Fiscal Theory of the Price Level

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Our Lab Economy

- Will need a common framework to think about several distinct policies
- Will build from explicit microfoundations, to understand incentives
- Want to talk about monetary/fiscal policy, will need:
  - A motive for the existence of money
  - Taxes
  - Government bonds

Notable elements from which we abstract

- Heterogeneity
- Nominal frictions (e.g., Phillips curve)

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- Heterogeneity
- Nominal frictions (e.g., Phillips curve)
- Are these elements important?
- For some questions, very important (e.g. cost of inflation, price impact of money increase)
- ... but not for those that we will ask:
  - Are monetary and fiscal policy connected?
  - How is the price level determined?

Our lab economy: commodity space

- Time: discrete and infinite: $t = 0, 1, 2, \ldots;$
- Large number ("continuum") of identical "islands;"
- A single good per period and island;
- "Money," a useless object

Agents

- Large number (“continuum”) of identical households;
- Monetary authority (CB, for “central bank”)
- Fiscal authority (“Treasury”)
- CB + Treasury = “government”

Household preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(c_{1t}) + c_{2t}] \tag{1}$$

- c_{1t} and c_{2t} will be acquired from different islands;
- We will call goods 1 “cash goods” and goods 2 “credit goods”
- Notice special assumption: quasilinear preferences (linear in credit goods)
- Assume also $\lim_{c_{1t} \to 0} u'(c_{1t}) = \infty$, $\lim_{c_{1t} \to \infty} u'(c_{1t}) = 0$ (“Inada” conditions)
- Last two assumptions simplify the algebra

Government behavior

- Will not describe what motivates government
- Will consider various strategies, implications

Technology, markets, information: First part of period

- A sunspot shock s_t and a potential fiscal shock τ_t are realized
- Each household has a home island, where it starts each period with y_t units of the good; the good cannot be stored
- Money can be produced for free by the monetary authorities, perfectly storable
- Asset markets open:
  - households and government trade money and nominally risk-free debt,
  - households trade state-contingent debt (in zero net supply),
  - Treasury levies taxes (in money): T_t
- Note: time-t variables are adapted to $\{s_s, y_s, \tau_s\}_{s=0}^{t}$.

Technology, markets, information: Second part of period

- Households divide: shopper, worker
- Worker stays on island, sells goods to others, buys credit goods
- Can promise to settle payment at beginning of next period
- Shopper travels to another island where she is anonymous
- Shopper buys cash good, but needs money

Household flow budget constraint

$$B_{t-1} + M_{t-1} + P_{t-1}(y - c_{1\,t-1} - c_{2\,t-1}) + A_t - T_t \geq \frac{B_t}{1+R_t} + M_t + E_t(z_{t,t+1}A_{t+1}) \quad (2)$$

- M_t: (nominal) money holdings (currency unit: "dollar")
- B_t: (nominal) debt (number of dollars promised at t to be paid at $t+1$)
- A_{t+1}: (nominal) state-contingent debt (number of dollars promised at t to be paid at $t+1$, contingent on $t+1$ shocks)
- P_t: price level in period t
- R_t: nominal interest rate between period t and $t+1$
- $z_{t,t+1}$: state-price deflator between periods t and $t+1$

Some convenient definitions

- Nominal wealth coming into the period:

$$W_t := B_{t-1} + M_{t-1} + P_{t-1}(y - c_{1\,t-1} - c_{2\,t-1}) + A_t - T_t$$

- Multi-period asset-pricing kernel:

$$z_{0,0} := 1, z_{0,t} := \prod_{s=0}^{t-1} z_{s,s+1}, t > 0$$

$$z_{t,s} := z_{0,s}/z_{0,t}, s \geq t$$

- Will take W_0 as an exogenous initial condition

Cash-in-advance constraint

$$M_t \geq P_t c_{1t} \tag{3}$$

Note: it implies $M_t \geq 0$

No-Ponzi condition

$$W_t \geq -\limsup_{n\to\infty} \sum_{s=t}^{n} E_t[z_{t,s+1}(P_s y_s - T_{s+1})]$$

Will mostly consider "sane" policies where we can replace the limsup with a regular limit.

