Project4

April 29, 2025

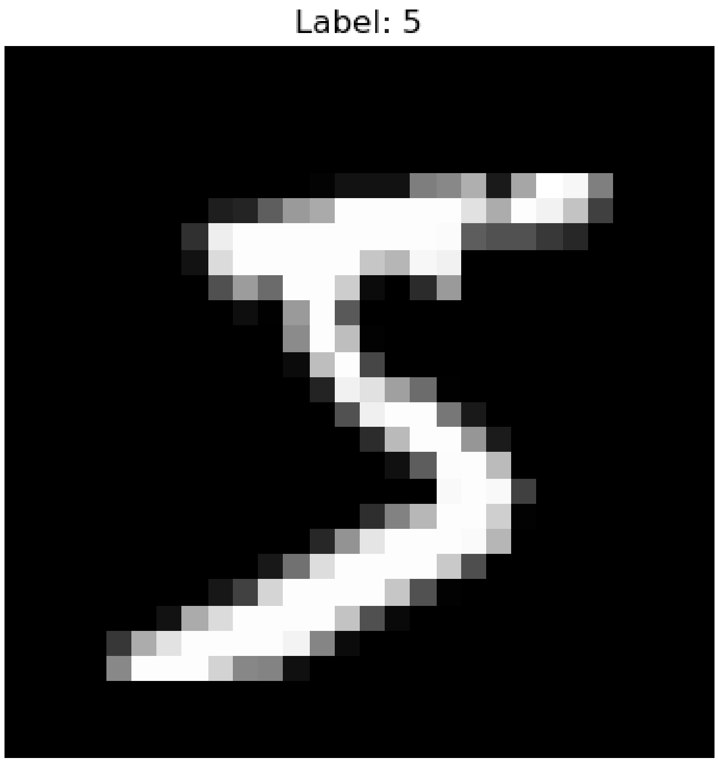
[1]:

```
import numpy as np
from sklearn.datasets import fetch_openml
import matplotlib.pyplot as plt

#Next we are normalizing pixel vals to 0/1
mnist = fetch_openml('mnist_784', version=1)
X = mnist.data.to_numpy().astype(np.float32) / 255.0
y = mnist.target.to_numpy().astype(int)
```

We importede the MNIST dataset, normalized the pixels. We will run quick test to make sure it is all working by pulling out a sample reshaping it back into 28x28 and plotting it in grayscale.

[4]:

```
#quick test
plt.imshow(X[0].reshape(28,28), cmap='gray')
plt.title(f"Label: {y[0]}")
plt.axis('off')
plt.show()
```



1 Task 1: PCA

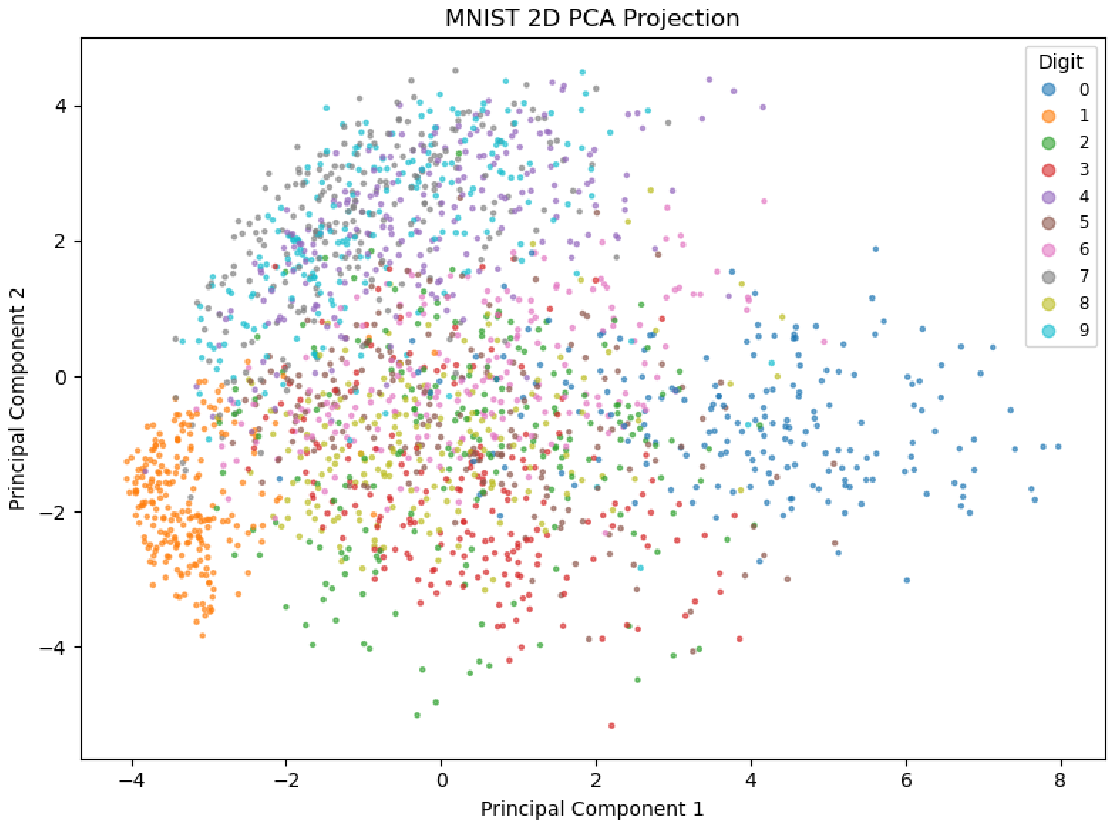
Here we are going to run PCA, which is a dimensionality-reduction technique where the greatest variance is recorded in our data. It then displays the MNIST vectors into K components to get a 2D visual that preserves as much of the variance as possible.

