Organizational Equilibrium with Capital

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Question

- Time inconsistency is a pervasive issue
  - taxation, government debt, consumption-saving problem, monetary policy, ...
- Two Benchmarks:
  - Markov equilibrium
  - Sequential equilibrium/sustainable plan

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  - Interesting comparative statics
  - Outcome determined by fundamentals
  - ... but can be largely improved upon

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- Markov equilibrium:
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  - Outcome determined by fundamentals
  - ... but can be largely improved upon
- Sequential equilibrium:
  - Can often attain very good outcomes (folk theorem)
  - Can also attain very bad outcomes (folk theorem again)
  - Relies on self-punishment as a threat
  - Weak predictions (big set of equilibria)

Our View

- Good institutions and social norms do not evolve overnight
- Collaboration across cohorts of decision makers builds slowly
- It probably also erodes slowly
- Look for equilibrium concept that captures this, and addresses shortcomings of Markov & Best Sequential Eq.

Equilibrium Properties

- Compare with Markov equilibrium
  - payoff only depends on state variables, like Markov equilibrium
  - action can depend on history, different from Markov equilibrium
- Compare with sequential equilibrium
  - no self-punishment
  - Refinement I: same continuation value on or off equilibrium path
  - Refinement II: no one wants to deviate and wait for a restart of the game
- New issues with state variables
  - how to induce stationary environment
  - Player preferences no longer purely forward-looking (new role for no-delaying condition)

Quantitative Findings

- Steady state
  - allocation is close to Ramsey outcome, much better than Markov equilibrium
- Transition
  - allocation starts similar to Markov, converges to similar to Ramsey

Related Literature

- Markov equilibrium and GEE
  - Currie and Levine (1993), Bassetto and Sargent (2005), Klein and Ríos-Rull (2003), Klein, Quadrini and Ríos-Rull (2005), Krusell and Ríos-Rull (2008), Krusell, Kuruscu, and Smith (2010), Song, Storesletten and Zilibotti (2012)
- Sustainable plan
  - Stokey (1988), Chari and Kehoe (1990), Abreu, Pearce and Stacchetti (1990), Phelan and Stacchetti (2001)
- Quasi-geometric discounting growth model
  - Strotz (1956), Phelps and Pollak (1968), Laibson (1997), Krusell and Smith (2003), Chatterjee and Eyigungor (2015), Bernheim, Ray, and Yeltekin (2017), Cao and Werning (2017)
- Refinement of subgame perfect equilibrium
  - Farrell and Maskin (1989), Kocherlakota (1996), Prescott and Ríos-Rull (2005), Nozawa (2014), Ales and Sleet (2015)

Plan

1. An example: a growth model with quasi-geometric discounting
2. Application to Foreign Trade
3. (General definition and properties)

Part I: A Growth Model

The Environment

- Preferences: quasi-geometric discounting

$$\Psi_t = u(c_t) + \delta \sum_{\tau=1}^{\infty} \beta^{\tau} u\left(c_{t+\tau}\right)$$

  - period utility function $u(c) = \log c$
  - $\delta = 1$ is the time-consistent case

- Technology

$$f(k_t) = k_t^{\alpha}, \qquad\qquad k_{t+1} = f(k_t) - c_t.$$

Benchmark I: Markov Perfect Equilibrium

- Take future $g(k)$ as given

$$\max_{k'} \; u[f(k) - k'] + \delta\beta\Omega(k'; g)$$

cont. value: $$\Omega(k; g) = u[f(k) - g(k)] + \beta\Omega[g(k); g]$$

- The Generalized Euler Equation (GEE)

$$u_c = \beta u'_c \left[\delta f'_k + (1-\delta)\, g'_k\right]$$

- The equilibrium features a constant saving rate

$$k' = \frac{\delta\alpha\beta}{1 - \alpha\beta + \delta\alpha\beta}\, k^\alpha = s^M\, k^\alpha$$

Benchmark II: Ramsey Allocation with Commitment

- Choose all future allocations at period 0

$$\max_{k_1} \; u[f(k_0) - k_1] + \delta\beta\Omega(k_1)$$

cont. value: $$\Omega(k) = \max_{k'} \; u[f(k) - k'] + \beta\Omega(k')$$

- The sequence of saving rates is given by

$$s_t = \begin{cases} s^M = \frac{\alpha\delta\beta}{1-\alpha\beta+\delta\alpha\beta}, & t = 0 \\ \\ s^R = \alpha\beta, & t > 0 \end{cases}$$