Government budget constraint

$$B_{t-1}^S + M_{t-1}^S - T_t = \frac{B_t^S}{1+R_t} + M_t^S$$

- B_t^S: bonds supplied by government
- M_t^S: money supplied by government

Does gov't have a no-Ponzi condition?

First-order conditions

Go to equilibrium definition

First-order conditions:

- Cash goods:

$$u'(c_{1t}) = (\beta E_t \lambda_{t+1} + \mu_t) P_t, \quad t \geq 0$$

- Credit goods:

$$1 = \beta E_t \lambda_{t+1} P_t, \quad t \geq 0$$

- Money:

$$\lambda_t = \mu_t + \beta E_t \lambda_{t+1}, \quad t \geq 0$$

- Government Bonds:

$$\frac{\lambda_t}{1 + R_t} = \beta E_t \lambda_{t+1}, \quad t \geq 0$$

- State-Contingent Bonds:

$$\lambda_t z_{t,t+1} = \beta \lambda_{t+1}, \quad t \geq 0$$

Transversality condition

One more necessary condition ($t \geq 0$):

$$E_t\left[\liminf_{T\to\infty} \beta^T \left[\lambda_T\left(W_T - \limsup_{n\to\infty} \sum_{s=T}^{n} E_t[z_{T,s+1}(P_s y_s - T_{s+1})]\right)\right]\right] = 0$$

- Weitzman, *Management Science*, 1973 (deterministic case)
- Coșar and Green, *Macroeconomic Dynamics*, 2016 (stochastic case)

The Friedman distortion

$$u'(c_{1t}) = 1 + R_t, \quad t \geq 0 \tag{4}$$

$$R_t > 0 \Longrightarrow M_t = P_t c_{1t}, \quad t \geq 0 \tag{5}$$

- Money is free to produce
- When $R_t > 1$, gov't charges for it
- Consumption tilted towards credit
- Only source of inflation cost in this model
- Cost related to expected inflation

The Fisher equation

$$1 = E_t\left[\beta(1 + R_{t+1})\frac{P_t}{P_{t+1}}\right] \quad t \geq 0 \tag{6}$$

- Positive relation between interest rates and inflation
- Holds across most models (and data) in the long run
- Here, in the short run as well

Gov't policy regime

- Description of competitive equilibrium cannot be completed without knowing what gov't does
- Will explore various options

A pure money supply rule

- Set $B_t^S = 0$;
- Set $M_t^S = (1+q)M_{t-1}^S$;
- Set $T_t = M_{t-1}^S - M_t^S$.

Equilibria under a money supply rule: the log case

Suppose $u(c_{1t}) = \log c_{1t}$, $q > \beta - 1$, use (4), (5), and (6):

- $$c_{1t} = \frac{\beta M_t}{M_{t+1}} = \frac{\beta}{1+q}, \quad t \geq 0$$
- $$\frac{P_{t+1}}{P_t} = \frac{M_{t+1}}{M_t} = 1 + q, \quad t \geq 0$$
- $$P_0 = \frac{M_0(1+q)}{\beta}$$
- Unique equilibrium, price level pinned down

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- We got control of prices using just money supply!

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- More problems:
  - Fiscal policy is more important than it seems
  - Works fine for log, but what about other preferences?

Power utility and money supply rules

When $\sigma < 1$ and $q \geq 0$ (most plausible case):

- SS equilibrium
- Continuum of equilibria where P_0 starts above SS, $P_t \to \infty$, so $c_{1t} = \bar{M}/P_t \to 0$
- No guarantee that money will have value!

