0.0329	0.0493
$\gamma = 3, \sigma_\epsilon = 0.5$	0.0896	0.2252	0.1327	0.1991

TABLE I: Percentage errors in policy functions in response to an one standard deviation unanticipated shock to aggregate TFP at date $t = 1$. The values reported are the maximum errors across states (b, ϵ) and time t relative to the true solution.

Main results

Calibrations

They calibrate the model to SCF 2007. They chose pareto weights so that average level of taxes are similar to US data.

TABLE II: FIT OF THE INITIAL DISTRIBUTION

Moments	
Fraction of pop. with zero equities	30%
Std. share of equities	2.63
Std. bond	6.03
Gini of financial wealth	0.82
Std. ln wages	0.80
Corr(share of equities, ln wages)	0.40
Corr(share of equities, bond holdings)	0.62
Corr(bond, ln wages)	0.33

Notes: Moments correspond to SCF 2007 wave after scaling wages, equity holdings, and debt holdings by the average yearly wage in our sample. The share of equities refers to the ratio of individual equity holdings to the total in our sample; the weighted sum of shares equals one. Financial wealth is defined as the sum of nominal and real claims.

Main results

TABLE III: RAMSEY ALLOCATION: MOMENTS

	RANK					HANK				
	Std.	Correlations				Std.	Correlations			
	Dev(%)	i_t	Π_t	W_t	$\ln Y_t$	Dev(%)	i_t	Π_t	W_t	$\ln Y_t$
Nominal Rate i_t	0.87	1				1.82	1			
Inflation Π_t	0.03	-0.01	1			0.46	-0.94	1		
Labor Share W_t	1.18	-0.09	-0.32	1		2.13	-0.78	0.78	1	
Log Output $\ln Y_t$	0.92	-0.98	-0.09	0.24	1	0.88	-0.31	0.10	0.12	1

Notes: Moments are computed using allocations under RANK (left) and HANK (right) optimal monetary policies.

Main results

TABLE IV: WELFARE DECOMPOSITION

	Efficiency	Redistribution	Insurance
Baseline			
(a) Optimal monetary policy	-122	9	213
(b) Optimal monetary and fiscal policy	-16	1	115
Extensions			
(c) Liquidity Frictions	-78	-4	182
(d) Mutual Fund	-154	-12	266
(e) Heterogeneous labor income exposures	-327	-7	334
Alternative Pareto Weights			
(f) High Labor Tax	-180	-125	405
(g) High Bond Tax	-115	52	163
(h) High Dividend Tax	-165	-1	266

Notes: We decompose welfare differences between optimal HANK and optimal RANK policies using the Bhandari et al. (2021) procedure. For all cases, the point of comparison (optimal RANK policy) is set so that expected levels of policy variables equal their optimal HANK counterparts in the absence of aggregate risk and stochastic processes for deviations of the policy variables from their means are optimal in the representative agent version. Lines (a) and (b) report results for our baseline calibration applied to both monetary and monetary-fiscal policies. Lines (c), (d), (e) report our decomposition of the optimal monetary policy for extensions that we describe in sections 6.2, 6.3, 6.4, respectively. Lines (f), (g), (h) consider alternative specifications of Pareto weights discussed in section 6.1.

Main results

With a positive aggregate markup shock, optimal monetary policy is:

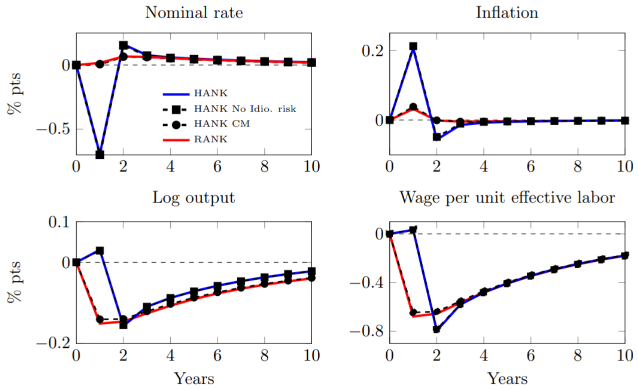

Figure II: Optimal monetary response to a markup shock. The bold blue and red lines are the calibrated HANK and RANK responses respectively. The dashed black lines with squares and circles are responses under HANK with idiosyncratic shocks shut down and with complete markets, respectively.

