

Truthmakers¹

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Philosophical Logic 2025
17 November 2025

¹Slides marked with * were not covered during the lecture, but they are included for completeness. You can skip them for what concerns the exam material of the course.

Readings

Suggested:

- ▶ Chapter 5 in lecture notes.
- ▶ Rodriguez-Pereyra, G. (2006). Truthmakers. *Philosophy Compass*, 1(2), 186-200.
- ▶ van Fraassen, B. C. (1969). Facts and tautological entailments. *The Journal of Philosophy*, 66(15), 477-487.
- ▶ Fine, K. (2017). A theory of truthmaker content I: Conjunction, disjunction and negation. *Journal of Philosophical Logic*, 46(6), 625-674.
- ▶ Fine, K. (2017). Truthmaker semantics. *A Companion to the Philosophy of Language*, 556-577.
- ▶ Fine, K., & Jago, M. (2019). Logic for exact entailment. *The review of symbolic logic*, 12(3), 536-556.

Plan

1. Truthmakers
2. Truthmaker Semantics
3. State Spaces*
4. Exact Truthmaking

Outline

1. Truthmakers

2. Truthmaker Semantics

3. State Spaces*

4. Exact Truthmaking

What is a truthmaker?

- ▶ Consider some ordinary sentences:
 1. *The grass is green.*
 2. *I am here.*
 3. *Dinosaurs existed.*
 4. *Unicorns do not exist.*
- ▶ All of these are (or at least seem to be) true.
- ▶ But what in reality **makes** them true?
- ▶ Is 'I am here' true because I am here in the world, or does my being here somehow depend on the truth of the sentence?
- ▶ Truthmaker theory takes seriously the idea that **the world makes our true sentences true**: the direction of explanation goes from world to word.

Truthbearers and truthmakers

- ▶ **Truthbearer**: something that can be true or false (sentences, beliefs, judgements, propositions, ...).
- ▶ **Truthmaker**: something in the world *in virtue of which* a truthbearer is true.
- ▶ Example:
 - Truthbearer: 'The cat is on the mat.'
 - Truthmaker: that very cat's being on that very mat.
- ▶ Truthmaker theorists hold (roughly): whenever a truthbearer is true, there is some portion of reality that makes it true.
- ▶ What kinds of things can be truthmakers, and do *all* truths have them?

Why care about truthmakers?

- ▶ Truthmaker theory sharpens the old correspondence idea:
true thoughts match reality.
- ▶ It forces us to connect:
 - claims about **truth** (logic, semantics),
 - with claims about **what there is** (metaphysics, ontology).
- ▶ Often used as an *argument for* or *against* certain entities:
 - If moral truths are real, what in the world makes them true?
 - Do modal truths ('It could have been otherwise') require possible worlds as truthmakers?

Truthmaking as entailment

- ▶ A tempting idea: connect truthmaking with logical entailment.

If x makes p true and $p \models q$, then x also makes q true.

- ▶ Consider $p =$ 'Amsterdam is in the Netherlands', and let x be *the fact that you are at the lecture*.
- ▶ $p \vee \neg p$ is always true.
- ▶ On a naive view, *any* existing x could serve as a truthmaker for $p \vee \neg p$.
- ▶ If we also assume that a disjunction is made true only by one of its disjuncts, we are pushed to say that your merely being here makes it true that p .
- ▶ That seems wrong: your presence is irrelevant to whether Amsterdam is in the Netherlands.
- ▶ This motivates more refined notions such as *relevant* entailment (more on this in Robert's lecture).

[Restall 1996]

Truthmaking as necessitation

- From a more metaphysical point of view, we can characterise truthmaking via **necessitation**:

An entity T is a truthmaker for a proposition p iff
 T *necessitates* p being true.

- So once the truthmaker is in place, the truth could not have failed.
- This builds into truthmaking an explanatory order: truthmakers are prior to (and more fundamental than) the truths they make.
- It also gives us a handle on what kinds of entities can plausibly serve as truthmakers.

Truthmakers as objects

- ▶ A first, naive view is to identify truthmakers with ordinary objects .
- ▶ Example: ‘Socrates exists’ is made true by Socrates himself.
- ▶ But under a strict necessitation condition, a bare particular rarely suffices:
 - The rose could exist without being red.
 - So the rose alone does not necessitate the truth of ‘The rose is red’.
- ▶ Conclusion: if we want necessitating truthmakers, we usually need entities *more fine-grained* than just objects.

Truthmakers as states of affairs

- ▶ Many truthmaker theorists prefer states of affairs as truthmakers.
- ▶ Rough idea: a state of affairs has the form

$\langle \text{object(s)}, \text{property/relation} \rangle$

actually instantiating that property or relation.

- ▶ The truth of 'The rose is red' is made true by the state of affairs:

the rose's being red.

- ▶ The truth of 'John greeted Mary' is made true by the relational state of affairs:

John's greeting Mary.