1.1 1-A

[8]:
```
from sklearn.decomposition import PCA
pca2 = PCA(n_components=2)
X_pca2 = pca2.fit_transform(X)
```

[9]:
```
import matplotlib.pyplot as plt

n_plot = 2000 #we are only plotting part of it for visibility and speed
plt.figure(figsize=(8,6))
scatter = plt.scatter(
    X_pca2[:n_plot, 0],
    X_pca2[:n_plot, 1],
    c=y[:n_plot],
    cmap='tab10',
    s=5,
```

```
    alpha=0.6
)
plt.legend(*scatter.legend_elements(), title="Digit", loc="upper right",␣
 ↪fontsize='small')
plt.title("MNIST 2D PCA Projection")
plt.xlabel("Principal Component 1")
plt.ylabel("Principal Component 2")
plt.tight_layout()
plt.show()
```



Here we are seeing clustering by digits, though you may notice that the clusterting is pretty loose and there are many overlaps, much of the overlap is due to PCA being linear and using only 2 components.

Even though there is overlap, this is also shows that meaningfull structure is being captured

1.2 1-B,C,D

Here we focus a bit more on compressing and reconstructing, we are now using 32 componenents meaning we will capture more of the variance. In the process, the 782 dim image will be compressed down to 32 dim. And then we reverse this compression to compute how much data was lost (MSE)

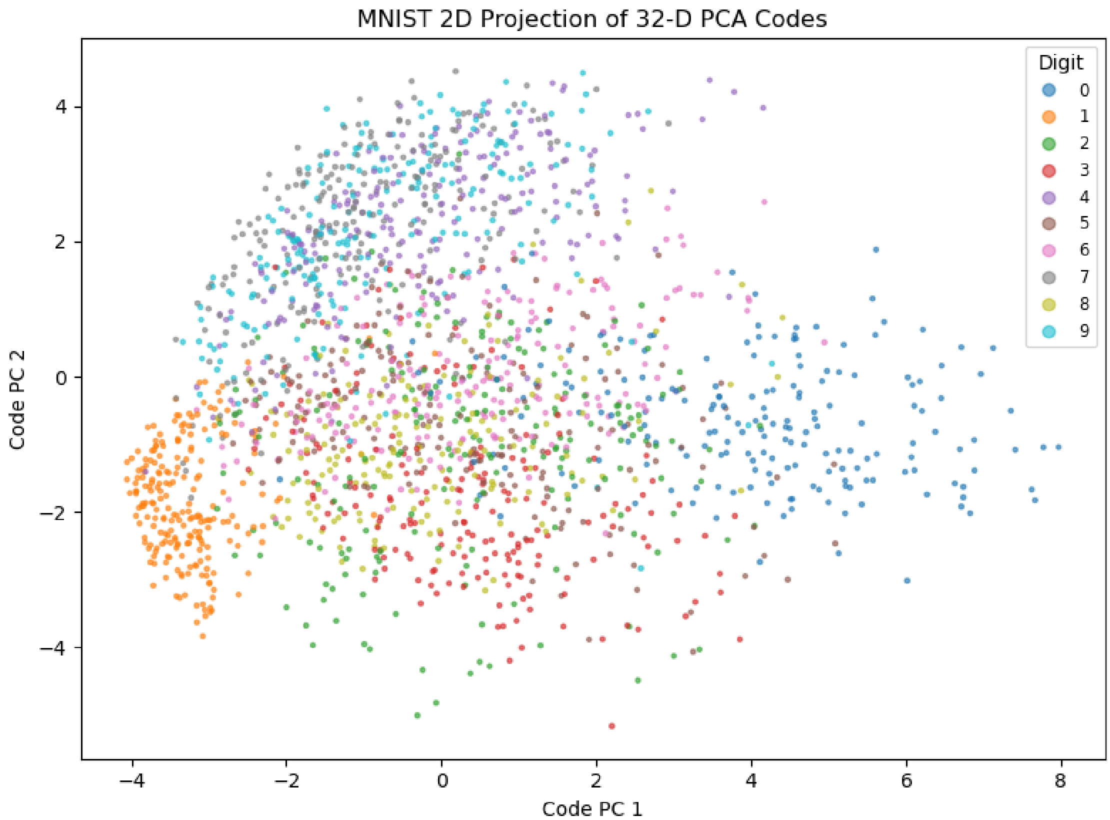
[12]:
```
from sklearn.decomposition import PCA

pca32 = PCA(n_components=32)

X_pca32 = pca32.fit_transform(X)
```

