# (Non-)specificity across languages: constancy, variation, *v*-variation

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# 1 Introduction

Many seminal controversies at the interface between linguistics and philosophy concerned the nominal domain. Well-known examples are the discussions around definite and indefinite descriptions, which led, and they are still leading, to pivotal turning points in formal semantics (Russell 1905; Fodor and Sag 1982; Heim 1982; Reinhart 1997; Kratzer 1998).

In the last two decades, starting from the seminal work by Haspelmath (1997), a great body of research has been devoted to describe systems of indefinite pronouns and determiners in a variety of languages. This led to the development of formal analyses aimed at capturing fine-grained distinctions which go beyond the simple definite/indefinite divide. (See among many others Farkas 2002; Kratzer and Shimoyama 2002; Chierchia 2013). In a recent introductory article Farkas and Brasoveanu (2020) distinguished between scopal and epistemic specificity in indefinites.¹ They argued that these notions are related to stability vs variation of reference across different assignments of the variable introduced by the indefinite. Their work ends with two challenges. First, new theoretical tools need to be developed or refined to rigorously study these differences in stability and variation. Second, the relevant linguistic phenomena underlying these distinctions need to be carefully studied. Our work addresses the first challenge and partly the second one by focusing on so-called epistemic indefinites.

We develop a two-sorted team semantics using tools from team logic and dependence logics (Hodges 1997; Väänänen 2007a,b; Galliani 2015; Lück 2020; Galliani 2021). We show that this framework captures the typological variety of indefinites within and across languages, explaining also why certain types of indefinites are unattested. We account for some diachronic developments of indefinites in terms of semantic weakening. And we put forward a clear distinction between stability and variation effects in plain indefinites and indefinites which mark these distinctions in their lexical meaning, which we will

<sup>&</sup>lt;sup>1</sup>In their work, they also introduce the notion of partitive specificity, which we do not address here.

refer as *marked indefinites*. In the last part of the paper, we focus on the class of epistemic indefinites (Alonso-Ovalle and Menéndez-Benito 2010, 2013; Aloni and Port 2015) and show how our framework is expressive enough to deal with subtle variations in their meaning.

This paper is structured as follows. In Section 2 we present the relevant linguistic data and the underlying questions to be addressed. We will focus on the exceptional scope of plain indefinites (Section 2.1) and on the properties of marked indefinites (Section 2.2), setting up a typological landscape which will be the basis of our formal analysis in the rest of the paper. In Section 3 we introduce the technical formalism behind team semantics and dependence logic, which we employ for our analysis in Section 4. Section 5 is dedicated to epistemic indefinites. Section 6 concludes.

# 2 A wealth of indefinites

In this section, we will outline the underlying empirical data of our formal investigation. As said, the core phenomena we want to investigate concern indefinite pronouns or determiners (i.e. functional elements of the nominal domain which express indefinite reference) and their differences in terms of specificity and epistemic variation - notions that we will carefully examine in the present section.

# 2.1 Plain indefinites and exceptional scope

The exceptional scope of plain indefinites has been studied from many different perspectives and formal semanticists often developed new theoretical tools to deal with this problem (Fodor and Sag 1982; Reinhart 1997; Winter 1997; Kratzer 1998; Brasoveanu and Farkas 2011; Charlow 2014). While our focus will be on the typological variety of marked indefinites, we want to understand the relationship between the exceptional scope of plain indefinites and indefinites which mark scope relationships in their meaning. To address this question, we start by examining the scopal proprieties of plain indefinites and review some important approaches to this problem.

A salient feature of plain indefinites is their ability of escaping syntactic islands, such as conditionals, embedded clauses, or in general intensionally-sensitive environments. To see this, consider the contrast below:

- (1) a. Two tourists visited the places that every tour guide recommended.
  - b. Two tourists visited the places that a tour guide recommended.

It is well-known (Ross 1967; Farkas 1981) that quantifying expressions occurring in a relative clause cannot escape the determiner phrase which governs that clause. In (1a), 'every tour guide' cannot escape the scope of 'the places'. By contrast, in (1b), an *exceptional scope* interpretation is available: there is a

particular tour guide such that two tourists visited the places that she recommended.<sup>2</sup>

This can lead to different possible readings when an additional operator (e.g. a quantifier) interacts with the indefinite. A well-known distinction is the narrow (NS) versus wide (WS) scope interpretation, exemplified also in (2):

- (2) Every student<sub>x</sub> read a book<sub>y</sub>.
  - a. WS  $[\exists y/\forall x]$ : *a book > every student* There is a specific book *y* such that for every student *x*, *x* read *y*.
  - b. NS  $[\forall x/\exists y]$ : every student > a book For each student x, there is a book y such that x reads y.

In (2), the indefinite *a book* can receive wide scope with respect to the universal quantifier *every student*, as in (2a), or narrow scope, as in (2b). Besides the wide versus narrow contrast, a third intermediate (IS) reading might also be available. The latter arises when an indefinite interacts with at least two other operators:

- (3) Every student<sub>x</sub> read every book<sub>z</sub> that a professor<sub>y</sub> recommended.
  - a. WS  $[\exists y / \forall x / \forall z]$ : a professor > every student > every book There is a professor y such that for every student x, for every book z that y recommended, x read z.
  - b. NS  $[\forall x/\forall z/\exists y]$ : every student > every book > a professor For every student x, for every book z such that there is a professor y that recommended z, x read z.
  - c. IS  $[\forall x/\exists y/\forall z]$ : every student > a professor > every book For every student x, there is a professor y such that x read every book z that y recommended.

(Example adapted from Brasoveanu and Farkas 2011)

In (3), we see a contrast similar to the previous example (2). A professor can receive wide scope w.r.t. to the two universal quantifiers *every student* and *every book* (3a); or narrow scope (3b), meaning that every student read every book that any professor recommended. However, a professor can also be interpreted in an intermediate position between *every student* and *every book*, as in (3c).

In classical first-order logic the wide vs narrow readings can be easily captured by swapping the existential and the universal quantifiers. To see this, let us consider again the simple example in (2):

 $<sup>^2</sup>$ The in-situ reading, according to which, two tourists visited all places x s.t. there is a tour guide which recommended x is also available, even though less salient given the context.

- (4) Every student<sub>x</sub> read a book<sub>y</sub>.
  - a. WS  $[\exists y / \forall x]$ : a book > every student  $\exists y (\text{book}(y) \land \forall x (\text{student}(x) \rightarrow \text{read}(x, y)))$
  - b. NS  $[\forall x/\exists y]$ : every student > a book  $\forall x (\text{STUDENT}(x) \rightarrow \exists y (\text{BOOK}(y) \land \text{READ}(x, y)))$

In (4), the in-situ position of the existential quantifier is below the universal quantifier and the configuration in (4b) is straightforwardly captured. The difficulty is how to derive the WS configuration in (4a). Relying on a naïve syntactic analysis is problematic, since the indefinite might occur within a syntactic island.

An influential view, championed by Fodor and Sag 1982, takes indefinites in WS configurations to be referring expressions and posits an ambiguity between a quantificational and referential sense of indefinites. In the latter case, indefinites behave like proper names (they are not existential quantifiers) and thus receive the widest scope possible. As observed by Farkas (1981), treating WS indefinites referentially does not predict intermediate scope configurations, as in (3c). Fodor and Sag (1982)'s ambiguity is thus not enough to solve the exceptional scope puzzle. A number of solutions have been proposed which also account for intermediate readings, including choice/Skolem function approaches (Reinhart 1997; Winter 1997; Kratzer 1998)<sup>3</sup>, alternative semantics (Charlow 2020), and Independence-Friendly Logic (Brasoveanu and Farkas 2011).

In the approach of Brasoveanu and Farkas (2011), different scopal readings of an indefinite can be viewed as (in)dependence relationships between the variables introduced by the different quantifiers. In particular, Brasoveanu and Farkas (2011) propose to enrich the definition of existentials with such relationships - where the wide vs narrow scope readings follow directly from the interpretation procedure. Indefinites are interpreted *in-situ*, and thus from a syntactic point of view an indefinite has access to the variables introduced by operators which syntactically scope over it. From a semantic point of view, the value of the indefinite can be independent of (or fixed relative to) the value of the variables in its syntactic scope. For a sentence like (3), their results are summarized in (5), where  $\exists^{U}z$  means that the values of z are (possibly) different for any different value assigned to the variables in *U*. In the narrow scope reading, the value of z can be fixed relative to no variable (i.e. its value can co-vary with both x and y); in the intermediate scope reading it can be fixed relative to *y*; and in the wide scope reading it can be fixed relative to both *x* and у.

