Assignment 5

Philosophical Logic 2025/2026

Instructions

- Discussion among students is allowed, but the assignments should be done and written individually.

- Late submissions will be accepted until one day after the original deadline, with a 0.5 penalty.

- Please be explicit and precise, and structure your answers in a way that makes them easy to follow.

- Please submit your answers as PDF and use *PL-2025-A5-⟨your-last-name⟩* as the name of your file.

- For any questions or comments, please contact {m.degano, t.j.klochowicz}@uva.nl

- **Deadline: Tuesday 2 December 2025, 9 pm**

Exercise 1 [25 points]

Lewis-style *similarity* analyses of counterfactuals have been challenged by a range of examples, including (among others):

- Tichý's hat case, where under fine weather Jones' hat-wearing is chancy, but similarity-based verdicts appear to make a determinate prediction.

- Fine's "Nixon button" case, where pressing the button might lead either to a holocaust or to a local malfunction, and it is disputed which worlds are "closest".

- Oswald/Kennedy cases, where it is argued that worlds in which someone else shoots Kennedy are allegedly more similar to the actual world than worlds in which Kennedy survives.

- Bizet-Verdi or Morgenbesser-style cases, which put pressure on how much of the actual course of events should be held fixed when evaluating counterfactuals.

- Fine's "matches" cases, which suggest that substitution of logically equivalent antecedents may fail for counterfactuals.

- Other examples you have come up with or found in the literature.

You do *not* need to discuss all of these. One or two well-developed cases are enough.

(i) Choose one or two of the above challenges and explain what general feature of similarity-based semantics it targets. Make this into a *general* argument (not just "this particular example gives this prediction") by identifying:

(a) the core assumptions of similarity-based accounts underlying the reason of the trouble (e.g. closest antecedent-worlds, centering, uniqueness, treatment of laws vs. particular facts), and/or

(b) independently plausible constraints on counterfactual reasoning (e.g. about chance, causation, or epistemic openness).

(ii) Now defend the similarity theorist. Sketch what you take to be the most promising reply to the argument you formulated in (i). For example, you might (carefully) adjust the similarity ordering, restrict which facts count as "fixed" in an improved version of Lewis's system of weights, weaken some logical principles, or appeal to context-sensitivity or pragmatic effects. Explain briefly whether your reply preserves the main desiderata of similarity semantics, or whether it introduces new challenges (for other cases, or for the logic of counterfactuals).

Use no more than 400 words.

Exercise 2 [10 points]

Let $p_1, p_2, \ldots$ be pairwise distinct propositional atoms and, for each $k \geq 1$, set

$$\alpha_k \ :=\ \bigvee_{i=1}^{k} p_i.$$

We define, for each $k \geq 1$,

$$\varphi_k \ :=\ (\alpha_{k+1} \rightsquigarrow \neg\alpha_k) \qquad \text{and} \qquad \psi_k \ :=\ \neg(\alpha_{k+1} \rightsquigarrow \alpha_k)$$

and let

$$\Delta \ :=\ \{\varphi_k,\ \psi_k \ :\ k \geq 1\}$$

Working with the *general* similarity semantics for $\rightsquigarrow$ (i.e. without assuming the Limit Assumption), show that Δ is satisfiable in the class of all similarity frames. That is, construct a similarity frame $\langle W, \prec \rangle$, a valuation V, and a world $w \in W$ such that in the model $M = \langle W, \prec, V \rangle$ we have for all $\delta \in \Delta$, $M, w \vDash \delta$.

Exercise 3 [40 points]

Frame condition (AC) at w:

$$\forall u, v, z \in W_w \quad (u \prec_w z \ \Rightarrow\ (u \prec_w v \ \lor\ v \prec_w z))$$

Axiom scheme (ASP):

$$(\neg(\phi \rightsquigarrow \neg\psi) \land (\phi \rightsquigarrow \chi)) \supset ((\phi \land \psi) \rightsquigarrow \chi)$$

1. Fix a base world w and write $u \simeq_w v$ iff neither $u \prec_w v$ nor $v \prec_w u$. Under AC, we can read $u \simeq_w v$ as "u and v are equally similar to w", and picture $\prec_w$ as a linear order of these equivalence classes.

   (a) Show that $\simeq_w$ is *reflexive* and *symmetric*, but not necessarily *transitive*.

   (b) Prove the equivalence:

   $$\text{AC holds at } w \quad \Longleftrightarrow \quad \simeq_w \text{ is transitive}$$

   (c) Let $[u]$ be the $\simeq_w$-class of u (i.e., $[u] \ :=\ \{v \in W_w \mid v \simeq_w u\}$). We define the following relation on classes:
   $$[u] \lhd_w [v] \quad :\Longleftrightarrow \quad u \prec_w v$$

   Show that under AC

      i. $\lhd_w$ is *well-defined*: If $u \simeq_w u'$ and $v \simeq_w v'$, then $(u \prec_w v \iff u' \prec_w v')$.

      ii. and that $\lhd_w$ is a *strict total order* on the set of classes (irreflexive, transitive, and total).

2. Prove that ASP is valid on a frame F iff AC holds at F.

Exercise 4 [25 points]

Consider the SDA principle

$$((\phi \lor \psi) \rightsquigarrow \chi) \supset ((\phi \rightsquigarrow \chi) \land (\psi \rightsquigarrow \chi))$$

1. Show that SDA is invalid in $\mathbf{P} + \text{ASP} + \text{MP}^{\rightsquigarrow}$.

2. Is SDA intuitively compelling from the point of view of natural language? If counterexamples are present, how to distinguish them from the cases where SDA is unproblematic?

3. Why not add SDA to $\mathbf{P} + \text{ASP} + \text{MP}^{\rightsquigarrow}$? Examine the resulting logic.