[14]:
```
import numpy as np

X_rec32 = pca32.inverse_transform(X_pca32)

mse32 = np.mean((X - X_rec32) ** 2)
print(f"Reconstruction MSE (32D PCA): {mse32:.6f}")
```

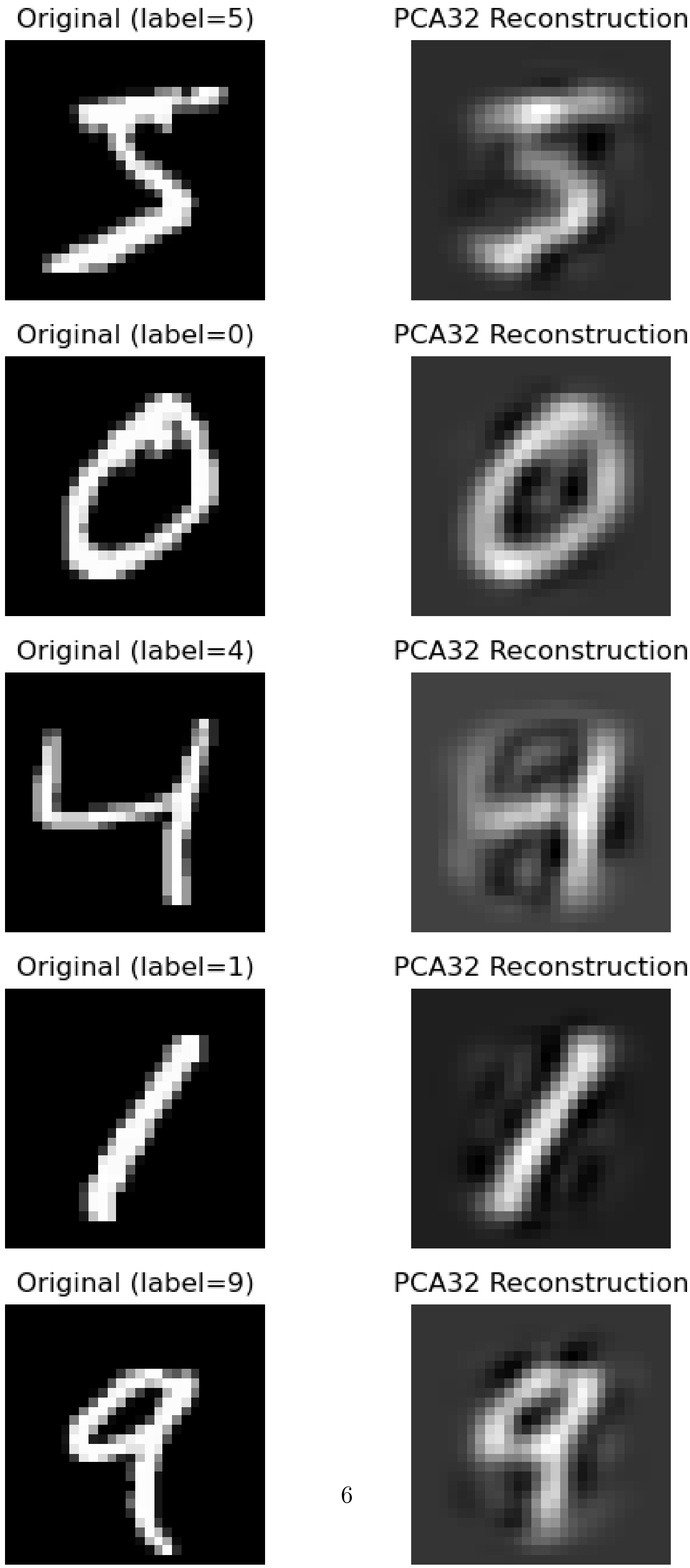
```
Reconstruction MSE (32D PCA): 0.017181
```

Pixel values range 0 to 1. So the average reconstructed pixel differs from the original by about 0.017180

