Risk aversion heterogeneity and contagion in endogenous financial networks

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Bank's balance sheet

- The model build on works of Aldasoro et. al (2017) and Bluhm and Krahnen (2014).
- It consists of N banks, each with the balance sheet satisfying following identity:

$$c_i + \underbrace{\sum_{j=1}^{N} l_{i,j}}_{l_i} + e_i = d_i + \underbrace{\sum_{j=1}^{N} b_{i,j}}_{b_i} + q_i$$

- Where asset side consists of cash c, external assets e and interbank loans $l_{i,j}$. On the other side there are deposits d, interbank liabilities $l_{i,j}$ and equity q_i.

Financial network representation

- The double indexation represents the interconectedness among banks. $l_{i,j}$ is the value of debt claim from bank i to bank j.
- Therefore, $\sum_{j=1}^{N} l_{i,j}$ is a portfolio of claims of bank i
- These assets are not necessarily limited to interbank loans, but can also include other contracts that posses credit risk.

Objective of banks

- The profit function of banks depend on external and interbank assets and their return, respectively r^e and r^l. As well as the cost of its funding, that depends on the default probability δ and loss given default ζ.

$$\pi_i = r_i^e e_i + r^l l_i - \left(\frac{1}{1-\zeta\delta}\right) r^l b_i$$

- The utility function of a bank is a standard CRRA function

$$U(\pi_i, \sigma_i) = \frac{\pi^{1-\sigma_i}}{1-\sigma_i}$$

Objective of banks - expected utility

$$U(\pi_i) \approx U(E[\pi_i]) + U'((\pi_i - E[\pi_i])) + \frac{1}{2}U''((\pi_i - E[\pi_i])^2)$$

Taking the expected value of both sides:

$$\begin{aligned} E[U(\pi_i)] &\approx E[U(E[\pi_i])] + U'(E[(\pi_i - E[\pi_i])]) + \frac{1}{2}U''(E[(\pi_i - E[\pi_i])^2]) \\ &\approx U(E[\pi_i]) + \frac{1}{2}U''(\pi_i)\sigma_\pi^2 \end{aligned}$$

The expected value of utility is following:

$$E[U(\pi, \sigma_i, \sigma_\pi)] = \frac{\pi^{1-\sigma_i}}{1-\sigma_i} - \frac{\sigma_i}{2}\pi^{-1-\sigma_i}\sigma_\pi$$

Objective of banks - profit variance

Where σ_i is the risk aversion of bank i and σ_π is the variance of the profit of bank i, given by:

$$\begin{aligned}\sigma_\pi^2 &= V[r_i^e e_i + r^l l_i - \frac{1}{1-\zeta\delta_i} r^l b_i] \\ &= e_i^2 \sigma_{r_i^e}^2 - (b_i r^l)^2 V[\frac{1}{1-\zeta\delta_i}] + 2 e_i r^l \mathbf{Cov}[r_i^e, \frac{1}{1-\zeta\delta_i}]\end{aligned}$$

Taking the first order Taylor approximation around expected value of δ_i:

$$V[\frac{1}{1-\zeta\delta_i}] = \zeta^2 (1-\zeta E[\delta_i])^{-4} \sigma_{\delta_i}^2$$

$$\sigma_\pi = e_i^2 \sigma_{r_i^e} - (b_i r_l)^2 \zeta^2 (1-\zeta \mathbf{E}[\delta])^{-4} \sigma_\delta$$

Regulatory requirements

- Banks have to satisfy the following capital requirements:

$$\frac{c_i + l_i + e_i - d_i - b_i}{\omega_e e_i + \omega_l l_i} \geq (\gamma + \tau)$$

- Where ω_e and ω_l are the weights of external and interbank assets, respectively. γ is the minimum capital requirement and τ is the buffer.
- Additionally, banks are required to hold a minimum amount of liquid assets, relative to their deposits:

$$c_i \geq \alpha \times d_i$$

Banks optimization problem

Each bank is deciding on the structure

$$\max_{c_i, e_i, l_i, b_i} \mathbf{E}[U(\pi_i, \sigma_i, \sigma_\pi)]$$

S.t.:

$$c_i + l_i + e_i = d_i + b_i + q_i$$

$$\frac{c_i + e_i + l_i - d_i - b_i}{\omega_e e_i + \omega_l l_i} \geq (\gamma + \tau)$$

$$c_i \geq \alpha \times d_i$$

Interbank market

- Both the interest at which the banks lend and borrow is in the equilibrium, that satisfy the condition:

$$\sum_{i}^{N} l_i^* = \sum_{i}^{N} b_i^*$$

- Where $*$ are optimal amount of lending and borrowing for each bank.
- The interest satisfying the condition is obtained Iteratively, using the tâtonnement process (next slide).

