APEC 8601: Natural Resource Economics Problem Set 1

Matt Braaksma

Table of contents

1	Conrad and	Clark	2.10.2													
2	Conrad and	Clark	2.10.3													į
3	Conrad and	Clark	2.10.4													8
4	Suckerville 1	Fishery	7													10

1 Conrad and Clark 2.10.2

Compute the MSY and corresponding effort levels for the following two examples, using the Schaefer model. Also find the level of effort just sufficient to drive the populations to extinction.

a) Antarctic baleen whales: r=.05 per year, K=400,000 BWU (bluewhale units), $q=1.6\times 10^3$ per whale-catcher year.

• Answer

For the logistic growth function, F'(X) = 0 at:

$$Y_{MSY} = \frac{rK}{4} = (0.05)(400000)/4 = 5000 \text{ BWU/yr}$$

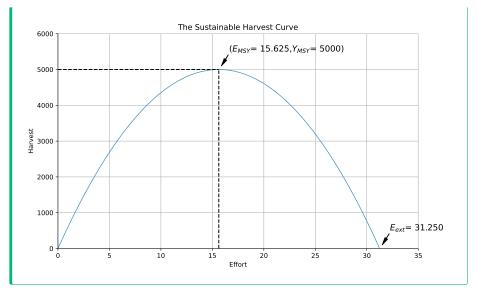
Then substitute $Y_{MSY}=\frac{rK}{4}, X_{MSY}=\frac{K}{2}$ into the sustained-yield function for the Schaefer model:

$$\begin{split} Y_{MSY} &= qEX_{MSY} \\ \frac{rK}{4} &= qE\frac{K}{2} \\ E &= \frac{r}{2q} \\ E &= \frac{(0.05)}{2(0.0016)} \\ E_{MSY} &= 15.625 \end{split}$$

If the relative rate of harvest qE exceeds the intrinsic growth rate of the population r, then the population will be driven to extinction.

$$E_{ext} = \frac{r}{q} = \frac{0.05}{0.0016} = 31.25$$

```
r = .05
K = 400000
q = 0.0016
# generate all possible values of stocks
S=np.arange(1,K + 1).reshape((K,1))
# find growth for each stock size
dSdt = r * S * (1 - S / K)
effort_ss = (r * S * (1 - S/K))/(q * S)
y_msy = r*K/4
e_msy = (r)/(2*q)
e_ext = (r)/(q)
# plot harvest as a function of stocks
plt.figure(figsize=(10, 6))
plt.subplot(111)
lw = 1
plt.plot(effort_ss,dSdt,lw=lw, c="#348ABD")
plt.plot((0, e_msy), (y_msy, y_msy), 'k--')
plt.plot((e_msy, e_msy), (0, y_msy), 'k--')
plt.annotate('($E_{MSY}$= %1.3f,$Y_{MSY}$= %i)' %(e_msy, y_msy), xy=(e_msy+.25, 5100), xy=
            arrowprops=dict(facecolor='black', width=.3, headwidth=5), fontsize=13)
plt.annotate('$E_{ext}$= %1.3f' %e_ext, xy=(e_ext+.25, 100), xytext=(e_ext+1, 500),
            arrowprops=dict(facecolor='black', width=.3, headwidth=5), fontsize=13)
plt.xlabel("Effort")
plt.ylabel("Harvest")
plt.title("The Sustainable Harvest Curve")
plt.xlim(0,35)
plt.ylim(0,6000)
sbn.despine()
plt.grid()
```



b) Pacific yellowfin tuna: r=2.6 per year, K=250,000 metric tons, $q=3.8\times 10^5$ per standard fishing day (SFD).

