RESEARCH ARTICLE

How indefinites choose their scope

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Abstract The paper proposes a novel solution to the problem of scope posed by natural language indefinites that captures both the difference in scopal freedom between indefinites and *bona fide* quantifiers and the syntactic sensitivity that the scope of indefinites does nevertheless exhibit. Following the main insight of choice functional approaches, we connect the special scopal properties of indefinites to the fact that their semantics can be stated in terms of choosing a suitable witness. This is in contrast to *bona fide* quantifiers, the semantics of which crucially involves relations between sets of entities. We provide empirical arguments that this insight should not be captured by adding choice/Skolem functions to classical first-order logic, but in a semantics that follows Independence-Friendly Logic, in which scopal relations involving existentials are part of the recursive definition of truth and satisfaction. These scopal relations are resolved automatically as part of the interpretation of existentials. Additional support for this approach is provided by dependent indefinites, a cross-linguistically common class of special indefinites that can be straightforwardly analyzed in our semantic framework.

Keywords Exceptional scope · (Dependent) indefinites · Choice/Skolem functions · Plural logic · Independence-friendly logic

1 Main challenges

This paper approaches the problem of the scope of indefinite noun phrases from a fresh perspective, aiming to address two inter-related concerns.

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The first problem is capturing the contrast between the intra-sentential scopal properties of simple indefinite DPs on one hand, and universals and other uncontroversially quantified DPs on the other. Simple indefinites, i.e., DPs whose D is the unmarked *a(n)* in English and its closest equivalents in other languages, contrast with *bona fide* quantificational DPs in that simple indefinites can have so-called 'exceptional' scope over a syntactically higher quantifier or operator independently of the complexity of the syntactic structure that separates them, while the inverse scope of universals and other *bona fide* quantificational DPs is limited to the clause in which they occur. Section 1.2 presents an overview of this problem. Section 3 presents our account and the way it handles the main hurdles brought to light by the vast literature on this topic.

The second problem we address concerns the question of how the account developed for simple indefinite DPs extends to cases of special indefinites, i.e., indefinites whose distribution and scopal properties are more circumscribed. In this paper we discuss the case of 'dependent' indefinites, introduced in Sect. 1.3 and analyzed in detail in Sect. 4.

A variety of tools have been used to deal with the first concern. In this paper, we work out a solution to the exceptional-scope problem that builds on the basic framework and insights of Independence-Friendly Logic (IFL, see Hintikka 1973; Sandu 1993; Hodges 1997; Hintikka and Sandu 1997; Väänänen 2007; Caicedo et al. 2009 among others). We modify this framework in various ways so as to better fit the needs of the analysis of simple indefinites in English and their counterparts in other languages.

The main novelty of this approach is that (in)dependence relationships between variables bound by different quantifiers is an explicit part of the interpretation procedure, i.e., of the recursive definition of truth and satisfaction, rather than being indirectly read off of the syntactic structure of the formula that is being interpreted. We argue that this framework provides an empirically and theoretically optimal solution to the exceptional scope problem posed by simple indefinites.

1.1 Basic distinctions: dependence versus independence, syntactic versus semantic scope

The essence of scope in natural language semantics can be characterized as follows: an expression e_1 takes semantic scope over an expression e_2 if the interpretation of the former affects the interpretation of the latter. More specifically for present purposes, we are interested in the special case in which the two expressions are DPs, as exemplified in (1):

(1)   Every^x student in my class read a^y paper about scope.

How can we tell whether the indefinite in (1) is in the scope of the universal or not? We can answer this question in two ways.

From a *dependence*-based perspective, a quantifier $\mathbf{Q}'y$ is in the semantic scope of another quantifier $\mathbf{Q}x$ iff the values of the variable y may covary with the values of x. The dependence-based perspective spells out the semantic consequences of

being *within* the semantic scope of a quantifier: when evaluating a sentence in which two quantifiers occur, the values given to y may covary with the values given to x.

In the case of (1), if the indefinite is within the semantic scope of the universal, the sentence is true in situations in which different students read (possibly) different papers. We have (possible) covariation of papers with students: the value given to y is allowed to change when the value given to x changes, and therefore the interpretation of x is allowed to affect the interpretation of y.

From an *independence*-based perspective, $\mathbf{Q}'y$ is outside the semantic scope of $\mathbf{Q}x$ iff y's value is fixed relative to the values of x. The independence-based perspective spells out the semantic consequences of being *outside* the scope of a quantifier: when evaluating a sentence in which the two quantifiers occur, the evaluation of y is not affected by the evaluation of x.

In the case of (1), if the indefinite is outside the semantic scope of the universal, the sentence is true in situations in which all the students read the same paper. There is no covariation of papers with students: the value given to y does not change when the value given to x changes.

Note that under both perspectives, $\mathbf{Q}'y$ is within the semantic scope of $\mathbf{Q}x$ if the values of y are allowed to covary with those of x, and $\mathbf{Q}'y$ is outside the semantic scope of $\mathbf{Q}'y$ if the values of x are fixed relative to those of y.

This brings us to the first question this paper raises, namely whether the scopal properties of simple indefinites should be characterized in terms of *dependence* or in terms of *independence*. A dependence-based approach establishes which quantifier(s) $\mathbf{Q}'y$ is dependent on, i.e., it establishes which variables y is allowed to covary with. An independence-based approach establishes which quantifier(s) $\mathbf{Q}'y$ is independent of, i.e., it establishes the variables relative to which the values of y have to be fixed.

Logical semantics has taken both paths to the notion of scope. The classical semantics of first-order logic (FOL) (or its Skolemized version) is dependence driven while the semantics of IFL is independence driven. Natural language semantics has only taken the dependence-based path. We argue here that there are advantages to following an independence-based approach to the scopal properties of natural language indefinites, and thus to importing the main insights from IFL into natural language semantics.

The second theoretical issue that arises is whether scope is a matter of syntax, semantics or both. And if the last answer is correct, one has to establish what the optimal division of labor is between syntax and semantics so as to best capture the scopal properties of natural language quantifiers in general and of simple and special indefinites in particular.

Once again, natural language semantics followed the lead of FOL and treated scopal phenomena as essentially syntactic. In standard approaches, scopal effects emerge as a consequence of the different ways in which multiple quantifiers occurring in a single sentence can be composed. In particular, there are two quantifier scopings intuitively associated with sentence (1) above and at the heart of various accounts that derive them, we find the two FOL formulas in (2) and (3) below. We adopt a restricted-quantification formalism here and use [ ] to indicate restrictor formulas and ( ) to indicate nuclear scope formulas.

(2) $\forall x[\text{STUD}(x)] \ (\exists y[\text{PAPER}(\mathbf{y})] \ (\text{READ}(x, y)))$

(3) $\exists y[\text{PAPER}(y)] \ (\forall x[\text{STUD}(x)] \ (\text{READ}(x, y)))$

According to classical FOL, formula (2) is true in case different students read (possibly) different papers, while formula (3) is true only in case the students read the same paper. Thus, the indefinite is within or outside the semantic scope of the universal depending on whether it is within or outside its syntactic scope.

In this paper, we depart from this tradition and propose an IFL-style account of scope that partially separates semantic scope from syntactic configuration. However, we change the IFL framework in several ways so that the syntax and semantics of scope are only partially divorced in the resulting logic. The separation has to be partial in order to account for the fact that the semantic scope of existentials is not totally insensitive to configurational matters: in natural language, syntactic configuration does not determine the semantic scope of indefinites, but it nonetheless places systematic constraints on it.

Our concern is primarily with natural language semantics rather than logical systems, so we define the formal system by incrementally introducing a series of changes to classical FOL. We point out connections with existing versions of IFL as we go along.[1]

The remainder of this section summarizes the main challenges raised by simple indefinites and provides a brief characterization of the special indefinites we deal with.

1.2 Special scopal properties of ordinary indefinites

A well-known problem for purely configurational approaches to scope in natural language is that simple indefinites enjoy free 'upward scope', disregarding not only clausal but also island boundaries. In contrast, the upward scope of universals is clause-bounded (see Farkas 1981; Fodor and Sag 1982; Abusch 1994 among others).

This is exemplified in (4) and (5) below.

(4)   John read a^x paper that every^y professor recommended.

(5)   Every^x student read every^y paper that a^z professor recommended.

In (4), the universal is in a relative clause restricting an indefinite and, crucially, the universal cannot scope over that indefinite: covariation between papers and professors is not possible here, which means that the existential cannot be within the semantic scope of the universal. Thus, this sentence can only be understood as describing situations in which John read a paper such that (s.t.) every professor

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[1] One of the initial motivations for developing IFL was the need to account for branching quantifiers in natural language (see Hintikka 1973; Hintikka and Sandu 1997 among others), exemplified by sentences like *Some book by every author is referred to in some essay by every critic*. The treatment of branching quantification remains outside the scope of this paper.

recommended that paper, an interpretation that corresponds to the case where the indefinite is outside the semantic scope of the universal. The sentence lacks a reading under which John read a set of papers s.t. for each professor, there is a paper in that set that the professor recommended—with possibly different papers for different professors. Such a reading would arise if the indefinite was in the semantic scope of the universal.

In contrast, the indefinite in (5) is in a relative clause restricting a universal which in its turn is in the syntactic scope of another universal. If the scoping possibilities of indefinites were parallel to those of universals, the indefinite in (5) would only have the narrowest possible scope and thus it would be interpreted as possibly covarying with both x and y. But in this case we have two more readings, showing that the indefinite can freely scope out of the relative clause, unlike the universal.

The three readings of (5) are given below. The intermediate scope (IS) reading in (7) is particularly important because it shows that exceptional scope cannot be analyzed away as a referential phenomenon (as argued in Farkas 1981 contra Fodor and Sag 1982).

(6) Narrowest Scope (NS):
      for every student x,
         for every paper y s.t.
▶   there is a professor z that recommended y,
               x read y.

(7) Intermediate Scope (IS):
      for every student x,
▶   there is a professor z s.t.,
         for every paper y that z recommended,
            x read y.

(8) Widest Scope (WS):
▶ there is a professor z s.t.,
      for every student x,
         for every paper y that z recommended,
            x read y.

The scopal freedom of indefinites illustrated above is problematic for theories in which semantic scope relations reduce to syntactic c-command relations. In order to account for (7) and (8), such theories have to assume that the indefinite c-commands one or both universals at some level of representation. This means that the indefinite has to move out of the relative clause, a problematic result since relative clauses are otherwise islands with respect to movement. In addition, universals and other *bona fide* quantificational DPs must be prevented from being thus moved, given the contrast between (4) and (5) above.

More generally, indefinites pose problems for any theory in which scopal relations can only arise out of different ways to proceed with semantic composition—whether semantic composition is exclusively syntax-driven or not. The reason is that such theories need a special composition rule for indefinites:

a rule that grants indefinites, but not universals, their observed freedom. Whether this rule is embedded in a Cooper-storage account of scope (as in Abusch 1994) or it simply states that syntactically covert movement for scope is upward free for indefinites (see Geurts 2000), there is no independent justification for the fact that such a rule can target only (in)definites, but not any *bona fide* quantifier.

To conclude, any adequate theory of DP scope must address the following two related questions:

Question 1: Why are the scopal properties of ordinary indefinites different from those of universals and other *bona fide* quantifiers?

Question 2: What explains the freedom of scope of indefinite DPs?

A movement-based account has problems answering both these questions since there is no principled reason to differentiate between the two types of quantifiers nor is there any reason to expect indefinites to move in a way that is unprecedented elsewhere in syntax.

Choice/Skolem-function accounts of indefinites (see Reinhart 1997; Winter 1997; Kratzer 1998; Matthewson 1999; Steedman 2007; Dekker 2008 among others) avoid having to stipulate a compositional rule for indefinites by encapsulating their scopal freedom into their lexical meaning or by special existential closure rules that apply to them.

The main idea is that the core semantics of indefinites is different from the semantics of *bona fide* quantifiers in that it involves choosing a witness that satisfies the restrictor and nuclear scope of the indefinite. The different ways in which a witness is chosen is responsible for the different semantic scopes the indefinite has. In particular, narrow scope of an indefinite relative to a quantifier Qx binding a variable x reduces to making the witness choice for the indefinite dependent on the values of the variable x. This amounts to Skolemization in FOL.

How choice/Skolem-function approaches answer the first question posed above depends on their details. But the basic insight, which we share, is the same: the scopal properties of ordinary indefinites are different from those of universals and other *bona fide* quantifiers because only indefinites can be interpreted in terms of witness choice.

Crucially, however, the way this insight is formally spelled out does not always preserve its initial appeal. Reinhart (1997), for example, formalizes it by means of existentially-bound functional variables. A special mechanism of existential closure needs to be postulated for such variables that acts long-distance and does not always take root-level scope. Since this mechanism is not otherwise motivated, the scopal freedom of indefinites is ultimately not explained.

In contrast, Kratzer (1998) assumes a single contextually-provided choice function, which immediately accounts for the widest-scope readings of indefinites without recourse to special binding mechanisms. This account, however, needs to posit implicit arguments for this choice function (which, in effect, make it a Skolem-function variable) in order to account for IS readings like (7) above.

Finally, the account proposed in Steedman (2007) derives the two possible scopes of the indefinite *a^y paper about scope* in our initial sentence (1) by always interpreting the indefinite in situ, but letting it contribute a Skolem function **f** of variable arity. If **f**'s arity is 0, the function is a constant and the choice of the witness is not dependent on the universal quantifier *every^x student in my class*. This yields the wide scope reading of the indefinite relative to the universal, given in (9) below. If **f**'s arity is 1, the witness is chosen in a way that is dependent on the values of the universally-quantified variable x. This yields the narrow scope reading of the indefinite relative to the universal, given in (10).[2]

(9) $\forall x[\text{STUD}(x)]\ ([\text{PAPER}(\mathbf{f})]\ (\text{READ}(x, \mathbf{f})))$
(10) $\forall x[\text{STUD}(x)]\ ([\text{PAPER}(\mathbf{f}(x))](\text{READ}(x, \mathbf{f}(x))))$

While the insight that different ways of choosing a witness yield different semantic scopes is an attractive one, the account in Steedman (2007) does not generalize to exceptional scope. The author suggests that an account along the lines of Kratzer (1998) and/or Schwarzschild (2002) should be pursued for cases of exceptional scope. This system, then, would need to be supplemented with additional assumptions to capture the IS reading of sentence (5) above.

We turn now to another challenge for theories of scope aiming to account for both indefinites and regular quantifiers. The issue is that although syntax does not determine semantic scope, it cannot be altogether disregarded even in the case of indefinites. The syntactic constraint one has to capture, and which we dub the Binder Roof Constraint, is formulated in (11) below and its effect is illustrated in (12):

(11)   Binder Roof Constraint: an indefinite cannot scope over a quantifier that binds into its restrictor.
(12)   Every^x student read every^y paper that one^z of its$_y$ authors recommended.

In (12), *one^z of its$_y$ authors* can have only narrowest scope because scoping over *every^y paper* would mean scoping over the quantifier that binds a pronoun in the restrictor of the indefinite—as observed in Abusch (1994), Chierchia (2001) and Schwarz (2001), among others. This kind of data raises the following question:

Question 3: What accounts for the existence of the Binder Roof Constraint?

Neither independence-friendly nor choice/Skolem-function approaches are able to answer this question. For a discussion of this problem in the context of choice-function approaches, see Chierchia (2001) and Schwarz (2001).

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[2] For ease of comparison, we preserve the syntax in (2) and (3) above rather than using the one in Steedman (2007).

We think that the essence of the answer to this question proposed in Steedman (2007) is right: the ways in which we can choose a suitable witness for an indefinite are constrained by the presence of bound variables in the restrictor of the indefinite. The restrictor constrains witness choice and the presence of bound variables in the restrictor renders witness choice dependent relative to the quantifiers binding these variables.

The account we propose departs from Steedman's, however, in that for us dependent choice is a matter of semantics rather than of syntax. Requiring the Skolem function contributed by an indefinite to be obligatorily indexed with a certain subset of the variables that the indefinite is indexed with, as Steedman does, accounts for the Binder Roof Constraint but obscures the intuition that (12) is scopally unambiguous because witness choice is dependent on the local context of interpretation/evaluation. In our view, the Binder Roof Constraint should follow from the definition of the interpretation function, which should require syntactically bound variables to be semantically dependent.

In sum, although simple indefinites exhibit exceptional upward scope, configurational issues cannot be disregarded altogether. Given that choice/Skolem-function approaches pack the scopal properties of indefinites into their lexical meanings, i.e., into the functions that they contribute, such accounts need additional syntactic constraints to limit the freedom of interpretation that they incorrectly predict for simple indefinites with bound pronouns in their restrictor.

We thus seek an account for the contrast between the scopal behavior of (in)definites and that of *bona fide* quantifiers, while still preserving a compositional interpretation procedure that is driven by and is sensitive to surface syntax.

1.3 Dependent indefinites

We turn now to the second general concern identified at the outset, namely how to extend the account of simple indefinites to the case of special ones.

In this paper we focus on one case, namely dependent indefinites, first discussed in Farkas (1997b). It was noted there that there is a class of indefinites marked by special morphology that have to be interpreted as covarying with an individual or event/situation variable bound by a *bona fide* quantifier, referred to as the licensor of the dependent indefinite. We exemplify below with Hungarian, where dependent indefinites are marked by reduplication, and with Romanian, where they are marked by the special morpheme *cîte*.[3]

Consider first the Hungarian example in (13) below.

(13) Minden   vonás   **egy-egy**   emlék.
      every   feature *a-a*   memory
      'Every feature is a memory.'