[56]:
```
from sklearn.decomposition import PCA
import matplotlib.pyplot as plt

pca_code2 = PCA(n_components=2)
X_code2   = pca_code2.fit_transform(X_pca32)

n_plot = 2000
plt.figure(figsize=(8,6))
scatter = plt.scatter(
    X_code2[:n_plot, 0], X_code2[:n_plot, 1],
    c=y[:n_plot], cmap='tab10', s=5, alpha=0.6
)
plt.legend(*scatter.legend_elements(), title="Digit", loc="upper right",␣
 ↪fontsize='small')
plt.title("MNIST 2D Projection of 32-D PCA Codes")
plt.xlabel("Code PC 1")
plt.ylabel("Code PC 2")
plt.tight_layout()
plt.show()
```



[17]:
```
import matplotlib.pyplot as plt

n_show = 5
fig, axes = plt.subplots(n_show, 2, figsize=(6, 2*n_show))

for i in range(n_show):
    # orig
    axes[i,0].imshow(X[i].reshape(28,28), cmap='gray')
    axes[i,0].set_title(f"Original (label={y[i]})")
    axes[i,0].axis('off')
    # reconcstructed
    axes[i,1].imshow(X_rec32[i].reshape(28,28), cmap='gray')
    axes[i,1].set_title("PCA32 Reconstruction")
    axes[i,1].axis('off')

plt.tight_layout()
plt.show()
```


Original (label=5)
PCA32 Reconstruction
Original (label=0)
PCA32 Reconstruction
Original (label=4)
PCA32 Reconstruction
Original (label=1)
PCA32 Reconstruction
Original (label=9)
PCA32 Reconstruction

PCA seems to do better with linear compression, but its still capturing a good amount of digit structures.

2 Task 2

2.1 Task 2-a

We will start by defining an encoder and decoder, then we will wrap those two into an autoencoder

We encode to compress the image into 32-dim, and we decode to reconstruct the 32-dim back into a 784 vector

[21]:
```
import torch.nn as nn

class Encoder(nn.Module):
     def __init__(self):
         super().__init__()
         self.encoder_layer = nn.Linear(784, 32) #784 inpute features and 32␣
 ↪latent
         self.activation = nn.ReLU() # we will use Relu

     def forward(self, x):
         x = self.encoder_layer(x)
         x = self.activation(x)
         return x

class Decoder(nn.Module):
    def __init__(self):
        super().__init__()
        self.decoder_layer = nn.Linear(32, 784)
        self.activation = nn.Sigmoid() #get input back in 1 and 0

    def forward(self, z):
        z = self.decoder_layer(z)
        z = self.activation(z)
        return z
```

[23]:
```
class Autoencoder(nn.Module):
    def __init__(self):
        super().__init__()
        self.encoder = Encoder()
        self.decoder = Decoder()

    def forward(self, x):
        z = self.encoder(x)
        recon = self.decoder(z)
```

```
        return recon
```

2.2 Task 2-b

Next we need to setup the compondents needed to train Device, we choose GPU (cuda) but if we cant we fall back to CPU autoencoder starts the model and moves it to the available device citerion stores the reconstruction error as we have seen in PCA optimizer ajusts teh encoder and decoder weights

[27]:
```
import torch
import torch.nn as nn
from torch.utils.data import TensorDataset, DataLoader

device = torch.device('cuda' if torch.cuda.is_available() else 'cpu')

autoencoder = Autoencoder().to(device)

criterion = nn.MSELoss()

optimizer = torch.optim.Adam(autoencoder.parameters(), lr=1e-3)
```

Here we are prepping the data loader to be used in the training, the tensorDataset is wrapping each image with itself as teh label and the dataloader feeds random batches of images(128 per iteration)

[29]:
```
X_tensor = torch.from_numpy(X).to(device)
dataset = TensorDataset(X_tensor, X_tensor)
dataloader = DataLoader(dataset, batch_size=128, shuffle=True)
```

[30]:
```
autoencoder.train()
running_loss = 0.0

for inputs, targets in dataloader:
    optimizer.zero_grad() # resets gradient for each batch
    recon = autoencoder(inputs)
    loss = criterion(recon, targets) #comp the MSE
    loss.backward()
    optimizer.step() #we need to update the weights

    running_loss += loss.item() * inputs.size(0) #tot loss

epoch_loss = running_loss / len(dataset)
print(f"Epoch 1 Loss: {epoch_loss:.6f}") #print the avg loss per img
```

