Assignment 2

Philosophical Logic 2025/2026

Instructions

- Discussion among students is allowed, but the assignments should be done and written individually.
- Late submissions will be accepted until one day after the deadline, with a 0.5 penalty per day.
- Please be explicit and precise, and structure your answers in a way that makes them easy to follow.
- For induction proofs, one or two cases besides the base step are usually enough. Always include the case of $\rightarrow$ if present in the language. If you feel safer, you can include the full induction.
- Please submit your answers as PDF and use *PL-2025-A2-⟨your-last-name⟩* as the name of your file.
- For any questions or comments, please contact {m.degano, t.j.klochowicz}@uva.nl
- **Deadline: Tuesday 11 November 2024, 9 pm**

Note. The assignment is three pages long, but it takes longer to read than to actually do! Most of it just explains the exercises.

Exercise 1 [25 points]

On one view, vagueness can be attributed to our lack of knowledge or the precision of our language in describing the world. On a different view, however, vagueness exists in the world itself - meaning that some things are inherently indeterminate.

For example, consider a vague object like a cloud. Its boundaries might be unclear, not because we lack sufficient knowledge about them, but because the boundaries themselves are objectively indeterminate.

(i) Do you find the idea that vagueness exists in the world itself plausible? Why or why not?

(ii) One might argue that vagueness can be applied also to moral or metaphysical cases (e.g., the sorites paradox and the moral status of a fetus during pregnancy; the gradual loss of identity in someone experiencing severe cognitive decline, …)? Are these also cases of vagueness-in-the-world? Do you find an appeal to vagueness (of any kind) satisfactory in addressing these kinds of moral or metaphysical cases? Why or why not?

Use no more than 350 words in your answer.

Exercise 2 [45 points]

This exercise explores higher-order vagueness, its potential issues, and different notions of validity in modalized supervaluationism. In particular, you are familiar with the *global* notion of truth. In this exercise, we consider *admissible truth* at a valuation.

Models and semantics. A model is $M = (V, R)$ where each $v \in V$ is a classical valuation (over a propositional base) and $R \subseteq V \times V$. Modal clause for Δ:

$$M, v \models \Delta\phi \quad \text{iff} \quad \forall v' \in V\,(vRv' \Rightarrow M, v' \models \phi).$$

Global (super)truth: $M \models^{1} \phi$ iff $\forall v \in V\ (M, v \models \phi)$.

Admissible truth from a base point. For $v \in V$, write

$$M \models^{\mathrm{ad}}_{v} \phi \quad \text{iff} \quad \forall v' \in V\,(vRv' \Rightarrow M, v' \models \phi).$$

Sorites setting. We consider a 5-stage Sorites sequence from "tall" (p_1) to "not tall" ($\neg p_5$). So we work with models M s.t.:

$$M \models^{1} p_1 \qquad \text{and} \qquad M \models^{1} \neg p_5,$$

with valuations intended to respect the usual monotone pattern along the sequence (see Slide 15 in *2.1 - Supervaluations*).

If someone is definitely tall, then the next element in the series should be not definitely not tall. A natural demand is thus to require **no sharp jumps at successive orders**:

(i) $\Delta^4 p_1 \to \neg\Delta\neg\Delta^3 p_2$

(ii) $\Delta^3 p_2 \to \neg\Delta\neg\Delta^2 p_3$

(iii) $\Delta^2 p_3 \to \neg\Delta\neg\Delta p_4$

(iv) $\Delta p_4 \to \neg\Delta\neg p_5$

[$\Delta^m \varphi := \Delta(\Delta^{m-1}\varphi)$ for $m \geq 2$]

Question 1 (Global truth)

Working with *global truth* ($\models^1$) and a universal Δ (i.e. vRv' for all $v, v' \in V$), show that accepting (i)-(iv) as valid in the Sorites setting forces a contradiction (e.g. derive $M \models^1 (\neg p_1 \wedge p_1)$).

Question 2 (Global vs. Admissible truth)

Now R is *not* assumed universal. Consider the following conditions on R:

(A) R is serial.

(B) R is reflexive.

(C) R is reflexive and transitive.

(D) R is reflexive and symmetric.

(E) R is reflexive, transitive, and symmetric.

For each class (A)-(E), answer both:

2.a (Global). Working with *global* truth, does the contradiction from Question 1 persist? If **yes**, motivate your answer and show why the contradiction still persists. If **no**, exhibit a model $M = (V, R)$ where (i)-(iv) are true in our Sorites setting *without* contradiction. For clarity, you would need to find a Sorites model M s.t.

1. $M \models^1 p_1$
2. $M \models^1 \neg p_5$
3. $M \models^1 \Delta^4 p_1 \to \neg\Delta\neg\Delta^3 p_2$
4. $M \models^1 \Delta^3 p_2 \to \neg\Delta\neg\Delta^2 p_3$
5. $M \models^1 \Delta^2 p_3 \to \neg\Delta\neg\Delta p_4$
6. $M \models^1 \Delta p_4 \to \neg\Delta\neg p_5$

2.b (Admissible at a valuation). Fix a base point $v \in V$ and work with *admissible* truth $\models^{\text{ad}}_{v}$. Do (i)-(iv) being *admissibly true at* v force a contradiction at v? If **yes**, explain why there is no such M, v in the frame class. If **no**, exhibit $M = (V, R)$ and $v \in V$ in the class such that (i)-(iv) are admissibly true at v in the Sorites setting *without* contradiction. Note that in this case, for the Sorites setting, we still have for p_1 and p_5, $M \models^{1} p_1$ and $M \models^{1} \neg p_5$ (not just $\models^{\text{ad}}_{v}$). For clarity, you would need to find a Sorites model M and $v \in V$ s.t.

1. $M \models^{1} p_1$
2. $M \models^{1} \neg p_5$
3. $M \models^{\text{ad}}_{v} \Delta^4 p_1 \to \neg\Delta\neg\Delta^3 p_2$
4. $M \models^{\text{ad}}_{v} \Delta^3 p_2 \to \neg\Delta\neg\Delta^2 p_3$
5. $M \models^{\text{ad}}_{v} \Delta^2 p_3 \to \neg\Delta\neg\Delta p_4$
6. $M \models^{\text{ad}}_{v} \Delta p_4 \to \neg\Delta\neg p_5$

Note: you should discuss 10 cases in total (5 for 2a and 5 for 2b), but some cases might follow immediately from one another.

Exercise 3 [30 points]

The logic of ST over the base language has the same consequence relation of classical logic (CL):

$$\Gamma \models_{ST} \phi \text{ iff } \Gamma \models_{CL} \phi$$

Recall that we consider the logic of ST as a three-valued propositional logic with $\neg, \vee, \wedge, \to$ as connectives with a Strong Kleene semantics, but with the following notion of logical consequence:

ST Consequence. Given a set of formulas Γ and a formula ϕ, we say that Γ entails ϕ and we write $\Gamma \models_{ST} \phi$ iff for any valuation v s.t. $v(\gamma) \in \{1\}$ for all $\gamma \in \Gamma$, then $v(\phi) \in \{1, i\}$.

ST is based on Strong Kleene semantics. While keeping the same notion of logical consequence of ST and a language over $\neg, \vee, \wedge, \to$, we consider the following two variants for the semantics of the connectives and whether the statement above (the fact that ST consequence is classical) holds: (i) Weak Kleene semantics; (ii) Łukasiewicz semantics. For each of these, if **yes**, prove the fact. If **no**, provide a counterexample.