# 1 Design Parameters

## 1.1 Design Load

$$M_u = 150.0 \text{ k-ft}$$

## 1.2 Design Parameters

| Steel Properties | | |
|---|---|---|
| $f_y$ (ksi) | $\epsilon_y$ | $E_s$ (ksi) |
| 60.0 | 0.002 | 29000.0 |

| Concrete Properties | | |
|---|---|---|
| $f'_c$ (ksi) | $\epsilon_c$ | $\beta$ |
| 5.0 | 0.003 | 0.8 |

| Section Properties | | |
|---|---|---|
| b (in) | h (in) | d (in) |
| 12.0 | 12.0 | 10.0 |

# 2 Calculate Balanced Reinforcement Ratio

## 2.1 Balanced Reinforcement Ratio

$$\xi_b = \frac{\epsilon_c}{\epsilon_c + \epsilon_y} = \frac{0.003}{0.003 + 0.002} = 0.6$$

$$\rho_b = 0.85 \cdot \beta \frac{f'_c}{f_y} \cdot \xi_b = 0.85 \cdot 0.8 \frac{5.0 \text{ ksi}}{60 \text{ ksi}} \cdot 0.6 = 0.034$$

## 2.2 Ensure Tension Controlled Section

$$\xi_{lim} = \frac{\epsilon_c}{\epsilon_c + 0.005} = \frac{0.003}{0.003 + 0.005} = 0.375$$

$$\rho_{lim} = 0.85 \cdot \beta \frac{f'_c}{f_y} \cdot \xi_{lim}$$
$$\rho_{lim} = 0.85 \cdot 0.8 \frac{5.0 \text{ ksi}}{60 \text{ ksi}} \cdot 0.375 = 0.02125$$

$$x_{lim} = \xi_{lim} \cdot d = 0.375 \cdot 10.0 \text{ in} = 3.75 \text{ in}$$

# 3 Determine Required Reinforcement

## 3.1 Find Maximum Strength of Singly Reinforced Section

$$M_{u,lim} = \phi \cdot 0.85 \cdot f'_c \beta \text{ b} x_{lim} \left( \text{d} - \beta \frac{x_{lim}}{2} \right)$$

$$M_{u,lim} = 0.9 \cdot 0.85 \cdot (5.0\text{ksi})(0.8)(12.0 \text{ in})(3.75 \text{ in}) \left( 10.0 \text{ in} - 0.8 \frac{3.75 \text{ in}}{2} \right) \cdot \frac{12 \text{ in}}{\text{ft}}$$

$$M_{u,lim} = 98.0 \text{ k-ft}$$

## 3.2 Calculate Required Reinforcement

$M_u \geq M_{u,lim} \Rightarrow$ Positive and Negative Reinforcement Required

**Positive Reinforcement**, $A_s$

$$A_s = \frac{0.85 f_c \beta x_{na} b}{\sigma_s} = \frac{0.85(5.0 \text{ ksi}) 0.8 (3.75 \text{ in})(12.0 \text{ in})}{(60 \text{ ksi})}$$

$$A_s = 4.01 \text{ in}^2$$

**Negative Reinforcement**, $A'_s$

$$A'_s = \frac{\phi 0.85 f_c \beta x_{na} \left( d - \beta \frac{x_{na}}{2} \right) b - M_u}{\phi \sigma'_s (d - d')}$$

$$A'_s = \frac{(0.9)(0.85)(5.0 \text{ ksi})(0.8)(3.75 \text{ in}) \left( 10.0 \text{ in} - 0.8 \frac{3.75 \text{ in}}{2} \right) (12.0 \text{ in}) - 150.0 \text{ k-ft} \frac{12 \text{ in}}{\text{ft}}}{0.9(-40.6 \text{ ksi})(10.0 \text{ in} - 2.0 \text{ in})} = 2.15 \text{ in}^2$$

Required Reinforcement by Bar Size

| Bar No. | $A_b$( in$^2$) | No. Pos. Bars | No. Neg. Bars | Pos. Bar Spacing (in) | Neg. Bar Spacing (in) |
|---|---|---|---|---|---|
| 3.0 | 0.11 | 37.0 | 20.0 | 0.32 | 0.60 |
| 4.0 | 0.20 | 21.0 | 11.0 | 0.57 | 1.09 |
| 5.0 | 0.31 | 13.0 | 7.0 | 0.92 | 1.71 |
| 6.0 | 0.44 | 10.0 | 5.0 | 1.20 | 2.40 |
| 7.0 | 0.60 | 7.0 | 4.0 | 1.71 | 3.00 |
| 8.0 | 0.79 | 6.0 | 3.0 | 2.00 | 4.00 |
| 9.0 | 0.99 | 5.0 | 3.0 | 2.40 | 4.00 |
| 10.0 | 1.23 | 4.0 | 2.0 | 3.00 | 6.00 |
| 11.0 | 1.48 | 3.0 | 2.0 | 4.00 | 6.00 |
| 14.0 | 2.41 | 2.0 | 1.0 | 6.00 | 12.00 |
| 18.0 | 3.98 | 2.0 | 1.0 | 6.00 | 12.00 |