Answer

For the logistic growth function, F'(X) = 0 at:

$$Y_{MSY} = \frac{rK}{4} = (2.6)(250000)/4 = 162500~\mathrm{BWU/yr}$$

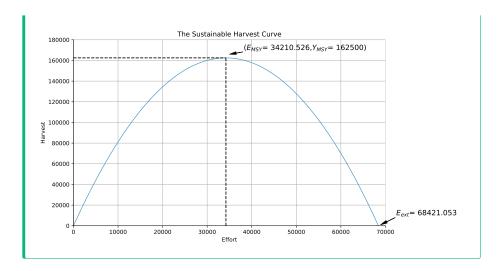
From part (a) we know,

$$E_{MSY} = \frac{r}{2q} = \frac{2.6}{2(0.000038))} = 34210.526$$

If the relative rate of harvest qE exceeds the intrinsic growth rate of the population r, then the population will be driven to extinction.

$$E_{ext} = \frac{r}{q} = \frac{2.6}{0.000038} = 68421.053$$

```
r = 2.6
K = 250000
q = 0.000038
# generate all possible values of stocks
S=np.arange(1,K+1).reshape((K,1))
# find growth for each stock size
dSdt = r * S * (1 - S / K)
effort_ss = (r * S * (1 - S/K))/(q * S)
y_msy = r*K/4
e_msy = (r)/(2*q)
e_ext = (r)/(q)
# plot harvest as a function of stocks
plt.figure(figsize=(10, 6))
plt.subplot(111)
lw = 1
plt.plot(effort_ss,dSdt,lw=lw, c="#348ABD")
plt.plot((0, e_msy), (y_msy, y_msy), 'k--')
plt.plot((e_msy, e_msy), (0, y_msy), 'k--')
plt.annotate('($E_{MSY}$= %1.3f,$Y_{MSY}$= %i)' %(e_msy, y_msy), xy=(e_msy+800, 166000),
            arrowprops=dict(facecolor='black', width=.3, headwidth=5), fontsize=13)
plt.annotate('$E_{ext}$= %1.3f' %e_ext, xy=(e_ext+500, 1000), xytext=(e_ext+4000, 10000),
            arrowprops=dict(facecolor='black', width=.3, headwidth=5), fontsize=13)
plt.xlabel("Effort")
plt.ylabel("Harvest")
plt.title("The Sustainable Harvest Curve")
plt.xlim(0,70000)
plt.ylim(0,180000)
sbn.despine()
plt.grid()
```



2 Conrad and Clark 2.10.3

Continuing with Problem 2.10.2, compute the optimal stock levels X^* , optimal sustained biological yields Y^* , optimal effort levels E^* , optimal sustained economic rents $R^* = pY^* - cE^*$, and optimal initial economic yields Y_0 assuming that X(0) = K (and that maximum effort is unconstrained). Use the values $\delta = 0$ and $\delta = 0.1$ % per year. Explain your results.

a) whales: p = 7,000 per BWU, c = 600,000 per whale-catcher year.

• Answer

The profit function is $\pi=pY_t-cE_t$. Since $Y_t=qE_tX_t$, the profit function can be rewritten as $\pi=pY_t-\frac{cY_t}{qX_t}$. Then the optimal harvesting problem is:

$$\max \int_0^\infty (pY_t - \frac{cY_t}{qX_t})e^{-\delta t}dt$$

subject to

$$\dot{X}_t = rX_t(1-\frac{X_t}{K}) - Y_t$$

The current value Hamiltonian is:

$$\tilde{H} = pY_t - \frac{cY_t}{qX_t} + \mu_t[rX_t(1-\frac{X_t}{K}) - Y_t]$$

Necessary conditions for an optimal solution:

$$\begin{split} \frac{\partial \tilde{H}}{\partial y} &= 0 \\ &\Longrightarrow p - \frac{c}{qX_t} - \mu_t = 0 \\ \frac{\partial \tilde{H}}{\partial X} &= -\dot{\mu}_t + \delta \mu_t \\ &\Longrightarrow \frac{cY_t}{qX_t^2} + \mu_t (1 - \frac{X_t}{K}) = -\dot{\mu}_t + \delta \mu_t \\ &\dot{\mu}_t = \mu_t (\delta - G'(X_t)) - U_X \\ \frac{\partial \tilde{H}}{\partial \mu} &= \dot{X}_t \\ &\Longrightarrow rX_t (1 - \frac{X_t}{K}) - Y_t = \dot{X}_t \end{split}$$