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[3] Most of the examples below are taken from the web or from naturally occurring conversation and sound quite natural to native speakers.

The reduplicated indefinite must covary with the variable bound by the licensor of the dependent indefinite, namely the universal determiner *minden* 'every'. Here the variable bound by the universal ranges over a non-singleton set of features. In order for the sentence to be true, each such feature must be associated with a memory (a memorable event). The reduplicated indefinite article *egy-egy* requires that there be some variation across these memories and thus rules out a situation in which each feature is associated with the same memory. Besides *bona fide* quantificational DPs, the licensor may be a distributively-interpreted plural, as in (14) below, or an adverb of quantification, as in (15).

(14)   Azzal   **egy-egy**   puszit   nyomott   *az   arcunkra*   és   beült
       with.that   a-a   kiss   planted   the   face.1pos.pl   and   sat-in
       a   taxiba.
       the   cab.in
       'With that [she] planted a kiss on our faces and took the cab.'

(15)   *Olykor-olykor*   **egy-egy**   ember   felkiáltott.
       occasionally   a-a   man   cried-out
       'Occasionally a man cried out.'

In (14), the licensor is the distributively-interpreted plural (in italics) and the reduplicated indefinite signals covariation between kisses and the faces they are planted on. In (15), the licensor is the adverb *olykor-olykor*—itself reduplicated and because of that, interpreted as 'on several occasions'. The reduplicated indefinite signals covariation between occasions and the persons crying out, ruling out a situation in which the same individual cried out every single time.

Farkas (2007) shows that the same effect is obtained in Romanian by having the item *cîte* precede an indefinite or numeral determiner.

(16)   *Fiecare   băiat*   a   recitat   **cîte**   un   poem.
       every   boy   has   recited   *cîte*   a   poem.
       'Every boy recited a poem.'

The addition of the morpheme *cîte* to the simple indefinite *un* imposes the requirement that there be covariation between boys and the poems they recited.

In (17) below, we have an example of a dependent indefinite in Romanian licensed by an adverbial construction with quantificational force and in (18), an example in which the licensor is a distributively-interpreted plural (both licensors are italicized).

(17)   *Din   cînd   în   cînd,*   trenul   se   opera   în   **cîte**   o   gară.
       from   when   in   when   train.the   refl   stopped   in   *cîte*   a   station
       'Occasionally, the train stopped in a station.'

(18)   Am   decis   să   lucrăm   *amîndoi*   **cîte**   un   album   solo.
       we-have   decided   to   work   both   *cîte*   an   album   solo
       'We have decided to each work at a solo album.'

In both languages, sentences like the ones above contrast with sentences that are identical except for the replacement of the dependent indefinite with a simple one (the non-reduplicated indefinite article *egy* in Hungarian and the simple indefinite article *un/o* in Romanian). The witnesses of the dependent indefinite have to covary with those of the licensor, while the simple indefinite allows both a covariation and a wide-scope/fixed-value interpretation—just as in the English translation of each example above.

Assuming that both the indefinite and the universal introduce a variable at LF, we will say that the variable introduced by the dependent indefinite has to covary with the variable introduced by the universal, while the simple indefinite is free to covary or not.

Dependent indefinites are not acceptable in the absence of a licensor, as illustrated in (19) below for Romanian.

(19)   *Cîte   un   student   a   plecat.
       cîte   a   student   has   left
       'A student left.'

Example (20) shows that dependent indefinites are not licensed by negation.

(20)   *Ana   nu   a   adus   **cîte**   o   umbrelă.
       Ana   not   has   brought   *cîte*   an   umbrella
       'Ana didn't bring an umbrella.'

Finally, (21) shows that a dependent indefinite cannot be licensed by another singular indefinite. Intuitively, the problem here is that the variable provided by the indefinite does not vary, so the dependent indefinite does not have a variable to covary with. The facts are parallel in Hungarian, as discussed in Farkas (1997b).

(21)   *Un   student   a   vorbit   cu   **cîte**   un   profesor.
       a   student   has   talked   with   *cîte*   a   professor
       'A student talked to a professor.'

Dependent indefinites may impose sortal restrictions on the variable they covary with. In Romanian and Hungarian, dependent indefinites may covary with individual variables or with event/situation variables, but not with world-level ones (Farkas 1997b, 2002). In contrast, this restriction does not hold in Russian (Pereltsvaig 2008). As a result, dependent indefinites in Romanian and Hungarian contrast with the relevant items in Russian in that the latter, but not the former, can be licensed by modals or intensional predicates. We exemplify with Romanian below.

(22)   *Trebuie   să   plece   **cîte**   un   student
       must   subj   leave   *cîte*   a   student.
       'A student must leave.'

The examples discussed so far show that dependent indefinites are *special* indefinites in that they involve special morphology (reduplication in Hungarian and the

presence of *cîte* in Romanian) accompanied by special restrictions on interpretation, which result in restricted distribution.

Pereltsvaig (2008) notes that the World Atlas of Language Structures Online[4] lists 189 languages as having such dependent nominals. These are all special indefinites in that they are marked by affixation (Basque, Turkish, Maori), a free standing morpheme (Russian, German, Latvian) or reduplication (Bengali, Georgian, Pashto). This suggests that we are dealing with a relatively widespread phenomenon here and any account of ordinary/simple indefinites should generalize to dependent indefinites in a fairly straightforward way.

In particular, we should account for the fact that the role of the special marking on dependent indefinites imposes covariation between the variable introduced by the indefinite and the variable introduced by the licensor. Our analysis will view the covariation requirement as a ban against having a fixed witness for the dependent indefinite relative to all the values of the licensor variable. The licensor, therefore, must have the special indefinite in its semantic scope and the semantics of the licensor must allow for variation in values so that covariation with it is possible.[5]

We take dependent indefinites to require actual and not just possible covariation. A case where covariation appears to be only possible but not actual is if (16) above were to be followed by a sentence claiming that, later on, it turned out that the poems were in fact identical. An account of such examples is left for future research.

In sum, dependent indefinites across languages are morphologically more complex than ordinary indefinites and the interpretation associated with this dependent morphology restricts the distribution of these indefinites. This raises the final question that we will be concerned with in this paper:

Question 4: In the case of dependent indefinites, what is the semantic contribution of the dependent morphology?

A satisfactory answer to this question would show in what way the special morphosyntax of dependent indefinites contributes to their special interpretive constraints, which in turn would explain their special distribution. More precisely, since the dependent morphology is added to the morphology associated with ordinary indefinites, the semantic contribution of dependent morphology should combine with the semantics of ordinary indefinites and constrain it in a way that explains the special properties of dependent indefinites.

It is not obvious how the above-mentioned previous approaches to ordinary indefinites and their exceptional scope can be extended to allow for an account of dependent indefinites. The most amenable to such an extension is Steedman (2007). In that kind of approach, the dependent morphology on indefinites could be taken to constrain the arity of the Skolem function contributed by indefinites, which would

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[4] http://wals.info/feature/description/54.

[5] An issue we leave open is the exact nature of the constraints on the material intervening between the quantificational licensor and the dependent indefinite. We know that the latter must be in the semantic scope of the former, but there seem to be limits on the 'downward' reach of such quantifiers, e.g., an attitude report with a quantificational licensor in subject position and a dependent indefinite in the subordinate clause is degraded, just as a sentence with a quantificational licensor in subject position and a dependent indefinite in the restrictive relative clause modifying an (in)definite direct object.

have to be greater than 0. We do not pursue this line here, however, given that Steedman (2007) does not account for the full range of scopal possibilities associated with ordinary indefinites.

In sum, we have identified four main issues that need to be addressed by an account of indefinites and their scopal properties:

  (i) capturing the contrast in scopal behavior between indefinites and *bona fide* quantifiers
 (ii) deriving the upwards freedom of scope exhibited by ordinary indefinites
(iii) explaining the sensitivity of this upwards freedom to syntactic considerations
(iv) allowing the semantics of ordinary indefinites to be further enriched so as to account for the special properties of dependent indefinites.

The rest of the paper is organized as follows. Section 2 provides an informal outline of our independence-based account of exceptional scope and dependent indefinites. Section 3 formalizes this account, thereby answering questions (i) through (iii) above. Section 4 answers question (iv) by providing a formal account of dependent indefinites in the logical framework introduced in the preceding section. Section 5 briefly considers the consequences of our account with respect to a variety of issues closely connected to the exceptional scope properties of ordinary indefinites and outlines some of the remaining open problems. Section 6 concludes.

2 Outline of the account

Following Steedman (2007) and Farkas (1997a), as well as choice-functional approaches, we interpret indefinites *in situ*, thus partially divorcing scope from configurational matters. But we depart from these previous accounts in conceptualizing the scope of indefinites as an *independence*-based notion. More concretely, we assume that the semantics of an indefinite specifies whether the witness it contributes is independent of (or fixed relative to) other quantifiers. Previous approaches are dependence-based in the sense that for them the semantic interpretation of the indefinite specifies possible dependency (or covariation) relative to other quantifiers.

In the approach developed below, the main role of syntactic structure is to provide constraints on witness choice: if a quantifier $\mathbf{Q}x$ syntactically scopes over an indefinite $\mathbf{Q}'y$, it becomes possible for the values of y to be required to stay fixed relative to the values of x. If this possibility is taken advantage of, y will be independent of x and thus $\mathbf{Q}'y$ will be outside the semantic scope of $\mathbf{Q}x$. In case this possibility is not taken advantage of, the values of x do not have to be fixed relative to those of y and thus covariation is possible, in which case $\mathbf{Q}'y$ will be inside the semantic scope of $\mathbf{Q}x$.

Our main proposal has two components:

(i) Just as in choice/Skolem-function approaches, we take the essence of the semantics of indefinites to be choosing a witness.
(ii) In contrast to choice/Skolem-function approaches, we follow IFL and take witness choice to be part of the interpretation procedure.

That is, indefinites choose a witness *at some point in the evaluation* and require its non-variation from that point on. In contrast to Skolemization, which ensures non-variation by means of a higher-order functional variable, we ensure non-variation by directly constraining the values assigned to the first-order variable contributed by the indefinite.

The first component, i.e., taking the core semantics of indefinites to be witness choice, is essential in accounting for the difference between existentials and *bona fide* quantifiers with respect to scope (Question 1 above).

The second component, i.e., building witness choice into the general interpretation procedure (that is, into the recursive definition of truth and satisfaction), is crucial for capturing the details of the scopal behavior of existentials (Questions 2 and 3).

As standardly assumed (albeit sometimes implicitly), we take the set of variables introduced by previous/higher quantifiers to be accessible at the point when an existential is interpreted. That is, this set of variables is part of the local context of evaluation for the existential. The existential is free to choose which of these variables the witness choice can be dependent on and which variables the witness choice is independent of.

Syntax (and, more generally, order of composition) determines which variables are accessible to a given existential, and therefore which variables an existential may in principle depend on. Selecting one of the various possibilities is, however, not syntactically determined but is a matter of interpretive choice.

More specifically, we assume that an existential accesses the set $\mathcal{V}$ of variables introduced by quantifiers taking syntactic scope over it and chooses a subset $\mathcal{U} \subseteq \mathcal{V}$ relative to which the values of its witness may covary (in the spirit of Steedman 2007). The variables in $\mathcal{U}$ are those the existential may be dependent on, while the variables in $\mathcal{V} \setminus \mathcal{U}$ are those that the existential is independent of.

In our approach then, an indefinite is syntactically indexed with the set of variables it is dependent on, and requires semantic non-variation relative to all the other previously introduced variables. That is, logical *representation* marks possible *dependence*: indefinites are indexed with the variables they possibly covary with. The *interpretation* rule, on the other hand, is stated in terms of *independence*: an indefinite contributes a witness that is required to be unique/fixed relative to all the variables that the indefinite is not indexed with.

This seems to be the correct way to capture the morphological makeup of indefinites on one hand and their semantics on the other hand. On the morphological side, additional morphological marking (such as reduplication or the use of a special morpheme accompanying the simple indefinite article) is correlated with dependence requirements rather than independence constraints. For example, the morphological marking occurring in dependent indefinites across languages requires them to covary with (that is, to be within the semantic scope of) other quantifiers: dependent morphology constrains the witness contributed by the indefinite to exhibit actual—rather than merely possible—covariation. But there seem to be no special morphological markers on indefinites requiring them to be independent of (that is, take semantic scope over) particular quantificational items. That is, the morphology of indefinites requires a representation that marks dependence.

On the semantic side, however, we take the essence of indefinite interpretation to be witness choice, which means that at some point in the interpretation, a referent for the indefinite is chosen and this choice remains fixed for/independent of the quantificational items that are subsequently interpreted.

Our core proposal concerning the scope of existentials can be illustrated with the formula in (23) below, involving an existential in the syntactic scope of two universals.

(23) $\forall x[\phi] \; (\forall y[\phi'] \; (\exists z[\phi''] \; (\psi)))$

The set $\mathcal{V}$ of variables accessible to the existential is $\{x, y\}$. Suppose that the set of values for x that satisfy the restrictor formula ϕ is $\{\alpha_1, \alpha_2\}$ and that the set of values for y that satisfy the restrictor formula ϕ' is $\{\beta_1, \beta_2\}$. That is, suppose that the universal $\forall x[\phi]$ quantifies over the set $\{\alpha_1, \alpha_2\}$ and the universal $\forall y[\phi']$ quantifies over the set $\{\beta_1, \beta_2\}$.

The existential $\exists z$ chooses a witness that satisfies its restrictor formula ϕ'' and its nuclear scope formula ψ. The witness choice can happen in three different ways, as shown in (24) below:

- it can be fixed relative to no variables (narrowest scope), in which case the subset $\mathcal{U} \subseteq \mathcal{V}$ chosen by the existential is $\{x, y\}$
- it can be fixed relative to the variable y contributed by the lower universal but possibly dependent on x (intermediate scope), in which case $\mathcal{U} = \{x\}$
- finally, it can be fixed relative to both x and y (widest scope), in which case $\mathcal{U} = \emptyset$

Syntactically, the scope of the existential is the same, namely narrowest scope. Semantically, however, the existential can have three possible scopes depending on how witnesses are chosen.

(24)   Narrowest scope (NS), $\mathcal{U} = \{x, y\}$: z is fixed relative to no variable, i.e., z (possibly) covaries with both x and y

        Intermediate scope (IS), $\mathcal{U} = \{x\}$: z is fixed relative to y and (possibly) covaries with x

        Widest scope (WS), $\mathcal{U} = \emptyset$: z is fixed relative to both x and y

The three semantic scopes are schematically depicted by the matrices in (25) below:

- in the NS case, the values of z are (possibly) different for any two different pairs of values for x and y
- in the IS case, the values of z are (possibly) different for the two values of x, i.e., α_1 is associated with witness γ and α_2 is associated with witness γ'; crucially, the witnesses are fixed relative to the two values of y, i.e., witness γ is associated with both β_1 and β_2 and so is witness γ'
- in the WS case, the values of z are fixed relative to any combination of values for x and y: γ is associated with all four pairs of values $\langle \alpha_1, \beta_1 \rangle, \langle \alpha_1, \beta_2 \rangle, \langle \alpha_2, \beta_1 \rangle$ and $\langle \alpha_2, \beta_2 \rangle$

(25)

NS				IS				WS		
x	y	z		x	y	z		x	y	z
α_1	β_1	γ		α_1	β_1	γ		α_1	β_1	
α_1	β_2	γ'		α_1	β_2			α_1	β_2	γ
α_2	β_1	γ''		α_2	β_1	γ'		α_2	β_1	
α_2	β_2	γ'''		α_2	β_2			α_2	β_2	

In principle, there is a fourth possibility: the witness choice could be dependent on y, but fixed relative to x, i.e., $\mathcal{U} = \{x\}$. Note that this reading is in fact expected under the independently needed assumption that some clause-bounded scoping mechanism (such as Q(uantifier) R(aising)) is needed to assign inverse scope to universals and other *bona-fide* quantifiers.

This fourth possibility then is equivalent to QR-ing the existential $\exists z[\phi'']$ to take scope over the topmost universal $\forall x[\phi]$, then QR-ing the other universal $\forall y[\phi']$ to take scope over the existential and, finally, giving the existential semantic scope in situ. The availability of this fourth possibility for this particular sentence is thus independent of how one treats the exceptional scope of indefinites. We return to this matter in Sect. 3.5, after introducing the full formal system and discussing our analysis in more detail.

Turning now to the problem of dependent indefinites, the essence of the requirement they impose is that witness choice must be dependent on some parameter of evaluation. Consequently, the WS matrix in (25) is not possible if the variable z is introduced by a dependent indefinite, nor is it possible to use a dependent indefinite in an environment that does not provide an appropriate variable for the indefinite to covary with, i.e., a variable that takes multiple values.

Thus, an account in terms of matrices like the ones in (25) above enables us to immediately connect the scopal properties of ordinary indefinites and the constraints on interpretation imposed by dependent indefinites.

The next section is dedicated to the formalization of this basic account. We will define the interpretation function $[\![\cdot]\!]$ in terms of matrices like the one in (25) above and show how to formalize the non-variation requirement contributed by existentials relative to them.

3 Scope in first-order logic with choice (C-FOL)

Our account is couched in a slightly modified version of the language of classical first-order logic (FOL): we keep the language first order, i.e., we have variables only over individuals (no choice/Skolem-function variables), but we add restricted quantification, just as we did in the discussion and various formulas above.[6]

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[6] The formulation of a Montague-style compositional translation procedure from English into (a higher-order version of) this language is left for future research. We might be able to provide such a translation procedure if we follow the general strategy used in Muskens (1995) to couch situation semantics into a relational version of type logic and in Muskens (1996) (following Janssen (1986), among others) to couch dynamic semantics into classical (many-sorted) type logic.

