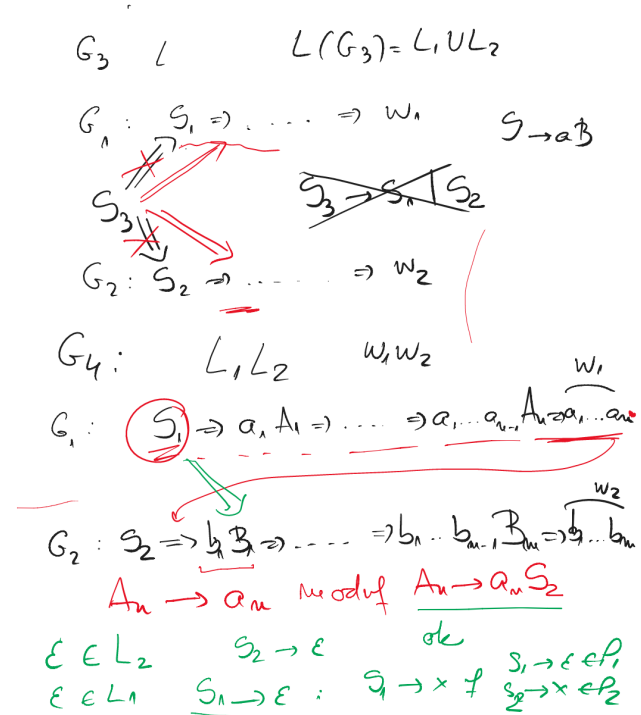


# Notes from Whiteboard – course 4

- $G_3$  – grammar for union
- $G_4$  – grammar for concatenation
- $G_5$  – grammar for  $*$  closure



$G_5: L^* \text{ (Kleene)}$

$S_1 \rightarrow aA \mid b$

$S_1 \Rightarrow a_1 A_1 \Rightarrow \dots \Rightarrow a_1 \dots a_n A_n \Rightarrow a_1 \dots a_n \overbrace{A_n \Rightarrow a_1 \dots a_n}^{w_1}$

$w_1 w_2$  is  $G_1$  concat. with  $G_2$

$A_n \rightarrow a_n \quad A_n \rightarrow a_n \underline{S_1}$

$L^0 = \{\epsilon\} \quad \underline{S_1 \rightarrow \epsilon}$

# Example for union, concatenation, \* closure

$$G_1: S_1 \rightarrow aA \mid aS_1$$

$$A \rightarrow bA \mid b$$

$$L(G_1) = \{a^m b^n \mid m, n \geq 0\}$$

$$G_2: S_2 \rightarrow aA_1$$

$$A_1 \rightarrow bA_1 \mid bB$$

$$B \rightarrow a \quad L(G_2) = \{ab^u a^v \mid u, v \geq 0\}$$

$$L(G_1) \cup L(G_2)$$

$$G_3: S_3 \rightarrow aA \mid aS_1 \mid aA_1$$

$$S_1 \rightarrow aA \mid aS_1$$

$$A \rightarrow bA \mid b$$

$$S_2 \rightarrow aA_1$$

$$A_1 \rightarrow bA_1 \mid bB$$

$$B \rightarrow a$$

$$L_1 L_2 \quad S_4 = S_1$$

$$G_4: S_1 \rightarrow aA \mid aS_1$$

$$A \rightarrow bA \mid bS_2$$

$$S_2 \rightarrow aA_1$$

$$A_1 \rightarrow bA_1 \mid bB$$

$$B \rightarrow a$$

$$G_5: L(G_1)^*$$

$$S_5 \rightarrow aA \mid aS_1 \mid \epsilon$$

$$S_1 \rightarrow aA \mid aS_1$$

$$A \rightarrow bA \mid bS_1$$

# Solving reg.exp. system of equations – system constructed from reg grammar

reg exp eq:

$$S = 0A + 1B + \epsilon$$

$$A = 0B + 1A$$

$$B = 0S + 1$$

$$X = aX + b$$

$$X = a^*b$$

$$A = 0(S+1) + 1A \equiv 00S + 01 + 1A$$

$$\Rightarrow A = 1^*(00S + 01)$$

$$S = 0A + 1B + \epsilon \quad \text{replace } A, B:$$

$$S = 01^*(00S + 01) + 1(0S + 1) + \epsilon$$

$$S = \underbrace{(01^*00 + 10)}_a S + \underbrace{01^*01 + 11 + \epsilon}_b$$

$$S = (01^*00 + 10)^*(01^*01 + 11 + \epsilon)$$

# Solving reg.exp. system of equations –system constructed from FA

$$Q_1 = Q_3 0 + \epsilon$$

$$Q_2 = Q_1 0 + Q_1 1 + Q_2 0 + Q_3 0$$

$$Q_3 = Q_2 1$$

$$Q_1 = Q_2 1 0 + \epsilon$$

$$Q_2 = \underbrace{(Q_2 1 0 + \epsilon) 0}_{\epsilon} + \underbrace{(Q_2 1 0 + \epsilon) 1}_{Q_2 1 0} + Q_2 0 + Q_2 1 0$$

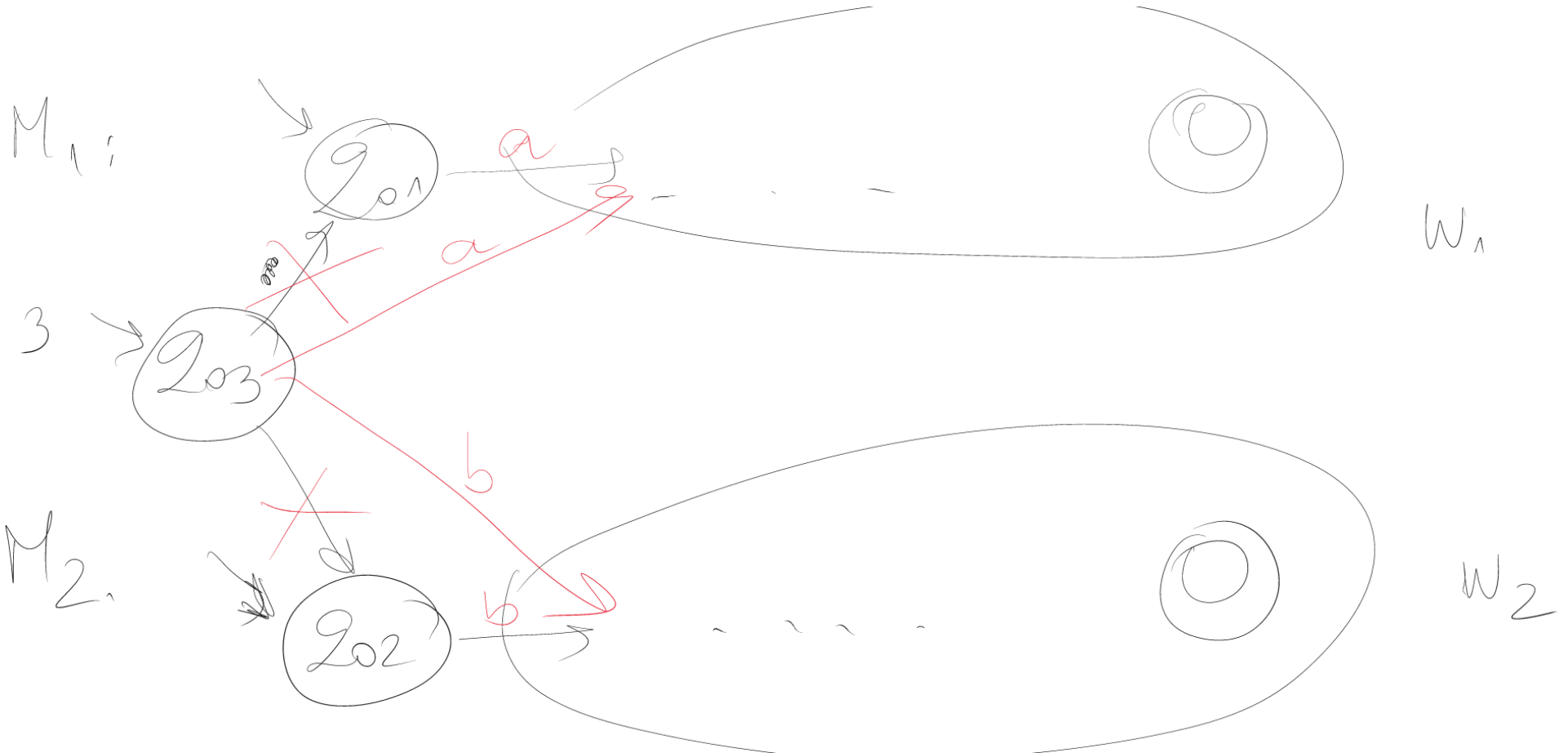
$$Q_2 = Q_2 1 0 0 + 0 + Q_2 1 0 1 + 1 + Q_2 0 + Q_2 1 0$$

