```
In [1]: #import necessary packages
   import matplotlib.pyplot as plt
   import numpy as np
   import sympy
   from IPython.display import Image
   from tqdm import trange

#Set up sympy to display outputs after a latex conversion.
   from sympy.interactive import printing
   printing.init_printing(use_latex=True)

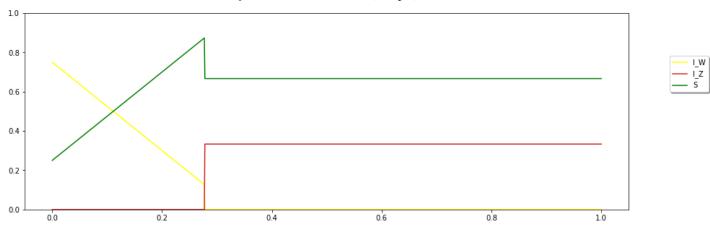
# Import custom module
   import SMB_Code_models_equilibria as models_equil
```

1 Where does an increase in ϵ cause a discontinuous increase in disease burden and what is the increase in disease burden.

```
In [2]:
        param values = {'N':1, 'epsilon':0.4, 'beta W': 0.04, 'beta Z': 0.015, 'gamma': 0.01, 'gamma
In [3]:
        epsilon vals = np. arange (0, 1.001, 0.001)
        S = []
        I W endem = []
        I_Z_{endem} = []
        for i in trange(len(epsilon vals)):
             param values['epsilon']=epsilon vals[i]
            prevs = models equil.replace inf end equil(param values)
             S endem.append(prevs[0]['S'])
             I W endem.append(prevs[0]['I W'])
             I Z endem.append(prevs[0]['I Z'])
        fig = plt.figure(figsize=(15,5))
        fig.suptitle('Proportion over Time (Days)',
                     fontsize=20)
        ax = plt.subplot(111)
        axes = plt.gca()
        axes.set ylim([0,1])
        ax.plot(epsilon vals,I W endem,color='yellow',label='I W')
        ax.plot(epsilon vals, I Z endem, color='tab:red', label='I Z')
        ax.plot(epsilon vals,S endem,color='green',label='S')
        chartBox = ax.get position()
        ax.legend(loc='upper center', bbox to anchor=(1.1, 0.8), shadow=True, ncol=1)
        plt.show()
```

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Proportion over Time (Days)



The jump in disease burden occurs when resistant strain endemic equilibrium becomes stable. So what is the first element of I_Z -endem to be >0.

```
In [4]:
           I Z endem = np.array(I Z endem)
           I Z endem
           np.where(I Z endem>0)
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The values of ϵ , S, I_W and I_Z at that point are equal to:

In [7]:

S endem[277] - S endem[278]

2 Deriving equations for ϵ at the discontinuous increase of disease burdan and the increase in Disease burden.

Lets look at the disease equalibria, assocaited Jacobian Matrix and Eigen values.