While the syntax of the language is fairly standard, the semantics is not. The main formal novelty is that, in contrast to standard Tarskian semantics where evaluation contexts are single assignments, our contexts of evaluation have a more articulated structure:

(i) following the (in)dependence logics in Hodges (1997) and Väänänen (2007), we evaluate formulas relative to *sets* of assignments $G, H, \ldots$ instead of single assignments $g, h, \ldots$

(ii) in the spirit of the main insight in Steedman (2007), we evaluate a quantifier relative to the set of variables $\mathcal{V}$ introduced by the syntactically-higher quantifiers, i.e., contexts of evaluation contain the set of variables $\mathcal{V} = \{x, y, \ldots\}$ introduced by all the previously interpreted quantifiers $\mathbf{Q}x, \mathbf{Q}'y$, etc.

Sets of assignments can be thought of as matrices like the ones in (25) above, with each assignment giving each variable a value. The advantage of having sets of assignments is that they enable us to treat scopal (in)dependence in terms of such matrices because we can talk of values that are the same or that are different across different rows of such matrices.

In order to say whether a variable x depends on a variable y, it is not enough to examine which value x takes relative to each value of y *separately*. We have to consider the *whole* relation between the values of x and the values of y. If x has the same value for all values of y, we can say that x does not depend on y. If the value of x varies relative to different values of y, we can say that x depends on y. Similar considerations apply to dependence on more variables. That is, (in)dependence is a global property of an *entire* set of assignments relative to which a formula is evaluated, so interpretation has to be relativized to such sets.[7] Adding to this setup the set of previously introduced variables as an explicit evaluation parameter has the advantage that it allows the existential to choose which of these variables its witness is (possibly) dependent on.

As already indicated, a set of assignments G can be represented as a matrix with assignments $g, g', g'', \ldots$ as rows, as shown in (26) below.

(26)

G	$\ldots$	x	y	z	$\ldots$
g	$\ldots$	$\alpha_1\ (= g(x))$	$\alpha_2\ (= g(y))$	$\alpha_3\ (= g(z))$	$\ldots$
g'	$\ldots$	$\beta_1\ (= g'(x))$	$\beta_2\ (= g'(y))$	$\beta_3\ (= g'(z))$	$\ldots$
g''	$\ldots$	$\gamma_1\ (= g''(x))$	$\gamma_2\ (= g''(y))$	$\gamma_3\ (= g''(z))$	$\ldots$
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$

or simply:

$\ldots$	x	y	z	$\ldots$
$\ldots$	α_1	α_2	α_3	$\ldots$
$\ldots$	β_1	β_2	β_3	$\ldots$
$\ldots$	γ_1	γ_2	γ_3	$\ldots$
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$

---

[7] We are indebted to Theo M.V. Janssen for the pertinent observations that form the bulk of this paragraph.

To keep the formalism as simple as possible and as close as possible to the standard Tarskian semantics for FOL, we work with total assignments.

Having sets of assignments as contexts of evaluation enables us to encode when a quantifier $\mathbf{Q}'y$ is independent of another quantifier $\mathbf{Q}''z$ by requiring the variable y to have a fixed value relative to the varying values of z. This requirement is given in (27) below; G is the set of assignments relative to which the formula is evaluated and y is the variable bound by the quantifier $\mathbf{Q}'$.

(27) Fixed value condition (basic version):
      for all $g, g' \in G$, $g(y) = g'(y)$.

The fixed value condition for y leaves open the possibility that the values of z vary from assignment to assignment, i.e., that $g(z) \neq g'(z)$ for some $g, g' \in G$, as shown below.

(28)

y	z
γ	β_1
	β_2

Thus, having sets of assignments enables us to state directly in the semantics that y does not covary with z, without having to go through the intermediary of syntax. We are therefore able to state that the quantifier $\mathbf{Q}'y$ is not in the *semantic* scope of $\mathbf{Q}''z$, although it may well be in its *syntactic* scope. Partially separating syntactic scope from semantic scope is one of the crucial features of our proposal. Note also that the condition we give is an *independence* condition rather than a dependence one since it require the values of a variable in a matrix to be fixed.

Let us suppose now that the quantifier $\mathbf{Q}'y$ is not in the semantic scope of $\mathbf{Q}''z$. In this case it is still possible that a third quantifier, $\mathbf{Q}x$, takes both syntactic and semantic scope over $\mathbf{Q}'y$. This is exactly what happens in the intermediate scope (IS) configuration in (25) above (for the formula in (23)), where the values of y are fixed relative to z but covary with x.

In such intermediate scope cases, the values of some variable y have to be fixed relative to the values of some other variable z, but they should be free to covary with the values of a third variable x. To allow for this possibility, we relativize the fixed value condition to the values of x, as shown in (29) below, where x and y are bound by the quantifiers $\mathbf{Q}$ and $\mathbf{Q}'$ respectively.

(29) Fixed value condition (relativized version):
      for all $g, g' \in G$, if $g(x) = g'(x)$, then $g(y) = g'(y)$.

The matrix below (very similar to the IS matrix in (25) above) satisfies precisely this kind of relativized fixed value condition.

(30)

x	y	z
α_1	γ	β_1
α_1		β_2
α_2	γ'	β_1
α_2		β_2

In sum, the advantage of working with sets of assignments instead of single assignments is that it enables us to formulate non-variation/fixed-value conditions relativized to particular variables.

Recall now that we also need to keep track of which variables are introduced by syntactically higher quantifiers, so that we can let existentials contribute fixed-value conditions relativized to (some of) these variables.

This brings us to the second way in which we add structure to our contexts of evaluation: they contain the set of variables $\mathcal{V} = \{x, y, \ldots\}$ introduced by the previous quantifiers. These are the variables an existential *could* in principle covary with—but, in contrast to the standard Tarskian semantics, the existential *does not have* to covary with them.

Existentials have a choice. They can choose which ones of the quantifiers that take syntactic scope over them also take semantic scope over them. As already mentioned, when we interpret an indefinite, we choose a subset of variables $\mathcal{U} \subseteq \mathcal{V} = \{x, y, \ldots\}$ containing the variables that the indefinite possibly covaries with, i.e., the variables that the indefinite is possibly dependent on. The complement set $\mathcal{V} \setminus \mathcal{U} = \{v \in \mathcal{V} : v \notin \mathcal{U}\}$ contains the variables relative to which the indefinite does *not* vary, i.e., the variables that the indefinite is independent of. We dub the resulting first-order language and its associated semantics Choice-FOL or C-FOL for short.

In the system we propose, an indefinite in the syntactic scope of a quantifier $\mathbf{Q}x$ binding a variable x is in its semantic scope iff $x \in \mathcal{U}$. This makes the following two correct predictions:

 (i) an indefinite may be in the semantic scope of a quantifier $\mathbf{Q}x$ only if $\mathbf{Q}x$ has syntactic scope over the indefinite
(ii) an indefinite may in principle be outside the semantic scope of a quantifier $\mathbf{Q}x$ that takes syntactic scope over it.

3.1 Existentials in first-order logic with choice (C-FOL)

We turn now to the treatment of existential quantification in C-FOL.

A model $\mathfrak{M}$ for C-FOL has the same structure as the standard models for FOL, i.e., $\mathfrak{M}$ is a pair $\langle \mathfrak{D}, \mathfrak{I} \rangle$, where $\mathfrak{D}$ is the domain of individuals and $\mathfrak{I}$ the basic interpretation function. An $\mathfrak{M}$-assignment g for C-FOL is also defined just as in FOL: g is a total function from the set of variables $\mathcal{VAR}$ to $\mathfrak{D}$, i.e., $g \in \mathfrak{D}^{\mathcal{VAR}}$.

The essence of quantification in FOL is pointwise (i.e., variablewise) manipulation of variable assignments. We indicate this by means of the abbreviation $h[x]g$, defined in (31) below. Informally, $h[x]g$ abbreviates that the assignments h and g differ at most with respect to the value they assign to x.

(31) $h[x]g :=$ for all variables $v \in \mathcal{VAR}$, if $v \neq x$, then $h(v) = g(v)$

Given that we work with sets of variable assignments, we generalize this to a notion of pointwise manipulation of sets of assignments, abbreviated as $H[x]G$ and defined in (32). This is the cumulative-quantification style generalization of $h[x]g$: any

$h \in H$ has to have an $[x]$-predecessor $g \in G$ and any $g \in G$ has to have an $[x]$-successor $h \in H$.

(32) $H[x]G :=$
$$\begin{cases} \text{for all } h \in H, \text{ there is a } g \in G \text{ s.t. } h[x]g \\ \text{for all } g \in G, \text{ there is a } h \in H \text{ such that } h[x]g \end{cases}$$

With these basic notions in place, we can define the interpretation function $[\![\cdot]\!]^{\mathfrak{M},G,\mathcal{V}}$. We will generally leave the model superscript $\mathfrak{M}$ implicit and represent the C-FOL interpretation function simply as $[\![\cdot]\!]^{G,\mathcal{V}}$, where G is a set of assignments and $\mathcal{V}$ a set of variables. Together, G and $\mathcal{V}$ form a context of evaluation.

Atomic formulas are interpreted as shown in (33) below; $\mathbb{T}$ and $\mathbb{F}$ stand for true and false respectively.

(33)   Atomic formulas:
$[\![R(x_1, \ldots, x_n)]\!]^{G,\mathcal{V}} = \mathbb{T}$ iff
   a. $G \neq \emptyset$
   b. $\{x_1, \ldots, x_n\} \subseteq \mathcal{V}$
   c. $\langle g(x_1), \ldots, g(x_n) \rangle \in \mathfrak{I}(R)$, for all $g \in G$

The condition $\{x_1, \ldots, x_n\} \subseteq \mathcal{V}$ in (33b) bans free variables. We assume that deictic pronouns require the discourse-initial set of variables $\mathcal{V}$ to be non-empty, much like the discourse-initial partial assignments in Discourse Representation Theory (DRT)/ File-Change Semantics (FCS) are required to have a non-empty domain.[8]

The condition in (33c) is the central one: the set of assignments G satisfies an atomic formula $R(x_1, \ldots, x_n)$ if each assignment $g \in G$ satisfies it. That is, we distribute over the set G and, in this way, relate the C-FOL notion of set-based satisfaction to the standard FOL notion of single-assignment-based satisfaction. The non-emptiness condition in (33a) rules out the case in which the distributive requirement in (33c) is vacuously satisfied.

A set of assignments $G = \{g, g', g'', \ldots\}$ that satisfies the atomic formula $R(x_1, \ldots, x_n)$ in such a distributive way is provided in (34) below.

(34)

G	$\ldots$	x_1	$\ldots$	x_n	$\ldots$
g	$\ldots$	$\alpha_1 \ (= g(x_1))$	$\ldots$	$\alpha_n \ (= g(x_n))$	$\ldots$
		$\underbrace{\qquad\quad \langle \alpha_1, \ldots, \alpha_n \rangle \in \mathfrak{I}(R) \qquad\quad}$			
g'	$\ldots$	$\beta_1 \ (= g'(x_1))$	$\ldots$	$\beta_n \ (= g'(x_n))$	$\ldots$
		$\underbrace{\qquad\quad \langle \beta_1, \ldots, \beta_n \rangle \in \mathfrak{I}(R) \qquad\quad}$			
g''	$\ldots$	$\gamma_1 \ (= g''(x_1))$	$\ldots$	$\gamma_n \ (= g''(x_n))$	$\ldots$
		$\underbrace{\qquad\quad \langle \gamma_1, \ldots, \gamma_n \rangle \in \mathfrak{I}(R) \qquad\quad}$			
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$

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[8] If we were to have a partial/trivalent semantics (avoided here only for presentational simplicity), failure to satisfy condition (33b)—or (33a), if that condition was still present, would yield undefinedness, not falsity.

The interpretation of conjunction is the expected one: we just pass the current context of evaluation down to each conjunct.

(35)   Conjunction:
$\llbracket \phi \wedge \psi \rrbracket^{G,\mathcal{V}} = \mathbb{T}$ iff $\llbracket \phi \rrbracket^{G,\mathcal{V}} = \mathbb{T}$ and $\llbracket \psi \rrbracket^{G,\mathcal{V}} = \mathbb{T}$.

We turn now to existential quantification, leaving negation for Sects. 3.4 and 5.2 and disjunction for Sect. 5.5.

The interpretation of existential quantification is provided in (36). The novelty here is that existentials choose a subset of the previously introduced variables and the elements of this subset are the variables that the witness contributed by the existential may be dependent on. This choice is encoded by the superscript $\mathcal{U}$:

- the set of variables $\mathcal{U}$ is a subset of the current set of variables $\mathcal{V}$ contributed by the previous/higher quantifiers
- the variables in $\mathcal{U}$ are those variables in $\mathcal{V}$ that the witness contributed by the existential is allowed to covary with[9]

(36)   Existential quantification:
$\llbracket \exists^{\mathcal{U}} x [\phi] \ (\psi) \rrbracket^{G,\mathcal{V}} = \mathbb{T}$ iff $\mathcal{U} \subseteq \mathcal{V}$ and $\llbracket \psi \rrbracket^{H,\mathcal{V} \cup \{x\}} = \mathbb{T}$, for some H s.t.
a. $H[x]G$
b. $\llbracket \phi \rrbracket^{H, \mathcal{U} \cup \{x\}} = \mathbb{T}$
c. $\begin{cases} \text{if } \mathcal{U} = \emptyset : h(x) = h'(x), \text{ for all } h, h' \in H \\ \text{if } \mathcal{U} \neq \emptyset : h(x) = h'(x), \text{ for all } h, h' \in H \text{ that are } \mathcal{U}\text{-identical} \end{cases}$

(37)   Two assignments h and h' are $\mathcal{U}$-identical iff for all variables $v \in \mathcal{U}$, $h(v) = h'(v)$.

The fixed-value conditions in (36c), which cover the case in which the superscript $\mathcal{U}$ is empty and the case in which it is non-empty, rule out sets of assignments that contain witnesses covarying with the variables in $\mathcal{V} \setminus \mathcal{U}$. These conditions formalize what it means to make one of the allowed choices:

- if $\mathcal{U} = \emptyset$, the value of the variable bound by the existential is fixed in absolute terms and the indefinite has widest scope; in this case, the variable must satisfy the unrelativized fixed-value condition in (27) above
- if $\mathcal{U} \neq \emptyset$, the values of the variable bound by the existential may vary, but only relative to the values of the variables in the superscript $\mathcal{U}$ of the existential; in this case, the variable must satisfy a fixed-value condition relativized to the variables in $\mathcal{U}$, as defined in (29) above.[10]

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[9] In a partial/trivalent framework, failure to satisfy condition $\mathcal{U} \subseteq \mathcal{V}$ would yield undefinedness, not falsity.

[10] As Philippe Schlenker points out, the two conditions in (36c) can be unified. They are in fact both subsumed under the second condition "$h(x) = h'(x)$, for all $h, h' \in H$ that are $\mathcal{U}$-identical". The reason is that if $\mathcal{U} = \emptyset$, it is trivially true that any two assignments $h, h' \in H$ are $\mathcal{U}$-identical since there are no variables $v \in \mathcal{U}$. The redundant formulation matches the previous informal discussion more closely, so we keep it here (and in the rest of the paper) for consistency.

Thus, an existential formula $\exists^{\mathcal{U}}x[\phi]\,(\psi)$ is interpreted relative to a set of variables $\mathcal{V}$ introduced by the sequence of quantifiers that take syntactic scope over the existential. The superscript $\mathcal{U}$ on the existential indicates that among the $\mathcal{V}$-quantifiers, only the $\mathcal{U}$-quantifiers also take semantic scope over the existential. In other words, $\mathcal{U}$ indicates the *non-variation* of the existential with respect to the quantifiers binding variables in $\mathcal{V} \setminus \mathcal{U}$.

Just like in IFL, our semantics involves independence: we specify how the witness is fixed/chosen. But unlike IFL, we index existentials with the set of variables the existential is possibly dependent on.

Existentials in IFL are syntactically marked for those variables that they have to be independent of. An IFL existential is represented as $\exists x_{/Y}$, where the 'slashed' set of variables Y contains the variables that x is independent of. The IFL semantic clause for existentials and our semantic clause are basically the same, but an IFL existential is syntactically indexed with the variables that it is independent of, while in our system it is indexed with those variables that it may depend on.

For our limited purposes, translating the IFL treatment into our notation amounts to superscripting existentials not with $\mathcal{U}$, but with the complement of $\mathcal{U}$, i.e., $\overline{\mathcal{U}} = \mathcal{V} \setminus \mathcal{U}$.[11] As we will see in detail below, dependent indefinites in natural language provide an empirical argument for keeping the IFL semantics of independence but marking dependence in the logical representation/syntax. Moreover, as we will see in Sect. 3.3, the Binder Roof Constraint follows from the fact that the restrictor of the indefinite is interpreted relative to a context containing the set of variables $\mathcal{U}$ and, crucially, not the complement set $\overline{\mathcal{U}} = \mathcal{V} \setminus \mathcal{U}$.

Despite this syntactic difference, our system and IFL converge on the idea that the superscript on existentials constrains witness choice by delimiting possible covariation. Existentials are compatible with such a superscript because their semantics involves choosing a witness, while universals and other *bona fide* quantifiers cannot meaningfully occur with such a superscript because their semantics cannot be given in terms of single-witness choice.