Tâtonnement process

Algorithm 1 tatonnement process

$r^l \leftarrow 0.05$
$r^l_{max} \leftarrow 0.1$
$r^l_{min} \leftarrow 0$
while $|\Delta| < tol$ **do**
  $\Delta \leftarrow \sum_i^N l_i^* - \sum_i^N b_i^*$
  if $\Delta > 0$ **then**
    $r^l_{max} \leftarrow r^l$
    $r^l \leftarrow \frac{r^l_{min} + r^l}{2}$
  else if $\Delta < 0$ **then**
    $r^l_{min} \leftarrow r^l$
    $r^l \leftarrow \frac{r^l_{max} + r^l}{2}$
  end if
end while

Matching funds

Following linear program allows to match the funds among banks:

$$
\begin{aligned}
\max_{A_i^{ib}} \quad & \sum_{i}^{N} \sigma_i (A_i^{ib})^T k_i & & \\
\text{s.t.} \quad & A_{i,i}^{ib} = 0 & \forall\, i \in N & \quad \text{No self-lending} \\
& A_i^{ib} \geq 0 & \forall\, i \in N & \quad \text{No short-selling} \\
& \sum_{i}^{n} A_i^{ib} = l_i & \forall\, i \in N & \quad \text{Matching aggregated loans} \\
& \sum_{j}^{n} A_j^{ib} = b_j & \forall\, j \in N & \quad \text{Matching aggregated borrowing} \\
& \frac{A_i^{ib}}{A_i^{\textbf{total}}} \leq \frac{1}{5} & \forall\, j \in N & \quad \text{Maximum exposure limit}
\end{aligned}
$$

Exogenous shock to the system

After setting the system in equilibrium, we introduce a shock in a form of a single random default. The process of defaulting is as follows:

- Defaulted bank repays the deposit with cash.
- If there are remaining deposits to be repaid, the bank is calling its interbank loans.
- Remainings from the called loans are distributed across interbank creditors.
- the rest of the debt is written down.
- Creditors with defaulted loans may have a negatie equity and are thus defaulting.

Risk aversion heterogeneity

- We want to inspect the effect of risk aversion heterogeneity on the contagion.
- In other words, what is the effect on a system of a single bank with substantially different risk-apetite?
- In order to do it, the following procedure is applied:

$$\sigma_{i\neq \mathrm{ss}} = 2 + \frac{\Delta_{\mathrm{ss}}}{N-1}$$
$$\sigma_{\mathrm{ss}} = 2 - \Delta_{\mathrm{ss}}$$

Simulation parameters

Parameter	Value
N	20
α	0.01
ω_e	1
ω_l	0.2
τ	0.01
ζ	0.6
$E[\delta]$	0.005
σ_δ	0.003
r_e	$U(0, 0.15)$
$V[r_e]$	$\frac{1}{12}(\max(r^e) - \min(r^e))^2$

Parameters were based on bank level balance sheet data (2019) from Orbis

Reproducing empirical data

Parameter	empirical value	model value
$cash/deposits$	5.56%	3.56%
interbank lend./$captial$	138.5%	159.3%
interbank lend./A	15.29%	22.17%
capital/A	7.2%	9.23%
Average degree	NA	2.8
r^l	-0.32%	2.24%

Simulation results

Δ_{ss}	mean # defaults	P(# defaults > 2)
0	1.68	0.19
0.5	1.71	0.2
1	1.78	0.24
1.5	1.78	0.25
2	1.80	0.27
2.5	1.93	0.28
3	2.06	0.31

Table: results based on 7×300 simulations

References

Thank you!

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- Bluhm, Marcel, and Jan-Pieter Krahnen. "Systemic risk in an interconnected banking system with endogenous asset markets." Journal of Financial Stability 13 (2014): 75-94.
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