Main results

With a positive aggregate markup shock, optimal monetary-fiscal policy is:

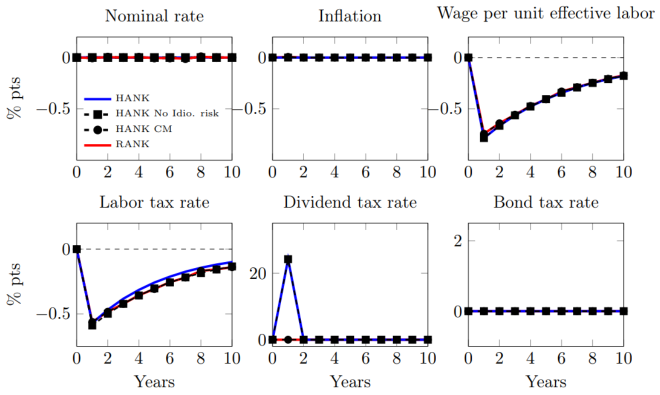

Figure III: Optimal monetary-fiscal response to a markup shock. The bold blue and red lines are the calibrated HANK and RANK responses, respectively. The dashed black lines with squares and circles are responses under HANK with idiosyncratic shocks shut down and with complete markets, respectively.

Main results

With a negative aggregate TFP shock, optimal monetary policy is:

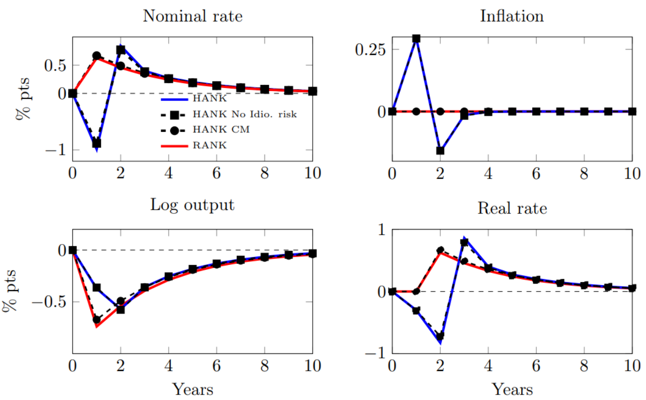

Figure IV: Optimal monetary response to a TFP shock. The bold blue and red lines are the calibrated HANK and RANK responses, respectively. The dashed black lines with squares and circles are responses under HANK with idiosyncratic shocks shut down and with complete markets, respectively.

Conclusions

With a negative aggregate TFP shock, optimal monetary-fiscal policy is:

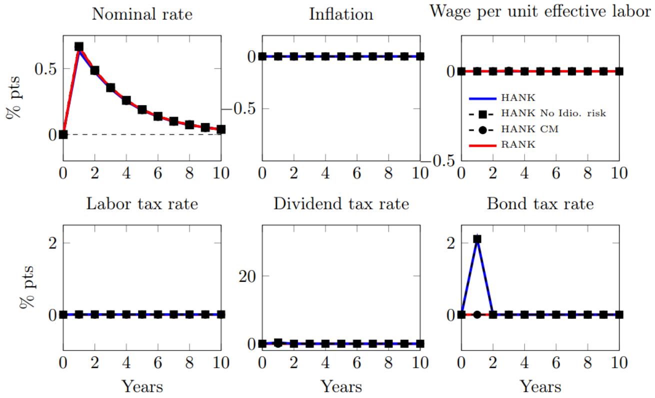

Figure V: Optimal monetary-fiscal response to a TFP shock. The bold blue and red lines are calibrated HANK and RANK responses, respectively. The dashed black lines with squares and circles are responses under HANK with idiosyncratic shocks shut down and with complete markets, respectively.

Conclusions

Conclusions

This paper develops a novel computation method to efficiently solve optimal policies in a generalized Aiyagari framework with aggregate risks.

Heterogeneity adds an insurance motive that quantitatively dominates the motives to stabilize nominal prices that have typically driven New Keynesian policy prescriptions.

The paper also provides several sensitivity tests and some interesting extensions.