- Steady state capital in Markov equilibrium is lower than Ramsey

$$s^M < s^R$$

Elements of Organization Equilibrium: Action Space

- Use saving rate as player t's action; equilibrium outcome is a sequence of saving rates $\{s_0, s_1, s_2, \ldots\}$

- Note $s \in [0, 1]$ always feasible, no matter what k is

- Given an initial capital k_0, the proposal induces a sequence of capital

$$
\begin{aligned}
k_1 &= s_0 k_0^\alpha \\
k_2 &= s_1 k_1^\alpha = k_0^{\alpha^2} s_1 s_0^\alpha \\
&\vdots \\
k_t &= k_0^{\alpha^t} \Pi_{j=0}^{t-1} s_j^{\alpha^{t-j-1}}
\end{aligned}
$$

Value Function and Separability

- The lifetime utility for player t is

$$
\begin{aligned}
&\underbrace{U(k_t, s_t, s_{t+1}, \ldots)}_{\text{total payoff}} \\
&= \log[(1-s_t)k_t^{\alpha}] + \delta \sum_{j=1}^{\infty} \beta^j \log\left[(1-s_{t+j})k_{t+j}^{\alpha}\right] \\
&= \frac{\alpha(1-\alpha\beta+\delta\alpha\beta)}{1-\alpha\beta} \log k_t + \log(1-s_t) \\
&+ \delta \sum_{j=1}^{\infty} \beta^j \log\left[(1-s_{t+j})\Pi_{\tau=0}^{j-1} s_{t+\tau}^{\alpha^{j-\tau}}\right] \\
&\equiv \underbrace{\phi \log k_t}_{\text{Contribution of the state}} + \underbrace{V(s_t, s_{t+1}, \ldots)}_{\text{action payoff}}
\end{aligned}
$$

Organizational Equilibrium

Proposition

A sequence $\{\bar{s}_t\}_{t=0}^{\infty}$ that satisfies the following properties is an organizational equilibrium:

1. *No-restarting:*

$$V(\bar{s}_t, \bar{s}_{t+1}, \bar{s}_{t+2}, ...) = \bar{V} \quad \forall t \geq 0;$$

2. *Optimality: No other sequence satisfying no-restarting achieves a higher constant value;*
3. *No-delay:*

$$V(\bar{s}_0, \bar{s}_1, \bar{s}_2, ...) \geq \max_s V(s, \bar{s}_0, \bar{s}_1, ...).$$

- It is a proposition, not a definition, because we will define OE in terms of a game
- Proposition in the paper has some assumptions, satisfied in our example

Where Do these Properties Come From?

- No-restarting:
  - akin to symmetry in Kocherlakota
  - From renegotiation proofness
  - If equilibrium is too generous to player 0, player 1 wants to forget the past.

- Optimality: no waste

- No-delay: who should start this game?
  - Comes from any ambiguity to the answer.
  - Many revolutions talk about "forgetting the past"
  - "This time's different"
  - Time 0 could be any time, and player 0 should not have an incentive to wait it out

Is the Ramsey Outcome an Organizational Equilibrium?

- Imagine the initial agent with k_0 proposes $\{s^M, s^R, s^R, \ldots\}$, which implies

$$k_1 = s^M k_0^{\alpha}$$

- By following the proposal, the next agent's payoff is

$$U\left(k_1, s^R, s^R, s^R, \ldots\right) = \phi \log k_1 + V\left(s^R, s^R, s^R, \ldots\right)$$

- By copying the proposal, the next agent's payoff is

$$\begin{aligned} U\left(k_1, s^M, s^R, s^R, \ldots\right) &= \phi \log k_1 + V\left(s^M, s^R, s^R, \ldots\right) \\ &> \phi \log k_1 + V\left(s^R, s^R, s^R, \ldots\right) \end{aligned}$$

- Copying is better than following, Ramsey outcome cannot be implemented (no-restarting fails)

Can a Constant Saving Rate be Implemented?