- ▶ Unlike bare objects, such states of affairs cannot exist without the corresponding sentence being true, so they are good candidates for necessitating truthmakers.

Other candidates

- ▶ Besides states of affairs, philosophers have proposed:
 - **Events**: particularly for truths about what happens or changes.
 - **Tropes**: particularised properties (e.g. this very redness) as truthmakers for predicative truths.
 - ...
- ▶ For today we will mostly talk in the neutral vocabulary of states of affairs/facts.

Which truths have truthmakers?

- ▶ Two broad positions:
- ▶ **Maximalism**: every truth has a truthmaker (Armstrong 2004).
- ▶ **Non-maximalism**: some truths lack truthmakers; there are *truthmaker gaps* (e.g. Cameron 2008).
- ▶ Maximalists get a very tight connection between truth and reality, but they also inherit demanding cases:
 - negative truths,
 - general and universal truths,
 - modal, mathematical, moral truths,
 - liar-style sentences.
- ▶ Non-maximalists avoid some metaphysical costs by denying that all truths need truthmakers.

Challenges to maximalism: hard truths

- ▶ Maximalism has intuitive appeal: truth is always grounded in how things are.
- ▶ But some kinds of truths look especially problematic:
- ▶ **Negative truths**
 - ‘Amsterdam is not in Italy.’
- ▶ **Non-existence truths**
 - ‘There are no unicorns.’
 - ‘Pegasus does not exist.’
- ▶ What in the world could *make* such truths true?

Strategies for negative truths

- ▶ Two influential maximalist strategies:
- ▶ **Negative facts**
 - For each true negative sentence, there is a corresponding negative fact.
 - Example: the fact that there are no unicorns.
 - Objection: this populates our ontology with many strange absence-like entities.
- ▶ **Totality facts**
 - Start with all the positive facts about what exists and how it is.
 - Add a *totality* fact saying: and that is everything.
 - From this, it follows that there are no further things (such as unicorns).
 - Objection: totality facts are metaphysically heavyweight.

A liar-style challenge to maximalism

Let M be the sentence:

‘This sentence has no truthmaker.’

- ▶ Milne (2005) argues:
- ▶ Suppose, for reductio, that M has a truthmaker T .
- ▶ Then M is true. So what M says is the case is the case: M has no truthmaker.
- ▶ On the supposition that M has a truthmaker, we deduce that M has *no* truthmaker.
- ▶ By reductio ad absurdum, M has no truthmaker.
- ▶ But that is exactly what M says. So M is true and yet has no truthmaker.
- ▶ Hence M is (apparently) a truth without a truthmaker: a counterexample to maximalism, analogous to liar-style paradoxes for truth.

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From metaphysics to semantics



Bas van Fraassen



Kit Fine

- ▶ So far, we have treated truthmaker theory as a **metaphysical** project:
 - what in reality makes a given sentence, belief, or proposition true?
 - which entities we must admit to be truthmakers?
- ▶ But there is also a more **formal** way of thinking about truthmakers:
 - build truthmakers *directly into the semantics* of a language,
 - so that the semantic value of a sentence tells us not just *when* it is true, but *what makes it true*.
- ▶ This brings us to what is now known as **truthmaker semantics**, first introduced by Bas van Fraassen and recently further developed and studied by Kit Fine.

Hyperintensionality

- ▶ In an **extensional** semantics, the meaning of a sentence is just its truth value in a model (relative to an assignment).
- ▶ **Intensional** semantics improves on this: meanings are functions from possible worlds to truth values (sets of worlds).
- ▶ But this is still not *fine-grained* enough.
- ▶ Consider:
 - (1) Venus is Venus.
 - (2) Venus is the morning star.
- (1) and (2) are necessarily equivalent (true in exactly the same worlds).
- ▶ Intensional semantics cannot distinguish them, but they clearly differ in *informational content* and in what would count as a truthmaker.
- ▶ We need a **hyperintensional** semantics: one that distinguishes necessarily equivalent sentences.

[Berto and Nolan 2023]

A note on formalisms and notation

- ▶ We will see a variety of ways to formalize truthmaker semantics.
- ▶ The point is not to confuse you. Rather, you should get used to the fact that the literature (not just on truthmaker semantics) exhibits:
 - different notations,
 - slightly different but equivalent formalizations,
 - different names for the same underlying notions.
- ▶ A useful skill:
 - be able to navigate between these presentations,
 - recognize when they amount to the *same* formalism,
 - and see which parts are notational choice vs. substantive.
- ▶ We start with the truthmaker semantics proposed by Bas van Fraassen (1969)²

²The presentation that follows is an adaptation of Bas van Fraassen (1969) as well as some 2020 posts in Bas van Fraassen blog.

Facts and their combination

We work with a countable set of *facts* Fct .

- ▶ Some facts are *atomic*.
- ▶ Other facts are *complex*, formed by *combination*.