 $<sup>^3</sup>$ In this accounts, the binding of the function variable can take place arbitrarily in the derivation - capturing also the wide and intermediate readings. Choice-functional analyses have their merits and led to many important developments (Szabolcsi 1997; Dayal 2002). We refer the reader to Brasoveanu and Farkas (2011, pp. 6-8) for some criticism of the choice-functional approach.

- (5) a. Every<sub>x</sub> student read every<sub>y</sub> paper that  $a_z$  professor recommended.
  - b. NS  $[\exists z/\forall x/\forall y]$ :  $\forall x\forall y\exists^{\{x,y\}}z\phi$
  - c. IS  $[\forall x/\exists z/\forall y]$ :  $\forall x\forall y\exists^{\{x\}}z\phi$
  - d. WS  $[\forall x/\forall y/\exists z]$ :  $\forall x\forall y\exists^{\varnothing}z\phi$

We have reviewed the exceptional scope of plain indefinites. We have also seen how analyses in the tradition of (in)dependence logics deal with the scopal properties of indefinites. In the next section, we will turn our attention to marked indefinites, where specificity and epistemic constraints are part of the lexical meaning of the indefinite. We will not only be concerned with scopal specificity, but we will also describe how indefinites mark different epistemic requirements in their meaning. This will lead to a comprehensive typological landscape, which the formal framework we develop in Section 3 is able to account for.

#### 2.2 Marked indefinites

So far we have only considered plain indefinites and examined their exceptional scope properties. We have seen that in their WS reading, plain indefinites are interpreted specifically (i.e. there is a *specific* individual which satisfies their existential claim). Languages mark this specificity requirement in different ways.

In what follows we consider indefinite pronouns expressions formed with an indefinite marker (e.g. English *some*- or *any*-), often occurring in series (e.g. *some-thing*, *some-where*, ...) (Haspelmath 1997). This excludes from our analysis expressions such as *a certain*. As we will see in Section 4.6, specificity markers like *certain* in combination with indefinite articles behave differently than the specific indefinites we are considering in the present section.

#### 2.2.1 Specific vs Non-specific

Some languages have dedicated morphological markers for specific and non-specific uses. For a sentence like (6), Russian distinguishes between a specific indefinite marker *-to* and a non-specific one *-nibud'*.<sup>4</sup> As we will see, *-to* admits also non-specific uses, but other languages like *Kazakh* or *Georgian* have indefinites which are used to convey exclusively specific uses.

- (6) Russian -to vs -nibud'
  - a. *Ivan xočet spet' kakoj-to romans.*Ivan wants sing which-INDEF. romance

'Ivan wants to sing some [specific] romance.'

<sup>&</sup>lt;sup>4</sup>To reiterate the point, here we distinguish between *specificity markers* like *certain*, which are used in combination with a plain indefinite; and specific indefinites which have a morphological specific marker as part of their make-up.

b. *Ivan xočet spet' kakoj-nibud' romans.*Ivan wants sing which-INDEF. romance

'Ivan wants to sing some [non-specific] romance.' (from Haspelmath 1997, p. 39, quoting Padučeva 1985, p. 211)

Some additional notes on the specific vs non-specific distinction are in order. First, indefinites which are unambiguously non-specific (e.g. Russian *-nibud*) cannot occur freely in episodic sentences, but they need to be licensed by an operator:

(7) \*Ivan včera kupil kakuju-nibud' knigu. Ivan yesterday bought which-indef. book.

'Ivan bought some book [non-specific] yesterday.'

(from Partee 2004)

Second, specific and non-specific indefinites interact with the scopal interpretations examined in the previous sections. Non-specific indefinites tend to receive narrow-scope and are incompatible with wide-scope, but they also admit intermediate readings:

- (8) a. Možet byť, Maša xočet kupiť kakuju-nibuď knigu. may be, Maša want buy which-INDEF. book.
  - b. Narrow Scope: It may be that Maša wants to buy some book [non-specific].
  - c. Intermediate Scope: It may be that there is some book [non-specific] which Maša wants to buy.
  - d. #Wide-scope: There is some book such that it may be that Maša wants to buy it.

On the other hand, specific indefinites seem to always require wide-scope:

(9) (Example from Georgian)

#### 2.2.2 Specific Known vs Specific Unknown

Specificity can come in different sorts. For instance, the Russian indefinite marker *koe*-, as in (10), conveys a stronger specificity requirement, signalling that the speaker has full knowledge about the identity of the referent the indefinite stands for.

(10) Masha xočet pročitať koe-kakuju knigu Masha wants read INDEF-WHICH book.

'Masha wants to read some specific book. I know exactly which one.'

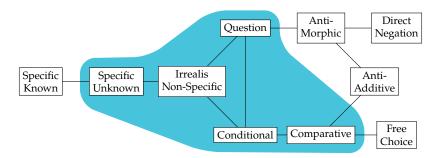


Figure 1: Haspelmath's map for Italian un qualche

The type of function expressed by -koe in 10 has been called by Haspelmath (1997) specific known. Haspelmath (1997) is a fundamental work about the typological landscape of indefinites. Haspelmath (1997) considered a variety of indefinites across languages and reduced the functional space of indefinites to an implicational map with nine main elements. Aguilar-Guevara et al. (2011) showed that an extension of the original map is better suited to capture more fine-grained distinctions cross-linguistically. Figure 1 shows an extended semantic map for the Italian indefinite determiner un qualche, where the colored area indicates the possible functions associated with un qualche:

In this map, functions on the right of the specific unknown function are non-specific whereas the specific functions are specific known and specific unknown. The relevant functions for our work will indeed be these latter two and the non-specific interpretations.

As regards the specific known vs specific unknown contrast, consider the simple case in (11):

- (11) a. Someone failed the course. He is called Marco.
  - b. Someone failed the course. I do not know who this person was.

An indefinite like *someone* in (11) exhibits a kind of *epistemic* variation. According to the reading in (11a), an individual, known to the speaker, failed the course. But the indefinite *someone* is also compatible with a interpretation where the speaker does not know the identity of the student, as in (11b). In Farkas and Brasoveanu (2020), the distinction is between epistemic specificity (known) and epistemic non-specificity (unknown), where epistemic specificity refers to uses where the speaker has one particular individual in mind. This terminology is appropriate to some degree, since what counts as known/unknown might differ and is context-dependent.<sup>5</sup> However, we will still use the categories of specific known and specific unknown, since the divide specific/non-specific and known/unknown is quite established in the typological literature.

 $<sup>^5</sup>$ For instance, I might be able to 'know' the referent of the indefinite by visual identifiability, but not by name. We will return to this issue in Section 5.

Even in the case of epistemic variation, languages developed lexicalized forms which encode such enriched meanings in particular pronouns or determiners. Example (12) illustrates the case of the Italian *un qualche*:

(12) Luca è andato in un qualche negozio. Luca is gone in a some store.

'Luca went to some shop.'

Indeed, the use of *un qualche* in (12) suggests that the speaker does not know which shop in particular Luca went to. Note also that lack of knowledge may occur at different levels. In (12), the speaker might continue with 'It is Amsterdam, but I do not know its name'. While the speaker knows the shop's location, she does not know its name, still maintaining some epistemic variation with respect to the referent of the indefinite.

Let us review what we discussed so far. Cross-linguistically, we witness indefinites which are not tied to a particular function. The English *someone* in (13) is a case in point:

- (13) a. Specific known: Someone called. I know who.
  - b. Specific unknown: Someone called. I do not know who.
  - c. Non-specific: John wants to hang out with someone.