- Consider $\{s, s, s \ldots\}$

- By following the proposal, the payoff for agent in period t is

$$U(k_t, s, s, \ldots) = \phi \log k_t + V(s, s, \ldots)$$

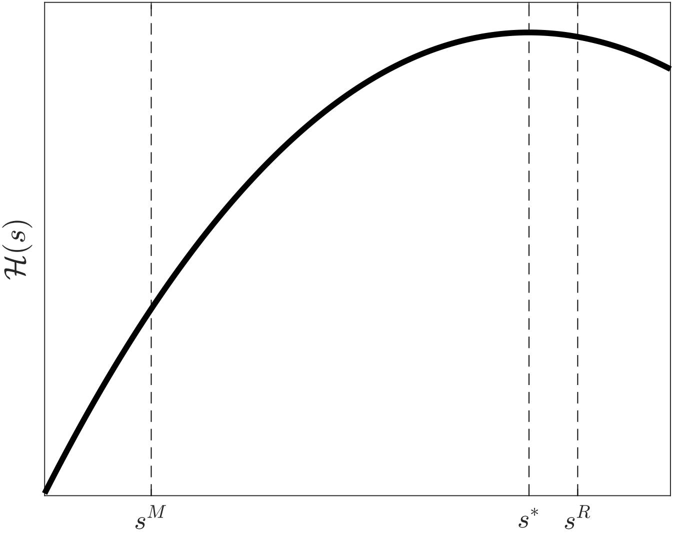
where

$$V(s, s, \ldots) \equiv \mathcal{H}(s) = \left(1 + \frac{\beta\delta}{1-\beta}\right) \log(1-s) + \frac{\delta\alpha\beta}{(1-\alpha\beta)(1-\beta)} \log(s$$

- No-restarting is fine
- Optimality: pick

$$s^* = \operatorname{argmax} \mathcal{H}(s)$$

Optimal Constant Saving Rate



$$s^M < s^* < s^R$$

Can $\{s^*, s^*, \ldots\}$ be Implemented?

- No-delay fails:

- Player 0 prefers to choose s^M, and wait the next to start $\{s^*, s^*, \ldots\}$

$$\begin{aligned} U\left(k_0, s^M, s^*, s^*, \ldots\right) &= \phi \log k_0 + V\left(s^M, s^*, s^*, \ldots\right) \\ &> \phi \log k_0 + V\left(s^*, s^*, s^*, \ldots\right) \end{aligned}$$

- But, something else can be implemented, which converges to s^*

Construct the Organizational Equilibrium

- Look for a sequence of saving rates $\{s_0, s_1, \ldots\}$

- Every generation obtains the same $\overline{V}$

$$V(s_t, s_{t+1}, \ldots) = V(s_{t+1}, s_{t+2}, \ldots) = \overline{V}$$

which induces the following difference equation

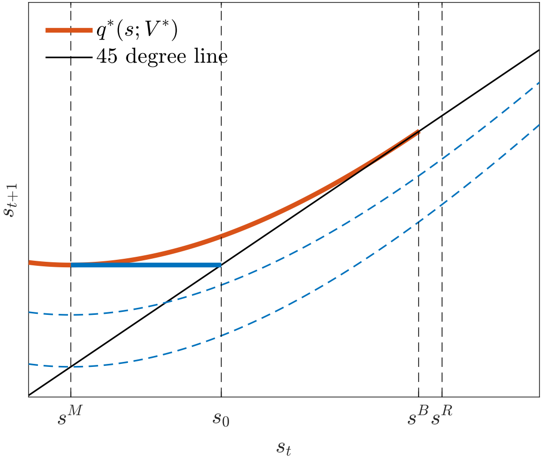
$$\beta(1-\delta)\log(1-s_{t+1}) = \frac{\delta\alpha\beta}{1-\alpha\beta}\log s_t + \log(1-s_t) - (1-\beta)\overline{V}$$

- We call this difference equation as the proposal function

$$s_{t+1} = q(s_t; \overline{V})$$

- The maximal $\overline{V}$ and an initial s_0 are needed to determine $\{s_\tau\}_{\tau=0}^{\infty}$

Determine V^*



- As $\overline{V}$ increases, the proposal function $q(s;\overline{V})$ moves upwards
- The highest $\overline{V} = V^*$ is achieved when $q(s;\overline{V})$ is tangent to the 45 degree line (at s^*)

Determine the Initial Saving Rate s_0

- The first agent should have no incentive to delay the proposal

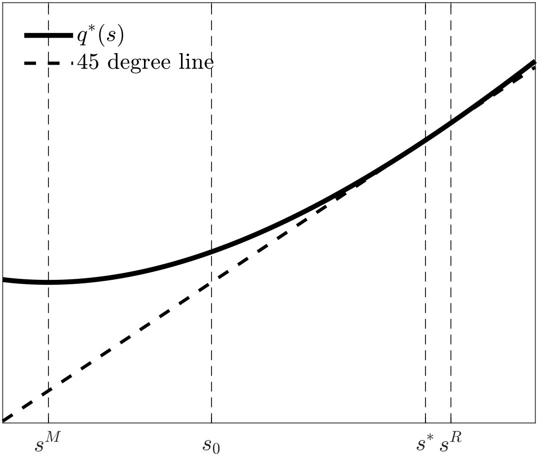
$$\max_s V(s, s_0, s_1, s_2, \ldots) = V(s^M, s_0, s_1, s_2, \ldots)$$