There is a binary operation ‘ \cdot ’ on facts:

$$b, c \in \text{Fct} \Rightarrow b \cdot c \in \text{Fct}.$$

It satisfies:

$$b \cdot b = b \quad (\text{idempotent})$$

$$b \cdot c = c \cdot b \quad (\text{commutative})$$

$$b \cdot (c \cdot d) = (b \cdot c) \cdot d \quad (\text{associative})$$

Atomic and complex facts

Facts: Every fact is a (possibly infinite) combination of atomic facts, and two facts are identical iff they are combinations of exactly the same atomic facts.

Atomic fact: A fact b is *atomic* iff for every fact c ,

$$b = b \cdot c \Rightarrow b = c$$

- ▶ Complex facts are like mereological sums or “bundles” of atomic facts.
- ▶ The order of components in a bundle is irrelevant.

Subordination of facts

For facts $b, c \in \text{Fct}$:

$$b \leq c \quad \text{iff} \quad \exists d \in \text{Fct} \text{ such that } b = d \cdot c$$

$b \leq c$ means: b is at least as *informative* as c , because b contains all the atomic facts that c contains (and maybe more).³

► \leq is reflexive:

$$b = b \cdot b \Rightarrow b \leq b$$

► \leq is transitive: if $b \leq c$ and $c \leq e$ then $b \leq e$.

► \leq is antisymmetric (recall identity of facts): if $a \leq b$ and $b \leq a$, then $a = b$.

► So it is a partial order.

³Keep in mind that we will reverse the conceptual order of $b \leq c$ when moving to Fine's formalization.

Propositions as closed sets of facts

For $X \subseteq \text{Fct}$, we define its closure under \leq as:

$$[X] = \{y \in \text{Fct} : \exists x \in X \text{ with } y \leq x\}$$

A set $Q \subseteq \text{Fct}$ is a *proposition* iff it is closed under \leq , i.e.

$$Q = [Q]$$

The relation $e \triangleright Q$ (*e makes Q true*) is defined by:

$$e \triangleright Q \quad \text{iff} \quad e \in Q$$

If $e \triangleright Q$ and $f \leq e$, then $f \triangleright Q$ as well, by closure.

The lattice of propositions*

Let Prop be the set of all propositions.

- ▶ Partial order: $Q \leq R$ iff $Q \subseteq R$
- ▶ Meet: $Q \wedge R = Q \cap R$
- ▶ Join: $Q \vee R = [Q \cup R]$

Any intersection of propositions is a proposition, and any union has a closure. Thus (Prop, \leq) is a complete lattice.

Special structure of this lattice*

In this specific construction, something stronger holds.

For any propositions Q, R :

$$Q \cap R = \{e \cdot f : e \in Q, f \in R\}$$

For any propositions Q, R :

$$[Q \cup R] = Q \cup R$$

So every union of two propositions is already closed.

Thus, for propositions:

$$Q \wedge R = Q \cap R \qquad Q \vee R = Q \cup R$$

Hence the lattice Prop is distributive.

Complementation on atomic facts

On the set of atomic facts there is an operation $b \mapsto \bar{b}$ such that:

- ▶ \bar{b} is atomic,
- ▶ $\bar{\bar{b}} = b$ for every atomic fact b .

Think of b as “the apple is red”, and \bar{b} as “the apple is non-red”.

Complex facts *do not* in general have complements (there are no disjunctive facts in the ontology).

Hence, there is no global Boolean complement on the set of propositions Prop . Negation must be handled indirectly, via the propositions expressed by sentences.

Language and primary bases

Fix a propositional language with connectives \neg, \wedge, \vee .

For each atomic sentence p , pick an atomic fact e :

$$\mathsf{T}(p) = \{e\} \quad \mathsf{F}(p) = \{\bar{e}\}$$

For $X, Y \subseteq \mathsf{Fct}$ we define:

$$X \cdot Y = \{e \cdot f : e \in X, f \in Y\}$$

Recursive definition of T- and F-bases

For all sentences A, B :

$$T(\neg A) = F(A) \quad F(\neg A) = T(A)$$

$$T(A \wedge B) = T(A) \cdot T(B) \quad F(A \wedge B) = F(A) \cup F(B)$$

$$T(A \vee B) = T(A) \cup T(B) \quad F(A \vee B) = F(A) \cdot F(B)$$

- ▶ $T(A)$: the set of primary facts that make A true.
- ▶ $F(A)$: the set of primary facts that make A false.

Examples

Fix atomic sentences p, q, r with basic facts e_p, e_q, e_r and complements $\overline{e_p}, \overline{e_q}, \overline{e_r}$. Then

$$\mathsf{T}(p) = \{e_p\}, \mathsf{F}(p) = \{\overline{e_p}\}, \quad \dots$$

(For sets X, Y of facts, $X \cdot Y = \{e \cdot f : e \in X, f \in Y\}$.)