As we have seen in the examples above, other indefinites have a more restricted functional distribution, and the contrasts in (13) are tied to their lexical meanings. Table 1 displays several examples cross-linguistically:

LANGUAGE INDEFINITE			FUNCTIONS	5	TYPE
LANGUAGE	INDEFINITE	SK	SU	NS	IIFE
Italian	un qualche	X	✓	✓	epistemic
	qualcuno	✓	✓	✓	unmarked
Russian	koe-	<b>√</b>	Х	Х	fully specific
	-to	X	✓	✓	epistemic
	-nibud	X	X	✓	non-specific
Japanese	-ka	✓	✓	✓	unmarked
Turkish	bir	✓	✓	✓	unmarked
	herhangi	X	✓	✓	epistemic
German	etwas	<b>√</b>	✓	✓	unmarked
	irgend	X	✓	✓	epistemic
Georgian	-ghats	✓	✓	Х	specific
	-me	X	X	✓	non-specific
Ossetic	-dær	<b>√</b>	✓	Х	specific
	is-	X	X	✓	non-specific
Kazakh	bir	<b>√</b>	✓	✓	unmarked
	älde	✓	✓	X	specific
Kannada	-00	Х	✓	Х	pure specific unknown
	-aadaruu	X	X	✓	non-specific

Table 1: Specificity and Epistemic variation cross-linguistically (data from Haspelmath 1997)

In the next section, we will introduce the technical tools (team semantics and dependence logics) which will form the basis for our formal implementation of the contrasts examined in this section.

# 3 Team semantics and dependence logics

Traditionally, formulas are interpreted with respect to a single evaluation point. In team semantics formulas are interpreted with respect to sets of evaluation points, rather than single ones. These evaluations points can be valuations (as in propositional team logic, Yang and Väänänen 2017), assignments (as in first-order team semantics, Lück 2020; Galliani 2021) or possible worlds (as in team-based modal logic, Lück 2020; Aloni 2021). This *set* of evaluations is usually called a *team*.<sup>6</sup> Before going in the details, let us just consider a simplified propositional example described in Table 2. The team  $Z = \{i, j\}$  is composed of two valuations (i and j), assigning truth values to propositions. As we will see, Z does not make p true, since p is not satisfied in all assignments of the team, but it makes q true, as q is satisfied in both i and j.

In what follows, we will work with a first-order language:

<sup>&</sup>lt;sup>6</sup>In the original Hodges (1997)'s account, a team is referred as a *trump*.

Z	р	q
i	1	1
j	0	1

Table 2: Simple Team over propositions

**Definition 1 (Language)** Given a first-order signature  $\sigma$  (composed of constants  $c \in C$  and predicates  $P^n \in P^n$  with  $n \in \mathbb{N}$ ) and variables  $v \in V$ , the terms and formulas of our language are defined as follows:<sup>7</sup>

$$t ::= c | v$$
 
$$\phi ::= P(\vec{t}) \mid \phi \lor \psi \mid \phi \land \psi \mid \exists v \phi \mid \forall v \phi$$

As said, a first-order team is just a set of assignments:

**Definition 2 (Team)** Given a first-order model  $M = \langle D, I \rangle$  and a sequence of variables  $\vec{v}$ , a team T over M with domain  $Dom(T) = \vec{v}$  is a set of variable assignments from  $\vec{v}$  to Dom(M) = D.

We now give precise rules for semantic clauses of the formulas of our language (Hodges 1997; Väänänen 2007a; Galliani 2012). We assume that our expression are in negation normal form.

#### **Definition 3 (Semantic Clauses)**

$$M,T \models P(x_{1},...,x_{n}) \iff \forall i \in T : \langle \llbracket x_{1} \rrbracket_{M,i},...,\llbracket x_{n} \rrbracket_{M,i} \rangle \in I(P^{n})$$

$$M,T \models \phi \land \psi \iff M,T \models \phi \text{ and } M,T \models \psi$$

$$M,T \models \phi \lor \psi \iff \text{there is a team } T = T_{1} \cup T_{2} \text{ s.t. } M,T_{1} \models \phi \text{ and } M,T_{2} \models \psi$$

$$M,T \models \forall y \phi \iff M,T[/y] \models \phi, \text{ where } T[/y] = \{i[x/d]|i \in T \text{ and } d \in D\}$$

$$M,T \models \exists_{\text{strict}} y \phi \iff \text{there is a function } h:T \to D \text{ s.t. } M,T[h/y] \models \phi, \text{ where } T[h/y] = \{i[h(i)/y]:i \in T\}$$

$$M,T \models \exists_{\text{lax}} y \phi \iff \text{there is a function } f:T \to \wp(D) \setminus \{\emptyset\} \text{ s.t. } M,T[f/y] \models \phi, \text{ where } T[f/y] = \{i[d/y]:i \in T,d \in f(i)\}$$

A first order literal is true in a team T iff it is true in all assignments in T. A team T satisfies a conjunction  $\phi \wedge \psi$  iff T satisfies  $\phi$  and satisfies  $\psi$ . A team T satisfies a disjunction  $\phi \wedge \psi$  iff T is the union of two subteams, each satisfying

 $<sup>^{7}\</sup>vec{t}$  stands for an arbitrary sequence  $t_1, \ldots, t_n$ .

one of the disjuncts.<sup>8</sup> A team T satisfies a universal quantifier  $\forall y \phi$  iff it is true in the universal y-extension of T (the extended team which considers all possible assignments which differ from the original team only with respect to the values assigned to y). To see this, consider the examples in Table 3. Given the initial team in (a) ranging only over x and with a domain of two individuals, the universal y-extension is depicted in (b).

There are two traditional ways of defining existentials in team semantics and dependence logics: strict and lax. In the strict definition, a team T satisfies an existential quantifier  $\exists_{strict} y \phi$  iff there is a function h from T to D such that  $\phi$  is supported in a team which extends T by assigning h(i) to y for each i in T. For instance, (c) below represents a possible example. In the case of the lax existential, the function f is from T to  $\wp(D) \setminus \emptyset$ . This means that we can assign multiple values to the new variable introduced by the existential within the same initial assignment, as in (d) below. In practice, while the strict definition avoids so-called branching (i.e. the possibility of mapping a previous assignment to more than one individual), the lax one allows for it.

Ruling out branching in the case of existential quantification has been convincingly defended by Aloni (2001) and it is a common approach in accounts which deal with multiple information states, as in dynamic semantics. In the following, we will see that the strict definition is suitable for existential quantification in the nominal domain, whereas so-called existential modals are better captured by the lax notion of existential.

T x	T[/y] $x$ $y$	T[h/y] $x$ $y$	T[f/y] $x$ $y$
$\overline{i_1}$ $d_1$	$i_{11}$ $d_1$ $d_1$	$i_{12}$ $d_1$ $d_2$	$i_{12}$ $d_1$ $d_2$
$i_2$ $d_2$	$i_{12}$ $d_1$ $d_2$	$i_{21}$ $d_2$ $d_1$	$i_{21}$ $d_2$ $d_1$
	$i_{21}$ $d_2$ $d_1$		$i_{22}$ $d_2$ $d_2$
	$i_{22}$ $d_2$ $d_2$		
(a)	(b)	(c)	(d)

Table 3: Initial Team (a), universal *y*-extension (b), strict functional *y*-extension for the strict existential (c), and lax functional *y*-extension for the lax existential (d).

Team semantics frameworks are often equipped with dependency atoms - expressions which impose conditions of dependence on the variable's values given by the different assignments. Dependencies can thus be thought as functions between variables and they come in very different sorts (see Galliani 2021 for an overview). In this work, we will adopt the following two atoms:

#### **Definition 4 (Dependence Atom)**

$$M, T \models dep(\overrightarrow{x}, \overrightarrow{y}) \Leftrightarrow for \ all \ i, j \in T : i(\overrightarrow{x}) = j(\overrightarrow{x}) \Rightarrow i(\overrightarrow{y}) = j(\overrightarrow{y})$$

<sup>&</sup>lt;sup>8</sup>This notion of disjunction, called split or tensor disjunction, is quite standard in team logic (Väänänen 2007b).

#### **Definition 5 (Variation Atom)**

$$M, T \models var(\overrightarrow{x}, \overrightarrow{y}) \Leftrightarrow there \ is \ i, j \in T : i(\overrightarrow{x}) = j(\overrightarrow{x}) \& i(\overrightarrow{y}) \neq j(\overrightarrow{y})$$

The first atom in Definition 4 asserts that if any two assignments agree on the value of  $\vec{x}$ , they also agree on the value of  $\vec{y}$  (i.e. the value of  $\vec{y}$  is *dependent* on the value of  $\vec{x}$ ). The variation atom in Definition 5 corresponds to the negation of the Dependence Atom above. It is valid when there is at least a pair of assignments for which the value of  $\vec{y}$  varies even if  $\vec{x}$  is the same. In Table 4, we have that dep(x,y), since for any assignment i,j and k, the value of x determines the value of x. However, we have that var(x,z) since var(x) = var(x) = var(x). A special case are constancy atoms of the form  $dep(\emptyset,x)$ , which is valid when x receives the same value across all assignments; and full variation atoms of the form  $var(\emptyset,x)$ , which is valid when x receives different values across at least a pair of assignments. In Table 4, we have  $dep(\emptyset,l)$ , since the value of the variable x is constant across all assignments. And  $var(\emptyset,y)$ , since the value of y differs in y and y.