- s_0 has to be such that

$$V^* = V(s_0, s_1, s_2, \ldots) \geq V(s^M, s_0, s_1, s_2, \ldots)$$

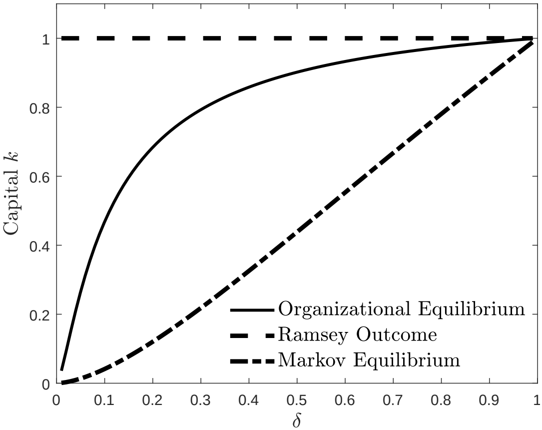
- We select $s_0 = q^*\left(s^M\right)$, which yields the highest welfare for period $t+1$

Transition Dynamics



- The equilibrium starts from s_0, and monotonically converges to s^*.

Comparison: Steady State



- Organizational equilibrium is much better than the Markov equilibrium

Part III: Trade Policy

Setup

- Two countries, home and foreign
- Two tradeable intermediate goods, 1 and 2
- One final good
- Two units of hands-to-mouth households per country, each unit has one unit of labor usable in one of the sectors (labor immobile across sectors and countries)
- A group of capitalists making saving decisions

Technology

- Home country in sector i

$$A_i K_t^{1-\alpha} l_{it}^{1-\alpha} k_{it}^{\alpha}$$

- $A_1 > A_2$
- Foreign: symmetric (A_1 TFP of intermediate 2)
- Final good (can be consumed or invested as capital):

$$y_t = [0.5^{1-\rho} m_{1t}^{\rho} + 0.5^{1-\rho} m_{2t}^{\rho}]^{\frac{\rho-1}{\rho}}$$

Government Policy

- A tariff τ_t on imports
- Study cooperative solution across the two countries

Preferences

- Workers:

$$\sum_{t=0}^{\infty} \beta^t \log c_{it}$$

- Capitalists:

$$\sum_{t=0}^{\infty} \beta^t \frac{c_{it}^{1-\sigma}}{1-\sigma},$$

  $\sigma < 1$

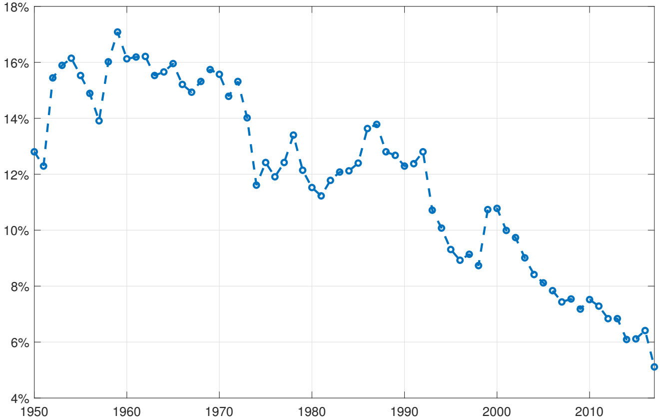
- Government:

$$U_t \equiv ((1-\theta)\log c_{1t} + \theta \log c_{2t}) + \beta E_t U_{t+1}$$

Time Inconsistency

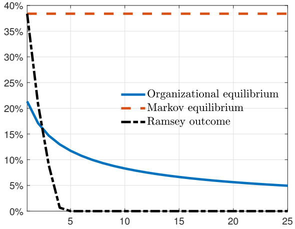
- A tariff protects the wages of sector-2 workers in the home country (and sector-1 workers in the foreign country)
- A tariff discourages saving, hurts everybody in the long run

World Average Tariff

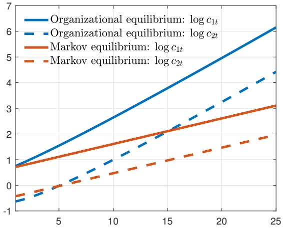


Source: Antras (2020)