1. $p \wedge \neg q$ and $\neg p \vee q$

$$\mathsf{T}(p \wedge \neg q) = \{e_p \cdot \overline{e_q}\} \quad \mathsf{F}(p \wedge \neg q) = \{\overline{e_p}, e_q\}$$

$$\mathsf{T}(\neg p \vee q) = \{\overline{e_p}, e_q\} \quad \mathsf{F}(\neg p \vee q) = \{e_p \cdot \overline{e_q}\}$$

2. $A = (p \wedge q) \vee \neg r$

$$\mathsf{T}(A) = \{e_p \cdot e_q, \overline{e_r}\} \quad \mathsf{F}(A) = \{\overline{e_p} \cdot e_r, \overline{e_q} \cdot e_r\}$$

3. $B = (p \wedge q) \vee (p \wedge r), C = p \wedge (q \vee r)$

$$\mathsf{T}(B) = \{e_p \cdot e_q, e_p \cdot e_r\} = \mathsf{T}(C)$$

From bases to propositions of sentences

For each sentence A , we determine two propositions:

$$T^*(A) = [T(A)] \quad F^*(A) = [F(A)]$$

So $T^*(A)$ is the proposition (closed set of facts) that A is true, and $F^*(A)$ the proposition that A is false.

(Later we will identify $T(A)$ with the set of *exact* truthmakers of A , and $T^*(A)$ with the set of *inexact* truthmakers of A)

We have:

$$T^*(A \wedge B) = T^*(A) \cap T^*(B) \quad T^*(A \vee B) = T^*(A) \cup T^*(B)$$

$$F^*(A \wedge B) = F^*(A) \cup F^*(B) \quad F^*(A \vee B) = F^*(A) \cap F^*(B)$$

Examples

Fix atomic sentences p, q, r with basic facts e_p, e_q, e_r and complements $\overline{e_p}, \overline{e_q}, \overline{e_r}$. For facts f, g , recall $f \leq g$ for “ f is subordinate to g ”. Then:

1. Atoms and their negations

$$\mathsf{T}^*(p) = [\{e_p\}] = \{f : f \leq e_p\} \quad \mathsf{T}^*(\neg p) = [\{\overline{e_p}\}] = \{f : f \leq \overline{e_p}\}$$

2. Conjunction and disjunction

$$\mathsf{T}(p \wedge q) = \{e_p \cdot e_q\} \Rightarrow \mathsf{T}^*(p \wedge q) = [\{e_p \cdot e_q\}] = \{f : f \leq e_p \cdot e_q\}$$

$$\mathsf{T}(p \vee q) = \{e_p, e_q\} \Rightarrow \mathsf{T}^*(p \vee q) = [\{e_p, e_q\}] = \{f : f \leq e_p \text{ or } f \leq e_q\}$$

3. A more complex formula

$$A = (p \wedge q) \vee \neg r$$

From $\mathsf{T}(A) = \{e_p \cdot e_q, \overline{e_r}\}$ we get

$$\mathsf{T}^*(A) = [\{e_p \cdot e_q, \overline{e_r}\}] = \{f : f \leq e_p \cdot e_q \text{ or } f \leq \overline{e_r}\}$$

Facts about double negation and order*

For every sentence A :

$$T^*(\neg\neg A) = T^*(A) \quad F^*(\neg\neg A) = F^*(A).$$

Indeed, $T(\neg\neg A) = F(\neg A) = T(A)$ and $F(\neg\neg A) = T(\neg A) = F(A)$, so their closures coincide:

$$T^*(\neg\neg A) = [T(\neg\neg A)] = [T(A)] \quad F^*(\neg\neg A) = [F(\neg\neg A)] = [F(A)]$$

If $T^*(A) \subseteq T^*(B)$, then

$$T^*(\neg B) \subseteq T^*(\neg A)$$

If every truth-maker for A is also a truth-maker for B , then every truth-maker for $\neg B$ is also a truth-maker for $\neg A$.

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State Spaces*

van Fraassen formulation identifies a proposition with a closed set of facts. Fine (2017) instead takes a proposition to be any set of facts and further discusses different notions of propositions.

There is a non-empty set S of *states* (facts, events, conditions, ...), partially ordered by parthood \leq and closed under *fusion*:

$$s_1, s_2, \dots \in S \Rightarrow s_1 \sqcup s_2 \sqcup \dots \in S$$

We work with a structured state space

$$\langle S, \leq, \sqcup \rangle$$

Distinguished extremal states:

- ▶ *null* state 0: fusion of no states; $0 \leq s$ for all $s \in S$,
- ▶ *full* state 1: fusion of all states; $s \leq 1$ for all $s \in S$.

Propositions and constraints*

A (unilateral) **proposition** is a set of verifying states:

$$P \subseteq S$$

Two propositions are identical iff they have the same verifiers.