If X^*,Y^*,μ^* are the optimal steady state values, we can solve for X^* (knowing $\dot{\mu}_t=0,\dot{X}_t=0$):

$$X^* = \frac{1}{4} \left(\frac{c}{pq} + K(1-\frac{\delta}{r}) + \sqrt{(\frac{c}{pq} + K(1-\frac{\delta}{r}))^2 + 8K\frac{c}{pq}\frac{\delta}{r}} \right)$$

Substitute this back into the third condition and the harvest equation:

$$Y^* = rX^*(1 - \frac{X^*}{K})$$

$$E^* = \frac{Y^*}{qX^*}$$

When $\delta = 0$:

```
def print_optimal(r, K, q, p, c, delta):
          X_{star} = \frac{1}{4} * (c/(p*q) + K*(1-delta/r) + np.sqrt((c/(p*q)+K*(1-delta/r))**2 + 8*K*c/(p*q) + K*(1-delta/r))**2 + 8*K*c/(p*q)**2 + 8*K*c/(p*q)**3 + 8*K*c/(p*q)**4 + 8*K*
          Y_star = r * X_star * (1 - X_star / K)
          E_star = Y_star / (q * X_star)
          R_star = p * Y_star - c * E_star
          Y_0 = p * (K - X_{star}) - (c / q) * np.log( K / X_{star})
          print("Optimal Stock Levels (X*): ", X_star.round(2))
          print("Optimal Sustained Biological Yields (Y*): ", Y_star,round(2))
          print("Optimal Effort Levels (E*): ", E_star.round(2))
          print("Optimal Sustained Economic Rents (R*): ", R_star.round(2))
          print("Optimal Initial Economic Yields (Y_0): ", Y_0.round(2))
r = .05
K = 400000
q = 1.6*(10)**(-3)
p = 7000
c = 600000
delta=0
print_optimal(r, K, q, p, c, delta)
Optimal Stock Levels (X*): 226785.71
Optimal Sustained Biological Yields (Y*): 4910.32
Optimal Effort Levels (E*): 13.53
Optimal Sustained Economic Rents (R*): 26252790.18
Optimal Initial Economic Yields (Y_0): 999702887.98
When \delta = 0.1:
r = .05
K = 400000
q = 1.6*(10)**(-3)
p = 7000
c = 600000
delta=0.1
print_optimal(r, K, q, p, c, delta)
Optimal Stock Levels (X*): 83479.21
Optimal Sustained Biological Yields (Y*): 3302.86
Optimal Effort Levels (E*): 24.73
Optimal Sustained Economic Rents (R*): 8283131.16
Optimal Initial Economic Yields (Y_0): 1628070423.09
```

b) tuna: p = 600 per metric ton, c = 2,500 per SFD.

```
• Answer
We can substitute the given tuna values into the equations solved above.
When \delta = 0:
r = 2.6
K = 250000
q = 3.8*(10)**(-5)
p = 600
c = 600000
delta=0.0
print_optimal(r, K, q, p, c, delta)
Optimal Stock Levels (X*): 13282894.74
Optimal Sustained Biological Yields (Y*):
                                            -1800391516.62
Optimal Effort Levels (E*): -3566897.51
Optimal Sustained Economic Rents (R*): 1059903594182.83
Optimal Initial Economic Yields (Y_0): 54908233556.63
When \delta = 0.1:
r = 2.6
K = 250000
q = 3.8*(10)**(-5)
p = 600
c = 2500
delta=0.1
print_optimal(r, K, q, p, c, delta)
Optimal Stock Levels (X*): 177978.79
Optimal Sustained Biological Yields (Y*): 133309.78
Optimal Effort Levels (E*): 19711.07
Optimal Sustained Economic Rents (R*):
                                         30708196.35
Optimal Initial Economic Yields (Y_0): 20857690.82
```

3 Conrad and Clark 2.10.4

Find the stock level X_{∞} , effort level E_{∞} , and sustained yield Y_{∞} corresponding to bionomic equilibrium for the whale and tuna examples above. Explain why $Y_{\infty} > Y^*$ ($\delta = 0$ or 0.1) for the case of tuna.