This idea, which we also share with choice/Skolem-function approaches, is central to our account and explains why we never have superscripts on universals or other *bona fide* quantifiers. The semantic scope of existentials is different from the semantic scope of *bona fide* quantifiers because of the essential difference between the interpretations of the two types of nominal phrases:

- The interpretation of existentials involves choosing a witness and that choice may depend on the values of other variables.
- The interpretation of *bona fide* quantifiers cannot be formulated in terms of single-witness choice, so the superscripting mechanism that constrains this choice cannot meaningfully apply to them.

While the semantic scope of existentials is determined by their superscripted set of variables $\mathcal{U}$, the scope of *bona fide* quantifiers is exclusively determined by

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[11] We are indebted to Theo M.V. Janssen for this observation and for emphasizing the importance of the "limited purposes" caveat. The general problem of C-FOL—IFL translation is definitely more complicated than complementizing superscripts and it is left as an open issue here.

(clause-bounded) quantifier raising or whatever other quantifier-scoping mechanism the reader favors.[12]

It is by means of this mechanism that the universal *every meeting* in sentence (38) below can take scope over the indefinite in subject position.

(38) A^x representative of our organization was present at every^y meeting.

If the universal *every^y meeting* is scoped above the indefinite in subject position, it is semantically evaluated before this indefinite. Hence, the variable y it contributes is an element of $\mathcal{V}$ when the indefinite is interpreted and it becomes available for the indefinite to covary with (i.e., y is a possible element of the superscripted set $\mathcal{U}$).

We return to the interpretation of universals below. Our aim here is to emphasize that the way we determine the semantic scope of existentials and universals is intrinsically different and, therefore, that there is no reason to expect the scopal properties of existentials to be similar to those of universals and other *bona fide* quantifiers. Thus, our answer to Question 1 (the contrast between existentials and universals) is semantic in nature.

Our answer to Question 2 (the free upward scope of existentials) is again essentially semantic: the scope of an existential is a matter of relating that existential to a set of previously introduced variables. There is no reason to think that this relation might be syntactically local, e.g., sensitive to syntactic islands, since there is no requirement for the existential to c-command all the quantifiers it is independent of.

The relation between $\mathcal{U}$ and $\mathcal{V}$, which is what determines the semantic scope of the existential, is similar to what we find in anaphoric relations. Both phenomena connect the interpretation of an expression to the interpretation of a previous expression. In the case of scope, we connect an existential to the previously evaluated quantifiers. In the case of anaphoric relations, we connect the anaphor to the previously evaluated antecedent. This parallel is responsible for the fact that both phenomena are insensitive to syntactic islands.

But although both phenomena involve some form of variable coindexation in syntax (an assumption we make in part for expository reasons; see Sect. 3.5 below for an alternative formulation), the associated semantic rules are fundamentally different. For scopal (in)dependence, the set of variables that the indefinite is indexed with provides the parameters relative to which we choose the *new* entity introduced by the indefinite. For referential dependencies, e.g., anaphoric or bound pronouns, the variable that the pronoun is indexed with provides the *old* entity that the pronoun refers back to.

Moreover, anaphoric links can be established simply by inspecting single assignments, i.e., we examine a given set of assignments one assignment at a time.

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[12] Recall that indexing an ordinary indefinite with a variable y contributed by a quantifier requires *possible*, not actual, covariation with y. For example, as Theo M.V. Janssen points out, the sentence *Every^y man is protected by a^x woman who prays for him$_y$* requires the indefinite to have narrow scope relative to the universal if the universal is coindexed with/binds the pronoun in the restrictor of the indefinite. But this does not exclude an interpretation of this sentence in which every man is protected by the same woman, e.g., Virgin Mary, who prays for everyone. In this case, the narrow-scope indefinite accidentally happens to select the same witness x for each value of y—a situation our semantic clause for existentials allows for.

In contrast, scopal (in)dependence is a global property of the whole set of assignments relative to which a formula is evaluated.[13]

Finally, although $\mathcal{V}$ keeps track of previously introduced variables in a way that is parallel to how DRT/FCS keeps track of previously introduced variables by means of (the domains of) their partial variable assignments, the two sets of variables are crucially different: the purpose of the set $\mathcal{V}$ is to store variables that are possible candidates for quantificational covariation, not anaphoric retrieval, so $\mathcal{V}$ keeps track only of the *intra-sententially*-introduced variables and, as a general rule, it is reset to the empty set $\emptyset$ every time a new sentence is interpreted. In contrast, the DRT/FCS partial variable assignments are preserved across sentential boundaries and monotonically increase as we incrementally process sentences in discourse.

3.2 Universals and exceptional scope in C-FOL

Defining the interpretation of universally quantified formulas $\forall x[\phi] \, (\psi)$ in C-FOL requires some care because existential quantifiers can occur both in the restrictor formula ϕ and in the nuclear scope formula ψ.

Let us disregard for the moment the possibility that existential quantifiers can occur in the restrictor of universals and give a simpler, preliminary definition as a stepping stone towards our final one. This preliminary definition is provided in (39) below. The main idea is that the nuclear scope formula ψ of a universal quantifier is evaluated relative to the set of all assignments that satisfy the restrictor formula ϕ. That is, we collect all assignments h s.t. ϕ is true relative to the singleton set of assignments $\{h\}$ and pass the set H consisting of all these assignments to the nuclear scope formula ψ.

(39) Universal quantification (preliminary):
$\llbracket \forall x[\phi] \, (\psi) \rrbracket^{G,\mathcal{V}} = \mathbb{T}$ iff $\llbracket \psi \rrbracket^{H,\mathcal{V} \cup \{x\}} = \mathbb{T}$, where H is the maximal set of assignments that satisfies ϕ relative to x, G and $\mathcal{V}$.

(40) H is the maximal set of assignments that satisfies ϕ relative to the variable x, the set of assignments G and the set of variables $\mathcal{V}$ iff
$$H = \bigcup\nolimits_{g \in G} \left\{ h : h[x]g \text{ and } \llbracket \phi \rrbracket^{\{h\},\mathcal{V} \cup \{x\}} = \mathbb{T} \right\}.$$

For example, consider the sentence in (41) below and its C-FOL translations in (42) and (43). We assume that the indefinite is interpreted *in situ* and is translated by an existential that can have any superscript licensed by the interpretation of the previous quantifiers in the sentence. Given that the indefinite a^y *paper* is evaluated after the universal quantifier *every*^{x} *student*, it is interpreted relative to the non-empty set of variables $\{x\}$, so there are two possible superscripts, namely $\emptyset$ and $\{x\}$.

(41) Every^x student read a^y paper.
(42) $\forall x[\text{STUD}(x)] \, (\exists^{\emptyset} y[\text{PAPER}(y)] \, (\text{READ}(x,y)))$

(43) $\forall x[\text{STUD}(x)]\, (\exists^{\{x\}} y[\text{PAPER}(y)]\, (\text{READ}(x, y)))$

The only difference between the C-FOL translations in (42) and (43) is the superscript on the existential quantifier:

- if the superscript is $\emptyset$, as in (42), the existential receives a wide-scope interpretation
- if the superscript is $\{x\}$, as in (43), the existential receives a narrow-scope interpretation

Thus, in contrast to the two FOL formulas in (2) and (3) above, C-FOL does not capture the wide vs narrow scope readings of sentence (41) by means of two syntactically different formulas. The indefinite always has narrow scope syntactically, but at the point when it is interpreted, we may choose to select a witness that is independent of the higher universal quantifier, effectively removing the indefinite from the semantic scope of the universal.

In more detail, assume that the C-FOL formulas in (42) and (43) are interpreted relative to an arbitrary non-empty set of assignments G and the empty set of variables $\emptyset$. This is what is required by the definition of truth for C-FOL, provided in (44) below.

(44)   Truth: a formula ϕ is true (in model $\mathfrak{M}$) iff $[\![\phi]\!]^{G,\emptyset} = \mathbb{T}$ for any non-empty set of assignments G, where $\emptyset$ is the empty set of variables.

Assume that there are exactly three students in our model $\mathfrak{M}$, namely $\{stud_1, stud_2, stud_3\}$. Then, the interpretation of the formulas in (42) and (43) proceeds as shown in (45) below. First, the universal quantifier $\forall x[\text{STUD}(x)]$ introduces the set of all students relative to the variable x and relative to each assignment $g \in G$. Then, the existential $\exists^{\emptyset/\{x\}} y[\text{PAPER}(y)]$ introduces a paper and chooses whether it is the same for every student (if the superscript is $\emptyset$) or whether it is possibly different from student to student (if the superscript is $\{x\}$). Finally, we check that, for each variable assignment in the resulting set of assignments, the x-student in that assignment read the y-paper in that assignment.

(45)

Our preliminary definition in (39) above accounts for the scopal interaction between universals and indefinites in their nuclear scope. But it makes incorrect predictions for indefinites in the restrictor of universals.

A typical case is our exceptional scope sentence in (5) above. Its C-FOL translation is provided in (46) below. The three readings, i.e., the WS, IS and NS readings, are obtained by letting the superscript on the indefinite be $\emptyset, \{x\}$ and $\{x, y\}$, respectively.

(46) $\forall x[\text{STUD}(x)]$
        $(\forall y[\text{PAPER}(y) \wedge$
            $\exists^{\emptyset/\{x\}/\{x,y\}} z[\text{PROF}(z)] \ (\text{RECOM}(z, y))]$
        $(\text{READ}(x, y)))$

Consider now the IS reading more closely. The superscript $\{x\}$ on the existential quantifier $\exists^{\{x\}} z[\text{PROF}(z)]$ indicates that only the first universal quantifier $\forall x[\text{STUD}(x)]$ takes semantic scope over it. That is, for each student x, we choose a professor z and require x to have read every paper that z recommended.

However, the interpretation of universal quantification in (39) and (40) above fails to capture this reading. This is because when we evaluate the restrictor formula of the universal quantifier $\forall y[\text{PAPER}(y) \wedge \exists^{\{x\}} z[\text{PROF}(z)] \ldots]$, we do not have access to the entire previous set H of assignments that stores all the x-students: according to definition (40), we only examine one assignment h in that set at a time.

Hence, we vacuously satisfy the relativized fixed-value condition contributed by the existential $\exists^{\{x\}} z$ because this condition has the form: for all $h', h'' \in \{h\}$, if $h'(x) = h''(x)$, then $h'(z) = h''(z)$. Fixed-value conditions are always satisfied by singleton sets of assignments. So we fail to ensure that a single z-professor is chosen for each x-student.

In order to account for our intuitions about exceptional scope in sentences that have indefinites in the restrictor of universals, we need to modify the C-FOL semantic clause for universally quantified formulas as in (47) below.

(47)   Universal quantification (final version):
        $[\![\forall x[\phi] \, (\psi)]\!]^{G, \mathcal{V}} = \mathbb{T}$ iff $[\![\psi]\!]^{H, \mathcal{V} \cup \{x\}} = \mathbb{T}$, for some H that is a maximal
        set of assignments relative to x, ϕ, G and $\mathcal{V}$
(48) H is a maximal set of assignments relative to a variable x, a formula ϕ,
        a set of assignments G and a set of variables $\mathcal{V}$ iff
        a. $H[x]G$ and $[\![\phi]\!]^{H, \mathcal{V} \cup \{x\}} = \mathbb{T}$
        b.   there is no $H' \neq H$ s.t. $H \subseteq H'$ and:
               $H'[x]G$ and $[\![\phi]\!]^{H', \mathcal{V} \cup \{x\}} = \mathbb{T}$

The crucial difference between (39) and (40) and (47) and (48) is that:

- in (39) and (40), we obtain the maximal set of assignments H in a distributive way, i.e., by evaluating the restrictor formula ϕ relative to each assignment $h \in H$

- in (47) and (48), we obtain a maximal set of assignments H in a *collective* way, i.e., by evaluating the restrictor formula ϕ relative to H as a whole

Note that the set H is *the* maximal set according to (39) and (40), while according to (47) and (48) it is merely *a* maximal set. This captures cases in which the restrictor formula ϕ contains existentials for which we could choose any individual from a non-singleton set of witnesses: for any choice of a single witness, we would obtain a possibly different maximal set H.[14]

The final definition in (47) and (48) above allows us to correctly capture the IS reading of indefinite $\exists^{\{x\}}z[\text{PROF}(z)]$ in the restrictor of the universal quantifier $\forall y[\text{PAPER}(y) \land \ldots]$ because we now have access to the entire set of assignments that stores the values of the variable x bound by the higher universal $\forall x[\text{STUD}(x)]$.

The new definition yields the same results as the old one for cases where there are no indefinites in the restrictor of the universal, such as (41) above.

3.3 Deriving the binder-roof constraint

We turn now to Question 3, namely to our account of the fact that an existential cannot have scope over a quantifier that binds a variable in its restrictor.

Note first that any account in which the syntactic scope of an existential fully determines its semantic scope immediately captures this constraint: if an indefinite takes semantic scope over a quantifier, it must c-command that quantifier and therefore the quantifier cannot c-command, much less bind, any variable in the restrictor of the indefinite.

But if existentials are interpreted in situ, as in our present account, the Binder Roof Constraint no longer follows. Thus, both independence-friendly and choice-function based approaches predict that indefinites with bound variables in their restrictors are able to take exceptional scope over the binders of those variables, and therefore cannot block the unattested IS or WS readings for sentence (12) above (see Chierchia (2001) and Schwarz (2001) for detailed discussions of this problem for choice-function approaches).

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[14] As Theo M.V. Janssen points out, the definition of universal quantification in (47) also comes with a built-in existential commitment: "for some H that..." effectively requires the restrictor formula ϕ to be satisfied by a non-empty set of individuals. That is, contrary to received wisdom (and FOL semantics), the sentence *Every female student passed the test* is false if there are no female students. While this restrictor non-emptiness requirement does seem to be part of the interpretation of at least some natural language quantifiers, it is definitely not obvious that it should be part of the at-issue/asserted content of quantificational items as opposed to being one of their presuppositions (or perhaps implicatures). We make it part of the assertion here just for presentational simplicity. As suggested by Theo M.V. Janssen, a more adequate treatment of quantification would involve a partial/trivalent logic and quantificational structures that do not satisfy the restrictor non-emptiness requirement would simply be undefined. This way of dealing with the case of empty quantificational domains has the added advantage of avoiding the problem of weak truth conditions for conditionals such as *If we invite a philosopher at our party, Alice will be happy* where the existential has wide scope over the conditional but the restrictor of the indefinite is interpreted in situ; for more discussion of this problem, see Abusch (1994) and Reinhart (1997) among others.

The C-FOL account does not suffer from this overgeneration problem. The interpretation clause for existentials in (36) above allows the available NS reading of (12) but not the problematic IS and WS interpretations. Crucially, our interpretation rule for existentials ensures that their restrictor formula ϕ is interpreted only relative to the variables that the indefinite possibly depends on. That is, the restrictor of an existential $\exists^{\mathcal{U}}$ is interpreted relative to the set of variables $\mathcal{U}$ and not relative to the full set of contextually-available variables $\mathcal{V}$. This captures the Binder Roof Constraint because it ensures that the semantic scope of the restrictor formula ϕ is the same as the semantic scope of the existential $\exists^{\mathcal{U}}$.

For example, making the indefinite *one^z of its$_y$ authors* in sentence (12) above independent from the universal *every^y paper* makes the variable y contributed by the pronoun *its$_y$* a free variable, which is ruled out by the interpretation clause for atomic formulas in (33) above.

To see in more detail how this works out formally, consider the simpler example in (49) below. The existential contributed by the indefinite in this example can in principle have two superscripts, as shown in (50).

(49) Every^x boy who talked to a^y friend of his$_x$ left.
(50) $\forall x[\text{BOY}(x) \wedge$
           $\exists^{\emptyset/\{x\}} y [\text{FRIEND-OF}(y, x)] \ (\text{TALK-TO}(x, y))]$
           $(\text{LEAVE}(x))$

Suppose now that the superscript is $\emptyset$. Then by clause (36b) in the semantic rule for existentials, the restrictor formula $\text{FRIEND-OF}(y, x)$ is interpreted relative to a context of evaluation whose set of variables is just $\{y\}$. Therefore, by the semantic rule for atomic formulas in (33) above (in particular, clause (33b)), $\text{FRIEND-OF}(y, x)$ is necessarily false.

This, in turn, means that the entire restrictor of the universal is false. Hence, by the semantic rule for universals in (47) and (48) above, the whole universal quantification is false because there is no set whatsoever of assignments H that satisfies the restrictor of the universal, so there cannot be any *maximal* set of assignments H that satisfies the restrictor.

In our approach, violating the Binder Roof Constraint yields falsity; in a partial/trivalent version, it would yield undefinedness. In contrast, the usual FOL way of combining unrestricted universal quantification and material implication would incorrectly yield trivially true formulas.

3.4 Negation, downward-entailing contexts and indefinites

This subsection discusses some aspects of the interaction between indefinites and negation (and other downward-entailing contexts) in the extensional C-FOL system introduced above. An intensional version of negation is introduced in Sect. 5.2 below.

3.4.1 Negation in extensional C-FOL

The semantic clause for negation is provided below.

(51) Negation:
      $[\![\neg\phi]\!]^{G,\mathcal{V}} = \mathbb{T}$ iff $G \neq \emptyset$ and $[\![\phi]\!]^{H,\mathcal{V}} = \mathbb{F}$, for any $H \subseteq G$ s.t. $H \neq \emptyset$

Note that closure under non-empty subsets, i.e., the requirement to evaluate the formula ϕ relative to every $H \subseteq G$ s.t. $H \neq \emptyset$, is necessary because a simpler semantic interpretation rule like the one in (52) below makes incorrect predictions for sentences in which universals take scope over sentential negation.[15]

(52) Negation without closure under subsets:
      $[\![\neg\phi]\!]^{G,\mathcal{V}} = \mathbb{T}$ iff $G \neq \emptyset$ and $[\![\phi]\!]^{G,\mathcal{V}} = \mathbb{F}$

To see this, consider the sentence in (53) below under the ''wide-scope universal'' reading provided in (54).