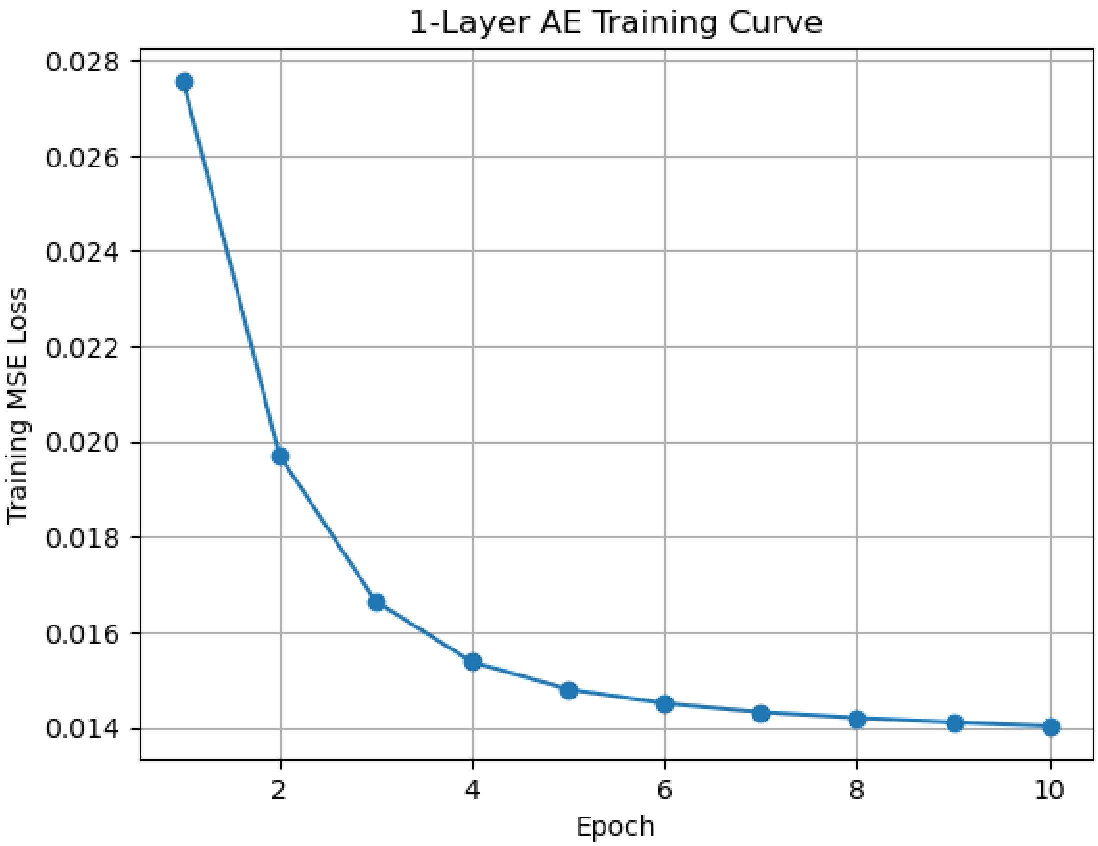
```
Epoch 1 Loss: 0.056862
```

since it ran well, we can move on to running multiple epochs

[32]:
```
num_epochs = 10
train_losses = []
```

```python
for epoch in range(1, num_epochs+1):
    autoencoder.train()
    running_loss = 0.0

    for inputs, targets in dataloader:
        optimizer.zero_grad()
        recon = autoencoder(inputs)
        loss = criterion(recon, targets)
        loss.backward()
        optimizer.step()

        running_loss += loss.item() * inputs.size(0)

    epoch_loss = running_loss / len(dataset)
    train_losses.append(epoch_loss)
    print(f"Epoch {epoch:2d} Loss: {epoch_loss:.6f}")

import matplotlib.pyplot as plt

plt.plot(range(1, num_epochs+1), train_losses, marker='o')
plt.xlabel("Epoch")
plt.ylabel("Training MSE Loss")
plt.title("1-Layer AE Training Curve")
plt.grid(True)
plt.show()
```

```
Epoch  1 Loss: 0.027567
Epoch  2 Loss: 0.019720
Epoch  3 Loss: 0.016649
Epoch  4 Loss: 0.015377
Epoch  5 Loss: 0.014804
Epoch  6 Loss: 0.014509
Epoch  7 Loss: 0.014324
Epoch  8 Loss: 0.014204
Epoch  9 Loss: 0.014110
Epoch 10 Loss: 0.014029
```



Here we have the training MSE over 10 epochs.We can see that over that after each epoch, the recontruction done by the AE is able to hold on to more fine details from the original image.