$$TR = TC$$

$$pY = cE$$

$$p(qEX) = cE$$

$$X_{\infty} = \frac{c}{pq}$$

Since $\dot{X} = 0$,

$$\begin{split} F(X_{\infty}) - Y &= \dot{X} \\ F(X_{\infty}) - Y &= 0 \\ Y &= Y_{\infty} = F(X_{\infty}) \\ E_{\infty} &= \frac{Y_{\infty}}{qX_{\infty}} \end{split}$$

Then,

$$E_{\infty} = \frac{Y_{\infty}}{qX_{\infty}}$$

When $\delta = 0$, the results for whales:

```
def print_infty(r, K, q, p, c, delta):
   X_{infty} = c / (p*q)
    Y_{infty} = r * X_{infty} * (1 - (X_{infty} / K))
    E_infty = Y_infty / (q * X_infty)
    print("Bionomic Stock Levels (X_infty): ", X_infty)
    print("Bionomic Sustained Biological Yields (Y_infty):", Y_infty)
    print("Bionomic Effort Levels (E_infty): ", E_infty)
r = .05
K = 400000
q = 1.6*(10)**(-3)
p = 7000
c = 600000
delta=0
print_infty(r, K, q, p, c, delta)
Bionomic Stock Levels (X_infty): 53571.428571428565
Bionomic Sustained Biological Yields (Y_infty): 2319.8341836734694
Bionomic Effort Levels (E_infty): 27.064732142857146
When \delta = 0, the results for tuna:
```

```
r = 2.6
K = 250000
q = 3.8*(10)**(-5)
p=600
c = 2500
delta=0
print_infty(r, K, q, p, c, delta)

Bionomic Stock Levels (X_infty): 109649.12280701754
Bionomic Sustained Biological Yields (Y_infty): 160049.2459218221
Bionomic Effort Levels (E_infty): 38411.8190212373
The 10% discount factor has a much stronger impact on the whale results compared to the tuna results due to the large difference in r. THe lower growth rate for whales means that the discounted results are much lower.
```

4 Suckerville Fishery

The people of Suckerville harvest the long-nosed sucker fish from Big Sucker Lake. Suckerville residents get utility both from harvesting fish and from the stock of fish in the lake, represented by a utility function $U(h_t, X_t)$, where h_t is fish harvest at time t and X_t is fish stock at time t. Assume that $U_h > 0$, $U_{hh} < 0$, $U_X > 0$, $U_{XX} < 0$, $U_{hX} = 0$, where these expressions represent the first and second derivatives of the utility function with respect to h or X. The long-nosed sucker fish has a biological growth function given by $G(X_t)$, with G(0) = G(K) = 0, where K is natural carrying capacity, $G(X_t) > 0$ and $G''(X_t) < 0$ for $X_t \in (0,K)$, $G'(X_t) > 0$, for $X_t \in [0,X_{msy})$, $G'(X_t) < 0$ for $X_t \in (X_{msy},K)$, where X_{msy} is the maximum sustainable yield stock level. The change in the fish population is given by $X_t = G(X_t) - h_t$. The initial stock of the fish is $X_0 > 0$. The people of Suckerville have a discount rate applied to utility of $\delta > 0$.

a) Write down the dynamic optimization problem where the goal is to maximize the present value of utility from harvest over an infinite time horizon. Please include relevant constraints in the statement of the problem.

Answer

Optimal harvesting problem:

$$\begin{split} V(S_0) &= \max \int_0^\infty U(h_t, X_t) e^{-\delta t} dt \\ \text{subject to} \\ \dot{X}_t &= G(X_t) - h_t \\ h_t &\geq 0; X_t \geq 0; X_0 \geq 0 \end{split}$$

b) Write down the current value Hamiltonian and derive the necessary conditions for an optimal path.