(53) Every^x boy didn't leave.
(54) $\forall x[\text{BOY}(x)] (\neg\text{LEAVE}(x))$

The interpretation of the formula proceeds as shown in (55) below. In particular, the nuclear scope formula $\neg\text{LEAVE}(x)$ is interpreted relative to the set of assignments storing the set of all boys under the variable x, i.e., $\{boy_1, boy_2, boy_3\}$ (we assume that there are only three boys in the model).

   If negation is interpreted as in (52) above, i.e., without requiring closure under subsets, we derive overly weak truth-conditions: we merely require at least one of the boys to not have left. However, if negation is interpreted as in (51) above, we derive the correct truth conditions: we require each boy to not have left.

(55)

The sentence in (53) can also have a reading under which the universal takes narrow scope relative to negation, represented in (56) below. Any quantifier-scoping mechanism in the literature can be used to derive these two readings; C-FOL does not require us to commit to any specific scoping mechanism here. The meaning for negation in (51) above derives the intuitively correct truth conditions for this reading too.

(56) $\neg\forall x[\text{BOY}(x)]\,(\text{LEAVE}(x))$

Similarly, the sentence in (57) below can have both a narrow-scope and a wide-scope indefinite reading, represented in (58) and (59). Once again, our interpretation for negation derives the correct truth conditions.

(57)   John didn't bring an^x umbrella.
(58) $\neg\exists^\emptyset x[\text{UMBRELLA}(x)]\,(\text{BRING}(\text{JOHN}, x))$
(59) $\exists^\emptyset x[\text{UMBRELLA}(x)]\,(\neg\text{BRING}(\text{JOHN}, x))$

Note that the indefinite can only have one superscript, namely $\emptyset$. The superscript, however, does not interact semantically with sentential negation since negation does not introduce any variable. Under this extensional version of negation, the treatment of the semantic scope of an indefinite relative to negation is identical to that found in classical FOL: syntactic and semantic scope go hand in hand.

   An obvious drawback is that the semantic scope of indefinites relative to negation has the same freedom as its scope relative to extensional or intensional quantifiers. We address this problem in Sect. 5.2 below, where we provide our final, intensional account of negation. Before doing that, we briefly discuss the issue of indefinites in other downward entailing contexts.

3.4.2 Downward-entailing contexts and exceptional scope

Armed with the semantic rule for negation given above, we can turn to the analysis of the much discussed issue of exceptional scope of indefinites in other downward-entailing contexts.

   Chierchia (2001) (see also Schwarz 2001) draws attention to these contexts and to the problem they pose for the ''free choice/Skolem-function variable'' approaches in Kratzer (1998) and Matthewson (1999). To see what the problem is, consider sentence (60) below. Its most salient reading, provided in (61), has the indefinite *some^z problem* taking exceptional scope intermediately between the two universal quantifiers.

(60)   Every^x linguist that studied every^y solution that some^z problem might have has become famous.

(61)   The most salient reading of (60) (in classical FOL):
       $\forall x(\text{LING}(x)\wedge$
            $\exists z(\text{PROB}(z)\wedge$
                 $\forall y(\text{SOL}(y)\wedge\text{MIGHT-HAVE}(z,y) \rightarrow \text{STUDY}(x,y)))$
       $\rightarrow \text{BECOME-FAMOUS}(x))$

As Chierchia (2001) observes, ''free choice-function variable'' approaches like the one in Kratzer (1998) represent sentence (60) as shown in (62) below, while the ''intermediate existential closure'' approaches in Reinhart (1997) and Winter (1997) represent it as shown in (63).[16] In both cases, **f** is a variable over choice functions, i.e., whenever it is applied to a set of entities X, the selected individual $x = \mathbf{f}(X)$ is s.t. $x \in X$.

(62) $\forall x(\text{LING}(x)\wedge$
            $\forall y(\text{SOL}(y)\wedge\text{MIGHT-HAVE}(\mathbf{f}(\text{PROB}),y) \rightarrow \text{STUDY}(x,y))$
       $\rightarrow \text{BECOME-FAMOUS}(x))$
(63) $\forall x(\text{LING}(x)\wedge$
            $\exists\mathbf{f}($
                 $\forall y(\text{SOL}(y)\wedge\text{MIGHT-HAVE}(\mathbf{f}(\text{PROB}),y) \rightarrow \text{STUDY}(x,y)))$
       $\rightarrow \text{BECOME-FAMOUS}(x))$

If we assume together with Chierchia (2001) that any choice function can in principle be assigned to a free choice-function variable (but see Kratzer (2003) for an argument against this assumption), then the former kind of approaches derive truth conditions that are too weak: (62) is verified by any problem for which some linguist didn't study every solution. Such a problem makes the restrictor formula $\text{LING}(x) \wedge \forall y(\text{SOL}(y) \wedge \ldots)$ false and the whole formula in (62) true. See also the argument in Schwarz (2001) that ''free choice-function variable'' approaches undergenerate.

Approaches like the one exemplified in (63) that countenance intermediate existential closure of choice-function variables enable us to give the indefinite exceptional scope semantically, while syntactically leaving it in situ. But allowing such non-local existential closure nullifies much of the initial motivation for choice-function approaches: if this kind of existential closure is needed, allowing for non-local existential closure of individual-level variables as in Abusch (1994), which obviates the need for choice functions, might prove to be the more parsimonious choice.

In contrast, C-FOL derives the correct intermediate-scope reading for example (60) without any additional stipulations. This reading is represented as shown in (64) below.

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[16] The representation we get in ''top level existential closure'' approaches like Matthewson (1999) is, for our current purposes, virtually identical to that given by ''free choice-function variable'' approaches.

(64) $\forall x[\text{LING}(x)\wedge$
  $\quad\forall y[\text{SOL}(y)\wedge$
  $\quad\quad\exists^{\{x\}}z[\text{PROB}(z)]\ (\text{MIGHT-HAVE}(z,y))]\ (\text{STUDY}(x,y))]$
  $\quad(\text{BECOME-FAMOUS}(x))$

The C-FOL analysis does not face the same problems as choice/Skolem-function analyses because the determiner *every* is not analyzed in terms of material implication:[17] we treat *every* in terms of restricted quantification, i.e., as contributing a maximal set of assignments satisfying the restrictor and passing it on to the nuclear scope. The nuclear scope is interpreted relative to this maximal set of assignments, which is tantamount to saying that the nuclear scope further elaborates on the quantificational dependencies introduced in the restrictor.

The superscript $\{x\}$ on the existential effectively ensures that the resulting reading is the same as the classical-FOL one in (61) above (with the addition that the sets of individuals satisfying the restrictors of the two universals need to be non-empty). There is no need for an additional existential-closure operator that takes intermediate scope because the indefinite itself has existential force.

The C-FOL analysis also generalizes to other kinds of downward-entailing contexts besides the restrictor of *every*. Consider, for example, the ''wide-scope negation'' sentence in (65) below, also from Chierchia (2001).

(65) Not every^x linguist studied every^y solution that some^z problem might have.

The intermediate-scope reading for this sentence, which is the most salient one, is provided in (66) below. As the second formula in (66) explicitly shows, this reading says that there is a sloppy unsystematic linguist that failed to study some solution of every single problem.

(66) a. The most salient reading of (65) (in classical FOL):
  $\neg\forall x(\text{LING}(x)\rightarrow$
  $\quad\exists z(\text{PROB}(z)\wedge$
  $\quad\quad\forall y(\text{SOL}(y)\wedge\text{MIGHT-HAVE}(z,y)\rightarrow \text{STUDY}(x,y))))$

  b. This is equivalent to the simpler formula below (again, in classical FOL):
  $\exists x(\text{LING}(x)\wedge$
  $\quad\forall z(\text{PROB}(z)\rightarrow$
  $\quad\quad\exists y(\text{SOL}(y)\wedge\text{MIGHT-HAVE}(z,y)\wedge\neg\text{STUDY}(x,y))))$

The C-FOL representation of the intermediate-scope reading is given in (67) below. Just as before, we interpret the indefinite *in situ* and specify its semantic scope by means of the superscript $\{x\}$.

(67) $\neg\forall x[\text{LING}(x)]\ (\forall y[\text{SOL}(y)\wedge\exists^{\{x\}}z[\text{PROB}(z)]\ (\text{MIGHT-HAVE}(z,y))]$
  $\quad\quad\quad\quad(\text{STUDY}(x,y)))$

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[17] The material-implication problem is not specific to choice/Skolem-function analyses (see Abusch (1994) for an early discussion).

Informally, the C-FOL formula in (67) is true iff the formula in the scope of negation is false. This means that, if we introduce all the linguists and store them under variable x, there is no way to introduce a problem z for each linguist x, store all the solutions of z under the variable y and satisfy the requirement that x studied every solution y.

The existential force is contributed by the indefinite itself, so once again we do not need an existential-closure operator with intermediate scope (as Reinhart 1997 or Winter 1997), a special storage mechanism (as Abusch 1994) or a special presupposition associated with specific indefinites that would have to be accommodated neither globally nor locally, but in an intermediate position (as Geurts 2010).

3.5 Further restrictions on exceptional scope

We turn now to the discussion of the fourth exceptional scope reading that we predict for sentence (8), namely the reading that we obtain if the indefinite is superscripted with $\{y\}$: the indefinite covaries with the direct object universal $every^y$ $paper$, but not with the subject universal $every^x$ $student$. This reading, provided in (68) below, is intuitively represented by the matrices in (69).

(68) $\forall x[\text{STUD}(x)]$
        $(\forall y[\text{PAPER}(y)\wedge$
            $\exists^{\{y\}}z[\text{PROF}(z)]\ (\text{RECOM}(z,y))]$
        $(\text{READ}(x,y)))$

(69)   The others IS reading

x	y	z
α_1	β_1	γ
α_1	β_2	γ'
α_2	β_1	γ
α_2	β_2	γ'

The others IS reading (rows recorded)

x	y	z
α_1	β_1	γ
α_2	β_1	
α_1	β_2	γ'
α_2	β_2	

As noted above, the existence of this reading is predicted by any system that adheres to the usual assumptions about the way in which universal quantifiers take scope: the entire direct object $every^y$ $paper$ $that$ … can be raised to take scope over the subject $every^x$ $student$. In the resulting logical form, if the indefinite takes the narrowest scope it can, we generate exactly this reading.

In this particular case then, C-FOL does not overgenerate—at least not more than any of the usual scoping mechanisms generally assumed in the literature. But more problematic cases can be constructed where the freedom of choosing superscripts for existentials, specific to IFL-based systems like ours, may in fact lead to overgeneration.

To illustrate, note that sentence (70) below appears to lack the reading provided in (71). That is, the indefinite a^z $gift$, whose syntactic scope is fixed by the double object construction, cannot scope over $every^x$ man and, at the same time, under $most^y$ $women$.

(70)   Every^x man decided to give most^y women a^z gift.

(71)   most women are s.t.
                there is a gift s.t.
                        every man
                                decided to give that gift to (each of) them

Another example supporting the same point is provided in (72). This sentence does not seem to have the reading in (73), where the indefinite *a^z paper* scopes over the matrix subject *most^x students*, but under the embedded subject *every^y professor*.[18]

(72)   Most^x students noticed that every^y professor recommended a^z paper about scope.

(73)   for every professor,
                there is a paper s.t.
                        most students
                                noticed that the professor recommended the paper

The missing readings are perfectly possible in the account developed above: the existential accesses the set of previously introduced variables and is free to choose any variables in this set as the variables it may depend on.[19]

Determining the available or missing readings of the sentences in (71) and (73) is very difficult. Let us assume that our claim about these missing readings is actually true. These readings then would be ruled out by a plausible constraint on the scopal properties of indefinites which we dub the No Skipping Constraint, formulated in (74) below. This constraint states that an indefinite cannot take scope over a quantifier **Q** without taking scope over everything below that quantifier:

(74)   No Skipping Constraint: if an indefinite is independent relative to
        a quantifier **Q**, it is independent relative to all quantifiers **Q**′ that are in the
        (syntactic and semantic) scope of **Q**.

Our current account does not capture this constraint. More careful empirical work needs to be done to ascertain whether the No Skipping Constraint is in fact operative, but we will assume in this subsection that it is.

The general principle behind it is the following: indefinites have to respect the evaluation order of the quantifiers in the sentence. Note that what has to be respected is evaluation order, i.e., the order of interpretation, not syntactic c-command or linear order.

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[18] In the paraphrase of (70), given in (71) above, and the paraphrase of (72), given in (73) below, the lower quantifier appears to scope over a higher non-clausemate quantifier violating the restriction delimiting the scope of *bona fide* quantifiers to their own clause. This is simply the consequence of our attempt to give a linear paraphrase of the intended readings.

[19] The same issues arise for systems where the existential is syntactically indexed with the variables it is independent of.

Thus, even when indefinites take exceptional scope, they should still respect the background evaluation order of the sentence in which they occur. Ultimately, this principle is just methodological parsimony at work: we relax the classical FOL interpretation procedure with its rigid evaluation order only as much as we need to capture the exceptional scope of indefinites, but not more than that.

We provide below a way of reformulating the semantics of C-FOL that reflects this more constrained theory of exceptional scope. Instead of merely keeping track of the *set* of variables $\mathcal{V}$ introduced by the previous/higher quantifiers, we keep track of the *sequence* in which these variables have been introduced (see Bittner (2003) and Schlenker (2005) for two recent related proposals and Dekker (1994) for one of the first relevant discussions in the linguistic literature).

Thus, the interpretation function has the sequence-based form $[\![\cdot]\!]^{G,\langle x_1,\ldots,x_n\rangle}$ instead of the simpler set-based form $[\![\cdot]\!]^{G,\{x_1,\ldots,x_n\}}$—or, using our more concise set-based notation, $[\![\cdot]\!]^{G,\mathcal{V}}$.

The set of variables $\mathcal{V} = \{x_1,\ldots,x_n\}$ contains the variables introduced by the previously evaluated quantifiers $\mathbf{Q}_1 x_1,\ldots,\mathbf{Q}_n x_n$. The sequence of variables $\langle x_1,\ldots,x_n\rangle$ contains the variables introduced by the previously evaluated quantifiers too but in addition, the sequence lists them in the order in which these quantifiers have been evaluated.

Given these finer-grained contexts of evaluation, the semantic contribution of existentials is as follows. An existential accesses the sequence of variables $\langle x_1,\ldots,x_n\rangle$ contributed by the previously-evaluated quantifiers and breaks this sequence into two subsequences, a left one and a right one. That is:

- the existential chooses a position m in the sequence of variables $\langle x_1,\ldots,x_n\rangle$,
  which we can depict as $\begin{array}{c}\langle x_1,\ldots,x_n\rangle\\ \leftarrow m \rightarrow\end{array}$
- then the existential breaks the sequence $\langle x_1,\ldots,x_n\rangle$ at position m, which yields two subsequences $\langle x_1,\ldots,x_m\rangle$ and $\langle x_{m+1},\ldots,x_n\rangle$.

The existential now contributes a fixed value condition relative to these subsequences. The witness choice is *independent* of the values of the variables in the right subsequence, i.e., the witness is chosen in such a way that it is invariant relative to the values of the variables $x_{m+1},\ldots,x_n$. In contrast, the left subsequence $\langle x_1,\ldots,x_m\rangle$ stores the variables that the indefinite possibly covaries with.

Under this view, an indefinite that is in the syntactic scope of a quantifier binding a variable x_n is in its semantic scope iff x_n is in the left sequence of variables.

The definitions below formally spell out this proposal. Note in particular the definition of existential quantification: witness choice is determined by the superscript m targeting the sequence of variables $\langle x_1,\ldots,x_n\rangle$ contributed by the previous/higher quantifiers:

- if $m = 0$, the value of the variable contributed by the existential is fixed in the absolute terms of (27), so the existential has widest scope
- if $m \neq 0$, the value is fixed relative to $\langle x_{m+1},\ldots,x_n\rangle$, i.e., it is fixed in the relativized terms of (29)

(75) $\left[\!\left[R\left(x_{i_1}, \ldots x_{i_{n'}}\right)\right]\!\right]^{G,\langle x_1,\ldots,x_n\rangle} = \mathbb{T}$ iff
    a. $G \neq \emptyset$
    b. $\{x_{i_1}, \ldots, x_{i_{n'}}\} \subseteq \{x_1, \ldots, x_n\}$
    c. $\langle g(x_{i_1}), \ldots, g(x_{i_{n'}})\rangle \in \Im(R)$, for all $g \in G$

(76) $\left[\!\left[\phi \wedge \psi\right]\!\right]^{G,\langle x_1,\ldots,x_n\rangle} = \mathbb{T}$ iff $\left[\!\left[\phi\right]\!\right]^{G,\langle x_1,\ldots,x_n\rangle} = \mathbb{T}$ and $\left[\!\left[\psi\right]\!\right]^{G,\langle x_1,\ldots,x_n\rangle} = \mathbb{T}$

(77) $\left[\!\left[\exists^m x[\phi]\,(\psi)\right]\!\right]^{G,\langle x_1,\ldots,x_n\rangle} = \mathbb{T}$ iff $0 \leq m \leq n$ and $\left[\!\left[\psi\right]\!\right]^{H,\langle x_1,\ldots,x_n,x\rangle} = \mathbb{T}$,
    for some H s.t.
    a. $H[x]G$
    b. $\left[\!\left[\phi\right]\!\right]^{H,\langle x_1,\ldots,x_m,x\rangle} = \mathbb{T}$ (if $m = 0$, this is simply $\left[\!\left[\phi\right]\!\right]^{H,\langle x\rangle} = \mathbb{T}$)
    c. $\begin{cases} \text{if } m = 0 : h(x) = h'(x), \text{ for all } h, h' \in H \\ \text{if } m \neq 0 : h(x) = h'(x), \text{for all } h, h' \in H \text{ that are } \{x_1, \ldots, x_m\}\text{-identical} \end{cases}$

(78) $\left[\!\left[\forall x[\phi]\,(\psi)\right]\!\right]^{G,\langle x_1,\ldots,x_n\rangle} = \mathbb{T}$ iff $\left[\!\left[\psi\right]\!\right]^{H,\langle x_1,\ldots,x_n,x\rangle} = \mathbb{T}$, for some H that is a
    maximal set of assignments relative to x, ϕ, G and $\langle x_1, \ldots, x_n\rangle$

(79) H is the maximal set of assignments relative to a variable x, a formula ϕ,
    a set of assignments G and a sequence of variables $\langle x_1, \ldots, x_n\rangle$ iff
    a. $H[x]G$ and $\left[\!\left[\phi\right]\!\right]^{H,\langle x_1,\ldots,x_n,x\rangle} = \mathbb{T}$
    b. there is no $H' \neq H$ s.t. $H \subseteq H'$ and:
       $H'[x]G$ and $\left[\!\left[\phi\right]\!\right]^{H',\langle x_1,\ldots,x_n,x\rangle} = \mathbb{T}$

(80) Truth: a formula ϕ is true (in model $\mathfrak{M}$) iff $\left[\!\left[\phi\right]\!\right]^{G,\langle\rangle} = \mathbb{T}$ for any non-empty
    set of assignments G, where $\langle\rangle$ is the empty sequence of variables.