Fine imposes structural conditions on such P :

- ▶ *Verifiability*: every proposition has at least one verifier:

$$P \neq \emptyset$$

- ▶ *Closure*: if $s_1, s_2, \dots \in P$, then their fusion is in P :

$$s_1 \sqcup s_2 \sqcup \dots \in P$$

- ▶ *Convexity*: if $s, u \in P$ and $s \leq t \leq u$, then $t \in P$

Different choices of constraints give different **domains** of propositions:

- ▶ **Full** domain: no structural constraints.
- ▶ **Semi-regular** domain: convexity only.
- ▶ **Regular** domain: closure + convexity (often also verifiability).

Regular propositions and subject-matter*

In a **regular** domain, each non-empty proposition $P \subseteq S$ is:

- ▶ *closed*: closed under (finite / arbitrary) fusions.
- ▶ *convex*: if $s, u \in P$ and $s \leq t \leq u$, then $t \in P$.

Such a P has a **maximal verifier**

$$p = \bigsqcup P$$

the fusion of all its verifiers. This p is the *subject-matter* of P : the largest state within which P is evaluated.

Let the *base verifiers* of P be the \leq -minimal elements of P :

$$L(P) = \{p_i \in P : \neg \exists t \in P (t < p_i)\}$$

In a regular domain we can recover P from $L(P)$ and p :

$$P = \{t \in S : \exists p_i \in L(P) (p_i \leq t \leq p)\}$$

This gives a restricted **monotonicity inside the subject-matter**:

$$s \in P \ \& \ s \leq t \leq p \Rightarrow t \in P$$

Verifiers must be “stable”: not below any p_i , and not above p .

Boolean operations on unilateral propositions*

We work first with *unilateral* propositions, identified with sets of verifiers $P, Q \subseteq S$.

Conjunction: exact fusion of verifiers:

$$P \wedge Q = \{p \sqcup q : p \in P, q \in Q\}$$

A state verifies $P \wedge Q$ iff it is (exactly) a fusion of a verifier of P and a verifier of Q .

Disjunction: pool verifiers and then close as required by the chosen domain:

$P \vee Q = P \cup Q$	(full domain)
$P \vee Q = (P \cup Q)^*$	(semi-regular: convex closure)
$P \vee Q = (P \cup Q)^{**}$	(regular: convex + fusion closure)

So \wedge corresponds to *mereological fusion* of truthmakers. \vee corresponds to *choice* between them, plus structural closure.

Bilateral propositions and negation*

- ▶ A **bilateral proposition** is an ordered pair

$$P = (P^+, P^-)$$

where $P^+ \subseteq S$ are its verifiers and $P^- \subseteq S$ its falsifiers.

- ▶ We extend the structural constraints componentwise (P is regular, semi-regular, etc. iff both P^+ and P^- are.)
- ▶ **Negation** just swaps positive and negative content:

$$\neg(P^+, P^-) = (P^-, P^+)$$

- ▶ For conjunction and disjunction we combine the unilateral operations on both sides:

$$(P^+, P^-) \wedge (Q^+, Q^-) = (P^+ \wedge Q^+, P^- \vee Q^-)$$

$$(P^+, P^-) \vee (Q^+, Q^-) = (P^+ \vee Q^+, P^- \wedge Q^-)$$

- ▶ This yields a compositional truthmaker semantics: the semantic value of a sentence A is a (bi)proposition. The value of complex sentences is obtained by these operations.

States, possibility, and worlds*

- We distinguish a set $S^\diamond \subseteq S$ of *possible* (consistent) states, assumed **downward closed**:

$$s \in S^\diamond \ \& \ t \leq s \Rightarrow t \in S^\diamond$$

- **Compatibility**: two states s, t are (jointly) *compatible* iff they can co-exist in some possible state

$$s \text{ and } t \text{ compatible} \iff \exists u \in S^\diamond (s \leq u \ \& \ t \leq u)$$

Thus: s and t can both obtain together without inconsistency.

- A *world* is a **maximal possible state**: a state $w \in S^\diamond$ such that any possible state compatible with w is already part of w :

$$w \text{ is a world} \iff w \in S^\diamond \text{ and } \forall s \in S^\diamond (s \text{ compatible with } w \Rightarrow s \leq w)$$

Bilateral propositions and truth / falsity at a world*

Let w be a world (maximal possible state).

Truth at w :

$$P \text{ is true at } w \iff \exists s \in P^+ \text{ with } s \leq w$$

Falsity at w :

$$P \text{ is false at } w \iff \exists s \in P^- \text{ with } s \leq w$$

Exclusivity, exhaustivity, and bivalence*

- **Exclusivity:** no verifier of P is compatible with any falsifier of P .

$$\forall s \in P^+ \forall t \in P^- \neg(s \text{ compatible with } t)$$

So at no world w is P both true and false (no gluts).

- **Exhaustivity:** every possible state is compatible with some verifier or some falsifier of P .

$$\forall s \in S^\diamond \exists t \in P^+ \cup P^- (s \text{ compatible with } t)$$

So at every world w , P is at least true or false (no gaps).