Comparing Different Equilibria



Tariff in Various Equilibria



Consumption of Workers in Various Equilibria

Introducing uncertainty (in progress)

- Shock to government preferences:

$$U_t \equiv ((1-\theta_t)\log c_{1t} + \theta_t \log c_{2t}) + \beta E_t U_{t+1}$$

- High $\theta_t \Longrightarrow$ higher tariff
- Gradual return to lower tariffs

Other Applications

- Climate change
- Capital-income taxation

Separable Economies

- Most economies do not satisfy separability condition
- Our strategy: use local approximations
- Linear or second-order approximation
  - satisfies separability
  - choose approximating point so that it's the steady state implied by OE

Conclusion

- New equilibrium concept
- Suitable for positive analysis of gradual policy transition under time inconsistency
- Easy to compute

Part II: Organizational Equilibrium for Weakly Separable Economies

General Definition: Game of Perfect Information

An infinite sequence of decision makers is called to act

- state $k \in K$
- action $a \in A$
- state evolves $k_{t+1} = F(k_t, a_t)$
- Player t preferences: $U(k_t, a_t, a_{t+1}, a_{t+2}, \ldots)$

Separability Assumption

Assumption

1. *At any point in time t, the set A is independent of the state k_t*
2. *U is weakly separable in k and in $\{a_s\}_{s=0}^{\infty}$*

$$U(k, a_0, a_1, a_2, \ldots) \equiv v(k, V(a_0, a_1, a_2, \ldots)).$$

   and such that v is strictly increasing in its second argument.

3. *Technical stuff: A is compact, convex, V is continuous and quasiconcave...*

On the Choice of Actions

- Weak separability and state independence of A depend on the specification of the action set
- Example: hyperbolic discounting. If the choice is c, feasible actions depend on k
- So, sometimes a problem may look nonseparable, but may become separable by rescaling actions appropriately

Requirements

Look for Subgame-Perfect Equilibria that satisfy:

1. State Independence: the strategy followed by any player is independent of the state k
2. No-restarting and optimality: Equilibria are symmetric, that is, the action payoff is independent of the past. Best among symmetric eq.
3. No Delay: Restarting the strategy profile from period 0 is a sufficient deterrent against any deviation:

$$\bar{V} = V(a_{0,\sigma}, a_{1,\sigma}, a_{2,\sigma}, ...) \geq V(a, a_{0,\sigma}, a_{1,\sigma}, a_{2,\sigma}, ...).$$

Definition

An Organizational Equilibrium is the outcome of any subgame perfect equilibrium that satisfies the requirements above.

Existence Results

- An optimally symmetric state-independent equilibrium exists
- If
$$V(a_0, a_1, a_2, ...) \equiv \widetilde{V}(a_0, \widehat{V}(a_1, a_2, \ldots)),$$
then an optimally symmetric state-independent equilibrium that satisfies no delay exists.

Organizational Equilibrium (OE) vs. Subgame-Perfect Equilibrium

1. OE is the equilibrium path of a sub-game perfect equilibrium
2. It can be implemented through various strategies. Examples:
    - restart from the beginning when someone deviates
    - use difference equation to make each player indifferent between deviating and following the equilibrium strategy (over a range)

Properties

- A sequence of actions satisfying no-restarting, optimality and no-delay is an organizational equilibrium

- Assume that continuation utility is recursive:

$$\widehat{V}(a_1, a_2, ...) = W(a_1, \widehat{V}(a_2, a_3, ...))$$

  Then:
  - OE admits a recursive structure

$$v_{t+1} = g(v_t)$$

  - Equilibrium converges to the best constant allocation $(\max V(a, a, a, ...))$
  - Convergence is not immediate (except in degenerate cases)

Alternative Game

- Record keeping not immediately possible: players do not observe past actions
- Becomes possible at random time $\hat{t}$ (not known)
- From time $\hat{t}$ on, player t chooses a_t and ρ_t:
  - $\rho_t = H$: Hide history from the past. Future players do not observe past actions.
  - $\rho_t = S$: Start record keeping. Future players only observe a_t.
  - $\rho_t = C$: Continue record keeping. Future players only observe history from last restart

Equivalence: Justifying No-Delay

Proposition

If

$$U(k, a_0, a_1, a_2, \ldots) \equiv \bar{v}(k) V(a_0, a_1, a_2, \ldots)) + \bar{\bar{v}}(k).$$

a state-independent sequential equilibrium which is optimally symmetric from $\hat{t}$ on satisfies no-delay

Intuition: can always pretend that $\hat{t}$ has not happened yet.