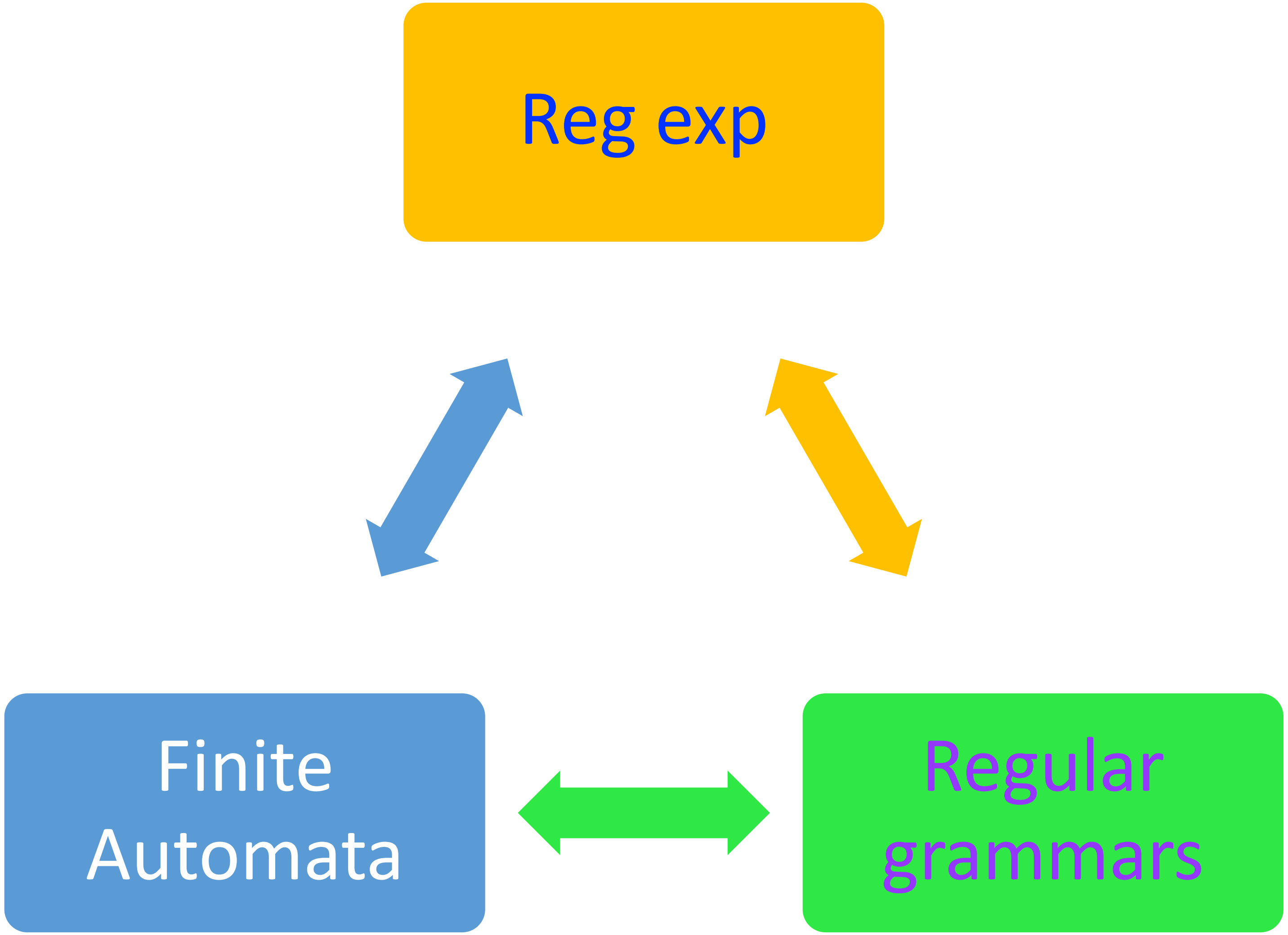
Course 4

-mă pun comod în pat-
Prof: Vă rog să porniți camerele

Source: Facebook – Viata de student


Reg exp
Finite Automata
Regular grammars

Prop:*Regular sets are right linear languages*

Lemma 1: $\boldsymbol{\Phi}$,{$\boldsymbol{\varepsilon}$}, {a},$\forall a \in \boldsymbol{\Sigma}$ are right linear languages