- Hence, if P satisfies both conditions, then for every world w exactly one of P is true at w and P is false at w holds: P is bivalent at worlds.

Extremal propositions*

From the extremal states 0 and 1 we get four **extremal unilateral propositions**:

$$T_0 = \{0\} \quad F_0 = \emptyset \quad T_1 = S \quad F_1 = \{1\}$$

- ▶ T_0 : trivially true, *nothing* beyond 0 is required.
- ▶ T_1 : trivial truth verified by *any* state.
- ▶ F_0 : trivial falsehood, no state can verify it.
- ▶ F_1 : trivial falsehood verified only by the “impossible” full state 1.

Two kinds of consequence: entailment and containment*

For verifiable *unilateral* propositions $P, Q \subseteq S$ Fine defines two distinct consequence relations:

- P **entails** Q (notation: $P \leq_d Q$) iff

$$P \subseteq Q$$

Every verifier of P is also a verifier of Q .

Disjunction $P \vee Q$ is the *least upper bound* of P and Q w.r.t. \leq_d : it is the strongest proposition entailed by both.

- Q **contains** P (notation: $P \leq_c Q$) iff:

$$(i) \quad \forall q \in Q \exists p \in P (p \leq q)$$

$$(ii) \quad \forall p \in P \exists q \in Q (p \leq q)$$

Conjunction $P \wedge Q$ is the *greatest lower bound* of P and Q w.r.t. \leq_c (in a regular domain).

Consequences for bilateral propositions*

For bilateral $P = (P^+, P^-)$ and $Q = (Q^+, Q^-)$, we define two consequence relations:

- [Containment] P is **contained in** Q iff

$$P^+ \leq_c Q^+ \quad \text{and} \quad P^- \leq_d Q^-$$

- [Entailment] P **entails** Q iff

$$P^+ \leq_d Q^+ \quad \text{and} \quad P^- \leq_c Q^-$$

These two consequence relations interact nicely with negation:

$$P \leq_d Q \Rightarrow \neg Q \leq_c \neg P$$

$$P \leq_c Q \Rightarrow \neg Q \leq_d \neg P$$

Fine: three notions of verification*

Let \mathbf{A} be the proposition corresponding to A . Depending on which closure conditions we impose, \mathbf{A} may be any of

$$|A|, \quad |A|^*, \quad |A|_*, \quad |A|^{**}$$

(no closure; fusion-closed; convex; regular = fusion-closed + convex).

Relative to a model \mathcal{M} , Fine distinguishes three verification relations on states s :

$$\text{Exact:} \quad s \models_{\text{ex}} A \iff s \in \mathbf{A}$$

$$\text{Inexact:} \quad s \models_{\text{in}} A \iff \exists s' \leq s (s' \models_{\text{ex}} A)$$

$$\text{Loose (classical):} \quad s \models A \iff \forall u (u \text{ comp } s \Rightarrow \exists t \in \mathbf{A} (u \text{ comp } t))$$

where $u \text{ comp } s$ means: u is *compatible* with s in the underlying modal state space.

Fine: notions of consequence*

From these verification relations we obtain corresponding consequence relations (between formulas A and C):

Exact $A \models_e C \iff \forall s(s \models_{\text{ex}} A \Rightarrow s \models_{\text{ex}} C)$

Inexact $A \models_i C \iff \forall s(s \models_{\text{in}} A \Rightarrow s \models_{\text{in}} C)$

Analytic (containment) $A > C \iff [A] \geq_c [C]$

Loose / classical $A \models_l C \iff \forall s(s \models A \Rightarrow s \models C)$

Each induces a different logic:

- ▶ \models_l : classical consequence.
- ▶ $>$: Angell's analytic entailment.
- ▶ \models_i : first-degree entailment.
- ▶ \models_e : exact truthmaker logic (Correia, Fine & Jago).

Outline

1. Truthmakers
2. Truthmaker Semantics
3. State Spaces*
4. Exact Truthmaking

Exact truthmaking

- ▶ On the *necessitation* picture, any state that guarantees A counts as a truthmaker for A .
- ▶ Often this is too coarse: we want to exclude states with *irrelevant* parts.
- ▶ **Exact truthmaker:** a state s which
 - guarantees that A is true, and
 - is wholly relevant to A 's truth: no proper part of s is “irrelevant” w.r.t. A .
- ▶ If s is an exact truthmaker for A , then every part of s either belongs to a truthmaker for A or is needed to fuse such parts together.
- ▶ Fine & Jago (2019) study the logic of *exact truthmaker semantics*.