# 4 Analysis

In this section, we outline how the framework presented in Section 3 handles the empirical phaenomena presented in Section 2.

# 4.1 The scope of plain indefinites

Dependence atoms allow us to easily capture the different scope readings associated with plain indefinites examined in Section 2. The general idea is to leave the existential quantifier in situ and model its scopal proprieties with the help of the dependency atoms introduced in the previous section. This approach is conceptually similar to Brasoveanu and Farkas (2011), even though in our framework dependency relations are not part of the meaning of the existential, but they are evaluated as separate clauses. This allows us to work with a uniform lexical entry for existentials and with a better behaved logical system.<sup>9</sup>

We examine the case of the wide, narrow and intermediate reading for the case of (14). As Table 5 shows, the presence of  $dep(\emptyset, y)$  yields a wide scope

<sup>&</sup>lt;sup>9</sup>A potential problem might be issues with compositionality when it comes to dependency atoms. (Janssen 2013).

$\overline{T}$	х	у	Z	l
i	$a_1$	$b_1$	$c_1$	$d_1$
j	$a_1$	$b_1$	$c_2$	$d_1$
k	$a_3$	$b_2$	$c_3$	$d_1$

Table 4: Assignments and atoms

$\boldsymbol{\mathcal{X}}$	z	y		$\chi$	Z	y		$\boldsymbol{x}$	z	y
		$b_1$		$\overline{a_1}$	$c_1$	$b_1$	-	$\overline{a_1}$		$b_1$
		$b_1$	•	$\overline{a_1}$	<i>c</i> <sub>2</sub>	$b_2$	-	$\overline{a_1}$		$b_1$
		$b_1$		$\overline{a_2}$	$c_1$	<i>b</i> <sub>3</sub>		$a_2$		$b_2$
		$b_1$		$\overline{a_2}$	<i>c</i> <sub>2</sub>	$b_4$		$a_2$		$b_2$
1110	1 (-			<b>.</b> 10	1	<i>(</i>			1 /	,
WS:	dep(2	(y)		NS:	: depi	(xz, y)		IS:	dep(x)	, y)

Table 5: Wide, Narrow and Intermediate Scope

interpretation where the value of y is constant; dep(xz, y) yields narrow scope where the value of y depends on both the values of the variables associated with the two universal quantifiers; dep(x, y) yields the intermediate reading where the value of y depends only on the first quantifier  $\forall x$ .

(14) Every student<sub>x</sub> read every book<sub>z</sub> that a professor<sub>y</sub> recommended.

- a. WS  $[\exists y/\forall x/\forall z]$ :  $\forall x\forall z\exists y(\phi \land dep(\emptyset, y))$
- b. NS  $[\forall x/\forall z/\exists y]$ :  $\forall x\forall z\exists y(\phi \land dep(xz, y))$
- c. IS  $[\forall x/\exists y/\forall z]$ :  $\forall x\forall z\exists y(\phi \land dep(x,y))$

As said in Section 2.1, in this approach we leave the indefinite *in-situ*. The indefinite might interact with other operators which are present above the *in-situ* position of the indefinite (i.e. in its syntactic scope). The indefinite has access to the variables introduced by these operators and dependence relationships can be established with the variable of the indefinite. This leads to the following generalization:

#### (15) Syntactic Scope and Dependence

An indefinite  $\exists x$  in syntactic scope of  $O_{\vec{z}}$  allows all  $dep(\vec{y}, x)$ , with  $\vec{y}$  included in  $\vec{z}$ .

Crucially, an analysis along the lines of (14) would also be able to capture the specific versus non-specific contrast in the functional map outlined in Section 2. Specific indefinites could be modelled via a constancy atom of the form  $dep(\emptyset, x)$ . Non-specific indefinites as  $var(\emptyset, x)$ . However, if we want to capture the known vs unknown distinction, we would need also some variation among epistemic possibilities, normally represented using possible worlds. This would imply that non-specificity can no longer be expressed in terms of  $var(\emptyset, x)$  and by the same token in  $dep(\emptyset, x)$  the value of x is always fixed, making it impossible to capture unknown uses.

To capture the known versus unknown distinction, we need to add an additional layer to our assignments, so that we can distinguish between assignments for individual variables and worlds. We propose to model these functional distinction by using a two-sorted team semantics with a world variable v. We

can then capture the functional variety of indefinites by imposing dependence conditions on the variables associated with the existential statement. We turn to this issue in the next subsection.

#### 4.2 Wealth of indefinites

We need a way to distinguish between full specificity (specific known) and what we called specific unknown function: a specific individual, but epistemically not determined. We can cast this difference in intensional terms. In the former case, the specific individual will be constant across all epistemically possible worlds, while in the latter it will vary across epistemically possible worlds. We translate this intuition formally with a two-sorted teams semantics with  $\boldsymbol{v}$  as a special variable for the actual world.

Our model will be a triple  $M = \langle D, W, I \rangle$ , where D is a set of individuals, W a set of worlds and I an interpretation function. We take a sentence to be felicitous/grammatical if there is an initial state which supports it, where an initial state is one where only v, the variable referring to the actual world, is defined.

## **Definition 6 (Initial Team)** A team T is initial iff $Dom(T) = \{v\}$ .

We can now model the functional variety of indefinites by working with dependency and variation atoms relativized to the actual world v. In particular, we introduce the conditions summarized in Table 6. As we will see, the contribution of our functions will be the result of different combinations of such conditions.

		v	x			v	х
constancy	$dep(\emptyset, x)$		$\overline{d_1}$	v-constancy	dep(v,x)	$\overline{w_1}$	$\overline{d_1}$
-			$\overline{d_1}$			$\overline{w_2}$	$\overline{d_2}$
		v	х			v	х
variation	$var(\emptyset, x)$		$\overline{d_1}$	v-variation	var(v, x)	$\overline{w_1}$	$\overline{d_1}$
			$\overline{d_2}$			$\overline{w_1}$	$\overline{d_2}$

Table 6: Constancy and variation conditions

Constancy means that the variable x is mapped to the same individual in every assignment, while *variation* guarantees that there is at least a pair of assignments in which x receives different values. Their v counterparts relativizes these notions to the world variable v: v-constancy means that the value of x is constant in v, whereas v-variation guarantees that there is at least a pair of assignment in which x receives different values in v.

We have now all the ingredients to capture the variety of marked indefinites discussed in Section 2. We summarize our results in Table 7, which we discuss below.

Unmarked indefinites, like English *someone*, don't have particular requirements. As we have seen, this kind of indefinites can in principle express all

TYPE		NCTI	ONS	REQUIREMENT	EVAMDLE	
ITE	SK	SU	NS	REQUIREMENT	EXAMPLE	
(i) unmarked	✓	1	1	none	Italian <i>qualcuno</i>	
(ii) specific	✓	1	X	dep(v,x)	Georgian ghats	
(iii) non-specific	X	X	1	var(v, x)	Russian -nibud	
(iv) epistemic	1	1	X	$var(\emptyset, x)$	German -irgend	
(v) specific known	1	X	X	$dep(\emptyset, x)$	Russian -koe	
(vi) ŠK + NS	✓	X	1	$dep(\emptyset, x) \vee var(v, x)$	unattested	
(vii) specific unknown	X	✓	X	$dep(v,x) \wedge var(\emptyset,x)$	Kannada -oo	

Table 7: Indefinites and dependency requirements

the functions that we considered. Specific indefinites are associated with 'v-constancy': the referent of the indefinite is the same in a given world, but it can possibly vary between worlds. The negation of this requirement, 'v-variation', forms the class of non-specific indefinites. Unknown indefinites require 'variation': the referent of the indefinite must vary, possibly within the same world. Its negation, 'constancy', leads to specific known: a specific individual within the same world.

Let us know turn to the last two types of Table 7, which require a more detailed explanation. The type 'specific known + non-specific' cannot be subsumed under a single atom. It requires that the referent satisfies either 'constancy' or 'v-variation', which are contradictory with each other. Therefore, in our framework this type can only be captured by means of a disjunctions of atoms, which explains the difficulty of finding a lexicalized indefinite encoding almost opposite meanings, with a clear violation of connectedness, normally assumed as a constraint of lexicalizations (Gardenfors 2014; Enguehard and Chemla 2021).