Proof: constructive

i. G = ({S}, $\boldsymbol{\Sigma}$, $\boldsymbol{\Phi}$, S) – regular grammar such that L(G) = $\boldsymbol{\Phi}$

ii. G = ({S}, $\boldsymbol{\Sigma}$,{S→$\boldsymbol{\varepsilon}$}, S) – regular grammar such that L(G) ={$\boldsymbol{\varepsilon}$}

iii. G = ({S}, $\boldsymbol{\Sigma}$,{S→a}, S) – regular grammar such that L(G) ={a}

Lemma 2: If L_1 and L_2 are right linear languages then:

$L_1 \cup L_2$, L_1L_2 and L_1* are right linear languages.

Proof: constructive

L_1, L_2 right linear languages => $\exists G_1, G_2$ such that

$G_1 = (N_1, \mathbf{\Sigma}_1, P_1, S_1)$ and $L_1 = L(G_1)$

$G_2 = (N_2, \mathbf{\Sigma}_2, P_2, S_2)$ and $L_2 = L(G_2)$ assume $N_1 \cap N_2 = \emptyset$

i. $G_3 = (N_3, \boldsymbol{\Sigma}, P_3, S_3)$

$N_3 = N_1 \cup N_2 \cup \{S_3\}$; $\Sigma_3 = \Sigma_1 \cup \Sigma_2$

$P_3 = P_1 \cup P_2 \cup \cancel{\{S_3 \to S_1 \mid S_2\}}$

$$\{S_3 \to \boldsymbol{\alpha}_1 \mid S_1 \to \boldsymbol{\alpha}_1 \in P_1\} \cup \{S_3 \to \boldsymbol{\alpha}_2 \mid S_2 \to \boldsymbol{\alpha}_2 \in P_2\}$$

G_3 – right linear language

and

$L(G_3) = L(G_1) \cup L(G_2)$

PROOF!!! Homework

ii. $G_4 = (N_4, \boldsymbol{\Sigma}, P_4, S_4)$

$N_4 = N_1 \cup N_2$; $S_4 = S_1$; $\Sigma_4 = \Sigma_1 \cup \Sigma_2$

$$
\begin{aligned}
P_4 = \; & \{A \to aB \mid \text{if } A \to aB \in P_1\} \cup \\
& \{A \to aS_2 \mid \text{if } A \to a \in P_1\} \cup \\
& \{S_1 \to x \mid \text{if } S_1 \to \boldsymbol{\varepsilon} \text{ and } S_2 \to x\} \cup P_2
\end{aligned}
$$

G_4 – right linear language

and

$L(G_4) = L(G_1)\, L(G_2)$

PROOF!!! Homework

iii. $G_5 = (N_5, \Sigma_1, P_5, S_5)$

//IDEA: concatenate L_1 with itself

$N_4 = N_1 U \{S_5\}$;

$P_5 = \quad P_1 \ U \ \{S_5 \rightarrow \boldsymbol{\varepsilon}\} \ U$
$\{S_5 \rightarrow \boldsymbol{\alpha}_1 \mid S_1 \rightarrow \boldsymbol{\alpha}_1 \in P_1\} \ U$
$\{A \rightarrow aS_1 \mid \text{if } A \rightarrow a \in P_1\}$

G_5 – right linear language

and

$L(G_5) = L(G_1)^*$

PROOF!!! Homework

Theorem: A language is a regular set if and only if is a right linear language

Proof:

”=>” Apply lemma 1 and lemma 2

”<=” construct a system of regular exp equations where:

- Indeterminants – nonterminals
- Coefficients – terminals
- Equation for A: all the possible rewritings of A

Example: G=({S,A,B},{0,1}, P, S)

P: S → 0A | 1B | **ε**

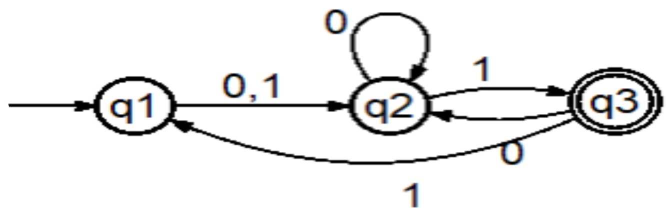
A → 0B | 1A

B → 0S | 1

$$\begin{cases} S = 0A + 1B + \boldsymbol{\varepsilon} \\ A = 0B + 1A \\ B = 0S + 1 \end{cases}$$