Frames and exact truthmaking models

- A **frame** is a structure

$$\langle S, \leq \rangle$$

where:

- S is a non-empty set of *states*
- \leq is a partial order (parthood / inclusion)
- every pair $s, t \in S$ has a least upper bound (fusion) $s \sqcup t$

- An **exact truthmaking model** is

$$\mathcal{M} = \langle S, \leq, I \rangle$$

where $I = (I^+, I^-)$ is a pair of functions $S \times P \rightarrow \{0, 1\}$ satisfying \sqcup -closure:

If $I^+(s, p) = 1$ and $I^+(s', p) = 1$, then $I^+(s \sqcup s', p) = 1$

If $I^-(s, p) = 1$ and $I^-(s', p) = 1$, then $I^-(s \sqcup s', p) = 1$

Exact truthmaking and falsemaking: clauses

Fix a model $\mathcal{M} = \langle S, \leq, I \rangle$. We define, by recursion on φ , two relations \models^+ and \models^- for positive and negative exact truthmaking:

$$s \models^+ p \text{ iff } I^+(s, p) = 1$$

$$s \models^- p \text{ iff } I^-(s, p) = 1$$

$$s \models^+ \neg\varphi \text{ iff } s \models^- \varphi$$

$$s \models^- \neg\varphi \text{ iff } s \models^+ \varphi$$

$$s \models^+ \varphi \wedge \psi \text{ iff there are } s', s'' \in S \text{ with } s' \sqcup s'' = s \text{ and } s' \models^+ \varphi \text{ and } s'' \models^+ \psi$$

$$s \models^- \varphi \wedge \psi \text{ iff } s \models^- \varphi \text{ or } s \models^- \psi \text{ or there are } s', s'' \in S \text{ with } s' \sqcup s'' = s \text{ and } s' \models^- \varphi \text{ and } s'' \models^- \psi$$

$$s \models^+ \varphi \vee \psi \text{ iff } s \models^+ \varphi \text{ or } s \models^+ \psi \text{ or there are } s', s'' \in S \text{ with } s' \sqcup s'' = s \text{ and } s' \models^+ \varphi \text{ and } s'' \models^+ \psi$$

$$s \models^- \varphi \vee \psi \text{ iff there are } s', s'' \in S \text{ with } s' \sqcup s'' = s \text{ and } s' \models^- \varphi \text{ and } s'' \models^- \psi$$

Logical Consequence⁴

$\Gamma \models_{\text{TM}} \varphi$ iff all models M and states s , if $s \models^+ \psi$ for all $\psi \in \Gamma$, then $s \models^+ \varphi$.

⁴We discuss here the *distributive* notion of logical consequence. Fine & Jago (2019) also consider a collective one, which takes the conjunction of all the premises.

Exactness and the failure of \wedge -elimination

- Exactness is built into the clauses for \models^+ : from

$$s \models^+ p$$

we do *not* infer that $s \sqcup t \models^+ p$ for arbitrary t . Adding irrelevant parts does not preserve exact truthmaking.

- Example: $\mathcal{M} = \langle S, \leq, I \rangle$:
 - States $S = \{s_1, s_2, s_3\}$ with $s_3 = s_1 \sqcup s_2$.
 - Atomic valuation:

$$I^+(s_1, p) = 1 \quad I^+(s_2, q) = 1 \quad I^+(s_3, p) = 0$$

- By the conjunction clause for \models^+ :

$$s_3 \models^+ (p \wedge q)$$

- But $s_3 \not\models^+ p$
- Hence we **lose** \wedge -elimination for exact entailment:

$$p \wedge q \not\models_{\text{TM}} p$$

Exact disjunction: “exclusive” vs “inclusive”

For conjunction, exact truthmaking works in a purely mereological way:

$$s \models^+ p \wedge q \text{ iff } \exists t, u (s = t \sqcup u, t \models^+ p, u \models^+ q)$$

For disjunction there are two natural options.

► Exclusive disjunction:

$$s \models_{\text{exc}}^+ p \vee q \text{ iff } s \models^+ p \text{ or } s \models^+ q$$

Only states that already make one disjunct true themselves count as exact truthmakers for $p \vee q$.

$$p \wedge q \not\models_{\text{TM}} p \vee q$$

► Inclusive disjunction:

$$s \models_{\text{inc}}^+ p \vee q \text{ iff}$$

$$s \models^+ p \text{ or } s \models^+ q$$

$$\text{or } \exists t, u (s = t \sqcup u, t \models^+ p, u \models^+ q)$$

$$p \wedge q \models_{\text{TM}} p \vee q$$

In-class examples

- ▶ Examples and models on blackboard
- ▶ $(p \vee q) \wedge (p \vee r) \not\models_{\text{TM}} p \vee (q \wedge r)$
- ▶ $(p \wedge q) \vee p \not\models_{\text{TM}} p$

The latter also shows that an exact verifier does not have to be minimal:

s minimally verifies φ if $s \models^+ \varphi$ and for any $s' \leq s$ s.t. $s' \models^+ \varphi$, $s' = s$.

From exact truthmaking to van Fraassen's primary bases

For each formula A define its exact truthmaking and falsemaking sets:

$$|A|^+ := \{s \in S : s \models^+ A\} \quad |A|^- := \{s \in S : s \models^- A\}$$

Using the clauses and fusion-closure of I^\pm , one shows by induction on A that $|A|^+$ and $|A|^-$ are closed under finite fusions $s \sqcup t$.