```
In [8]:
                #Setup Symbols for Sympy
               N, S, I W, I Z, beta W, beta Z, gamma, gamma T, epsilon = sympy.symbols('N S I W I Z beta
               Eqn S = -beta W/N*I W*S-beta Z/N*I Z*S+gamma*(1-epsilon)*I W+gamma T*epsilon*I W+gamma*I Z
               Eqn I W = beta W/N*I W*S-gamma*(1-epsilon)*I W-gamma T*epsilon*I W+(beta W/N-beta Z/N)*I W
               Eqn I Z = beta Z/N*I Z*S-gamma*I Z+(beta Z/N-beta W/N)*I W*I Z
 In [9]:
               ODEs = sympy.Matrix([Eqn S, Eqn I W, Eqn I Z])
                ODEs
                    I_W \epsilon \gamma_T + I_W \gamma \left(1 - \epsilon
ight) - rac{I_W S eta_W}{N} + I_Z \gamma - rac{I_Z S eta_Z}{N}
               igg|I_WI_Z\left(rac{eta_W}{N}-rac{eta_Z}{N}
ight)-I_W\epsilon\gamma_T-I_W\gamma\left(1-\epsilon
ight)+rac{I_WSeta_W}{N}
                            I_W I_Z \left( -rac{eta_W}{N} + rac{eta_Z}{N} 
ight) - I_Z \gamma + rac{I_Z S eta_Z}{N}
In [10]:
               equilibria = sympy.solve(ODEs,[S, I W, I Z])
               equilibria
Out[10]:
               \left|\left(S,\ 0,\ 0\right)\right.,
                 \left(rac{I_{W}eta_{W}-I_{W}eta_{Z}+N\gamma}{eta_{Z}},\;I_{W},\;rac{-I_{W}eta_{W}^{2}+I_{W}eta_{W}eta_{Z}-Neta_{W}\gamma-Neta_{Z}\epsilon\gamma+Neta_{Z}\epsilon\gamma_{T}+Neta_{Z}\gamma}{eta_{Z}\left(eta_{W}-eta_{Z}
ight)}
ight),
                 \left(rac{N\left(-\epsilon\gamma+\epsilon\gamma_T+\gamma
ight)}{eta_W},\;I_W,\;0
ight),\;\left(rac{N\gamma}{eta_Z},\;0,\;I_Z
ight)
ight|
In [11]:
               endem equil = equilibria[-2:]
               endem equil
Out[11]: \left[\left(\frac{N\left(-\epsilon\gamma+\epsilon\gamma_T+\gamma\right)}{\beta_{W}},\ I_W,\ 0\right),\ \left(\frac{N\gamma}{\beta_Z},\ 0,\ I_Z\right)\right]
In [12]:
               I W equil = N-endem equil[0][0]
               I Z equil = N-endem equil[1][0]
               display(I W equil, I Z equil)
              N-rac{N\left(-\epsilon\gamma+\epsilon\gamma_T+\gamma
ight)}{eta_{W}}
              N-\frac{N\gamma}{\beta_Z}
In [13]:
               J of ODEs = ODEs.jacobian(X=[S,I W,I Z])
               J of ODEs
```

Out[13]:

$$\begin{bmatrix} -\frac{I_W\beta_W}{N} - \frac{I_Z\beta_Z}{N} & \epsilon\gamma_T + \gamma\left(1 - \epsilon\right) - \frac{S\beta_W}{N} & \gamma - \frac{S\beta_Z}{N} \\ \frac{I_W\beta_W}{N} & I_Z\left(\frac{\beta_W}{N} - \frac{\beta_Z}{N}\right) - \epsilon\gamma_T - \gamma\left(1 - \epsilon\right) + \frac{S\beta_W}{N} & I_W\left(\frac{\beta_W}{N} - \frac{\beta_Z}{N}\right) \\ \frac{I_Z\beta_Z}{N} & I_Z\left(-\frac{\beta_W}{N} + \frac{\beta_Z}{N}\right) & I_W\left(-\frac{\beta_W}{N} + \frac{\beta_Z}{N}\right) - \gamma + \frac{S\beta_Z}{N} \end{bmatrix}$$

Substitute the 2nd non-disease free equilibrium (non-DFE) into the Jacobian matrix and determine Eigen values.

```
In [14]: non_DFE_2 = {'S':equilibria[3][0],'I_W':equilibria[3][1],'I_Z':equilibria[3][2]}

J_of_ODEs_non_DFE_2 = J_of_ODEs.subs(non_DFE_2)
J_of_ODEs_non_DFE_2

eigs_J_non_DFE_2 = J_of_ODEs_non_DFE_2.eigenvals()
# The above code line enters the Eigen values as keys to a dictionary.
# It would be more useful to have them as elements in a vector.
eigs_J_non_DFE_2 = [k for k in eigs_J_non_DFE_2.keys()]
eigs_J_non_DFE_2
```

$$\begin{array}{c} \text{Out[14]:} & \left[\frac{I_Z \beta_W \beta_Z - I_Z \beta_Z^2 + N \beta_W \gamma + N \beta_Z \epsilon \gamma - N \beta_Z \epsilon \gamma_T - N \beta_Z \gamma}{N \beta_Z}, \; - \frac{I_Z \beta_Z}{N}, \; 0 \right] \end{array}$$

2.1 Deriving equation for ϵ at the discontinuous increase of disease burdan

Model switches over between equilibrium when the 2nd equation in the set of eigen values = 0 so..

$$0 = rac{I_Z eta_W eta_Z - I_Z eta_Z^2 + N eta_W \gamma + N eta_Z \epsilon \gamma - N eta_Z \epsilon \gamma_T - N eta_Z \gamma}{N eta_Z}$$

Out[16]:
$$\left[\frac{-I_Z\beta_W\beta_Z+I_Z\beta_Z^2-N\beta_W\gamma+N\beta_Z\gamma}{N\beta_Z\left(\gamma-\gamma_T\right)}\right]$$

Out[17]:
$$\frac{-N\beta_W\gamma+N\beta_Z\gamma-\beta_W\beta_Z\left(N-\frac{N\gamma}{\beta_Z}\right)+\beta_Z^2\left(N-\frac{N\gamma}{\beta_Z}\right)}{N\beta_Z\left(\gamma-\gamma_T\right)}$$