Detail for $\sigma \neq 1$

A different way of running monetary policy

- Since the mid-1980s, central banks only use interest rates as the main policy tool
- Caveat: until the great recession, they did manage money on a day-to-day basis
- 2nd caveat: they now use QE too ("bonds-in-advance?")

The simplest case: A fixed interest rate peg

- New specification of monetary-fiscal policy:
- Central bank sets $R_t = \bar{R}$ in every period.
- Gov't budget constraint:

$$B^S_{t-1} + M^S_{t-1} - T_t = \frac{B^S_t}{1+\bar{R}} + M^S_t$$

Infinitely elastic supply for M^S_t and B^S_t (but sum of two is set)

Computing the set of equilibria under an interest rate peg (Sargent-Wallace, 1975)

- Friedman distortion:

$$u'(c_{1t}) = 1 + \bar{R} \implies \text{ get } c_{1t}!$$

- Fisher equation:

$$1 = \beta(1 + \bar{R})E_t\left(\frac{P_t}{P_{t+1}}\right)$$

- Cash-in-advance (if $\bar{R} > 0$): $M_t = P_t c_{1t}$

The good news

- The real allocation (consumption of cash vs. credit goods) pinned down
- Expected (inverse) inflation pinned down

The bad news

- For now, no way to pin down P_0
- Why is this?

$$M_0 = P_0 c_{10}$$

  and $M_0 = M_0^S$, but M_0^S can be anything
- Price level is "indeterminate"
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- ... but they would in a richer model
... what about our old friend, the transversality condition? [Suspense]

Taylor rules

- So far, interest rate was unconditional commitment
- What if the interest rate responds to the past?
- E.g., to past inflation?
- Note: look only at deterministic equilibria, but we have sunspot equilibria every time there are multiple deterministic equilibria

Taylor rules

- Consider

$$R_{t+1} = \frac{\bar{\pi}^{1-\alpha}}{\beta} \left(\frac{P_t}{P_{t-1}} \right)^{\alpha} - 1, \tag{7}$$

  where $\bar{\pi}$ is some target inflation rate
- Define $\pi_t := P_t / P_{t-1}$. Substitute (7) into Fisher equation:

$$\pi_{t+1} = \bar{\pi}^{1-\alpha} (\pi_t)^{\alpha} .$$

- Steady state:

$$\pi^{SS} = \bar{\pi}^{1-\alpha} \left(\pi^{SS} \right)^{\alpha} \implies \quad \pi^{SS} = \bar{\pi}$$

The Taylor principle

- Are there equilibria outside of SS?
- Take logs:

$$(\log \pi_{t+1} - \log \bar{\pi}) = \alpha(\log \pi_t - \log \bar{\pi})$$

- If $|\alpha| < 1$, many equilibria converging to SS (local indeterminacy): name P_0, get R_1, which affects P_1 and so on
  - With uncertainty, can have sunspot equilibria
- If $|\alpha| > 1$, SS is locally unique: any path that starts away from SS will move further away, local determinacy, success?

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- Is there something wrong with paths where $\log \pi$ diverges?
- Also, ZLB (Benhabib, Schmitt-Grohé, and Uribe, 2001)

Playing with the timing of Taylor rules

- What if

$$R_{t+1} = \frac{\bar{\pi}^{1-\alpha}}{\beta}\left(\frac{P_{t+1}}{P_t}\right)^{\alpha} - 1? \tag{8}$$

- Get

$$\pi_{t+1} = \bar{\pi}^{1-\alpha}\left(\pi_{t+1}\right)^{\alpha} \Longrightarrow \pi_{t+1} = \bar{\pi}$$

- Success (other than P_0)?

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- Success (other than P_0)?
- How does central bank know P_{t+1} when setting R_{t+1}?

Taylor rule: targeting rule or reaction function?

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- Reaction function: "specifies the central bank's instrument as a function of predetermined endogenous or exogenous variables observable to the central bank at the time that it sets the instrument."