$$Q_2 = Q_2 (1 0 0 + 1 0 1 + 0 + 1 0) + 0 + 1$$

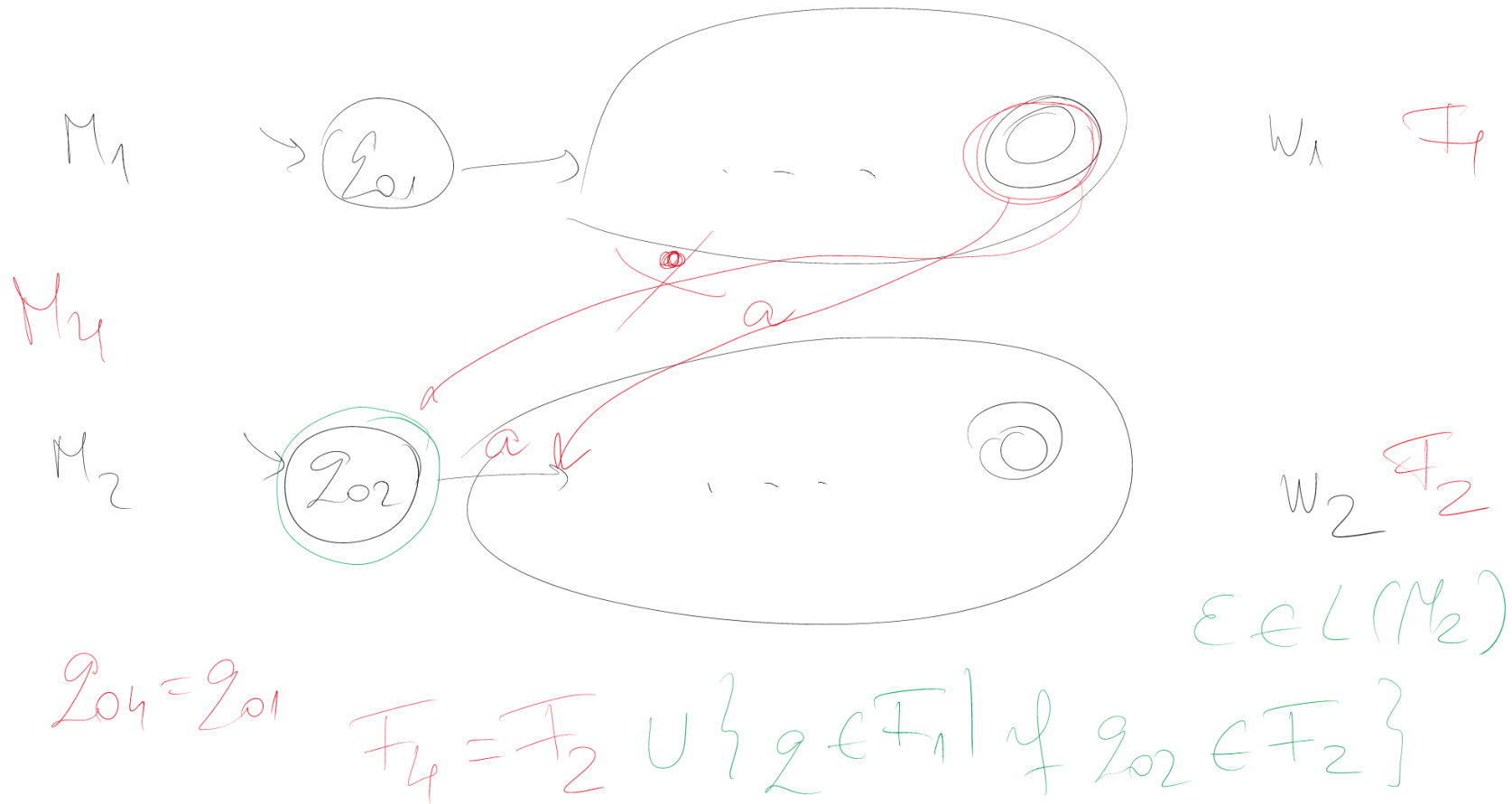
$$Q_2 = (0 + 1) (1 0 0 + 1 0 1 + 0 + 1 0)^*$$