• Answer

Current value Hamiltonian:

$$\tilde{H} = U(h_t, X_t) + \mu_t(G(X_t) - h_t)$$

Necessary conditions for an optimal solution:

$$\begin{split} \frac{\partial \tilde{H}}{\partial h} &= 0 \\ & \Longrightarrow U_h - \mu_t = 0 \\ \frac{\partial \tilde{H}}{\partial S} &= -\dot{\mu}_t + \delta \mu_t \\ & \Longrightarrow U_X + \mu_t G'(X_t) = -\dot{\mu}_t + \delta \mu_t \\ & \dot{\mu}_t = \mu_t (\delta - G'(X_t)) - U_X \\ \frac{\partial \tilde{H}}{\partial \mu} &= \dot{X}_t \\ & \Longrightarrow G(X_t) - h_t = \dot{X}_t \end{split}$$

c) Solve for conditions that define an optimal steady state.

Answer

Differentiate the first condition with respect to t and substitute out $\dot{X}_t, \dot{\mu}_t, \mu_t$:

$$\begin{split} U_{hh}\dot{h}_t - U_{hX}\dot{X}_t &= \dot{\mu}_t \\ U_{hh}\dot{h}_t - U_{hX}\dot{X}_t &= \mu_t(\delta - G'(X_t)) - U_X \\ U_{hh}\dot{h}_t - U_{hX}(G(X_t) - h_t) &= \mu_t(\delta - G'(X_t)) - U_X \\ U_{hh}\dot{h}_t - U_{hX}(G(X_t) - h_t) &= U_h(\delta - G'(X_t)) - U_X \\ \dot{h} &= \frac{U_h(G'(X_t) - \delta) + U_X + U_{hS}(G(X_t) - h_t)}{-U_{hh}} \end{split}$$
 At steady state $G(X) - h = \dot{X} = 0$, so the third term in the numerator

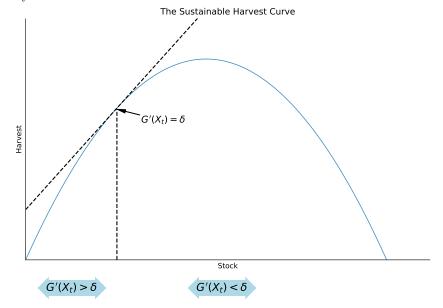
At steady state $G(X_t)-h_t=\dot{X}_t=0$, so the third term in the numerator is zero. Since $U_X>0$, it must be true that $G'(X_t)-\delta<0$ so that $\dot{h}_t=0$ since $U_h>0$.

d) How does the stock level in the optimal steady state compare to stock level that satisfies the golden rule of growth? Is it greater than, equal, less

than, or ambiguous?

Answer

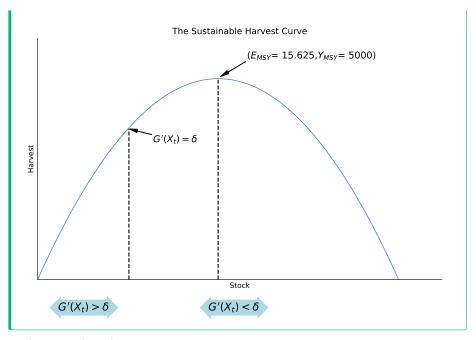
When the stock level satisfies the golden rule of growth then the growth rate equals the discount rate: $G'(X_t) = \delta$. Since the steady state requires that $G'(X_t) < \delta$, it must be the case that the stock level in the optimal steady is greater than the stock level that satisfies the golden rule: $X_t^{ss} > X_t^{gr}$.



e) How does the stock level in the optimal steady state compare to the maximum sustained yield stock level? Is it greater than, equal, less than, or ambiguous?

Answer

At the maximum sustained yield, the growth rate is equal to zero: $G'(X_t)=0$. This means that it is ambiguous compared to the optimal steady state stock level. The optimal steady state require only that $G'(X_t)<\delta$, but this could occur to either side of the maximum sustained yield point.



f) Clark (1973) showed that it may be optimal to harvest stock to extinction. Write down conditions that would give rise to optimal extinction and explain the economic logic for your result.

• Answer

Since $U_h>0$, there are positive returns from harvest. Then we can consider the case where $G'(0)<\delta$. Then $G'(X_t)<\delta$ and $\dot{h}_t<0$ for all $X_t>0$. The only steady state that exists is extinction where $h_t=X_t=0$. The intuition behind this is that the discount rate is so much higher than the growth rate that it is optimal to harvest all the stock and invest that capital elsewhere.