To our knowledge there is no language which encodes this meaning in a particular form. It would be unusual to express, even ambiguously, these two requirements in a single indefinite: in the former the referent of the indefinite is a specific individual across all worlds whereas; in the latter the referent is not always constant within worlds.<sup>11</sup>

The last type, pure specific unknown, requires two atoms: 'v-constancy' for specificity and 'variation' for unknown. Crucially, only one language among the ones examined by Haspelmath (1997) was purely specific unknown. The reason might be due to complexity. Pure specific unknown requires two atoms and a possible lexicalization is therefore less likely to occur.

We note that Haspelmath (1997) discusses the case of so-called DUNNO indefinites, which arise from an expression with a meaning similar to 'I do

<sup>&</sup>lt;sup>10</sup>Note in fact that  $dep(\emptyset, x)$  implies dep(v, x), which is contradicts var(v, x).

<sup>&</sup>lt;sup>11</sup>A possible criticism might be the ad-hoc nature of our atoms. One may say that the conditions we are examining are simple (i.e. expressed by just one atom) *because* we defined such atoms in a particular view. We think that this line of argument is not fair. The definition of dependency and variation atoms (the latter usually called non-dependence atom) are quite common in the literature and they neutrally described standard notions of dependence.

not know (which)'. A clear example is French *je ne sais quel* ('some kind of') from *je ne sais* (*pas*) *quel* (lit. 'I do not know which'). Haspelmath (1997) conjectures that such indefinite might emerge from specific unknown uses and then acquire other functions. However, we note that Haspelmath (1997) himself acknowledges that non-specific uses might be already present in the early form. For instance, we point out that in the case of Italian, an expression like *non so quale* (lit. 'not know which'), even though not grammaticalized, has non-specific readings like in (16). This means that even so-called DUNNO indefinite should be classified as 'epistemic', our type (iv).

(16) *Ogni bambino ha non so quale gioco preferito.* every child has not know which toy favourite.

'Every child has some favourite toy.'

Moreover, even if specific unknown readings are the most salient due to the emphatic nature of these constructions, we note that DUNNO indefinites are only weakly grammaticalized cross-linguistically<sup>12</sup>, confirming our hypothesis that indefinites which trigger the activation of two atoms do not lexicalize easily.

# 4.3 Non-specific indefinites & Licensing

In Section 2, we have pointed out that non-specific indefinites cannot occur freely in episodic sentences, but they need to be licensed by an operator. Example (7), repeated below, illustrates the case of Russian *nibud*.

(7) \**Ivan včera kupil kakuju-nibud' knigu.*Ivan yesterday bought which-INDEF. book.

'Ivan bought some book [non-specific] yesterday.'

In the previous section, we have defined what counts as an initial state and the conditions under which a sentence is grammatical. Crucially, this, together with the requirement for non-specific indefinites, is enough to explain cases like (7).

To see this, suppose that we have an initial state where v is assigned to two worlds (case (a) in Table 8). Remember that non-specific indefinites triggers the v-variation condition:  $\exists x (\phi(x,v) \land var(v,x))$ . In order to satisfy var(v,x), there must be a pair of assignments in which x differs and v is fixed. Remember also that our definition of the strict existential rules out branching. It follows that in a condition like (a), the variation requirement of non-specific indefinites cannot be satisfied. By defining a sentence as felicitous if it can be supported by an initial team, our analysis predicts the infelicity of (7).

Let us examine what happens when an operator (e.g. a universal quantifier) licences the non-specific indefinite:  $\forall y \exists x (\phi(x, v) \land var(v, x))$ . The universal

<sup>&</sup>lt;sup>12</sup>For instance, they do not usually undergo morphological compounding.

quantifier leads to a universal y-extension of the initial team with v (b). In these assignments var(v, x) can be then satisfied (c).

$\overline{v}$	$\overline{v}$ $y$	$\overline{v}$ $y$ $x$
$\overline{w_1}$	$\frac{3}{w_1}$	$\overline{w_1}$ $a_1$ $d_1$
$w_2$	$w_1$ $a_2$	$w_1$ $a_2$ $d_2$
	$w_2  a_1$	$w_2$ $a_1$ $d_2$
	$w_2$ $a_2$	$w_2$ $a_2$ $d_2$
(a)	(b)	(c)

Table 8: Examples

So far, we discussed the case of universal quantifiers and explained how they licence non-specific indefinites. Licensing however is possible also via other operators, like modals. We will dedicate the next section to modality. Our discussion will not only be relevant to licensing of non-specific indefinites, but it will also be important for epistemic indefinites and free choice phenomena, which we discuss in Section 5.

# 4.4 Modality

Many accounts of modality stemming from the work of Kratzer (Kratzer 1981, 2008) assume that modals vary with respect to two main components: the modal force (necessity vs possibility) and the modal base (epistemic vs root). Root modals are further distinguished by their ordering source (deontic, bouletic, stereotypical).

In Kratzer's analysis, modals are quantifiers over possible worlds and their modal force corresponds to existential (possibility) or universal (necessity) quantification. The modal base determines an initial set of worlds upon which modals quantify over. And the ordering source provides an ordering among such worlds (e.g. speaker's wishes in case of a bouletic modal).<sup>13</sup> In what follows, we will mostly be concerned with the first two components, since the ordering source can then be implemented on top of our formal analysis and does not affect the results we are concerned with.

To see the contrast between epistemic and root modals, consider the examples in (17). In the epistemic case, the statement in (17a) can be rephrased as 'in all worlds compatible with the available evidence (i.e. the fact that Anthony is not home), Anthony is at the gym'. While (17b) means that 'in all worlds where the current law is obeyed, Anthony is in prison'. In other words, epistemic modality refers to the knowledge state of the speaker, while root (aka circumustantial) modality refers to the circumstances which cause or allow an event to happen (Hacquard 2011).

<sup>&</sup>lt;sup>13</sup>Note that the the ordering source applies only to root modals.

- (17) a. *Epistemic must:* Anthony is not home. He must be at the gym.
  - b. *Root (deontic) must:* Anthony is under house arrest. He must stay in his house.

We will treat modals as quantifiers ranging over possible worlds, where the new variable introduced by the quantifier is responsible for their modal base:

#### **Definition 7 (Modals)**

$$\Diamond_w \phi \quad \Leftrightarrow \quad \exists_{lax} w \phi \\
\Box_w \phi \quad \Leftrightarrow \quad \forall w \phi$$

Modals introduce a new world variable w, where for possibility modal, we adopt a lax definition of existential (i.e. for some worlds accessible from the actual world, the complement holds); and for necessity modals we require that the complements holds in all worlds accessible from the actual world.

We observe that modelling non-specificity with v-variation allows for a variety of readings.

- (18) a. It is possible to take a book (non specific).
  - b.  $\Diamond_w \exists x (\phi(w, x) \land var(v, x))$ A sentence like (18a is true both in cases of partial variation, as in team (a) below, and total variation, as in team (b) below.

v	w	х	$\overline{v}$	w	х
$v_1$	$w_1$	$\overline{a_1}$	$v_1$	$w_1$	$a_1$
$v_1$	$w_2$	$a_1$	$v_1$	$w_2$	$a_2$
$v_2$	$w_1$	$a_2$	$v_2$	$w_1$	$a_1$
$v_2$	$w_2$	<i>a</i> <sub>2</sub>	$v_2$	$w_2$	$a_2$
	(a)			(b)	

Note also that in the case of multiple modalities, we should evaluate the variation atom with respect to the inner modal (i.e. the world variable preceding the world where  $\phi$  is evaluated). For instance, a sentence like (19a), where *a mask* is used non-specifically, would be satisfied in a team like (c), where the variation occurs between w and x.

- (19) a. It is possible that it is mandatory to wear a mask in the university.
  - b.  $\Diamond_w \Box_w' \exists x (\phi(x, w') \land var(w, x))$

#### 4.5 Exceptional scope and marked indefinites

In the previous section, we have seen that non-specific indefinites require an operator to be licensed. In this section, we explore in more detail the relationship between marked indefinites and their scope with respect to other possible operators.

As a case study, we examine the narrow, intermediate and wide scope readings examined in Section 2.1. We will assume in-situ configuration of the kind  $\forall z \forall y \exists x (\phi(x,v) \land ATOM)$ , where each indefinite form triggers the activation of certain dependence/variation atoms.