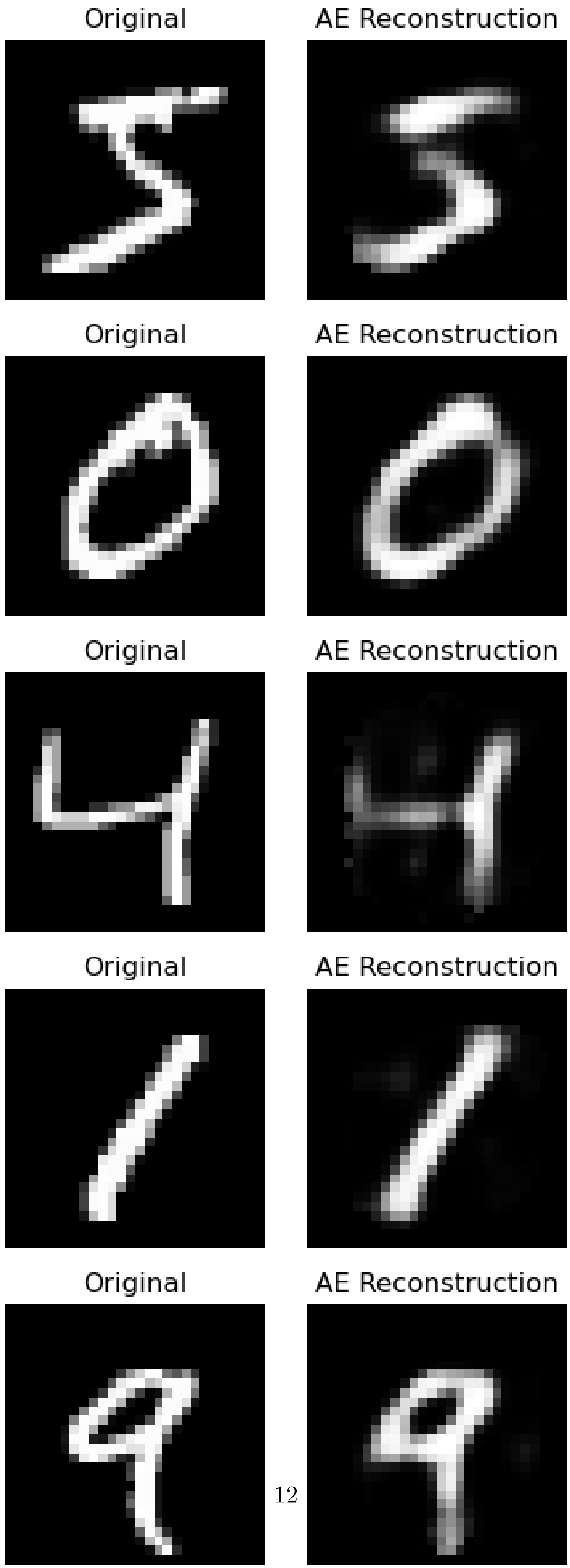
On average AE was able to reconstruct images better than PCA considering they both had the same bottle neck of32 components

2.3 Task 2-c

Next we are going to compare the original and reconstructed images side by side

[35]:
```
import matplotlib.pyplot as plt
import torch

autoencoder.eval()
with torch.no_grad():
    sample = X_tensor[:5]
    recon = autoencoder(sample)

#cpu will by used for plotting
orig_np = sample.cpu().numpy()
recon_np = recon.cpu().numpy()
```

```
#plot
n_show = orig_np.shape[0]
fig, axes = plt.subplots(n_show, 2, figsize=(4, 2*n_show))

for i in range(n_show):
    axes[i,0].imshow(orig_np[i].reshape(28,28), cmap='gray')
    axes[i,0].set_title("Original")
    axes[i,0].axis('off')

    axes[i,1].imshow(recon_np[i].reshape(28,28), cmap='gray')
    axes[i,1].set_title("AE Reconstruction")
    axes[i,1].axis('off')

plt.tight_layout()
plt.show()
```


Original
AE Reconstruction
Original
AE Reconstruction
Original
AE Reconstruction
Original
AE Reconstruction
Original
AE Reconstruction

Now that we have a visual, clearly the AE reconstruction using the single layer is doing better than teh PCA. But before we conclude anything we will compute tge full dataset MSE and compare that to PCA

[37]:
```
autoencoder.eval()

with torch.no_grad():
    recon_all = autoencoder(X_tensor)
    ae_mse = criterion(recon_all, X_tensor).item()

print(f"1-Layer AE Reconstruction MSE: {ae_mse:.6f}")
```

```
1-Layer AE Reconstruction MSE: 0.013966
```

Here we got a reconstruction MSE of 0.014800 vs the 0.01718 that we got from PCA Ignoring the numbers, I noticed in PCA that there was a lot of ghosting. Meaning i saw phantom strokes and just a ton of blur, the background was not black like in the reconstruction using AE. I did notice that the AE reconstruction of 4 resembles more of a 9, but other than that AE performed better both in the numbers and visually.