```
In [18]: epsilon_at_change_v2.subs(param_values)
```

Out[18]: 0.2777777777778

Lets check this is the correct value

```
In [19]: epsilon_vals[278]
```

```
Out[19]: 0.278
In [20]:
             param_values['epsilon'] = epsilon_at_change_v2.subs(param_values)
             param values
            {'N': 1,
Out[20]:
              'epsilon': 0.2777777777778,
              'beta W': 0.04,
              'beta Z': 0.015,
              'gamma': 0.01,
              'gamma T': 0.1}
            Increase in disease burden in replacement model is:
In [21]:
             models equil.replace inf end equil(param values)
             [{'s': 0.87500000000000, 'I W': 0.1250000000000, 'I Z': 0}]
Out[21]:
            This is indeed the correct value of \epsilon.
            2.2 Deriving Equation for the Increase in Disease burden.
In [22]:
             inc burd = I Z equil - I W equil.subs(epsilon,epsilon at change v2)
             inc burd
                       -\frac{N\left(\gamma-\frac{\gamma\left(-N\beta_W\gamma+N\beta_Z\gamma-\beta_W\beta_Z\left(N-\frac{N\gamma}{\beta_Z}\right)+\beta_Z^2\left(N-\frac{N\gamma}{\beta_Z}\right)\right)}{N\beta_Z(\gamma-\gamma_T)}+\frac{\gamma_T\left(-N\beta_W\gamma+N\beta_Z\gamma-\beta_W\beta_Z\left(N-\frac{N\gamma}{\beta_Z}\right)+\beta_Z^2\left(N-\frac{N\gamma}{\beta_Z}\right)\right)}{N\beta_Z(\gamma-\gamma_T)}}{\beta_W}
Out[22]:
In [23]:
             inc burd=inc burd.simplify()
             inc burd
Out[23]: N - \frac{N\gamma}{\beta_Z} - \frac{N\beta_Z}{\beta_W} + \frac{N\gamma}{\beta_W}
In [24]:
             inc burd.subs(param values)
Lets double check.
```

3 Checking working with different set of parameters.

S = mdem[277] - S = mdem[278]

```
I W endem = []
I Z endem = []
for i in trange(len(epsilon vals)):
    param values['epsilon']=epsilon vals[i]
    prevs = models equil.replace inf end equil(param values)
    S endem.append(prevs[0]['S'])
    I W endem.append(prevs[0]['I W'])
     I Z endem.append(prevs[0]['I Z'])
fig = plt.figure(figsize=(15,5))
fig.suptitle('Proportion over Time (Days)',
              fontsize=20)
ax = plt.subplot(111)
axes = plt.gca()
axes.set ylim([0,1])
ax.plot(epsilon vals,I W endem,color='yellow',label='I W')
ax.plot(epsilon vals,I Z endem,color='tab:red',label='I Z')
ax.plot(epsilon vals,S endem,color='tab:green',label='S')
chartBox = ax.get position()
ax.legend(loc='upper center', bbox to anchor=(1.1, 0.8), shadow=True, ncol=1)
plt.show()
100%|
                                                                                      | 1001/
1001 [01:39<00:00, 10.04it/s]
                           Proportion over Time (Days)
1.0
0.8
0.6
```

The jump in disease burden occurs when resistant strain endemic equilibrium becomes stable. So what is the first element of I_Z -endem to be >0.

0.4

0.4

0.2

0.2

```
In [27]:
           I Z endem = np.array(I Z endem)
          I_Z_{-}endem
          np.where(I Z endem>0)
          (array([ 112,
                           113,
                                  114,
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                                               116,
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Out[27]:
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332,								340,		
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673,	674,	675,	676,	677,	678,	679 ,	680,		682,	683,
684,	685,	686,	687,	688,		690,	691,		693,	694,
695,	696,				700,	701,	702,	703,		705,
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728,	729,	730,	731,	732,	733,	734,	735,	736,	737,	738,
739,	740,	741,	742,	743,	744,	745,	746,	747,	748,	749,
750,	751 ,	752,	753 ,	754,	755 ,	756 ,	757,	758,	759,	760,
761,	762,	763,	764,	765,	766,	767,	768,	769,	770,	771,
772,	773,	774,	775,	776,	777,	778,	779,	780,	781,	782,
783,	784,	785,	786,	787,	788,	789,	790,	791,	792,	
794,	795 ,	796,	797 ,	798,	799 ,	800,	801,	802,	803,	804,
805,	806,	807,	808,	809,	810,	811,	812,	813,	814,	815,
816,	817,	818,	819,	820,	821,	822,		824,	825,	
827,	828,		830,	831,	832,	833,		835,	836,	
838,	839,		841,		843,			846,	847,	
				842,		844,				848,
849,	850,		852,	853,	854,	855,		857,	858,	
860,	861,		863,	864,	865,	866,	867,		869,	
871,	872,	873,	874,	875,	876,	877,	878,	879,	880,	881,
882,	883,	884,	885,	886,	887,	888,	889,	890,	891,	892,
893,	894,	895,	896 ,	897 ,	898,	899 ,	900,		902,	
904,	905,	906,	907,	908,	909,	910,	911,		913,	
915,	916,	917,	918,	919,	920,	921,	922,		924,	
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948,	949,	950,	951,	952,	953,	954,	955,	956,	957,	958,
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           984,
                                           989,
                                                 990, 991,
                       997, 998, 999, 1000], dtype=int64),)
                 996,
```

The values of ϵ , S, I_W and I_Z at that point are equal to:

0.66666666666666

Just before that point the values of ϵ , S, I_W and I_Z are equal to:

```
In [29]: display(epsilon_vals[111],S_endem[111],I_W_endem[111],I_Z_endem[111])

0.111
0.49975
0.50025
0
```

Meaning the jump is disease burden is:

```
In [30]: S_endem[111]-S_endem[112]
```

Out[30]: 0.16641666666667

By derived calculation the the jump in deisease burden should occur when $\epsilon=$

```
In [31]: epsilon_at_change_v2.subs(param_values)
```

Out[31]: 0.1111111111111111

The increase in disease burden should be:

```
In [32]: inc_burd.subs(param_values)
```

Out[32]: 0.16666666666667

All seems correct, minor differences can be put down to rounding errors and that epsilon was altered in increments of 0.001 when ploting graphs of stable endemic equilria.

4 Plots of cost of antimicrobial resistance (reduction in transmission $1-\frac{\beta_Z}{\beta_W}$) against discontinuous increase in disease burden as a proportion of the population. (Figure 3)

```
In [33]: param_values = {'N':1,'beta_W': 0.04 , 'beta_Z': 0.015,'gamma': 0.01,'gamma_T': 0.1} dpi =96 # dpi for this screen is according to: https://www.infobyip.com/detectmonitordpi.pl
```

Remember the I_W strain's $R_0=rac{eta_Z}{\gamma}$. When $eta_Z=0.25eta_W=0.01$ and I_W strain's $R_0=1$, therefore we will not plot below $eta_Z=0.25eta_W=0.01$.

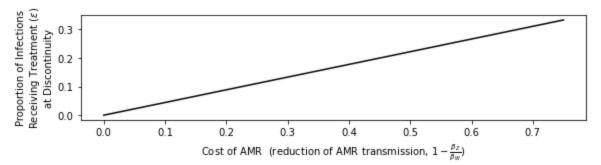
```
In [34]:
    beta_Z_props = np. arange (int(25), int(101), int(1))
    beta_Z_props = np.divide(beta_Z_props,100)
    level_of_treatment = []

for prop in beta_Z_props:
    param_values['beta_Z']= param_values['beta_W']*prop
    epsilon = epsilon_at_change_v2.subs(param_values)
    level_of_treatment.append(epsilon)

fig_epsilon = plt.figure(figsize=(870/dpi, 185/dpi))

ax = plt.subplot(111)
    axes = plt.gca()

ax.plot(np.subtract(1,beta_Z_props),level_of_treatment,color='black')
    ax.set_xlabel(r'Cost of AMR (reduction of AMR transmission, $1-\frac{\beta_Z}{\beta_W}$)
    ax.set_ylabel('Proportion of Infections\nReceiving Treatment ($\epsilon$)\nat Discontinuit plt.show()
```



```
In [35]:
         fig epsilon.savefig('Epsilon at disease jump v AMR to AMS transmission.png', bbox inches=
In [36]:
         param values = {'N':1,'beta W': 0.04 , 'beta Z': 0.015,'gamma': 0.01}
In [37]:
         increases in burden = []
         for prop in beta Z props:
             param values['beta Z']= param values['beta W']*prop
             increase = inc burd.subs(param values)
             increases in burden.append(increase)
         fig increase burden = plt.figure(figsize=(870/dpi, 185/dpi))
         ax = plt.subplot(111)
         axes = plt.gca()
         ax.plot(np.subtract(1,beta Z props),increases in burden,color='black')
         ax.set xlabel(r'Cost of AMR (reduction of AMR transmission, $1-\frac{\beta Z}{\beta W}$)
         ax.set ylabel ('Increase in Disease Burden\nat Discontinuity,\nas a Proportion of Total Por
         plt.show()
```