Besides being more restrictive, this sequence-based semantics for C-FOL makes clear the fact that the formalization of scopal (in)dependence is different from the formalization of anaphoric dependencies: anaphoric dependencies are captured by variable coindexation, but existentials are *not* indexed with variables.

That is, since we work with variable sequences now, witness choice can be constrained without making reference to any particular variables in the sequence: we only need to select a particular position m and break the sequence of variables relative to which the existential is interpreted into two subsequences.

All our previous results are preserved if we give C-FOL a finer-grained semantics along these lines. Consider, for example, the sentence in (41) above, where an existential is in the scope of a universal. We represent the two possible scopes of the indefinite by means of the two possible superscripts 0 (wide scope) and 1 (narrow scope).

(81) $\forall x[\text{STUD}(x)]\,(\exists^0 y[\text{PAPER}(y)]\,(\text{READ}(x,y)))$
(82) $\forall x[\text{STUD}(x)]\,(\exists^1 y[\text{PAPER}(y)]\,(\text{READ}(x,y)))$

The novelty is that the system now captures the No Skipping Constraint because scopal independence is established relative to the *sequence of previously introduced variables*. The order of the variables in this sequence is the order in which the quantifiers were evaluated, i.e., the order in which the quantifiers take semantic scope.

The superscript on the existential breaks this sequence of variables into two subsequences. We are free to choose this superscript, i.e., the position where we

break the sequence, but we cannot choose an arbitrary subsequence or an arbitrary subset of variables. Existentials still have a choice, i.e., they can still take exceptional scope, but the choices are more constrained.

For example, our exceptional scope sentence in (5) above can only have the three intuitively available readings WS, IS and NS that we want to capture. The superscript on the existential can only be 0 (WS), 1 (IS) or 2 (NS), no other choices are possible relative to the sequence of variables $\langle x, y \rangle$.

(83) $\forall x [\text{STUD}(x)]$
          $(\forall y [\text{PAPER}(y) \wedge$
              $\exists^{0/1/2} z [\text{PROF}(z)] \; (\text{RECOM(Z,Y)})]$
          $(\text{READ}(x, y)))$

Theo M.V. Janssen points out that incorporating the No Skipping Constraint into the system can also be motivated on logical grounds. As Hodges (1997) observes (see Hintikka 1997 and Janssen 2002 for more discussion), the IFL formula $\forall x \exists y_{/x}(x = y)$ is (generally) not true, but the formula $\forall x \exists z \exists y_{/x}(x = y)$ is true if we always choose the value of x for z and then for y, we choose the value of z. The No Skipping Constraint rules out cases like this in which independence conditions are undermined by 'signalling'.

While the more restrictive, sequence-based semantics for C-FOL seems appealing, we continue to work in this paper with the simpler, less restricted system based on sets of variables. This is both for expository simplicity and because we leave a systematic empirical investigation of the No Skipping Constraint for future research.

4 Dependent indefinites in C-FOL

The C-FOL analysis of ordinary indefinites has two crucial ingredients:

(i) the superscript on the existential that stores the set of parameters relative to which the indefinite may covary
(ii) the fixed-value constraint that makes use of this superscript and that constrains the values of the indefinite stored in the resulting matrix

We therefore expect the existence of special indefinites that target the same superscript and enforce further constraints on the values stored in the matrix. We provide here an account of dependent indefinites in which they do exactly this. While simple indefinites contribute a fixed value condition relativized to their superscript, dependent indefinites add a non-fixed value condition relativized to the same superscript.

The interpretation rule for dependent indefinites is provided in (84) below. It is identical to the interpretation rule for ordinary indefinites in (36) above except for the last clause in (84d), which is contributed by the dependent morphology.

(84) $\llbracket \textbf{dep-}\exists^{\mathcal{U}}x[\phi](\psi) \rrbracket^{G,\mathcal{V}} = \mathbb{T}$ iff $\mathcal{U} \subseteq \mathcal{V}$ and $\llbracket \psi \rrbracket^{H,\mathcal{V}\cup\{x\}} = \mathbb{T}$, for some H s.t.
    a. $H[x]G$
    b. $\llbracket \phi \rrbracket^{H,\mathcal{U}\cup\{x\}} = \mathbb{T}$
    c. $\begin{cases} \text{if } \mathcal{U} = \emptyset : h(x) = h'(x), \text{for all } h, h' \in H \\ \text{if } \mathcal{U} \neq \emptyset : h(x) = h'(x), \text{ for all } h, h' \in H \text{ that are } \mathcal{U}\text{-identical} \end{cases}$
    d. $h(x) \neq h'(x)$, for at least two $h, h' \in H$ that are not $\mathcal{U}$-identical

The clause in (84d) requires covariation because it requires the set of variables $\mathcal{U}$ that contains parameters of possible covariation to be non-empty: there have to be at least two assignments $h, h' \in H$ that are not $\mathcal{U}$-identical, which means that there has to be at least one variable $v \in \mathcal{U}$ s.t. $h(v) \neq h'(v)$. Therefore, the fixed-value condition in (84c) is effectively reduced to the second case in which $\mathcal{U} \neq \emptyset$.

Furthermore, we also correctly predict that dependent indefinites require a *bona fide* quantifier in order to be licensed. This is because only such quantifiers can introduce multiple values for the same variable. If all the variables in $\mathcal{V}$ are introduced by (in)definites, each variable stores only one value throughout G, and thus there can be no assignments that are $\mathcal{U}$-non-identical (since $\mathcal{U} \subseteq \mathcal{V}$). For an analysis of the covariation requirement contributed by dependent indefinites in a related dynamic framework, see Wang et al. (2006).

Consider, for example, the sentence in (85) below, which is the Romanian counterpart of the English sentence in (41) above, with the addition of the dependent-indefinite marker *cîte*. This example is represented in C-FOL as shown in (86) below.

In (86), we indicate that the empty set $\emptyset$ is not a possible superscript for the existential by starring it: $^*\emptyset$. We indicate that the singleton set $\{x\}$ is a possible superscript for the existential by adding a checkmark: $^{\checkmark}\{x\}$.

(85)   Fiecare^x student a citit *cîte* un^y articol.
(86) $\forall x[\textsc{stud}(x)] (\textbf{dep-}\exists^{*\emptyset/^{\checkmark}\{x\}}y[\textsc{paper}(y)] (\textsc{read}(x,y)))$

(87)
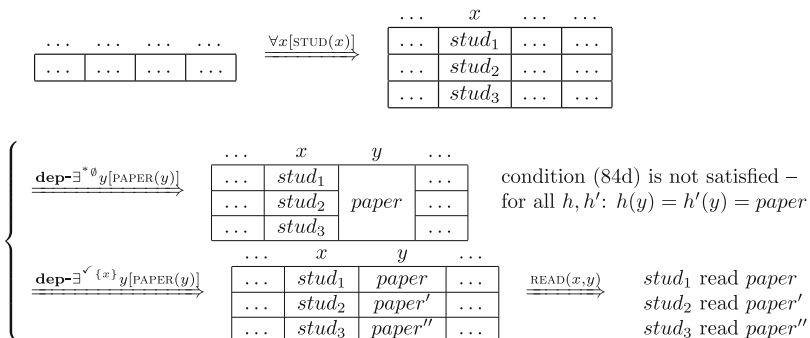
The way the evaluation proceeds is depicted in (87) above. First, the restrictor of the universal introduces the set of all students in column *x*. We then evaluate the dependent existential. If the existential is superscripted with the empty set $\emptyset$, we fail

to satisfy the variation condition (84d) contributed by the morpheme *cîte*: the variable *y* introduced by the existential has a unique value, namely the entity *paper*, which makes any variation or covariation impossible.

Thus, the dependent existential can only have the superscript $\{x\}$. This makes it *possible* for the variable *y* introduced by the existential to covary with the variable *x* introduced by the universal. The variation condition (84d) contributed by *cîte* requires this covariation to *actually* be realized. That is, at least one of the following three inequalities has to be true: *paper* $\neq$ *paper'* or *paper* $\neq$ *paper''* or *paper'* $\neq$ *paper''*. Finally, the nuclear scope of the indefinite checks that each *x*-student read the corresponding *y*-paper.

The semantic contribution of dependent morphology given above is too permissive for languages like Romanian and Hungarian, where modals cannot license dependent indefinites. For these languages, the covariation condition in (84d) has to be refined so as to be sensitive to the distinction between world and non-world variables.

To capture this, we need to identify the sort of the variables in the set of variables $\mathcal{U}$. We will take $\mathcal{U}_{\mathfrak{D}}$ to be the set of variables over individuals in $\mathcal{U}$ and $\mathcal{U}_{\mathfrak{W}}$ to be the set of variables over worlds in $\mathcal{U}$. In order to rule out modal licensors for dependent indefinites in Hungarian and Romanian we assume that the condition contributed by dependent morphology is not the general one in (84d) above, but rather the more specific one given below:

(88) $h(x) \neq h'(x)$, for at least two $h, h' \in H$ that are $\mathcal{U}_{\mathfrak{W}}$-identical but not
        $\mathcal{U}_{\mathfrak{D}}$-identical

In this way, we require dependent indefinites to covary with a variable over individuals rather than a variable over worlds. For languages such as Russian, where sortal distinctions on licensors are not relevant, dependent morphology brings in the general condition given in (84d). Note that this account correctly predicts that dependent indefinites are licensed by *bona fide* quantifiers but not by other indefinites or by negation.

The account of dependent indefinites given here treats them as necessarily being 'evaluation plurals' in the sense of Brasoveanu (2007, 2008), where two notions of plurality are distinguished: domain plurality and evaluation (or discourse) plurality.

Domain/ontological plurality is the usual notion of plural reference, i.e., reference to a non-atomic individual, which in our system involves a possibly non-atomic value for the relevant variable stored in one of the cells of the current matrix G of variable assignments.

Evaluation/discourse plurality, on the other hand, involves non-atomic reference relative to the whole matrix G of variable assignments, i.e., the column in the matrix G that stores the values of the relevant variable has to store a non-singleton set of entities (each of which can be atomic or non-atomic).[20]

Evaluation plurality involves precisely the type of quantificational dependency encoded by dependent morphology. The existence of this type of special

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[20] See Farkas (2001) for a connection between an intuitively characterized notion of evaluation plurality and dependent indefinites.

morphology provides independent support for the distinction between these two types of plurality.[21]

Moreover, connecting dependent indefinites with this generalized, two-faceted notion of plurality helps us understand why the morphological process of reduplication is used in some languages to encode domain plurality, while in others it is used to encode evaluation plurality.

Finally, note that the treatment of dependent indefinites given above allows us to make a welcome connection between indefinite article reduplication in Hungarian and another reduplicative process in this language that involves verbal morphology. As discussed in Farkas (2001), Hungarian also uses reduplication in the verbal realm to signal the repetition of an event. This reduplicative process involves the verbal particle that accompanies many verbs in Hungarian. It is illustrated below, where the particle is boldfaced.

(89) Az   éjszaka   folyamán   a   gyerek   **felé**bredt.
      the   night   during   the   child   PART.woke
      'During the night, the child woke up.'

(90) Az   éjszaka   folyamán   a   gyerek   **fel-fel**   ébredt.
      the   night   during   the   child   PART-PART   woke
      'During the night, the child kept waking up.'

The crucial distinction between these two examples is that (89) is true in case there is a single event of the child waking up during the night whereas in order for (90) to be true, there must be several such events. Without going into the details here, we suggest that the semantics of verbal particle reduplication requires covariation between an event variable and a temporal variable in a manner that is analogous to the covariation requirement imposed by the indefinite-determiner reduplication.

To conclude, we have proposed here an account of dependent indefinites in C-FOL that isolates the contribution of dependent morphology and treats it as imposing a covariation requirement between the variable associated with the indefinite and another variable. The formal system developed to account for the exceptional-scope properties of ordinary indefinites was shown to extend in a natural way to account for the existence, distribution and some special properties of dependent indefinites.

5 Consequences, extensions and open questions

This section is devoted to further issues in the semantics of indefinites, each of which deserves a lengthier treatment than what we are able to provide here. Our discussion is hopefully sufficient to provide the gist of what a full account would look like.

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[21] Brasoveanu (2007, 2008), building on van den Berg (1996) and Nouwen (2003) among others, provided arguments for this distinction based on intra- and cross-sentential anaphora.

5.1 Indefinites in *there*-existentials

The fact that indefinites have free upwards scope as part of their lexical meaning does not mean that certain constructions cannot trap their scope. There are several such scope-freezing environments of which we briefly discuss only one here, namely *there*-existentials. Indefinites in *there*-existentials, exemplified in (91) below, are always interpreted as taking narrow scope relative to quantifiers in the coda (for a recent discussion, see Francez 2009, p. 33).[22]

(91)   There was a^x linguist at every^y conference.

We can derive this generalization under our current account of indefinite scope if we assume that *there*-existentials contribute an operator δ over pivots that distributes over the set of assignments in the context of evaluation. As Theo M.V. Janssen points out, this distributivity operator is closely related to the flattening operator introduced in Hodges (1997) (flattening because it 'flattens' the interpretation to first-order level).

(92)   Distributivity over sets of assignments:
        $[\![\delta(\phi)]\!]^{G,\mathcal{V}} = \mathbb{T}$ iff $G \neq \emptyset$ and $[\![\phi]\!]^{\{g\},\mathcal{V}} = \mathbb{T}$, for every $g \in G$

The fact that the indefinite in (91) can only have narrow scope is a consequence of the semantics contributed by the existential construction, which forces the pivot to be interpreted distributively, rather than collectively, relative to the set of assignments in the context of evaluation.

While such a distributive evaluation procedure was undesirable for the restrictors of universal quantifiers (as discussed in Sect. 3.2 above), it is exactly what we need for *there*-existentials.

The distributivity operator δ conflates all the scoping possibilities for the indefinite pivot to narrow scope, irrespective of the superscript on the existential contributed by the indefinite. This is because fixed-value conditions, whether relativized or not, are vacuously satisfied relative to singleton sets of assignments.

This approach allows us to scopally trap an indefinite without constraining the gamut of superscripts that it can have. This is so because, crucially, the superscript does not have an interpretation by itself but rather, it is simply a way of expressing constraints on the structure of matrices/sets of assignments. We can limit or nullify its contribution by means of other, independent constraints on the structure of these matrices.

The example in (91) above is analyzed as shown in (93) below. We consider only the superscript $\emptyset$ on the existential quantifier, which would give the indefinite wide scope in the absence of the operator δ.

(93) $\forall y[\text{CONFERENCE}(y)]\ (\delta(\exists^\emptyset x[\text{LING}(x)]\ (x = x)))$

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(94)

$$\begin{array}{|c|c|c|c|}\hline \ldots & \ldots & \ldots & \ldots \\ \hline \ldots & \ldots & \ldots & \ldots \\ \hline \end{array} \quad \xrightarrow{\forall y[\text{CONFERENCE}(y)]} \quad \begin{array}{c|c|c|c} \ldots & y & \ldots & \ldots \\ \hline \ldots & conf_1 & \ldots & \ldots \\ \ldots & conf_2 & \ldots & \ldots \\ \ldots & conf_3 & \ldots & \ldots \\ \end{array}$$

$$\overset{\delta}{\Rightarrow} \left\{ \begin{array}{l} \begin{array}{c|c|c|c} \ldots & y & \ldots & \ldots \\ \hline \ldots & conf_1 & \ldots & \ldots \\ \end{array} \xrightarrow{\exists^{\emptyset} x[\text{LING}(x)]} \begin{array}{c|c|c|c} \ldots & y & x & \ldots \\ \hline \ldots & conf_1 & ling_1 & \ldots \\ \end{array} \\[2em] \begin{array}{c|c|c|c} \ldots & y & \ldots & \ldots \\ \hline \ldots & conf_2 & \ldots & \ldots \\ \end{array} \xrightarrow{\exists^{\emptyset} x[\text{LING}(x)]} \begin{array}{c|c|c|c} \ldots & y & x & \ldots \\ \hline \ldots & conf_2 & ling_2 & \ldots \\ \end{array} \\[2em] \begin{array}{c|c|c|c} \ldots & y & \ldots & \ldots \\ \hline \ldots & conf_3 & \ldots & \ldots \\ \end{array} \xrightarrow{\exists^{\emptyset} x[\text{LING}(x)]} \begin{array}{c|c|c|c} \ldots & y & x & \ldots \\ \hline \ldots & conf_3 & ling_3 & \ldots \\ \end{array} \end{array} \right\}$$

This analysis enables us to combine the account of exceptional scope of indefinites proposed here and the account of existential constructions with quantified codas in Francez (2009).