3 Task 3

We will need our encoder to: 784 -> 256 -> 64 -> 32 and our decoder to reverse it 32 -> 64 -> 256 -> 784

We will wrap this in the Deepencoder class

First we start with defining the deep AE classes

[42]:
```
class DeepEncoder(nn.Module):
     #activation is a class that acts a place holder untill we make a call with:
 ↪ ReLU(), Tanh(), LeakyReLU()
     #by doing we can avoid rewriting layers
    def __init__(self, activation):
        super().__init__() #linear blocks stacked to compress 784d to 32-d
        self.model = nn.Sequential(nn.Linear(784,256), activation(), nn.
 ↪Linear(256,64), activation(), nn.Linear(64,32), activation())

    def forward(self, x):
        return self.model(x)

class DeepDecoder(nn.Module):
    def __init__(self, activation):
        super().__init__() #we end with sigmoid so output stay within the 0 to␣
 ↪1 range
```

```
        self.model = nn.Sequential(nn.Linear(32,64), activation(), nn.
↪Linear(64,256), activation(), nn.Linear(256,784), nn.Sigmoid(),)

    def forward(self, z):
        return self.model(z)

#ties both encoder and decoder togaehr
class DeepAutoencoder(nn.Module):
    def __init__(self, activation):
        super().__init__()
        self.encoder = DeepEncoder(activation)
        self.decoder = DeepDecoder(activation)

    def forward(self, x):
        return self.decoder(self.encoder(x))
```

Next we activate each and train each. Each activation is trained for 10 epochs. For each activation we use a new deepAutoEncoder, move it to the selected device GPU or CPU, and we setup a MSE loss and optimize.

We switch to evaluation mode when all 10 epochs are done.

[44]:
```
import torch
import torch.nn as nn

activations = [nn.ReLU, nn.Tanh, nn.LeakyReLU]

num_epochs = 10

results = {}

for act in activations:
    print(f"\nTraining Deep AE with {act.__name__}…")

    model = DeepAutoencoder(act).to(device)
    criterion = nn.MSELoss()
    optimizer = torch.optim.Adam(model.parameters(), lr=1e-3)

    for epoch in range(1, num_epochs+1):
        model.train()
        running_loss = 0.0

        for inputs, targets in dataloader:
            optimizer.zero_grad()
            recon = model(inputs)
            loss = criterion(recon, targets)
            loss.backward()
            optimizer.step()
```

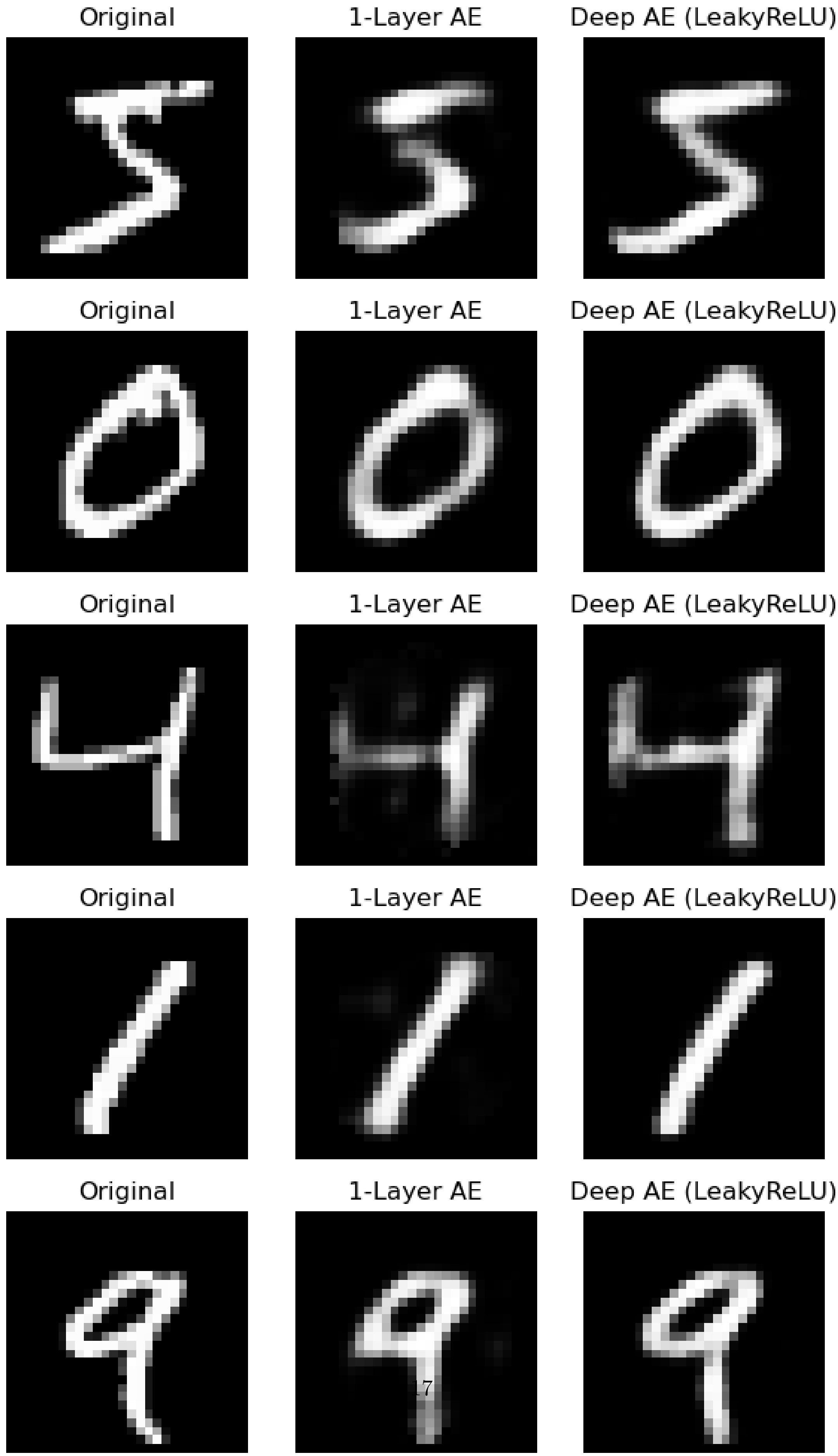
```
            running_loss += loss.item() * inputs.size(0)

        epoch_loss = running_loss / len(dataset)
        if epoch % 5 == 0 or epoch == 1:
            print(f"  Epoch {epoch:2d} Loss: {epoch_loss:.6f}")

    #here we compute a final MSE after all training is complete. MSE is done on
↪all 70k images
    model.eval()
    with torch.no_grad():
        recon_all = model(X_tensor)
        mse = criterion(recon_all, X_tensor).item()

    results[act.__name__] = mse
    print(f"{act.__name__} -> Final MSE: {mse:.6f}")

print("\n=== Activation comparison ===")
for name, mse in results.items():
    print(f"{name:10s} : MSE = {mse:.6f}")
```

```
Training Deep AE with ReLU…
  Epoch  1 Loss: 0.052649
  Epoch  5 Loss: 0.016968
  Epoch 10 Loss: 0.012890
ReLU -> Final MSE: 0.012785

Training Deep AE with Tanh…
  Epoch  1 Loss: 0.064899
  Epoch  5 Loss: 0.021297
  Epoch 10 Loss: 0.011507
Tanh -> Final MSE: 0.011016

Training Deep AE with LeakyReLU…
  Epoch  1 Loss: 0.048953
  Epoch  5 Loss: 0.012824
  Epoch 10 Loss: 0.008918
LeakyReLU -> Final MSE: 0.008534

=== Activation comparison ===
ReLU       : MSE = 0.012785
Tanh       : MSE = 0.011016
LeakyReLU  : MSE = 0.008534
```