We take the semantic clauses in their **exclusive** version, and we now recover van Fraassen's primary T - and F -bases from $|A|^+$ and $|A|^-$.

Fact-like assumption: For each propositional atom p there are distinguished *atomic* states $e_p, \overline{e_p} \in S$ such that $|p|^+ = \{e_p\}$ and $|p|^- = \{\overline{e_p}\}$

From exact truthmaking to van Fraassen's primary bases

For every formula A set

$$T(A) := |A|^+ \quad F(A) := |A|^-$$

By induction on A , the $T(A)$, $F(A)$ just defined satisfy van Fraassen's recursive primary bases:

$$T(p) = \{e_p\}$$

$$F(p) = \{\overline{e_p}\}$$

$$T(\neg A) = F(A)$$

$$F(\neg A) = T(A)$$

$$T(A \wedge B) = \{s \sqcup t : s \in T(A), t \in T(B)\} \quad F(A \wedge B) = F(A) \cup F(B)$$

$$T(A \vee B) = T(A) \cup T(B) \quad F(A \vee B) = \{s \sqcup t : s \in F(A), t \in F(B)\}$$

van Fraassen's (inexact) propositions are the closures $T^*(A) = \text{cl}(T(A))$, $F^*(A) = \text{cl}(F(A))$ as before.

Metalogical features

- ▶ Although the exact truthmaker consequence relation is unusual, it is very well behaved:
 - **Compact:** if $\Gamma \models_{\text{TM}} \varphi$, then some finite $\Delta \subseteq \Gamma$ already satisfies $\Delta \models_{\text{TM}} \varphi$.
 - **Decidable:** there is an effective procedure to determine, for finite Γ , whether $\Gamma \models_{\text{TM}} \varphi$.
- ▶ Fine & Jago (2019) also provide a sequent calculus for \models_{TM} , which is sound, complete, and admits cut-elimination.

Application: Imperatives (Fine 2017)*

- ▶ There is an intuitive notion of *imperative consequence*:
 - From *Turn on the light and shut the door* one can infer *Turn on the light*.
- ▶ Naive idea: reduce imperative consequence to indicative consequence:
 - Let X correspond to an indicative A , and Y to B .
 - Say: Y follows from X iff B follows (classically) from A .
- ▶ But this gives Ross's paradox:
 - Indicative: from *You turn on the light* we can infer *You turn on the light or burn the building down*.
 - Imperative: from *Turn on the light* we cannot infer *Turn on the light or burn the building down*.
- ▶ Truthmaker semantics suggests a different route: give a *direct* semantics for imperatives in terms of actions, parallel to the semantics for indicatives in terms of states.

Imperatives: exact compliance*

- ▶ Extend the state space so that **actions** are a special kind of state.
- ▶ For an imperative sentence X :
 - An action α is in *exact compliance* with X (intuitively: exactly doing what X requires, no more and no less).
 - An action α is in *exact contravention* of X (intuitively: exactly violating X).
- ▶ Analogy with indicatives:
 - Indicative: s exactly verifies *You shut the door* iff s is *just* your shutting the door.
 - Imperative: α is in exact compliance with *Shut the door* iff α is *just* your shutting the door (shutting the door *and* turning on the light is not exact compliance).

Imperative content*

- For a conjunctive imperative $X \wedge Y$:

α is in exact compliance with $X \wedge Y$

iff there are actions β, γ such that:

1. $\alpha = \beta \sqcup \gamma$
2. β exactly complies with X
3. γ exactly complies with Y

- **Content of an imperative X :**

$$|X|^{\text{imp}} = \{\alpha : \alpha \text{ is in exact compliance with } X\}$$

Imperative consequence as necessary means*

- ▶ Given contents $|X|^{\text{imp}}$ and $|Y|^{\text{imp}}$, Fine defines **imperative consequence** by a *containment* (or necessary-means) condition.
- ▶ Obeying X always involves, and can be decomposed into, obeying Y plus (possibly) more.
- ▶ Formally, Y follows from X iff the content of Y is a *conjunctive part* of the content of X :
 - (i) For every $\alpha \in |X|^{\text{imp}}$, there is some $\beta \in |Y|^{\text{imp}}$ with $\beta \leq \alpha$.
 - (ii) For every $\beta \in |Y|^{\text{imp}}$, there is some $\alpha \in |X|^{\text{imp}}$ with $\beta \leq \alpha$.
- ▶ Then Y is a *necessary means* to X .
- ▶ This dissolves Ross's paradox:
 - The content of *Turn on the light or burn the building down* is not a conjunctive part of the content of *Turn on the light*. So the imperative inference fails.
 - But the content of *Turn on the light* is a conjunctive part of the content of *Turn on the light and shut the door*. So that inference is validated.