All indefinites are associated with a dependence atom of the form  $dep(\vec{y}, x)$  where y is a subset of variable in the syntactic scope of x. Marked indefinites induce restrictions on the variables for  $\vec{y}$  or the activation of variation atoms. We propose the following generalizations:

```
(20) a. Plain: dep(\vec{y}, x)
```

- b. SK:  $dep(\vec{y}, x)$  with  $\vec{y} = \emptyset$
- c. Specific:  $dep(\vec{y}, x)$  with  $\vec{y} \subseteq \{v\}$
- d. Epistemic:  $dep(\vec{y}, x) \wedge var(\vec{z}, x)$  with  $\vec{z} \subseteq Var(W)$
- e. Non-specific:  $dep(\vec{y}, x) \wedge var(\vec{z}, x)$  with  $\vec{z} \subseteq Var(W)$  and  $\vec{z} \neq \emptyset$

Unmarked plain indefinites (e.g. English *someone*) are of course compatible with all possible readings, since they do not require particular constraints. Specific known indefinites (e.g. Russian *-koe*) trigger  $dep(\emptyset, x)$ . For the case under consideration, only the wide scope reading is available. For specific indefinites  $\vec{y}$  can be  $\emptyset$ , giving rise to a known reading, or v, giving rise to an unknown readings. They thus do not allow for intermediate or narrow scope readings:

#### (21) (Examples from Georgian)

Epistemic indefinites trigger the activation of the variation atom  $var(\vec{z},x)$ , where z is a subset of the world variables. Epistemic indefinites admit both su uses, where  $\vec{z} = \emptyset$  and non-specific uses, where  $\vec{z} = v.^{14}$  For the case under discussion, sx uses are compatible with all scope readings, while non-specific readings do not allow wide-scope readings, as var(v,x) is not compatible with dep(v,x). As discussed in Section 2, non-specific indefinites are incompatible with wide-scope.

 $<sup>^{14}\</sup>mathrm{As}$  discussed, in the case of embedded modalities, the world variable for the variation does not have to be v.

	wide scope $dep(v, x)$	intermediate scope $dep(vyz, x)$	narrow scope $dep(vy, x)$
unmarked	✓	✓	✓
specific $dep(v, x)$	✓	×	×
non-specific $var(v, x)$	×	✓	✓
epistemic $var(\emptyset, x)$	✓	✓	✓
specific known $dep(\emptyset, x)$	1	X	×
specific unknown $dep(v, x) \wedge var(\emptyset, x)$	✓	×	×

Table 9: Interaction with scope

# 4.6 Specificity Markers and Functional Dependence

In the discussion in Section 2 concerning specificity, we focused on specific indefinites, and not specificity markers. The latter is a class of items that together with indefinite article they give rise to an expression with a 'specific flavour'. This includes English *a certain*, Dutch *een zeker*, German *ein bestimmt* and *ein gewiss*, Italian *un certo* and Russian *opredelennyj*. This class of markers is by no means homogeneous and different markers are associated with variations in meaning and distribution. Considering only *English*, we have *a given*, *a particular*, *a specific*, . . .

In the case of Italian un certo, the latter admits both what we called known and unknown uses. <sup>15</sup>

(22) a. Ho bisogno di un certo microchip per aggiustare questo I-have need of a certain microchip to fix this television.

'I need a certain microchip to fix this television.

b. *Ad un certo punto la candela si spegnerà*. At a certain point the candle REFL. blow-out.

'At a certain point the candle will blow out.'

German has two specificity markers *gewiss* and *bestimmt* which can combine with the indefinite article *ein*. Ebert et al. (2013) observe that the former is compatible with what they call 'speaker identifiability', while the latter might not be.

<sup>&</sup>lt;sup>15</sup>For the moment we exclude idiomatic expressions like 'a certain Mr. Oliver'.

(23) a. Peter sucht schon seit Stunden nach einer gewissen CD – #
Peter searches already since hours after a gewiss CF keine Ahnung, welche genau er sucht.
no idea which exactly he searches.

'Peter is looking for a specific CD - no idea which one exactly.'

Peter sucht schon seit Stunden nach einer bestimmten CD – Peter searches already since hours after a bestimmt CD - keine Ahnung, welche genau er sucht.
 no idea which exactly he searches.

'Peter is looking for a certain CD - no idea which one exactly.' (Ebert et al. 2013)

In (23a) the use of *gewiss* indicates that the speaker can identify the referent of the indefinite. Hence the continuation 'I have no idea which one exactly he is looking for.' is not possible. This requirement does necessarily hold for *bestimmt*, and (23b) is acceptable. This suggests that *gewiss* should be analyzed as specific known with atom  $dep(\emptyset, x)$ , while *bestimmt* as specific with atom dep(v, x). While this analysis might be correct for *gewiss*, it seems that the behaviour of *bestimmt* and specificity markers in general is different than the specific marked indefinites we discussed in the previous section.

Plain indefinites with specific markers like *a certain* or *ein bestimmt* can also display intermediate readings, as in (24):

(24) Every professor rewarded every student who read a certain book he had recommended. (Abusch 1993)

In example (24), a possible, perhaps most salient, interpretation of *a certain* is an intermediate scope configuration: for every professor, there is a specific book such that that professor rewards every student who reads that book.

We also note that for an example like (24), a narrow-scope interpretation, even though quite marginal, is possible. The availability of these readings suggests again that plain indefinites with specificity markers cannot be treated as cases of specific indefinites, which do not admit intermediate or narrow-scope readings.

The picture is further complicated by the interaction of specificity markers with different attitude verbs (Farkas and Brasoveanu 2020):

- (25) a. John wants to marry a certain woman.
  - b. John believes that a certain man is following him.

The use of *a certain* under bouletic predicates, like *want* in (25a) commits the speaker to the existence of the referent (i.e. wide-scope reading). This is not the case for doxastic predicates, as in (25b). The attitude report in (25b) does not commit speakers to the existence of that man in the actual world.

Additional support from not identifying specificity markers like *a certain* with specific marked indefinites comes from cases like (26) discussed by Hintikka (1986).

(26) a. Every husband had forgotten a certain date - his wife's birthday.

b. 
$$\forall x H(x) \rightarrow \exists f(dep(\emptyset, f) \land F(x, f(x))$$
 (from Hintikka 1986)

Examples like (26) suggest that the correct analysis of *a certain* is a functional one, as argued by Hintikka (1973). In our framework, a unique function is associated with each possibility (dep(v, x)), but we can capture the intermediate and narrow scope of *a certain* by letting the function apply to the relevant variables, as in (26b).

As said, specificity markers are not homogeneous. It might be possible that other specificity markers display different patterns. Such differences might be captured by the *type* of functions associated with the marker (e.g. only speaker relative, restricted to animate referents, . . . ). The landscape of specificity markers is complex, and it is not our aim to fully describe it. Here we simply observe that our framework has the right expressive power to deal with such cases.

In this section, we have reviewed some of the properties of specificity markers. Our main point is that plain indefinites with specificity markers (e.g. English *a certain*) cannot treated as cases of specific indefinites (e.g. Georgian *-ghats*) as analyzed in this work. Specificity markers do not form an homogeneous class and their specificity requirements might differ within and across languages. In this section, we have also tried to explain how our analysis can be properly extended to deal with such cases, even though our focus was to show that they do not behave similarly to specific indefinites.

# 4.7 A diachronic perspective

In this section, we lay out some general considerations regarding the diachronic evolution of indefinites and its relationship with the formal systems discussed here. The diachronic pathways of indefinites are rather complex and a single section will not do justice to it. Indefinites constitute a complex grammatical environment which interacts with polarity, scalarity, questions, . . . Furthermore, their distribution might be restricted due to competition with equivalent expressions or other indefinites. It follows the diachronic evolution of a particular indefinite should be carefully examined case by case.

Despite its complexity, some generalization can still be made. First, we can study what processes trigger the formation of a new indefinite form. Second, we can determine how indefinites acquire or lose their functions. We will be mostly concerned with the second question here, and particularly with functions related to specificity and epistemic variation.

In historical linguistics and semantic change, weakening phenomena are quite common. Words or expression might lose or reduce part of their meaning and become more general. But what counts as weakening or loss of meaning is not always immediate and change can occur along several dimensions.