Regular exp = solution corresponding to S

Theorem: *A language is a regular set if and only if is accepted by a FA*



Proof:

=> Apply lemma 1' and lemma 2' (to follow, similar to RG)

<= construct a system of regular exp equations where:

- Indeterminants – states
- Coefficients – terminals
- Equation for A: all the possibilities that put the FA in state A
- Equation of the form: X=Xa+b => solution **X=ba***

$$\begin{cases} q_1 = q_3 0 + \boldsymbol{\varepsilon} \\ q_2 = q_1 0 + q_1 1 + q_2 0 + q_3 0 \\ q_3 = q_2 1 \end{cases}$$

Regular exp = union of solutions corresponding to final states

Lemma 1’: $\boldsymbol{\Phi}$,{$\boldsymbol{\varepsilon}$}, {a},$\forall a \in \boldsymbol{\Sigma}$ are accepted by FA

Reg exp	FA
$\boldsymbol{\Phi}$	$M = (Q, \boldsymbol{\Sigma}, \delta, q_{0,} \boldsymbol{\Phi})$
$\boldsymbol{\varepsilon}$	$M = (Q, \boldsymbol{\Sigma}, \boldsymbol{\Phi}, q_{0,} \{q_0\})$
a,$\forall a \in \boldsymbol{\Sigma}$	$M = (\{q_0,q_1\}, \boldsymbol{\Sigma}, \{\delta(q_0,a) = q_1\}, q_{0,} \{q_1\})$

Lemma 2': If L_1 and L_2 are accepted by a FA then: $L_1 \cup L_2$, L_1L_2 and L_1^* are accepted by FA

Proof:

$M_1 = (Q_1, \boldsymbol{\Sigma}_1, \delta_1, q_{01}, F_1)$ such that $L_1 = L(M_1)$

$M_2 = (Q_2, \boldsymbol{\Sigma}_2, \delta_2, q_{02}, F_2)$ such that $L_2 = L(M_2)$

$M_3 = (Q_3, \boldsymbol{\Sigma}_{1U}, \delta_3, q_{03}, F_3)$

$Q_3 = Q_1 \cup Q_2 \cup \{q_{03}\}$; $\Sigma_3 = \Sigma_1 \cup \Sigma_2$

$F_3 = F_1 \cup F_2 \cup \{q_{03} \mid \text{if } q_{01} \in F_1 \text{ or } q_{02} \in F_2\}$

$\delta_3 = \delta_1 \cup \delta_2 \cup \{\delta_3(q_{03},a) = p \mid \exists \delta_1(q_{01},a) = p\} \cup$
$\{\delta_3(q_{03},a) = p \mid \exists \delta_2(q_{02},a) = p\}$

$L(M_3) = L(M_1) \cup L(M_2)$

PROOF!!! Homework

$M_4 = (Q_4, \mathbf{\Sigma}_4, \delta_4, q_{04}, F_4)$

$Q_4 = Q_1 \cup Q_2$; $q_{04} = q_{01}$;

$F_4 = F_2 \cup \{q \in F_1 \mid \text{if } q_{02} \in F_2\}$

$$\delta_4(q,a) = \begin{cases} \delta_1(q,a), \text{ if } q \in Q_1 - F_1 \\ \delta_1(q,a) \cup \delta_2(q_{02},a) \text{ if } q \in F_1 \\ \delta_2(q,a), \text{ if } q \in Q_2 \end{cases}$$

$L(M_3) = L(M_1)L(M_2)$

PROOF!!! Homework

$M_5 = (Q_5, \boldsymbol{\Sigma}_1, \delta_5, q_{05}, F_5)$

$Q_5 = Q_1;$ $\qquad q_{05} = q_{01}$

$F_5 = F_1 \cup \{q_{01}\}$

$$\delta_5(q,a) = \begin{cases} \delta_1(q,a), \text{ if } q \in Q_1 - F_1 \\ \delta_1(q,a) \cup \delta_1(q_{01},a) \text{ if } q \in F_1 \end{cases}$$

$L(M_3) = L(M_1)^*$

PROOF!!! Homework