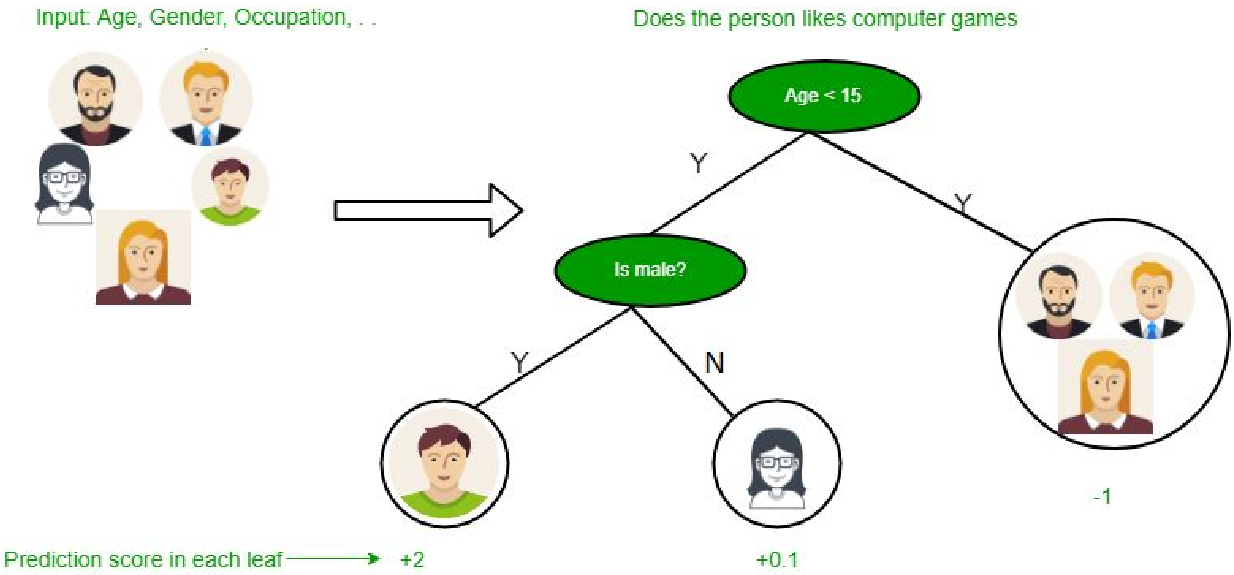
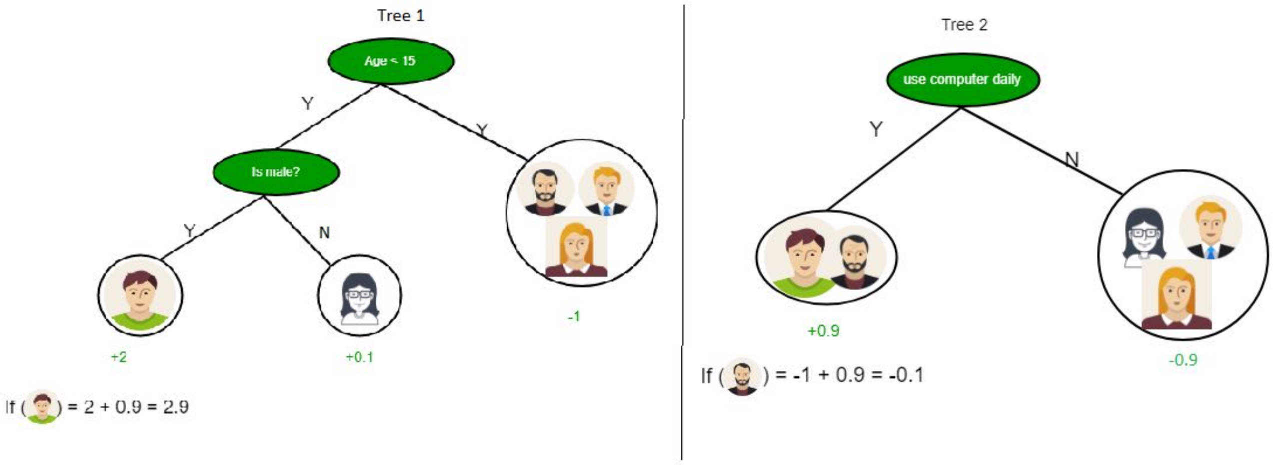
Example:

- Decision tree algorithms fall under the category of supervised learning. They can be used to solve both regression and classification problems.
- Decision trees use the tree representation to solve the problem in which each leaf node corresponds to a class label and attributes are represented on the internal node of the tree.
- We can represent any boolean function on discrete attributes using the decision tree.


