Digital Media Signal Processing: Summer Term 2022
Exercise Sheet 8: Stability and DTFT
(Hand-in Assignment II — Deadline: 20.06.2022)

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Important:

- **Deadline:** Monday, 20.06.2022 at 10:00.
- This is not group work. **Each student** has to solve the assignment and hand it in.
- The assignment will be handed in as a **single PDF file** through Moodle.
- Please make sure your solution is tidy and readable. If you hand in a scanned document, please make sure the quality is adequate.
- Please include all relevant steps of derivation/computation. It must be clear how final results are obtained.

1 Stability Analysis of LTI Systems (10 Points)

Show whether the following systems are stable or not. *Hint:* Check whether some poles and zeros cancel each other out.

a) $y(n) = \frac{3}{2}y(n-1) + y(n-2) + \frac{1}{2}x(n) - x(n-1)$ (7P)

b) $H(z) = \frac{z^{-1} + \frac{1}{3}}{z^{-2} + \frac{2}{3}z^{-1} + \frac{1}{9}}$ (3P)

2 Discrete Time Fourier Transform (DTFT) (30 Points)

Compute and sketch the impulse response of the following systems. Also compute the DTFT of the impulse response and sketch the corresponding magnitude and phase spectra from $\omega = -2\pi$ to $\omega = 2\pi$. Furthermore, label the axis also with the respective frequencies for a sampling rate of $F_\mathrm{s} = 8$ kHz.

a) $y(n) = x(n+1) + 2x(n) + x(n-1)$ (10P)

b) $y(n) = x(n-1) + 2x(n-2) + x(n-3)$ (10P)

c) $y(n) = x(n) - y(n-1)$ *Hint:* First obtain $H(z)$, then deduct $H(\omega)$ from it. (10P)

3 Delay Element (20 Points)

One of the simplest building blocks in digital signal processing is the delay element. An N-sample delay system is described by the difference equation $y(n) = x(n-N)$.

a) Determine the system function $H(z)$ of the N-sample delay system. (2P)

b) Find an expression for the magnitude spectrum $|H(\omega)|$ for the N-sample delay system. (5P)

c) Find an expression for the phase spectrum $\arg\{H(\omega)\} = \theta(\omega)$ for the N-sample delay system. (5P)

d) Sketch the magnitude and phase spectra for $N = 1, 2, 4$. (5P)

e) The group delay of a system is defined as $\tau_G = -\frac{d\theta}{d\omega}$. What is the group delay for an N-sample delay element? (3P)