$$Q_3 = (0 + 1) (1 0 0 + 1 0 1 + 0 + 1 0)^* 1$$

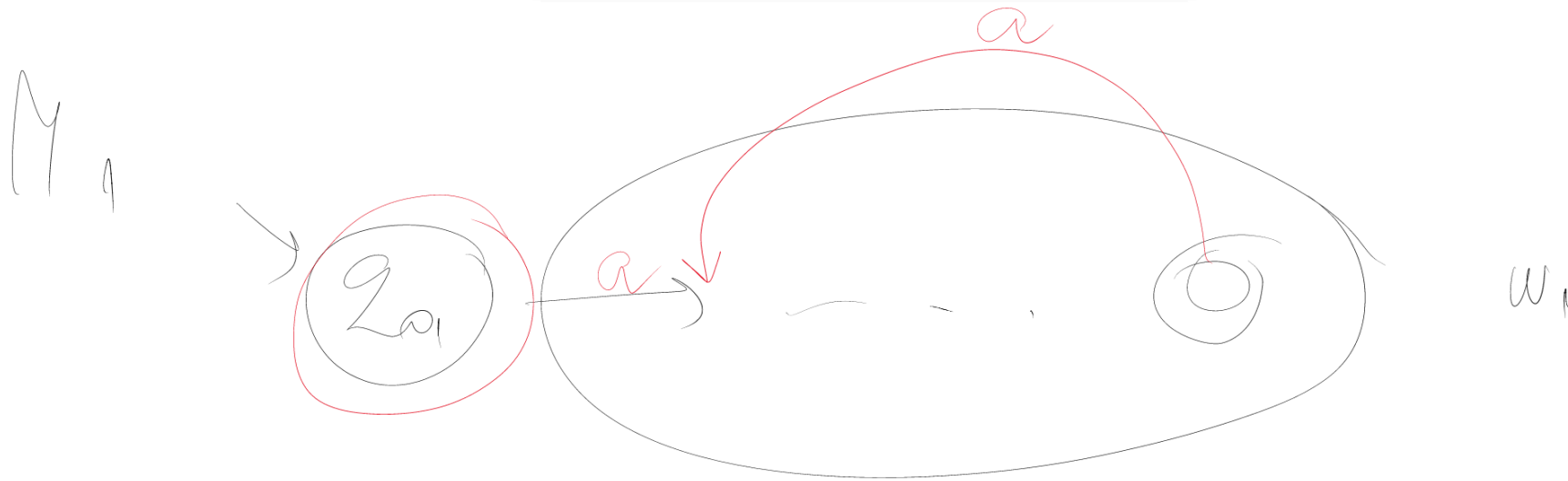
FA  $M_3$  corresponding to  $L(M_1) \cup L(M_2)$



# FA $M_4$ corresponding to $L(M_1)L(M_2)$



FA  $M_5$  corresponding to  $L(M_1)^*$



$L(M_1)^*$        $\varepsilon \in L(M_5) \Rightarrow 2a1 \in F_5$

$w_1 w_1$  - concat  $M_1$  with  $M_1$