Input: Age, Gender, Occupation, . .
Does the person likes computer games
Age < 15
Y
Y
Is male?
Y
N
-1
Prediction score in each leaf
+2
+0.1

Below are some assumptions that we made while using decision tree:

- At the beginning, we consider the whole training set as the root.
- Feature values are preferred to be categorical. If the values are continuous then they are discretized prior to building the model.
- On the basis of attribute values records are distributed recursively.
- We use statistical methods for ordering attributes as root or the internal node.



As you can see from the above image that Decision Tree works on the Sum of Product form which is also known as *Disjunctive Normal Form*. In the above image, we are predicting the use of computers in the daily life of the people.

In the Decision Tree the major challenge is to identify the attribute for the root node in each level. This process is known as attribute selection. We have two popular attribute selection measures:

1. Information Gain
2. Gini Index

1. Information Gain

When we use a node in a decision tree to partition the training instances into smaller subsets the entropy changes. Information gain is a measure of this change in entropy.

Definition: Suppose S is a set of instances, A is an attribute, S_v is the subset of S with A = v, and Values (A) is the set of all possible values of A, then

$$Gain(S,A) = E(S) - \sum_{v \in Values(a)} \frac{|S_v|}{|S|} \cdot E(S_v)$$

Entropy

Entropy is the measure of uncertainty of a random variable, it characterizes the impurity of an arbitrary collection of examples. The higher the entropy more the information content.

Definition: Suppose S is a set of instances, A is an attribute, S_v is the subset of S with A = v, and Values (A) is the set of all possible values of A, then

$$Gain(S,A) = E(S) - \sum_{v \in Values(a)} \frac{|S_v|}{|S|} \cdot E(S_v)$$

Example:

For the set $X = \{a, a, a, b, b, b, b, b\}$
Total instances: *8*
Instances of b: *5*
Instances of a: *3*

$$E(X) = -\left[(\tfrac{3}{8})log_2(\tfrac{3}{8}) + (\tfrac{5}{8})log_2(\tfrac{5}{8})\right] = -[0.375 * (-1.415) + 0.625 * (-0.678)] = -(-0.53 - 0.424) = 0.954$$

Building Decision Tree using Information Gain

The essentials:

- Start with all training instances associated with the root node
- Use info gain to choose which attribute to label each node with
- *Note:* No root-to-leaf path should contain the same discrete attribute twice
- Recursively construct each subtree on the subset of training instances that would be classified down that path in the tree.

- **The border cases:** If all positive or all negative training instances remain, label that node “yes” or “no” accordingly
- If no attributes remain, label with a majority vote of training instances left at that node
- If no instances remain, label with a majority vote of the parent’s training instances

Example:

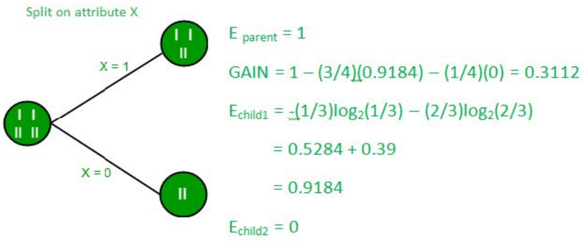
Now, let's draw a Decision Tree for the following data using Information gain.

Training set: 3 features and 2 classes

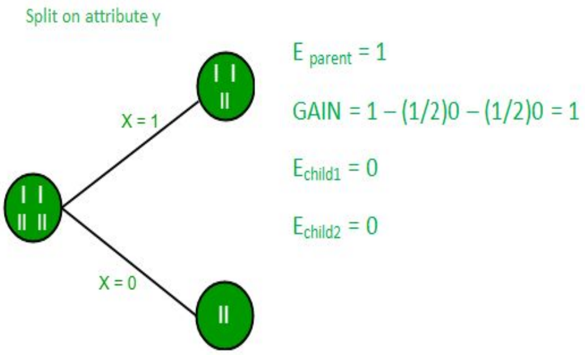
X	Y	Z	C
1	1	1	I
1	1	0	I
0	0	1	II
1	0	0	II

Here, we have 3 features and 2 output classes.