Roll household flow budget constraint forward to period $J+1$

$$W_0 = \sum_{s=0}^{J} E_0 z_{0,s} \frac{R_s}{1+R_s} M_s + \sum_{s=0}^{J} E_0 \left[z_{0,s+1} \left(T_{s+1} - P_s (y_s - c_{1s} - c_{2s}) \right) \right]$$
$$+ E_0[z_{0,J+1} W_{J+1}]$$

... and to infinity

Use no-Ponzi to replace $E_0[z_{0,J+1}W_{J+1}]$ and take limit as $J \to \infty$:

- $E_0[z_{0,J+1}W_{J+1}] \geq -E_0[\sum_{s=J+1}^{\infty} z_{0,s+1}(P_s y_s - T_{s+1})]$
- $$W_0 \geq \frac{R_0}{1+R_0}M_0 + \sum_{s=0}^{\infty} E_0\left[z_{0,s+1}\left(T_{s+1} - P_s y_s\right)\right] + \sum_{s=0}^{\infty} E_0\left[z_{0,s+1}\left(p_s(c_{1s}+c_{2s}) + \frac{R_{s+1}}{1+R_{s+1}}M_{s+1}\right)\right]$$

The government side

- Up to $J+1$:

$$W_0 = \sum_{s=0}^{J} E_0 z_{0,s} \frac{R_s}{1+R_s} M_s^S + \sum_{s=0}^{J} E_0 z_{0,s+1} T_{s+1} +$$
$$+ E_0[z_{0,J+1} \left(\frac{B_{J+1}}{1+R_{J+1}} + M_{J+1}^S \right)]$$

- Does gov't have a transversality condition
- Does gov't have a no-Ponzi condition?

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- Suppose real bound is imposed on taxes
- Debt is still a promise to money
  Can gov't print unlimited quantities of money?

The transversality condition under a money supply rule

- Under money supply rule, gov't cannot print unlimited money
- Then, taxes must adjust to meet budget constraint
- Limit on taxes $\implies$ no-Ponzi condition, at least for debt:

$$\implies \lim_{J \to \infty} E_0[z_{0,J} B_J^S] = 0$$

- Our example tax policy satisfied this (we had $B_t^S \equiv 0$)
- There are many others that would work, but, given prices, gov't must meet obligations

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- There are many others that would work, but, given prices, gov't must meet obligations
- Even here, no need to back money with tax revenue

The transversality condition under an interest-rate rule

- Under interest-rate rule, money supply infinitely elastic
- Gov't can meet debt obligations by printing money
- $\Longrightarrow$ no-Ponzi constraint absent, transversality condition not imposed on gov't

A first peek at CB independence

- Right now, we have only a consolidated gov't budget constraint
- Treasury can't print money to repay debt
- CB can (though usually done in a round-about way)
- Independent CB $\implies$ Treasury may face limit

One last important point

- B_t^S: nominal debt
- no-Ponzi applies to real debt (denominated in gold, foreign currency)

Does this matter?

- Households will exhaust their net worth, PVBC will hold as equality

$$\lim_{J\to\infty} E_0[z_{0,J+1}W_{J+1}] = 0$$

- Substitute market clearing:

$$\lim_{J\to\infty} E_0[z_{0,J+1}\left(\frac{B^S_{J+1}}{1+R_{J+1}} + M^S_{J+1}\right)] = 0$$

- Voilà: transversality condition obtained, write PVBC:

$$W_0 = \frac{R_0}{1+R_0}M^S_0 + \sum_{s=0}^{\infty} E_0\left[z_{0,s+1}\left(T_{s+1} + \frac{R_{s+1}}{1+R_{s+1}}M^S_{s+1}\right)\right]$$

It does matter!

- Market clearing is an equilibrium condition
- Does not have to hold for all prices
- If gov't PVBC does not hold, what adjusts: prices or taxes?