We might wonder if weakening can also explain the diachronic development (acquisition or loss of functions) of indefinites. Haspelmath (1997)'s himself acknowledged this fact and proposed that indefinites gradually acquire new functions on his map (see Figure 1) from the right (non-specific) region to the left (specific) region due to weakening. Based on the functions relevant to the present work, this amounts to the following:

# (27) Weakening of functions

(a) Non-specific  $\longrightarrow$  (b) Specific unknown  $\longrightarrow$  (c) Specific known

Haspelmath's explained the passage from (a) to (b) by observing that non-specific indefinites are also necessarily 'unknown', which is what remains when an indefinite allows for both non-specific and specific unknown readings. This seems indeed quite intuitive. The passage from (b) to (c) might be less so. One might argue that in this case the feature of 'unknowness' is lost, resulting in the specific known reading. However, while specific known and unknown share a feature together (being specific), specific know and non-specific do not.

In our framework, this alleged weakening would correspond to the following: (a) NS: var(v, x) > (b) SU:  $dep(v, x) \wedge var(\emptyset, x) >$  (c) SK:  $dep(\emptyset, x)$ .

It is interesting to note how our dependency atoms make the notion of weakening more precise. In particular, we can cast weakening in logical terms. Weaking amounts to a semantic entailment from a stronger to a weaker form. In this regard, we indeed observe a weakening from (a) to (b), since var(v, x) implies  $var(\emptyset, x)$ . But not from (b) to (c), where we actually have the opposite:  $dep(\emptyset, x)$  implies dep(v, x).

Looking at historical data concerning the functional development of indefinites seems to support these predictions. Loss of non-specificity together with the possibility of specific unknown uses occurs quite often. For instance, in the case of French *quelque*, which acquired specific unknown uses from an original non-specific meaning (Foulet 1919). Another case is the *irgend*- series in German: *irgend* started as locative particle with non-specific and then acquired specific unknown uses (Jäger 2008; Port and Aloni 2015).<sup>16</sup>

<sup>&</sup>lt;sup>16</sup>Another type of weakening which our framework predicts is from specific known to specific (from  $dep(\emptyset,x)$  to dep(v,x)). We might hypothesize that this is what underlies the formation of indefinite articles from numerals encoding the number 1. There is quite substantive evidence that some indefinite articles (e.g. English one > a(n), Italian un(o) > un(o) - and similarly for other Romance languages from Latin unus) have such diachronic source and that this development is cross-linguistically independent (Kouteva et al. 2019, pp. 299–301). Crucially, Givón (1981), Heine (1997, pp. 71–76) showed that the numeral stage, where the item identifies a single object with some proprieties, precedes specific uses with possible unknown meanings (what Givón 1981 calls 'the presentative stage'.). The numeral one is mostly used to single out a unique object/individual and hence the similarity to the specific known function. However, we note that the formation of indefinite articles from the numeral one is a quite different phenomenon, which does not involve indefinites from the start.

# 4.8 Negation

In the semantic clauses in Definition 3, we have assumed that all the formulas are in negation normal form and this allowed us to not explicitly add a clause for negation. Indeed, from a logic point of view negation is somehow problematic for dependence logics, as we have observed in Section 3. We then need to deal with cases like (28a). The latter admits two readings: there is a particular house which John does not own (28b) or there is no house which John owns (28c).

- (28) a. John does not have a book.
  - b.  $\exists x \text{ book}(x) \land \neg \text{have}(j, x)$
  - c.  $\neg \exists x \text{ book}(x) \land \text{have}(j, x)$

Assuming that all formulas are in negation normal formal would allows us to capture (28b), but it would require some ad-hoc solution for (28c). To capture both readings, we adopt an intensional notion of negation, which has been shown to work well with logics of dependence (Brasoveanu and Farkas 2011).<sup>17</sup> The definition of intensional negation requires adding a semantic clause for implication:

#### **Definition 8 (Intensional Negation)**

$$\neg \phi(v) \Leftrightarrow \forall w(\phi(w) \to v \neq w)$$

#### Definition 9 (Semantic Clauses[Conditional])

$$M, X \models \phi \rightarrow \psi \iff \text{ for some } X' \subseteq X \text{ s.t. } M, X' \models \phi \text{ and } X' \text{ is maximal } (\text{i.e. for all } X'' \subseteq X, \text{ if } M, X'' \models \phi, \text{ then } X'' \subseteq X'), \text{ we have } M, X' \models \psi.$$

The example in (28) will be then captured as follows:

- (29) a. John does not have a book.
  - b.  $\forall w(\exists x \phi(x, v) \land dep(\emptyset, x) \rightarrow v \neq w)$
  - c.  $\forall w(\exists x \phi(x, v) \rightarrow v \neq w)$

To show that our analysis makes the correct prediction, we will work with a toy model consisting of two books, a and b. and the initial world v will encode a particular state of affairs (i.e.  $w_{\varnothing}$  correspond to a world where John does not have any book,  $w_a$  to a world where John has only book a, and so on).

 $<sup>^{17}</sup>$ In (8), we of course need to specify when two worlds v and w are different. The proper implementation might differ depending on our conception of possible worlds. For our purposes, we can take two worlds v and w to be equivalent when for all n-place predicates of our language  $\pi$ , the extension of  $\pi$  at w is equal to the extension of  $\pi$  at v.

In the first reading (29b), the speaker commits herself to the fact that John does not have a particular book. This can happen if John does not have any book at all or if it John has a different book. So the initial state for v can be only  $w_{\emptyset}$  or one of  $w_a$  and  $w_b$ . Since we require  $dep(\emptyset, x)$  the value of x is constant. Now, suppose that we are in an initial team where v can be  $w_{\emptyset}$  or  $w_a$ , as in (a) below. In this context, we can have two maximal states which support the antecedent of  $w_a$ , depicted in blue below. The first one does not satisfy the consequent since  $v \neq w$ . The second one, corresponding to the situation in which the value of v is v is v satisfies (29b), as desired. Note, however, that adding v or v is v would make the sentence false, since both generated maximal teams will have an assignment where v = v.

Regarding the second reading, we want (29b) to be true when v gets assigned only to  $w_{\emptyset}$ . Indeed, we see that in that case, the maximal state will comprise assignments including  $w: w_a, w_b, w_{ab}$  as maximal team and this are all cases where  $v \neq w$ . With a similar reasoning, adding any other world to v would make (29b) false, since we can always find a maximal state where v = w.

w	v	$\boldsymbol{x}$
$v_{\varnothing}$	$w_{\varnothing}$	а
$v_{\varnothing}$	$w_a$	а
$v_{\varnothing}$	$w_b$	а
$v_{\varnothing}$	$w_{ab}$	а
$w_a$	$w_{\varnothing}$	а
$v_a$	$w_a$	а
$v_a$	$w_b$	а
$v_a$	$w_{ab}$	а
	(a)	

Table 10: Examples

# 5 Epistemic Indefinites and Irgendein (under construction)

The class of epistemic indefinites has received great attention in the formal semantics literature (Alonso-Ovalle and Menéndez-Benito 2010; Chierchia 2013; Aloni and Port 2015). As we have seen in Section 2, these kind of indefinites admit both specific unknown readings, where speakers signal that there is a particular individual the indefinite refers to, but they do not know of the identity of that individual; and non-specific uses, where there is some variation among the epistemic alternatives associated with the indefinite. In the sections that follow, we will return to these readings with the help of some examples. The German epistemic indefinite determiner *irgend*- (Kratzer and Shimoyama 2002) differs from other epistemic indefinites as it also allows for free choice readings, which we are going to analyze in the section.

Towards this end, we generalize the variation atom as follows:

#### **Definition 10 (Variation)**

```
M, T \models var_n(\vec{y}, x) \text{ iff } \forall d \in D^* \subseteq D \text{ with } |D^*| \ge n, \text{ for all } i \in T, \text{ there is a } j \in T_{i,\vec{y}} \text{ s.t. } j(x) = d, \text{ where } T_{i,\vec{y}} = \{j \in T | i(\vec{y}) = j(\vec{y})\}
```

We have redefined the variation atom to allow for splitting within worlds, as in Definition  $10^{.18}$  Note also that we generalize the atom also with the cardinality n of the variation. While this is not necessary for non-specific indefinites. In fact, variation allows the combination of further constraints on epistemic indefinites, allowing for a variety of epistemic indefinites crosslinguistically. To make the discussion concrete, in the next section we focus on the German indefinite determiner irgend-.

