

## GRAMMARS

1. Given the grammar  $G = (N, \Sigma, P, S)$

$$N = \{S, C\}$$

$$\Sigma = \{a, b\}$$

$$P : S \rightarrow ab \mid aCSb$$

$$C \rightarrow S \mid bSb$$

$$CS \rightarrow b,$$

prove that  $w = ab(ab^2)^2 \in L(G)$ .

Sol.:

$$a^2b^2 = aabb \neq (ab)^2 = abab$$

//Oana Nourescu

$$\begin{array}{ccccccc} S & \Rightarrow & aCSb & \Rightarrow & abSbSb & \Rightarrow & ababbabb = w \\ & & (2) & & (4) & & (1) \end{array}$$

$$\begin{array}{c} 4 \\ \Rightarrow S \Rightarrow w \Rightarrow w \in L(G) \end{array}$$

2. Given the grammar  $G = (N, \Sigma, P, S)$

$$N = \{S\}$$

$$\Sigma = \{a, b, c\}$$

$$P : S \rightarrow a^2S \mid bc,$$

find  $L(G)$ .

// Paun Tudor

$$L = \{a^{2k}bc, \text{ where } k \in N\}$$

**L = L(G) ?**

1.  $L \subseteq L(G)$

2.  $L(G) \subseteq L$

**//Ariana Hategan**

1.  $\forall k \in N, a^{2k}bc \in L(G)$

$P(k) : a^{2k}bc \in L(G)$  and prove that  $P(k)$  true for  $\forall k \in N$

**a) Verification step**

$P(0) : a^0bc \in L(G) \Leftrightarrow bc \in L(G)$

**S  $\Rightarrow$  bc**

**So, P(0) is true**

**b)  $P(n) \text{ true} \rightarrow P(n+1) \text{ true}, n \in N$**

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$P(n) \text{ true} \Rightarrow a^{2n}bc \in L(G) \Rightarrow S \Rightarrow a^{2n}bc$  (induction hypothesis)

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$S \Rightarrow a^2S \Rightarrow a^2a^{2n}bc = a^{2(n+1)}bc$

**(1) (ind hypo)**

**$\Rightarrow P(n+1) \text{ true}$**

**a) + b)  $\Rightarrow$  (1)**

**//Razvan Neta**

$S \Rightarrow bc$

$\Rightarrow a^2S \Rightarrow a^2bc$

$\Rightarrow a^4S \Rightarrow a^4bc$

$\Rightarrow a^6S \Rightarrow \dots$

$\Rightarrow \dots$

It may be noticed that starting from S and using **all productions** in **all possible combinations**, we only get, as sequences of terminals, sequences of the shape  $a^{2^n}bc$ ,  $n \in \mathbb{N}$   
It follows that the grammar generates nothing else.

3. Find a grammar that generates  $L = \{0^n 1^n 2^m \mid n, m \in \mathbb{N}^*\}$

Petcu Dragos

$A \rightarrow 01 \mid 0A1$

$B \rightarrow 2 \mid 2B$

$S \rightarrow AB$

Oana Nourescu

$S \rightarrow AB$

$A \rightarrow 01 \mid 0A1$

$B \rightarrow 2 \mid B2$

Diaconu Bogdan

$S \rightarrow 0A1B$

$A \rightarrow 0A1 \mid \epsilon$

$B \rightarrow 2 \mid 2B$

Moldovanu Dragos

$A \rightarrow 01 \mid 0A1$

$B \rightarrow 2 \mid 2B$

$S \rightarrow AB$

**//Moldovan Vasilica**

**S -> 0S1C | 012**

**C -> 2C | epsilon**

**Pascotescu Iuliana**

**S -> 0K1 | 01 | T | ST**

**T -> 2T | 2**

**K -> 0K1 | 01**

**//Onita Andrei**

**P: S = A B**

**A = 0 S 1 | 01**

**B = S 2 | 2**

**G = ({S,A,B}, {0,1,2}, P, S)**

**P:**

**S -> AB**

**A -> 01 | 0A1**

**B -> 2 | B2**

**?  $L(G) = L$**

1. ?  $L \subseteq L(G)$

?  $\forall n, m \in N^*, 0^n 1^n 2^m \in L(G)$

**Let  $n, m \in N^*$  fixed**

$$\begin{array}{ccccc} & \mathbf{n} & & \mathbf{m} & \\ S \Rightarrow AB & \Rightarrow 0^n 1^n B & \Rightarrow & 0^n 1^n 2^m & \\ \mathbf{(1)} & \mathbf{(i)} & & \mathbf{(ii)} & \end{array}$$

$$\begin{array}{c} \mathbf{n+m+1} \\ \Rightarrow S \Rightarrow 0^n 1^n 2^m \Rightarrow 0^n 1^n 2^m \in L(G) \end{array}$$

$$\begin{array}{c} \mathbf{n} \\ \mathbf{(i)} \ A \Rightarrow 0^n 1^n, \ \forall n \in N^* \end{array}$$

$$\begin{array}{c} \mathbf{m} \\ \mathbf{(ii)} \ B \Rightarrow 2^m, \ \forall m \in N^* \end{array}$$

2. ?  $L(G) \subseteq L \dots$





L<sup>A</sup>T<sub>E</sub>X

Lol

Lol x2 - aproape ai reusit