Assignment 3

Philosophical Logic 2025/2026

Instructions

- Discussion among students is allowed, but the assignments should be done and written individually.
- Late submissions will be accepted until one day after the deadline, with a 0.5 penalty.
- Please be explicit and precise, and structure your answers in a way that makes them easy to follow.
- For regular induction proofs, one or two cases besides the base step are usually enough. If you feel safer, you can include the full induction.
- Please submit your answers as PDF and use *PL-2025-A3-⟨your-last-name⟩* as the name of your file.
- For any questions or comments, please contact {m.degano, t.j.klochowicz}@uva.nl
- **Deadline: Tuesday 18 November 2025, 9 pm**

Note. The assignment may look a bit intimidating because of the transfinite induction, but once you understand the concept, the exercises are quite manageable. Good luck!

Exercise 1 [20 points]

Let t_0 be a time. At t_0, John utters:

(J) "What John says at time t_0 is not true."

Later, at a later time $t_1 > t_0$, Bob utters:

(B) "What John says at time t_0 is not true."

Intuitively, (J) is often judged "paradoxical", while (B) is not, although your judgment might differ.

1. Is John's utterance (J) paradoxical? Briefly explain why or why not. Is Bob's utterance (B) paradoxical in the same sense? Why (or why not)? In what sense are (J) and (B) "the same sentence", and in what sense do they differ?
2. Can we assign a truth value to (J) and (B)? Are there differences in interpreting "is not true" as "is false" versus "is not true" as "fails to be true"?

Use no more than 350 words in your answer.

Exercise 2 [40 points]

Fix D and I, and let J be the revaluation operator (relative to D, I). Let $\mathcal{T}, \mathcal{T}'$ be *coherent* valuations with $\mathcal{T} \subseteq \mathcal{T}'$, and $M = \langle D, I, \mathcal{T} \rangle$ and $M' = \langle D, I, \mathcal{T}' \rangle$. For each initial valuation $\mathcal{T}$ and $\mathcal{T}'$, let $\mathcal{T}^*$ and $\mathcal{T}'^*$ be the resulting fixed points in the Kripke transfinite iteration. Show that:

1. $J(\mathcal{T}) \subseteq J(\mathcal{T}')$
2. $\mathcal{T}^* \subseteq \mathcal{T}'^*$

Exercise 3 [40 points]

1. Show that consistency is preserved along the iteration. Let $(\mathcal{T}_\alpha)_\alpha$ be the Kripke sequence generated from an *initial valuation* $\mathcal{T}$ that is coherent and consistent:

$$\mathcal{T}_0 = \mathcal{T}, \qquad \mathcal{T}_{\alpha+1} = J(\mathcal{T}_\alpha), \qquad \mathcal{T}_\lambda = \bigcup_{\beta<\lambda} \mathcal{T}_\beta \ \ (\lambda \text{ limit})$$

   Prove that every $\mathcal{T}_\alpha$ is consistent.

2. Let τ be the truth-teller sentence ($\tau := T(t)$ with $I(t) = \tau$). Consider the bottom sequence $\mathcal{T}_0 = \emptyset$, $\mathcal{T}_{\alpha+1} = J(\mathcal{T}_\alpha)$, and $\mathcal{T}_\lambda = \bigcup_{\beta<\lambda} \mathcal{T}_\beta$. Show that for all ordinals α, and in particular at $\emptyset^*$, neither $\langle \tau, 1\rangle$ nor $\langle \tau, 0\rangle$ belongs to $\mathcal{T}_\alpha$.

3. Find a formula ϕ that is false in some but not all fixed points and true in no fixed point, and prove this. You can use the results in (1) and (2) to justify your claim.