# 5.1 Irgendein

The German prefix *-irgend* can be used to form indefinites in combination with *wh*-phrases (e.g. *irgendwie*, *irgendwo*, ...), the pronouns *welcher* (as in *irgendwelcher*) and *jemand* (as in *irgendjemand*), and the indefinite article *ein* (as in *irgendein*). The general contribution of *irgend* is to reinforce a sense of uncertainty with respect to the identity of the referent. In what follows, we will focus on the case of *irgendein*, since a recent corpus study by Aguilar-Guevara et al. (2011) showed that these items have similar distributions.

The German indefinite *irgendein* is mostly used to express the specific unknown function in episodic contexts (30a), as well as under epistemic modals (30b):

(30) a. *Irgendein Student hat angerufen*. irgendein student has called.

'Some student called. The speaker does not know who this student was.'

b. *Maria muss irgendeinen Artz geheiraten hebben.*Maria must irgendein doctor married have.

'Maria must have married some doctor or other.'

When licensed, non-specific readings are also available:

- (31) a. Jeder Student hat irgendein Buch gelesen. every student has irgendein book read.
  - b. specific unknown: Every student has read a specific book. The speaker does not know which one.
  - c. Non-specific: For every student x, there is a book y s.t. x read y. The speaker does not know which book y.

<sup>&</sup>lt;sup>18</sup>The previous variation atom  $var(\emptyset, x)$  corresponds to  $var_2(\emptyset, x)$ .

	episodic	epistemic modal	root modal
specific unknown	<b>√</b>	✓	✓
non-specific	X	✓	✓
free choice	<b>√-</b> X	X	✓

Table 11: Overview of irgendein uses

A sentence like (31a) admits a specific unknown reading (i.e. there is a specific book which every student read, and the speaker does not know which one.) A non-specific reading, where there is some co-variation between the students and the books, is also available.

Under negation *irgendein* has mostly non-specific readings and behaves like an NPI. Given our treatment of negation in Section 4.8, this amounts to a non-specific reading licensed by the universal quantification over worlds induced by negation.

(32) Niemand hat irgendein Buch gelesen. no-one has irgendein book read.

'No one has read any book.'

Under root modals, *irgendein* gives also rise to a so-called free choice readings. This is particularly relevant for deontic modals, where *irgendein* is ambiguous between specific unknown reading and a free choice one:

- (33) a. *Mary musste irgendeinen Mann heiraten.*Mary had-to irgend-one man marry.
  - b. Specific unknown: There was some man Mary had to marry, the speaker doesn't know or care who it was.
  - c. Free choice: Mary had to marry a man, any man was a permitted marriage option for her. (from Aloni and Port 2015, ex. 2)

In (33), *irgendein* can be interpreted in two salient ways. The speaker might suggest that there is a specific man which Mary had to marry, even though she does not know or care who this man was. In another reading *irgendein* suggests that Mary was given freedom of choice for her husband - with no ignorance or indifference inferences in this case. Free choice inferences are also available under epistemic modals, even though they are less salient (Kratzer and Shimoyama 2002; Aloni and Port 2015).

Table 11 summarizes the empirical data we have discussed so far. The generalized variation atom in Definition 10 allows us to capture all the associated readings. *Irgend*- indefinites obligatory triggers the activation of  $var_2(\vec{y}, x)$ , where  $\vec{y} \subseteq Var(W)$ . This atom models the variation component typical of *irgend*. To illustrate this, consider again the example in (34a):

- (34) a.  $Jeder_y$  Student hat  $irgendein_x$  Buch gelesen. every student has irgendein book read.
  - b. Specific unknown:  $\forall y \exists x \phi \land dep(v, x) \land var_2(\emptyset, x)$
  - c. Non-specific:  $\forall y \exists x \phi \land dep(v \vec{y}, x) \land var_2(v, x)$

In the specific unknown reading (as in team (a) below), the atom dep(v, x) ensures specificity (one individual for each epistemic possibility) and the variation component.  $var_2(\emptyset, x)$  ensures that the value of x is not constant (i.e. it is not determined for the speaker).

In the non-specific reading (as in team (b) below), the first atom  $dep(v\vec{y}, x)$  where is a *non-empty* sequence of variables, is responsible for different scope readings (e.g. intermediate vs narrow) in case of interactions with other operators.  $var_2(v, x)$  ensures that variation holds for all epistemic possibilities.

Under epistemic and root modals, free choice readings are also possible. We are going to capture free choice the total variation atom  $var_{|D|}(v,x)$ . The latter ensures that all individuals are considered within the epistemic possibilities in v. For instance, if the relevant domain is formed by three individuals, the desired reading is the one depicted in team (c) below. We will first outline why free choice is best captured by total variation and what might trigger this atom.

v	ų	<u>x</u>	$\overline{v}$	V	$\overline{x}$	$\overline{v}$	w	_
$\overline{w_1}$	$\frac{3}{a_1}$	$b_1$	$\overline{w_1}$	$\frac{3}{a_1}$	$b_1$	$\overline{v_1}$	$\overline{w}_1$	_
$v_1$	$a_2$	$b_1$	$w_1$	$a_2$	$b_2$	$v_1$	$w_1$	
2	$a_1$	$b_2$	$w_2$	$a_1$	$b_3$	$v_1$	$w_2$	
2	$a_2$	$b_2$	$w_2$	$a_2$	$b_1$	$v_2$	$w_1$	
						$v_2$	$w_2$	
						$v_2$	$w_2$	
	(a)			(b)			(c)	

Table 12: Examples

In the work of Alonso-Ovalle and Menéndez-Benito (2010), specific unknown readings like (33b) and free choice readings like (33c) can be explained in terms of domain variation. While (33b) allows for partial variation, free choice readings are associated with total variation, as in (c) in Table 12.<sup>19</sup> This seems to be in line with our treatment of non-specificity: in the original Haspelmath map, free choice occupied the most right position (hence the 'most' non-specific). Here we are requiring that the variation of free choice readings is in this sense fully non-specific.

Note that in fact that free choice inferences like (33c) are often taken to be the result of domain widening effects: the domain associated with *irgend*-indefinites is widened to include all possible salient individuals (Kadmon and

<sup>&</sup>lt;sup>19</sup>Note that this analysis might work well for *irgendein*, but not for so-called universal free choice items (e.g. Italian *qualunque*), which have a restrictive distribution and a different diachronic origin.

Landman 1993) and therefore requiring total variation. Crucially, free choice readings are associated with some kind of prominence (prosodic or contextual) which could trigger this widening effect.

What remains unexplained is why free choice readings of *irgend*- indefinites tend to be more salient with root modals, as opposed to epistemic ones. After all, in our definitions, epistemic and root modals differ only with respect to the modal base they associate with. A possible explanation might be the modal variability hypotheses (MVH), which has been proposed by Aloni and Franke (2013) and finds application in many phenomena. According to MVH, root and epistemic modals have different free choice potentials. The former allow for incorporation of free choice in the compositional semantics, while epistemic modals do not. This implies that the total variation atom  $var_{|D|}(v,x)$  is more likely to occur under a root modal rather than an epistemic one. Independently from Aloni and Franke (2013), Keshet (2012) observes that epistemic modals, unlike root ones, display a lack in focus sensitivity. Similarly, the kind of prominence effect induced by domain widening might not be available for epistemic modals.

We conclude this section with the relationship of *irgend*- indefinites with specificity markers. As mentioned in Section 4.6, German has two well-known specificity markers: *bestimmt* and *gewiss* (Ebert et al. 2013). Ebert et al. (2013) observe that *irgendein* can combine only with *bestimmt*, and not with *gewiss*. Example (35) illustrates a case of such combination:

(35) Was liest Du zur Inspiration? Irgendeinen bestimmten Autor oder What read you for inspiration? Irgendein bestimmt author or Dichter?

poet?

'What do you read to inspire you? Any authors or poets in particular?'

Given our analysis of *bestmitt* and *gewiss* put forward in (Section 4.6), the difference is easily explained. In the latter case, *gewiss* is associated with  $dep(\emptyset, x)$ , which contradicts the core meaning of *irgendein*.

### 6 Conclusion

In this paper, we have reviewed the exceptional proprieties of plain indefinites and the typological landscape of marked indefinites. We have developed a framework which comprehensively characterized indefinites with specific known, specific unknown and non-specific uses. We have showed how our formal system deals with licensing of non-specific indefinites, scope interactions, diachronic observations and accounts for epistemic indefinites.

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