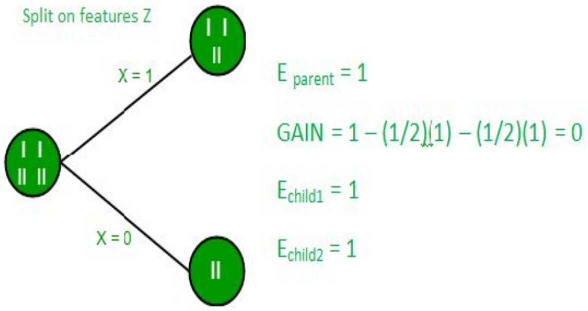
To build a decision tree using Information gain. We will take each of the features and calculate the information for each feature.



Split on feature X



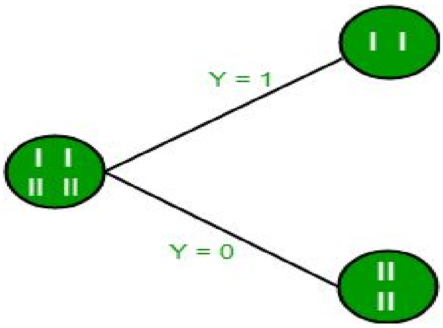
Split on feature Y



Split on feature Z

From the above images we can see that the information gain is maximum when we make a split on feature Y. So, for the root node best suited feature is feature Y. Now we can see that while splitting the dataset by feature Y, the child contains a pure subset of the target variable. So we don't need to further split the dataset.

The final tree for the above dataset would be look like this:



2. Gini Index

- Gini Index is a metric to measure how often a randomly chosen element would be incorrectly identified.
- It means an attribute with lower Gini index should be preferred.
- Sklearn (**Scikit-learn -** is a free software machine learning library for the Python programming language) supports "Gini" criteria for Gini Index and by default, it takes "gini" value.
- The Formula for the calculation of the Gini Index is given below.

$$GiniIndex = 1 - \sum_j p_j^2$$

Example:

Let's consider the dataset in the image below and draw a decision tree using gini index.

Index	A	B	C	D	E
1	4.8	3.4	1.9	0.2	positive
2	5	3	1.6	1.2	positive
3	5	3.4	1.6	0.2	positive
4	5.2	3.5	1.5	0.2	positive
5	5.2	3.4	1.4	0.2	positive
6	4.7	3.2	1.6	0.2	positive
7	4.8	3.1	1.6	0.2	positive
8	5.4	3.4	1.5	0.4	positive
9	7	3.2	4.7	1.4	negative
10	6.4	3.2	4.7	1.5	negative
11	6.9	3.1	4.9	1.5	negative
12	5.5	2.3	4	1.3	negative
13	6.5	2.8	4.6	1.5	negative
14	5.7	2.8	4.5	1.3	negative
5	6.3	3.3	4.7	1.6	negative
16	4.9	2.4	3.3	1	negative1

In the dataset above there are *5* attributes from which attribute *E* is the predicting feature which contains *2* (Positive & Negative) classes. We have an equal proportion for both the classes.

In the Gini Index, we have to choose some random values to categorize each attribute. These values for this dataset are:

A	B	C	D
≥ 5	≥ 3	≥ 4.2	≥ 1.4
< 5	< 3	< 4.2	< 1.4

Calculating Gini Index for Var A:

Value ≥ 5**:** 12 samples

$Attribute\ A\ \geq 5\ \&\ class\ =\ positive : \frac{5}{12}$
$Attribute\ A\ \geq 5\ \&\ class\ =\ negative : \frac{7}{12}$

$Gini(5,7) = 1 - \left[\left(\frac{5}{12}\right)^2 + \left(\frac{7}{12}\right)^2\right] = 0.4860$

Value < 5 **:** 4 samples

$Attribute\ A\ < 5\ \&\ class\ = \ positive : \frac{3}{4}$
$Attribute\ A\ < 5\ \&\ class\ = \ negative : \frac{1}{4}$

$Gini(3,1) = 1 - \left[\left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2\right] = 0.375$