What adjusts?

- Money supply rule, real debt, independent CB: taxes
- Otherwise: maybe taxes, maybe prices
- When prices adjust, fiscal theory of the price level (FTPL)

The FTPL with an interest rate peg

- We saw that setting a constant interest rate $\bar{R}$ would deliver indeterminate P_0 (and sunspots)
- Suppose now we set T_0 fixed, $T_t = \bar{T} P_{t-1}$ for some constant $\bar{T}$.
- Check transversality condition (or gov't PVBC): what prices are consistent with competitive equilibrium?

The FTPL in action

- Use Friedman distortion and Euler equation to substitute prices into the gov't PVBC
- Also, use market clearing
- Get:

$$W_0 = \frac{P_0}{(1-\beta)(1+\bar{R})}[\bar{c}\bar{R} + \bar{T}]$$

- W_0 given (initial condition)
- $\bar{T}, \bar{R}$ given, $\bar{c}$ determined by $\bar{R}$
- $\Longrightarrow$ at most one P_0 will work!
- Solution exists if $\text{sign}(W_0) = \text{sign}(\bar{T} + \bar{R}\bar{c})$

Economic intuition on the FTPL

- Initial nominal government liabilities: W_0
- Gov't PVBC: PV of gov't surpluses = liabilities
- Given fiscal policy, PV of taxes fixed real amount
- Seigniorage ($\bar{R}\bar{c}$) also fixed real amount
- Price level must be the ratio of nominal liabilities to real surpluses

More economic intuition on the FTPL

- Under the FTPL, bonds are claims to money
- Money (and thus bonds) is an entitlement to tax revenues, cannot be worthless
- Suppose P_0 above equilibrium level and $W_0 > 0$
- Gov't has excess resources $\Longrightarrow$ households do not have enough wealth to support their consumption
- $\Longrightarrow$ households cut back on their consumption
- Excess supply $\Longrightarrow$ prices go down, until equilibrium attains

Why intuition in previous slide is loose

- Previous slide describes a process by which the equilibrium is attained
- But this is really a mental process
- The economy is always in equilibrium
- "Starting from P_0 low" is a thought experiment, we do not model this
- ...but we can (Bassetto, Econometrica, 2002)

Microsoft Theory of Prices vs. AIG Theory of Prices

- Cochrane (2005): Gov't bonds like Microsoft stock
- Microsoft produces dividends, stock price adjusts
- Same with government bonds: gov't produces (primary) surpluses, price must adjust
- Complication: Gov't bonds like AIG stock
- What do you do if you have negative dividends (primary deficits) in the short run?

The FTPL and CB independence

- FTPL is the only complete theory of what pins down the price level
- Unpleasant feature: it's not CB, it's Treasury that matters!!

Patching up CB independence

- Is that a death knell for CB independence?

Patching up CB independence

- Is that a death knell for CB independence?
- CB retains control of inflation after time 0 (it is deterministic and equal to $\beta(1 + \bar{R})$)
- Also, no more pesky sunspots

Combining local determinacy with FTPL

- Start from an active Taylor rule ($\alpha > 1$)
- Add a fiscal policy that prunes equilibria with very high or very low inflation
- On a day-to-day basis inflation responds the way it would in the locally-unique equilibrium

Active and Passive Monetary Policy Rules

- Recall Taylor principle for interest-rate rules:
  - $\alpha > 1$: strong response to inflation, Fisher equation is a divergent difference equation (except for SS)
  - $\alpha < 1$: weak response to inflation, Fisher equation is convergent
- When Fisher equation is divergent, we say that monetary policy is active
- When Fisher equation is convergent, we say that monetary policy is passive

Active and Passive Fiscal Policy Rules

- Same “active” and “passive” language applies to fiscal policy rules, but what is the relevant difference equation?