5.2 Negation in intensional C-FOL

The extensional account of the interaction of indefinites and negation given in subsection 3.4 above left unsolved the issue of the relative scope of negation and indefinites. The generalization to be captured is that an ordinary indefinite within the syntactic scope of negation can freely escape its semantic scope in a manner parallel to the interaction of indefinites and other quantifiers or modal operators.

We provide an intensional treatment of negation here which accounts for the scopal interaction of indefinites and negation. Note that an intensional system has to provide a suitable intensional meaning for sentential negation quite independently of any matters related to the exceptional scope of indefinites, so we simply take advantage of that independently-needed intensional treatment of negation to account for the fact that indefinites can take exceptional scope relative to negation.

Consider again the sentence in (95) below, repeated from above.

(95)   John didn't bring an umbrella.

The standard way of interpreting this sentence in intensional logic involves two steps. First, we extract the set of possible worlds that satisfy the sentence radical *John bring an umbrella* (we ignore tense throughout this paper for simplicity), i.e., we λ-abstract over the sentence radical. Second, we check that the actual world $w^{@}$ is not among the worlds in this set.

(96) $w^{@} \notin \lambda w.$ John brought an umbrella in w

When we work with a semantics based on sets of assignments (as opposed to single assignments), we have to decide how to formalize this kind of λ-abstraction.

Recall that the main motivation for sets of assignments was that they enable us to encode both sets of entities and the quantificational dependencies between them,

and λ-abstraction is fundamentally quantificational in this sense. This is because it enables us to extract a maximal set of entities that satisfy certain properties and enter particular dependencies with the values of the other variables that are in the scope of the λ-abstractor.

Thus, λ-abstraction should be formalized in our system as maximization over sets of variable assignments. This enables us to extract and store both maximal sets of entities and the various quantificational dependencies these entities are part of.

In fact, we have already implicitly defined this very notion of λ-abstraction as part of the semantic rule for universal quantification: the nuclear scope of a universal is interpreted relative to the maximal set of assignments that satisfies its restrictor.

We therefore revise our analysis of negation and analyze it as universal quantification over possible worlds, where the sentence in the syntactic scope of negation provides the restrictor formula. The nuclear scope formula requires any world satisfying the restrictor to be distinct from the actual world $w^@$.

(97) $\textbf{not}_{w^@}\ \phi \rightsquigarrow \forall w[\phi]\ (w \neq w^@)$
(98) $w \neq w^@ := \neg w = w^@$
     a. $\neg$ is interpreted as in (51) above
     b. $w = w^@$ is interpreted as an ordinary atomic formula, i.e., following the format in (33) above

Given that we now make use of a designated world variable $w^@$, the value of which is contextually-specified to be the actual world, we need to change our context-independent definition of truth in (44) above to the notion of truth-in-a-context defined in (99) below.[23]

(99)   A formula ϕ is true relative to a context of evaluation consisting of a set of assignments G and a set of variables $\mathcal{V}$ iff $[\![\phi]\!]^{G,\mathcal{V}} = \mathbb{T}$.

We take all the relevant contexts of evaluation consisting of a set of assignments G and a set of variables $\mathcal{V}$ to be such that: $w^@ \in \mathcal{V}$ and for all $g, g' \in G, g(w^@) = g'(w^@)$.

The sentence in (95) above is represented as shown in (100) below.

(100)   a. $\forall w[\exists^{\{w^@\}} x[\text{UMBRELLA}(w^@, x)]\ (\text{BRING}(w, \text{JOHN}, x))]\ (w \neq w^@)$
       b. $\forall w[\exists^{\{w\}} x[\text{UMBRELLA}(w, x)]\ (\text{BRING}(w, \text{JOHN}, x))]\ (w \neq w^@)$

If the superscript on the existential is the singleton set $\{w^@\}$ containing the actual-world variable, as in (100a), the indefinite *an umbrella* is interpreted *de re* relative to the modal quantification contributed by negation, i.e., the indefinite has wide scope. In this case, the restrictor formula of the indefinite has to be relativized to the actual world $w^@$, given the ban against free variables built into the semantic rule for atomic formulas.

If the superscript on the existential is the singleton set $\{w\}$, as in (100b), the indefinite *an umbrella* is interpreted *de dicto* relative to the modal quantification

contributed by negation, i.e., the indefinite has narrow scope. In this case, the restrictor formula of the indefinite has to be relativized to the variable w contributed by negation.[24]

Finally, there is another way to give the indefinite narrow scope relative to the modal quantification contributed by negation, i.e., to interpret the indefinite *de dicto*, given in (101) below. In both formulas in (101), the superscript on the existential contains both the actual-world variable $w^@$ and the variable w contributed by negation. But the restrictor formula of the existential can be relativized either to the variable w contributed by negation, i.e., UMBRELLA(w, x), as shown in (101a), or to the actual-world variable $w^@$, i.e., UMBRELLA$(w^@, x)$, as shown in (101b).

(101)   a. $\forall w [\exists^{\{w^@, w\}} x [\text{UMBRELLA}(w, x)] \, (\text{BRING}(w, \text{JOHN}, x))] \, (w \neq w^@)$
         b. $\forall w [\exists^{\{w^@, w\}} x [\text{UMBRELLA}(w^@, x)] \, (\text{BRING}(w, \text{JOHN}, x))] \, (w \neq w^@)$

The formula in (101a) is simply an alternative way to represent the *de dicto* reading for the indefinite *an umbrella*. The formula in (101b) interprets the existential in a *de dicto* way, i.e., the existential has narrow scope relative to negation, but the restrictor formula is relativized to the actual world $w^@$, so it has 'scope' outside the modal quantification contributed by negation.

Thus, the semantic scope of the existential relative to a possible-world variable and the 'scope' of the restrictor of the existential relative to that same world variable are distinct: the restrictor formula can always be interpreted relative to a 'higher' world variable and, therefore, appear to take 'exceptional scope'.

Importantly, this is *not* an actual case of exceptional scope. It is simply non-local binding of the world variable on the predicate *umbrella*, much like the pronoun his_x is non-locally bound by the subject quantifier $every^x$ *boy* in the example below.

(102)   Every^x boy gave every^y girl the^z gift that his$_x$ mother suggested.

The intensional treatment of negation, unlike the extensional version, enables us to account for the fact that indefinites can have exceptional scope relative to negation, just as they can take exceptional scope relative to quantifiers. Consider the example in (103) below. (We are indebted to Philippe Schlenker for this example and relevant discussion.)

(103)   Joe will not hire any applicant who worked with a (certain) linguist.

The most salient reading of (103) gives the indefinite *a (certain) linguist* exceptional scope relative to the sentential negation *will not*: there is a certain linguist y s.t. Joe will not hire any applicant x who worked with y. This reading is obtained by making *a (certain) linguist* independent from negation, as shown below.

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[24] As Theo M.V. Janssen points out, Hintikka (1996) was the first to suggest that the *de re/de dicto* ambiguity could be analyzed in terms of IFL-style (in)dependence.

(104) $\forall w[\exists^{\{w\}}x[\text{APP}(w, x) \wedge \exists^{\{w^@\}}y[\text{LING } (w^@, y)] \, (\text{WORK-WITH}(w,x,y))]$
          $\big(\text{HIRE}(w,\text{JOE}, x))\big]$
        $(w \neq w^@)$

Summarizing, the treatment of negation given here accounts for the scopal inter-actions between indefinites and negation (in general, between indefinites and downward-entailing contexts). We also correctly predict that dependent indefinites in Romanian and Hungarian will not be licensed by negation given that negation contributes a modal variable that is not an appropriate variable for the dependent indefinite to covary with.

   As Philippe Schlenker points out, however, we also predict that negation will be an appropriate licensor for dependent indefinites in Russian, since dependent in-definites in this language may covary with modal variables bound by modal oper-ators. This prediction appears to be incorrect, as shown by the example in (105) below from Pereltsvaig (2008).

(105)   *Vanya ne pročital kakoe-nibud' stixotvorenie.
        Vanya didn't read a poem.

More work needs to be done to give the right intensional treatment of negation and capture all the details of dependent indefinites in Russian, but for the time being we leave this important problem open.

5.3 The scope of cardinal indefinites

Cardinal (plural) indefinites have two interpretations, a collective one and a distributive one. To analyze such indefinites, we build on Link (1983) and Schwarzschild (1996) among others and assume that the domain of individuals $\mathfrak{D}$ is the powerset of a designated set of entities IN minus the empty set, i.e., $\mathfrak{D} = \wp^+(\text{IN}) = \wp(\text{IN}) \setminus \emptyset$.

   The cardinality of an individual x, i.e., the number of atoms it consists of, is symbolized as $|x|$. Atomic individuals are the singleton sets in $\wp^+(\text{IN})$, identified by a predicate **atom**, defined below. Individuals that contain two atoms are the dou-bleton sets in $\wp^+(\text{IN})$, identified by a predicate **2.atoms** etc. The 'part of' relation $\leq$ over individuals is set inclusion over $\wp^+(\text{IN})$.

(106) $[\![\textbf{atom}(x)]\!]^{G,\mathcal{V}} = \mathbb{T}$ iff $G \neq \emptyset, x \in \mathcal{V}$ and $|g(x)| = 1$, for all $g \in G$
(107) $[\![\textbf{2.atoms}(x)]\!]^{G,\mathcal{V}} = \mathbb{T}$ iff $G \neq \emptyset, x \in \mathcal{V}$ and $|g(x)| = 2$, for all $g \in G$
(108) $[\![x \leq y]\!]^{G,\mathcal{V}} = \mathbb{T}$ iff $G \neq \emptyset, \{x, y\} \subseteq \mathcal{V}$ and $g(x) \subseteq g(y)$, for all $g \in G$

Collective cardinal indefinites are interpreted as shown in (109) below for the cardinal numeral *three*. They are interpreted exactly like we interpreted singular indefinites up until now, with the addition of a cardinality requirement $\textbf{3.atoms}(x)$ in their restrictor.

(109) $[\![\exists^{\mathcal{U}} x[\mathbf{3}.\mathbf{atoms}(x) \wedge \phi]\,(\psi)]\!]^{G,\mathcal{V}} = \mathbb{T}$ iff $\mathcal{U} \subseteq \mathcal{V}$ and $[\![\psi]\!]^{H,\mathcal{V} \cup \{x\}} = \mathbb{T}$, for some H s.t.

    a. $H[x]G$

    b. $[\![\mathbf{3}.\mathbf{atoms}(x) \wedge \phi]\!]^{H,\mathcal{U} \cup \{x\}} = \mathbb{T}$

    c. $\begin{cases} \text{if } \mathcal{U} = \emptyset : h(x) = h'(x), \text{ for all } h, h' \in H \\ \text{if } \mathcal{U} \neq \emptyset : h(x) = h'(x), \text{ for all } h, h' \in H \text{ that are } \mathcal{U}\text{-identical} \end{cases}$

Thus, if a cardinal indefinite is interpreted collectively, the cardinality requirement specifies the number of atoms that the chosen witness consists of. The number of atoms is the only difference between collectively-interpreted cardinal indefinites and ordinary singular indefinites, the semantic clause for which is repeated in (110) below for convenience. This clause is identical to the one provided in (36) above except for the addition of the $\mathbf{atom}(x)$ conjunct in the restrictor. Hence, all the scopal properties of ordinary indefinites are predicted to be shared by collective cardinal indefinites.

(110) $[\![\exists^{\mathcal{U}} x[\mathbf{atom}(x) \wedge \phi]\,(\psi)]\!]^{G,\mathcal{V}} = \mathbb{T}$ iff $\mathcal{U} \subseteq \mathcal{V}$ and $[\![\psi]\!]^{H,\mathcal{V} \cup \{x\}} = \mathbb{T}$, for some H s.t.

    a. $H[x]G$

    b. $[\![\mathbf{atom}(x) \wedge \phi]\!]^{H,\mathcal{U} \cup \{x\}} = \mathbb{T}$

    c. $\begin{cases} \text{if } \mathcal{U} = \emptyset : h(x) = h'(x), \text{ for all } h, h' \in H \\ \text{if } \mathcal{U} \neq \emptyset : h(x) = h'(x), \text{ for all } h, h' \in H \text{ that are } \mathcal{U}\text{-identical} \end{cases}$

We take the distributive interpretations of cardinal indefinites to be due to the presence of a covert distributive operator δ_x^y adjoined at the VP-level. Such operators are needed to derive the intuitively correct truth conditions of VP-conjunction sentences like (111) below (see Winter 2000 and references therein), where the first VP-conjunct contains a collective predicate and the second one contains an indefinite that covaries with the atomic individuals that are part of the subject.

(111)   Three^x girls met and δ_x^y(had an^z espresso).

Such distributive operators are simply universal quantifiers over atoms, as shown in (112) below. The universal quantification over atoms contributed by distributive operators is interpreted in the usual C-FOL way. For convenience, the semantic rule is repeated in (114) and (115) below.

(112) $\delta_x^y(\phi) := \forall y[\mathbf{atom}(y) \wedge y \leq x]\,(\phi_x^y)$

(113) ϕ_x^y is the formula obtained by substituting every occurrence of the variable x in ϕ with the variable y, where y is a fresh variable.

(114) $[\![\forall x[\mathbf{atom}(x) \wedge \phi]\,(\psi)]\!]^{G,\mathcal{V}} = \mathbb{T}$ iff $[\![\psi]\!]^{H,\mathcal{V} \cup \{x\}} = \mathbb{T}$, for some H that is a maximal set of assignments relative to x, $\mathbf{atom}(x) \wedge \phi$, G and $\mathcal{V}$.

(115) H is a maximal set of assignments relative to a variable x, a formula $\mathbf{atom}(x) \wedge \phi$, a set of assignments G and a set of variables $\mathcal{V}$ iff

    a. $H[x]G$ and $[\![\mathbf{atom}(x) \wedge \phi]\!]^{H,\mathcal{V} \cup \{x\}} = \mathbb{T}$

b.   there is no $H' \neq H$ s.t. $H \subseteq H'$ and:
     $H'[x]G$ and $[\![\textbf{atom}(x) \wedge \phi]\!]^{\overline{H',\mathcal{V}\cup\{x\}}} = \mathbb{T}$

By definition, covert distributors $\delta_x^y(\phi)$ have the same scopal properties as universals, so we correctly predict that they cannot exhibit exceptional scope. This is shown by the example in (116) below (see Winter 1997: p. 420), which can have a wide-scope reading for the cardinal indefinite relative to the conditional—but even then, the distributor has to scope within the conditional. This reading is paraphrased in (117) below.

(116)   If three workers on our staff have a baby soon, we will have to face some hard organizational problems.

(117)   There are three workers such that, if *each* of them has a baby soon, we will have to face some hard organizational problems.

In C-FOL, we capture the fact that collective cardinal indefinites can take exceptional scope out of conditionals in much the same way in which we capture exceptional scope out of the restrictors of universal quantifiers. For simplicity, we will formalize the simpler Ruys conditional in (118) below (see Ruys 1992) rather than the example in (116) above.

(118)   If three relatives of mine die, I will inherit a house.

We are interested in the wide-scope collective reading provided in (119) below. Importantly, the Ruys conditional in (118) above does not have a wide-scope distributive reading to the effect that there are three relative of mine s.t., for each of them, if s/he dies, I will inherit a house (see Winter 1997, p. 416 and references therein for more discussion)—a fact that we correcty predict.

(119)   There are three relatives of mine s.t., if they (all) die, I will inherit a house.

We analyze indicative conditionals as universal quantifiers over the worlds in the current context set $W^{@}$ (see Stalnaker 1978), i.e., over the set of live candidates for the actual world.

As it is usually done, we expand our models with a set of worlds $\mathfrak{W}$ and we let variables $w, w', w^{@}, \ldots$ over possible worlds take entities in $\mathfrak{W}$ as values. We use variables $W, W', W^{@}, \ldots$ over sets of worlds to restrict modal quantification.

The definition of universal modal quantification is provided in (120) and (121) below. Given that we make use of contextually-provided restrictors, we need the notion of truth-in-context provided in (99) above.

(120) $[\![\forall w \in W[\phi]\,(\psi)]\!]^{G,\mathcal{V}} = \mathbb{T}$ iff $[\![\psi]\!]^{H,\mathcal{V}\cup\{w\}} = \mathbb{T}$, for some H that is a maximal set of assignments relative to $w \in W, \phi, G$ and $\mathcal{V}$.

(121) H is a maximal set of assignments relative to a variable $w \in W$,
    a formula ϕ, a set of assignments G and a set of variables $\mathcal{V}$ iff
    a. $H[w]G$ and for all $h \in H$, $h(w) \in h(W)$ and $[\![\phi]\!]^{H,\mathcal{V}\cup\{w\}} = \mathbb{T}$
    b.   there is no $H' \neq H$ s.t. $H \subseteq H'$ and:
        $H'[w]G$ and for all $h \in H', h(w) \in h(W)$ and $[\![\phi]\!]^{H',\mathcal{V}\cup\{w\}} = \mathbb{T}$

The wide-scope reading of the Ruys conditional is represented as shown in (122) below. The variable $w^@$ is the designated variable for the actual world, the variable $x^@$ is the designated variable for the speaker and $W^@$ is the designated variable for the context set.

