DMSP Partial Solution for Exercise Sheet 8

Merlin Brandt

1 a)

$$y(n)=\frac{3}{2}y(n-1)+y(n-2)+\frac{1}{2}x(n)-x(n-1)$$

$$H(z)=\frac{z^0+\frac{3}{2}z^{-1}+z^{-2}}{\frac{1}{2}z^0-z^{-1}}=z^{(1-2)}\frac{z^2+\frac{3}{2}z^1+z^0}{\frac{1}{2}z^1-z^0}=z^{-1}\frac{z^2+\frac{3}{2}z^1+1}{\frac{1}{2}z^1-1}$$

$$z_{p1}=0;z_{p2}=2$$

Since y(n) is a causal signal, the ROC of H(z) is the exterior of a circle. Since H(z) has a pole at z_{p1} = 0 and z_{p2} = 2, its ROC is outside the range |z| < 2. Thus it does not contain the unit circle |z|=1. Thus the system is not BIBO-stable.

2a)

$y(n)=x(n+1)+2x(n)+x(n-1)$

Impulse response $h(n)$:

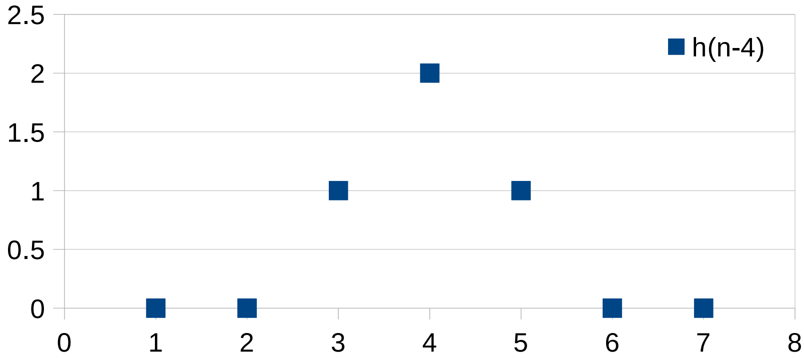
$h(m) = 0$ for all $m < -1$

$h(-1) = 1$

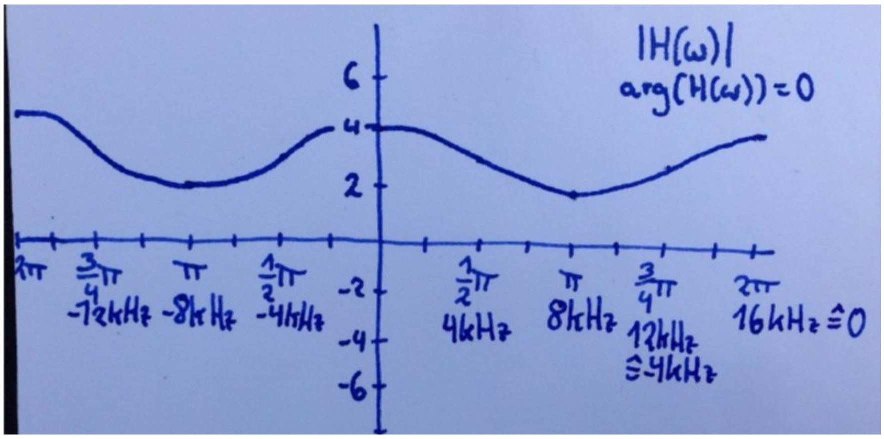
$h(0) = 2$

$h(1) = 1$

$h(m) = 0$ for all $m > 1$



$H(\omega) = 1e^{-j\omega(-1)} + 2e^{-j\omega(0)} + 1e^{-j\omega(1)} = e^{j\omega} + 2 + e^{-j\omega} = 2 + 2\cos(\omega)$



2b)

$y(n) = x(n-1) + 2x(n-2) + x(n-3)$

Impulse response $h(n)$:

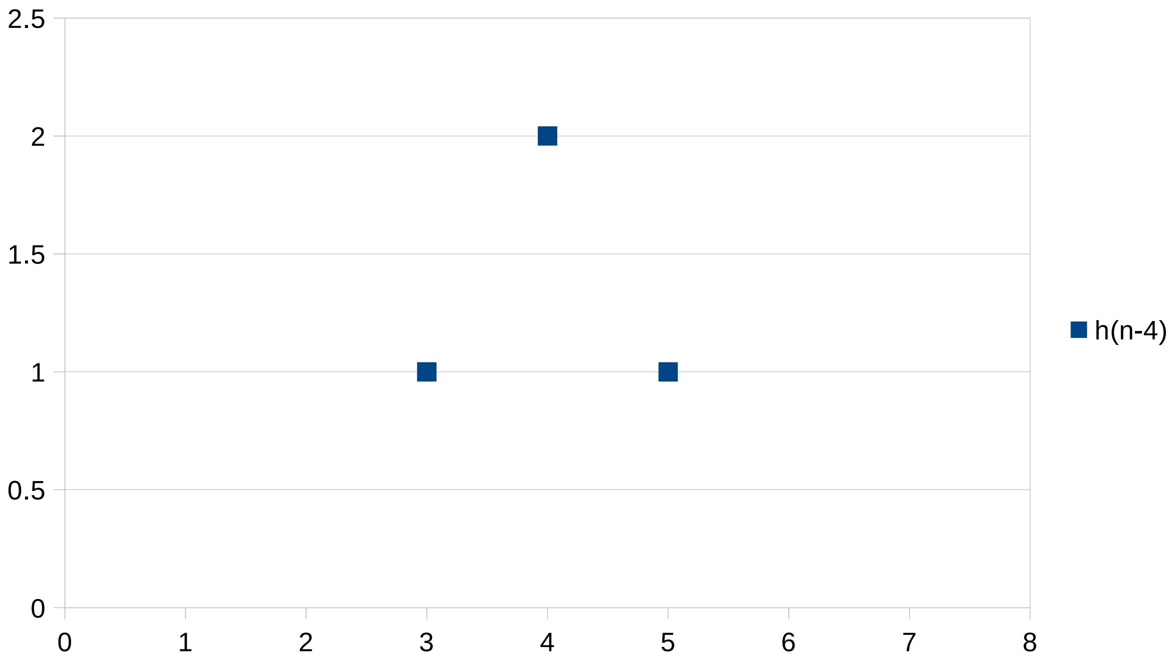
$h(m) = 0$ for $m < 1$

$h(1) = 1$

$h(2) = 2$

$h(3) = 1$

$h(m) = 0$ for $m > 3$



$H(\omega) = 1e^{-j\omega(1)}+2e^{-j\omega(2)}+1e^{-j\omega(3)} = e^{-j\omega}+2e^{-2j\omega}+e^{-3j\omega} = \cos(\omega)+j\sin(\omega)+2\cos(2\omega)+j2\sin(2\omega)+\cos(3\omega)$
$+j\sin(3\omega)$

$= \cos(\omega)+2\cos(2\omega)+\cos(3\omega)+j\sin(\omega)+j2\sin(2\omega)+j\sin(3\omega)$

3)

$y(n) = x(n-N)$

a)

$H(z) = z^{-N}$