Active and Passive Fiscal Policy Rules

- Same “active” and “passive” language applies to fiscal policy rules, but what is the relevant difference equation?
- The gov’t budget constraint!
- Define $H_t := E_0\left[z_{0,t}\left(\frac{B_t^S}{1+R_t} + M_t^S\right)\right]$
- Get

$$H_t = H_{t-1} + E_0[M_{t-1}^S(z_{0,t} - z_{0,t-1})] - E_0[z_{0,t}T_t] = H_{t-1} - E_0\left[\frac{z_{0,t-1}M_{t-1}^S R_{t-1}}{1+R_{t-1}}\right] - E_0[z_{0,t}T_t] \quad (9)$$

Initial condition

$$H_0 = \frac{B_0^S}{1+R_0} + M_0^S = W_0$$

Why do we write the budget constraint as in the previous slides?

Because it is the left-hand side that has to converge to 0 for PVBC to hold

Example of a Passive Fiscal Policy Rule

- Passive fiscal policy rule = difference equation converges to 0 no matter what P_0 is
- Technical assumption : $\lim_{R\to\infty} R(u')^{-1}(1+R) < \infty$
- $\implies$ implies $R(u')^{-1}(1+R) < \bar{S}$, i.e., seigniorage revenues are bounded

$$T_t = \gamma(M_{t-1}^S + B_{t-1}^S)$$

With $\gamma \in (0,1)$: taxes cover at least fraction γ of nominal liabilities

Active Fiscal Policy Rule

- Active fiscal policy rule = difference equation does not converge to 0, except for one value of P_0
- Example: $T_t = \bar{T} P_{t-1}$ (with $R_t = \bar{R}$)

A Difference between Exploding Paths

- Paths where inflation explodes according to

$$(\log \pi_{t+1} - \log \bar{\pi}) = \alpha(\log \pi_t - \log \bar{\pi})$$

  are not ruled out by any equilibrium conditions

- Paths where debt explodes according to

$$H_t = H_{t-1} = H_{t-1} - E_0 \left[\frac{z_{0,t-1} M^S_{t-1} R_{t-1}}{1 + R_{t-1}} \right] - E_0[z_{0,t} T_t]$$

  violate the households' transversality condition, ruled out by equilibrium

A Weaker Notion of Active Fiscal Policy

- Compute equilibrium difference equation for W_t/P_t
- Fiscal policy active if the difference equation is explosive
- Adopted by Bianchi, Melosi and coauthors, sometimes Leeper and coauthors,...
- Rationale: equilibria with exploding household wealth are weird...
- ... but they are still equilibria, unless the explosion is sufficiently fast to violate transversality

A Final Classification

	Fiscal Policy	
	Passive	Active (strong sense)
Interest-rate rule: Passive	Indeterminacy	Uniqueness
Interest-rate rule: Active	Local uniqueness	Uniqueness, explosive

Regime switching

- Davig and Leeper (2007, 2010)
- Bianchi and Melosi (2014, 2019, 2022), Bianchi, Faccini and Melosi (2023)

Incomplete information

- Bassetto and Galli (2019), Bassetto and Miller (2022)
- Angeletos and Lian (2023)

Low Interest Rates

- Bassetto and Cui (2018), Blanchard (2019), Brunnermeier, Merkel, and Sannikov (2020)
- Jiang et al. (2022, 2023), Elenev et al. (2022)

Endogenous Policy

- Camous and Matveev (2023)
- Barthélemy and Plantin (2018), Barthélemy, Mengus, and Plantin (2021)

Equilibrium concept: Competitive equilibrium: Elements

- An allocation $(c_{1t}, c_{2t}, M_t, B_t, A_{t+1})_{t=0}^{\infty}$
- A price system $(P_t, R_t, z_{t,t+1})_{t=0}^{\infty}$
- A government policy $(T_t, B_t^S, M_t^S)_{t=0}^{\infty}$