(122) $\forall w \in W^@ [\exists^{\{w^@,x^@\}} x [\mathbf{3}.\mathbf{atoms}(x) \wedge \text{RELATIVES-OF}(w^@, x, x^@)]$
             $(\text{DIE}(w,x))]$
       $(\exists^{\{w\}} y [\mathbf{atom}(y) \wedge \text{HOUSE}(w, y)] \; (\text{INHERIT}(w, x^@, y)))$

The representation of the Ruys conditional in (122) is interpreted as shown in (123) below.

(123)

...	$x^@$	$w^@$	$W^@$	...
...	me	@	$conx\text{-}set$	...

$\underrightarrow{\forall w \in W^@ [\exists^{\{w^@,x^@\}} x [\mathbf{3}.\mathbf{atoms}(x) \wedge \text{RELATIVES-OF}(w^@,x,x^@)] \; (\text{DIE}(w,x))]}$

...	$x^@$	$w^@$	$W^@$	w	x	...
...				u_1		...
...	me	@	$conx\text{-}set$	u_2	$3\text{-}relatives$	...
...				u_3		...
...				...		...

$\underrightarrow{\exists^{\{w\}} y [\mathbf{atom}(y) \wedge \text{HOUSE}(w,y)] \; (\text{INHERIT}(w,x^@,y))}$

...	$x^@$	$w^@$	$W^@$	w	x	y	...
...				u_1		$house_1$	...
...	me	@	$conx\text{-}set$	u_2	$3\text{-}relatives$	$house_2$	...
...				u_3		$house_3$	...
...				...		...	...

First, we assume that the context of evaluation has already brought to salience the designated variables for the speaker $x^@$, actual world $w^@$, and context set $W^@$, i.e., $\mathcal{V} = \{x^@, w^@, W^@\}$ and the initial set of assignments G provides suitable values for

these designated variables, i.e., the actual speaker *me*, the actual world @ and the current context set *conx-set*.

Then, we interpret the antecedent of the conditional relative to this context of evaluation. Since the cardinal indefinite *three relatives of mine* takes exceptional scope outside the antecedent of the conditional, as indicated by its superscript $\{w^@, x^@\}$, we store only one individual relative to x consisting of three atoms that are relatives of the speaker $x^@$. Also, we then introduce all the worlds $w \in W^@$, namely $\{u_1, u_2, u_3, \ldots\}$, in which this particular group of relatives dies.

Finally, we interpret the consequent of the conditional: for each world w, there has to be a house y (possibly different from world to world) that the speaker $x^@$ inherits in w.

Importantly, we do not predict that the conditional is vacuously satisfied if the speaker has no relatives. This is because C-FOL does not interpret conditionals (and universal quantifiers in general) in terms of material implication.

The predicate *die* in the nuclear scope of the cardinal indefinite *three relatives of mine* is lexically distributive, so there is no need for the (optional) insertion of a distributive operator of the form δ_x^z to derive the intuitively correct interpretation. However, if the nuclear scope contained an indefinite like in (111) or (116) above, inserting a distributive operator as shown in (124) below would be necessary to derive the correct truth conditions.

(124) $\forall w \in W^@ [\exists^{\{w^@, x^@\}} x [\mathbf{3.atoms}(x) \wedge \text{RELATIVES-OF}(w^@, x, x^@)]$
$(\delta_x^z(\text{DIE}(w, z)))]$
$(\exists^{\{w\}} y [\mathbf{atom}(y) \wedge \text{HOUSE}(w, y)] (\text{INHERIT}(w, x^@, y)))$

5.4 Anaphora to dependencies and apparent cases of exceptional scope

Schlenker (2006, p. 299 et seq.) introduces the example in (125a) below as a counterargument to the Binder Roof Constraint. The broader argument is that the full power of Skolem functions is useful and the limited form of quantification over Skolem functions implicit in IFL and, therefore, C-FOL is insufficient for natural language semantics.

(125)   [Context: Every student in my syntax class has one weak point—John doesn't understand Case Theory, Mary has problems with Binding Theory etc. Before the final, I say:]
a.   If each student makes progress in some/a⟨n⟩ (certain) area, nobody will flunk the exam.
b.   Intended Reading: There is a certain distribution of fields per student s.t. if each student makes progress in the field assigned to him/her (say, the one that he is weakest in), nobody will flunk the exam.
c.   # If each student makes progress in at least one area, nobody will flunk the exam.

As Schlenker (2006) notes, in a situation in which every student made progress in some area he was already good in but I still flunked some of the students, (125a) can still be uttered truthfully, but (125c) cannot.

That is, (125a) has the reading given in (125b), while (125c) doesn't. On the reading we are interested in, the choice of the area is crucially dependent on the student. This reading is preserved if *a certain area* is replaced with *a certain area he is weak in* (as Philippe Schlenker pointed out to us).

We take the strong need for contextual support in this kind of examples to be the crux of the matter. That is, we think that examples like (125) above are not instances of exceptional scope, but simply instantiate anaphora to previously established quantificational dependencies of the kind needed for the analysis of quantificational and modal subordination (see Brasoveanu 2007, 2010 for a recent discussion and an analysis in a closely related dynamic framework).

We only outline here how the C-FOL analysis of anaphora to dependencies would be executed. The main addition is the introduction of domain-restricting variables for universals and existentials, which is a rather uncontroversial enrichment of our semantic representations (see von Fintel 1994 among many others).

The new definitions are given below. They are minimally different from the previous ones: we just require the quantified-over variables to take values from a restricted domain. Domain-restricting variables $r, r', \ldots$ store possibly non-atomic individuals relative to each assignment $g \in G$ and the values of a newly-introduced variable x are required to be parts of these individuals: $x \leq r$. Adding this domain-restricting requirement is the only thing we need to account for examples like (125a) above.

(126) $\llbracket \exists^{\mathcal{U}} x \leq r[\phi] (\psi) \rrbracket^{G,\mathcal{V}} = \mathbb{T}$ iff $\mathcal{U} \subseteq \mathcal{V}$ and $\llbracket \psi \rrbracket^{H,\mathcal{V} \cup \{x\}} = \mathbb{T}$, for some H s.t.
    a. $H[x]G$ and $h(x) \leq h(r)$, for all $h \in H$
    b. $\llbracket \phi \rrbracket^{H,\mathcal{U} \cup \{x\}} = \mathbb{T}$
    c. $\begin{cases} \text{if } \mathcal{U} = \emptyset : h(x) = h'(x), \text{ for all } h, h' \in H \\ \text{if } \mathcal{U} \neq \emptyset : h(x) = h'(x), \text{ for all } h, h' \in H \text{ that are } \mathcal{U}\text{-identical} \end{cases}$

(127) $\llbracket \forall x \leq r[\phi] (\psi) \rrbracket^{G,\mathcal{V}} = \mathbb{T}$ iff $\llbracket \psi \rrbracket^{H,\mathcal{V} \cup \{x\}} = \mathbb{T}$, for some H that is a maximal set of assignments relative to $x \leq r, \phi, G$ and $\mathcal{V}$.

(128) H is a maximal set of assignments relative to a variable $x \leq r$, a formula ϕ, a set of assignments G and a set of variables $\mathcal{V}$ iff
    a. $H[x]G$ and for all $h \in H, h(x) \leq h(r)$ and $\llbracket \phi \rrbracket^{H,\mathcal{V} \cup \{x\}} = \mathbb{T}$
    b.   there is no $H' \neq H$ s.t. $H \subseteq H'$ and:
         $H'[x]G$ and for all $h \in H', h(x) \leq h(r)$ and $\llbracket \phi \rrbracket^{H',\mathcal{V} \cup \{x\}} = \mathbb{T}$

Importantly, domain restriction happens in an assignment-wise way: for every assignment $h \in H$, we require $h(x) \leq h(r)$. Therefore, if we have two domain restrictors r and r' that are correlated in a particular way, e.g., the binary relation $\{\langle g(r), g(r') \rangle : g \in G\}$ encodes a contextually-specified dependency between

students and areas—the restricted variables $x \leq r$ and $y \leq r'$ will preserve this correlation/dependency and further elaborate on it.

The resulting representation of sentence (125a) is provided in (130) below. The interpretation procedure is depicted in (131), where we omit the world variable w for simplicity.

(129) If^w each$^{x \leq r}$ student makes progress in an$^{y \leq r'}$ area, nobody$^{z \leq x}$ will flunk.

(130) $\forall w \in W^@ [\forall x \leq r[\text{STUD}(w,x)] \ (\exists^{\{w,x\}} y \leq r'[\text{AREA}(w,y)] \ (\text{PROG}(w,x,y)))]$
$(\forall z \leq x[\text{PERSON}(w,z)] \ (\neg(\text{FLUNK}(w,z))))$

(131)

...	$W^@$	r	r'	...
...	$\mathfrak{u}_1$	$stud_1$	$area_1$	...
...	$\mathfrak{u}_2$	$stud_2$	$area_2$	...
...	...	...	...	...

$$\xrightarrow{\forall w \in W^@[\forall x \leq r[\text{STUD}(w,x)] \ (\exists^{\{w,x\}} y \leq r'[\text{AREA}(w,y)] \ (\text{PROG}(w,x,y)))]}$$

...	$W^@$	r	r'	$x \leq r$	$y \leq r'$	...
...	$\mathfrak{u}_1$	$stud_1$	$area_1$	$stud_1$	$area_1$	...
...	$\mathfrak{u}_2$	$stud_2$	$area_2$	$stud_2$	$area_2$	...
...	...	...	...	...	...	...

$$\xrightarrow{\forall z \leq x[\text{PERSON}(w,z)] \ (\neg(\text{FLUNK}(w,z)))}$$

...	$W^@$	r	r'	$x \leq r$	$y \leq r'$	$z \leq x$	...
...	$\mathfrak{u}_1$	$stud_1$	$area_1$	$stud_1$	$area_1$	$stud_1$	...
...	$\mathfrak{u}_2$	$stud_2$	$area_2$	$stud_2$	$area_2$	$stud_2$	...
...	...	...	...	...	...	...	...

Thus, our account treats the dependency between students and areas not as a scopal matter, but rather as a matter of contextual domain restriction in a framework that allows anaphora to contextually-established dependencies between variables.

5.5 Disjunction as existential

Following the IFL analysis of disjunction and the arguments in Schlenker (2006, p. 303 et seqq), we take disjunction to exhibit scopal properties that are similar to indefinites.

For example, the sentence in (132) below is identical to our initial sentence in (5) above, except that the indefinite a^z *professor* has been substituted with the

disjunction *Mary or^z Jane*. This sentence also has three readings—widest scope, intermediate scope and narrowest scope, paraphrased below.

(132) Every^x student read every^y paper that Mary or^z Jane recommended.

(133) a. Narrowest Scope (NS): for every student x, for every paper y s.t. Mary or Jane recommended y, x read y.

  b. Intermediate Scope (IS): for every student x, there is a person that is either Mary or Jane s.t., for every paper y that she recommended, x read y.

  c. Widest Scope (WS): one of Mary or Jane is s.t., for every student x, for every paper y that she recommended, x read y.

We capture the parallel behavior of disjunctions and indefinites directly by taking disjunction to introduce an existential quantifier, just like indefinites do. The existential can quantify over individuals or over possible worlds.

Let us consider disjunction over individuals first. The translation schema for such disjunctions is provided in (134) below. It relies on the sentential connective $\lor$, whose semantic clause is given in (135).[25] This semantic clause is just the static counterpart of the dynamic definition of disjunction independently proposed in Brasoveanu (2008, p. 205, fn. 94) to account for cases of donkey anaphora to disjunctive antecedents.

(134) NAME$_1$ **or^x** NAME$_2$ $\phi \rightsquigarrow \exists^{\mathcal{U}} x[x = \text{NAME}_1 \lor x = \text{NAME}_2]\,(\phi)$

(135) C-FOL disjunction:

$[\![\phi \lor \psi]\!]^{G,\mathcal{V}} = \mathbb{T}$ iff at least one of the three cases below obtains

  a. $[\![\phi]\!]^{G,\mathcal{V}} = \mathbb{T}$

  b. $[\![\psi]\!]^{G,\mathcal{V}} = \mathbb{T}$

  c. there exist G' and G'' s.t. $G = G' \cup G''$ and $[\![\phi]\!]^{G',\mathcal{V}} = \mathbb{T}$ and $[\![\psi]\!]^{G'',\mathcal{V}} = \mathbb{T}$

Thus, in the NP disjunction translation schema in (134) above, we take the existential over individuals to be contributed by the NP part, not by the disjunction itself.

Supporting evidence for the hypothesis that the nature of the disjuncts (in this case, the fact that they are both singular, referential NPs) contributes semantic information available at the level of the whole disjunction is provided by the fact that such disjuncts can bind singular pronouns. For example, if we find a purse after a party and Mary and Jane were the only guests with purses, we can say: *Mary or Jane forgot her purse*.

According to the translation schema in (134), the sentence in (136) below is represented as shown in (137).

(136) Mary or^x Jane smiled.

(137) $\exists^{\emptyset} x[x = \text{MARY} \lor x = \text{JANE}]\,(\text{SMILE}(x))$

Disjunction can also introduce an existential quantifier over possible worlds, as shown in (138) below. For example, the disjunctive sentence in (139) is translated as shown in (140).

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[25] We are indebted to Theo M.V. Janssen for pointing out an error in a previous version of this clause.

(138) $\phi \ \mathbf{or}^w \psi \rightsquigarrow \exists^{\mathcal{U}} w[\phi \vee \psi] \ (w = w^@)$

(139)   It rained or it snowed.

(140) $\exists^\emptyset w[\text{RAIN}(w) \vee \text{SNOW}(w)] \ (w = w^@)$

The two sentential disjuncts restrict the existential over possible worlds contributed by *or* and the nuclear scope requires the actual world $w^@$ to satisfy at least one of these disjuncts.

The sentence in (132) above exemplifying exceptional scope for disjunction is represented as shown in (141) below. The three readings of the sentence correspond to the three different superscripts $\emptyset$, $\{x\}$ and $\{x, y\}$ on the existential, just as they did for the parallel example with an indefinite instead of a disjunction.

(141) $\forall x[\text{STUD}(x)]$
          $(\forall y[\text{PAPER}(y) \wedge$
          $\quad \exists^{\emptyset/\{x\}/\{x,y\}} z[z = \text{MARY} \vee z = \text{JANE}] \ (\text{RECOM}(z, y))]$
          $(\text{READ}(x, y)))$

Thus, our account captures the parallel behavior of existentials and disjunctions by treating disjunction as an indefinite over individuals or over worlds. The scopal similarity between the two follows by definition.

As Theo M.V. Janssen points out, disjunctions are analyzed in IFL in parallel to existentials (see Hodges 1997; Hintikka and Sandu 1997; Caicedo et al. 2009—their clause for disjunction is basically the final case in (135c) above). However, in contrast to the analysis sketched above, no actual existential (over individuals or possible worlds) is stipulated as part of the translation of natural language disjunctions into IFL. The exceptional scope of disjunctions is obtained in the same way as it is for existentials—we 'slash' disjunctions with the variables we want them to be semantically independent of.

We leave it as an open problem whether a C-FOL account of disjunction that is parallel to the simpler IFL one is possible and if it is, whether such an account is empirically more adequate than the one we sketched above.

6 Conclusion

This paper has proposed a novel solution to the problem of scope posed by natural language indefinites that captures both the fact that, unlike other quantifiers, indefinites have free upwards scope, and the fact that the scopal freedom of indefinites is nonetheless syntactically constrained.

As in independence-friendly logic, the special scopal properties of indefinites are attributed to the fact that their semantics can be stated in terms of choosing a suitable witness at a certain point during semantic evaluation. This is in contrast to *bona fide* quantifiers, the semantics of which cannot be given in terms of single witnesses because their semantics necessarily involves relations between sets of entities.

The syntactic constraints on the interpretation of indefinites follow from the fact that witness choice arises as a natural consequence of the process of (syntax-based)

compositional interpretation of sentences and it is not encapsulated into the lexical meaning of indefinites, as choice/Skolem-function approaches would have it.

We therefore expect an unmarked, ordinary indefinite to have free upwards scope, exactly as we find in English and other languages that have an unmarked indefinite article.

One way an indefinite can be special is by having constraints on its scopal independence, i.e., by requiring it to covary with some other quantifier. The cross-linguistic typology of indefinites appears to support this picture. In particular, dependent indefinites use additional morphology on top of the ordinary indefinite morphology to mark scopal dependence—e.g., in Hungarian, the indefinite article is reduplicated, while in Romanian the particle *cîte* is added immediately before the indefinite article. We have provided an approach in which this special morphology makes a specific semantic contribution which predicts the scopal restrictions that these special indefinites are subject to.

The question that arises now is how the tools introduced here can be used to capture the rich variety of cross-linguistically attested special indefinites. In particular, an important issue that remains open is the interaction of the topical status of indefinites with exceptional scope discussed recently in Endriss (2009). Another issue that remains open is how the perspective adopted here can be broadened to deal with various other types of specificity and special indefinites discussed in the literature (see, for instance Bende-Farkas and Kamp 2006 and references therein). Finally, as mentioned above, more work needs to be done to account for the interaction of dependent indefinites and negation as well as for the various scope-freezing constructions.

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