By adding weight and sum each of the gini indices:

$Gini(target,\ A) = \left(\frac{12}{16}\right) * (0.4860) + \left(\frac{4}{16}\right) * (0.375)$

Calculating Gini Index for Var B:

Value ≥ 3 **:** 12 samples

$Attribute\ B\ \geq\ 3\ \&\ class\ = \ positive : \frac{8}{12}$
$Attribute\ B\ <\ 3\ \&\ class\ = \ negative : \frac{4}{12}$

$Gini(8,4) = 1 - \left[\left(\frac{8}{12}\right)^2 + \left(\frac{4}{12}\right)^2\right] = 0.4460$

Value > 3 **:** 4 samples

$Attribute\ B\ <\ 3\ \&\ class\ = \ positive : \frac{0}{4}$
$Attribute\ B\ <\ 3\ \&\ class\ = \ negative : \frac{4}{4}$

$Gini(0,4) = 1 - \left[\left(\frac{0}{4}\right)^2 + \left(\frac{4}{4}\right)^2\right] = 0$

By adding weight and sum each of the gini indices:

$Gini(target,\ B) = \left(\frac{12}{16}\right) * (0.4460) + \left(\frac{0}{16}\right) * (0) = 0.3345$

Using the same approach we can calculate the Gini index for C and D attributes.

	Positive	Negative
For *A*|>= *5.0*	*5*	*7*
|< *5*	*3*	*1*

Gini Index of *A = 0.45825*

	Positive	Negative
For B|>= *3*	*8*	*4*
|< *3*	*0*	*4*

Gini Index of *B= 0.3345*

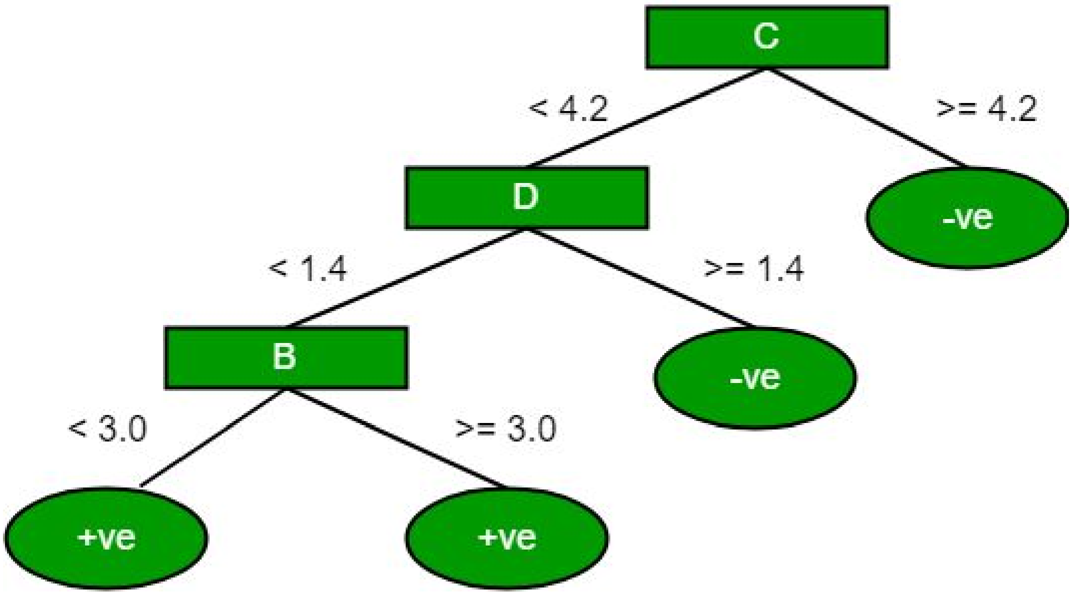
	Positive	Negative
For C|>= *4.2*	*0*	*6*
|< *4.2*	*8*	*2*

Gini Index of *C= 0.2*

	Positive	Negative
For D|>= *1.4*	*0*	*5*
|< *1.4*	*8*	*3*

Gini Index of *D= 0.273*

Decision tree for above dataset


C
< 4.2
>= 4.2
D
-ve
< 1.4
>= 1.4
B
-ve
< 3.0
>= 3.0
+ve
+ve