Equilibrium concept: Competitive equilibrium: Requirements

- Allocation maximizes household utility subject to their budget constraint, cash-in-advance, and no-Ponzi, taking gov't policy and prices as given (notice rational expectations);
- Markets clear:

$$B_t = B_t^S, M_t = M_t^S, c_{1t} + c_{2t} = y_t, A_{t+1} \equiv 0$$

- Gov't budget constraint is met period by period

Back to first-order conditions

Household Lagrangean

$$E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ u(c_{1t}) + c_{2t} +$$

$$\lambda_t \Big[B_{t-1} + M_{t-1} + A_t + P_{t-1}(y - c_{1\,t-1} - c_{2\,t-1})$$

$$- T_t - \frac{B_t}{1+R_t} - M_t - E_t(z_{t,t+1} A_{t+1}) \Big] + \mu_t (M_t - P_t c_{1t}) \Big\}$$

- Bewley, *Journal of Economic Theory*, 1972
- Luenberger, Optimization by Vector Space Methods, 1969
- Stokey, Lucas, and Prescott (dealing with the issue of LM being in L_1)

Equilibria under a money supply rule: other power cases

- Now $u(c_{1t}) = c_{1t}^{1-\sigma}/(1-\sigma)$.
- To keep it simple, set $q = 0$: $M_t = M_{t-1} = \bar{M}$, and study deterministic equilibria.

Use again (4), (5), and (6):

$$\bar{M}^{-\sigma} = E_{t-1} \frac{\beta P_{t-1}}{P_t^{1-\sigma}}, \quad t \geq 1$$

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so (for deterministic equilibria)

$$\log P_t = \left(\frac{1}{1-\sigma}\right)^t \left[\log P_0 - \log(\bar{M}\beta^{-\frac{1}{\sigma}})\right] + \log(\bar{M}\beta^{-\frac{1}{\sigma}}), \quad t \geq 1$$

Unique SS at $P = \bar{M}\beta^{-\frac{1}{\sigma}}$

Deterministic equilibria under a money supply rule: $\sigma > 2$

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- Name any $P_0 > 0$, solve for sequence that converges to SS

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- ... but at least it converges back to SS?

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- Name any $P_0 > 0$, solve for sequence that converges to SS
- Disturbing: price level can be anything
- ... but at least it converges back to SS?
- With randomness, it could bounce away from SS all the time

Deterministic equilibria under a money supply rule: $\sigma \leq 2$

- For $\sigma = 2$, we get a period-2 cycle outside of SS
- For $1 < \sigma < 2$, we get paths where prices oscillate, exploding to $+\infty$ every other period and going to 0 every other period
- For $\sigma < 1$, we get either prices going to $+\infty$ or to 0
- Are these equilibria?
- One more condition to check: transversality condition

Checking the transversality condition

Use (4), (5), and (6) once more, get

$$\prod_{s=0}^{t}(1+R_s)^{-1} = (1+R_0)^{-1}\beta^t \frac{P_0}{P_t}$$

Substitute into transversality condition, use constant money supply rule; transv. will hold if

$$\bar{M}(1+R_0)^{-1}P_0 \liminf_{t\to\infty} \frac{\beta^t}{P_t} = 0$$

Take logs, use difference equation: transv. will hold if

$$\liminf_{t\to\infty}\left[t\log\beta - (1-\sigma)^{-t}\left(\log P_0 - \log\left(\bar{M}\beta^{-\frac{1}{\sigma}}\right)\right)\right] + \log\left(\bar{M}\beta^{-\frac{1}{\sigma}}\right) = -\infty.$$

Conclusion from checking transversality condition

- If $\sigma = 2$, bounded oscillations, OK
- If $1 < \sigma < 2$, explosive oscillations (converging to 0 in either period and to $+\infty$ every other period), OK according to current transv
- If $\sigma < 1$, prices blow up to ∞ or go down to 0, transv. OK only when they go to ∞

Back to main slides