

1 Design Parameters

1.1 Design Load

$$M_u = 150.0 \text{ k-ft}$$

1.2 Design Parameters

Steel Properties		
f_y (ksi)	ϵ_y	E_s (ksi)
60.0	0.002	29000.0

Concrete Properties		
f'_c (ksi)	ϵ_c	β
5.0	0.003	0.8

Section Properties		
b (in)	h (in)	d (in)
12.0	12.0	10.0

2 Calculate Balanced Reinforcement Ratio

2.1 Balanced Reinforcement Ratio

$$\xi_b = \frac{\epsilon_c}{\epsilon_c + \epsilon_y} = \frac{0.003}{0.003 + 0.002} = 0.6$$

$$\rho_b = 0.85 \cdot \beta \frac{f'_c}{f_y} \cdot \xi_b = 0.85 \cdot 0.8 \frac{5.0 \text{ ksi}}{60 \text{ ksi}} \cdot 0.6 = 0.034$$

2.2 Ensure Tension Controlled Section

$$\xi_{lim} = \frac{\epsilon_c}{\epsilon_c + 0.005} = \frac{0.003}{0.003 + 0.005} = 0.375$$

$$\rho_{lim} = 0.85 \cdot \beta \frac{f'_c}{f_y} \cdot \xi_{lim}$$

$$\rho_{lim} = 0.85 \cdot 0.8 \frac{5.0 \text{ ksi}}{60 \text{ ksi}} \cdot 0.375 = 0.02125$$

$$x_{lim} = \xi_{lim} \cdot d = 0.375 \cdot 10.0 \text{ in} = 3.75 \text{ in}$$

3 Determine Required Reinforcement

3.1 Find Maximum Strength of Singly Reinforced Section

$$M_{u,lim} = \phi \cdot 0.85 \cdot f'_c \beta x_{lim} \left(d - \beta \frac{x_{lim}}{2} \right)$$

$$M_{u,lim} = 0.9 \cdot 0.85 \cdot (5.0 \text{ ksi}) (0.8) (12.0 \text{ in}) (3.75 \text{ in}) \left(10.0 \text{ in} - 0.8 \frac{3.75 \text{ in}}{2} \right) \cdot \frac{12 \text{ in}}{\text{ft}}$$

$$M_{u,lim} = 98.0 \text{ k-ft}$$

3.2 Calculate Required Reinforcement

$M_u \geq M_{u,lim} \Rightarrow$ Positive and Negative Reinforcement Required

Positive Reinforcement, A_s

$$A_s = \frac{0.85 f'_c \beta x_{na} b}{\sigma_s} = \frac{0.85 (5.0 \text{ ksi}) (0.8) (3.75 \text{ in}) (12.0 \text{ in})}{(60 \text{ ksi})}$$

$$A_s = 4.01 \text{ in}^2$$

Negative Reinforcement, A'_s

$$A'_s = \frac{\phi 0.85 f'_c \beta x_{na} \left(d - \beta \frac{x_{na}}{2} \right) b - M_u}{\phi \sigma'_s (d - d')}$$

$$A'_s = \frac{(0.9)(0.85)(5.0 \text{ ksi})(0.8)(3.75 \text{ in}) \left(10.0 \text{ in} - 0.8 \frac{3.75 \text{ in}}{2} \right) (12.0 \text{ in}) - 150.0 \text{ k-ft} \frac{12 \text{ in}}{\text{ft}}}{0.9(-40.6 \text{ ksi})(10.0 \text{ in} - 2.0 \text{ in})} = 2.15 \text{ in}^2$$

Required Reinforcement by Bar Size

Bar No.	A_b (in ²)	No. Pos. Bars	No. Neg. Bars	Pos. Bar Spacing (in)	Neg. Bar Spacing (in)
3.0	0.11	37.0	20.0	0.32	0.60
4.0	0.20	21.0	11.0	0.57	1.09
5.0	0.31	13.0	7.0	0.92	1.71
6.0	0.44	10.0	5.0	1.20	2.40
7.0	0.60	7.0	4.0	1.71	3.00
8.0	0.79	6.0	3.0	2.00	4.00
9.0	0.99	5.0	3.0	2.40	4.00
10.0	1.23	4.0	2.0	3.00	6.00
11.0	1.48	3.0	2.0	4.00	6.00
14.0	2.41	2.0	1.0	6.00	12.00
18.0	3.98	2.0	1.0	6.00	12.00