3.1 Task 3-C

Next we will be comparing LeakyReLU Reconstruction with a MSE of 0.008839 to the 1 layer AE reconstruction.

[46]:
```
import matplotlib.pyplot as plt
import torch

# both set to eval mode
autoencoder.eval()
model.eval()

with torch.no_grad():
    # get same 5 images
    sample = X_tensor[:5]
    recon1 = autoencoder(sample) # 1-layer AE reconstruction
    recon2 = model(sample) # eakyReLU reconstruction

orig_np   = sample.cpu().numpy()
ae1_np    = recon1.cpu().numpy()
deep_np   = recon2.cpu().numpy()

# plot
n_show = orig_np.shape[0]
fig, axes = plt.subplots(n_show, 3, figsize=(6, 2*n_show))

for i in range(n_show):
    axes[i,0].imshow(orig_np[i].reshape(28,28), cmap='gray')
    axes[i,0].set_title("Original")
    axes[i,0].axis('off')

    axes[i,1].imshow(ae1_np[i].reshape(28,28), cmap='gray')
    axes[i,1].set_title("1-Layer AE")
    axes[i,1].axis('off')

    axes[i,2].imshow(deep_np[i].reshape(28,28), cmap='gray')
    axes[i,2].set_title("Deep AE (LeakyReLU)")
    axes[i,2].axis('off')

plt.tight_layout()
plt.show()
```


Original
1-Layer AE
Deep AE (LeakyReLU)
Original
1-Layer AE
Deep AE (LeakyReLU)
Original
1-Layer AE
Deep AE (LeakyReLU)
Original
1-Layer AE
Deep AE (LeakyReLU)
Original
1-Layer AE
Deep AE (LeakyReLU)

Here we compare the first 5 images for the original, 1layer AE, and LeakyReLU. There are 3 things that stood out to me the most when looking at these side by side. 1. Deep AE was able to recreate strokes and backgrounds more accurately than the 1-layer AE. Specifically the 4 looks more like a 4 than a 9 in the deep AE. 2. While 1-layer AE already had reduced ghosting when compared to PCA, the Deep AE extended this and reduced it more. 3. Overall quality of the nunber is actualy better in Deep AE compared to the original in some cases which I found surprising.