Assignment 4

Philosophical Logic 2025/2026

Instructions

- Discussion among students is allowed, but the assignments should be done and written individually.
- Late submissions will be accepted until one day after the deadline, with a 0.5 penalty per day.
- Please be explicit and precise, and structure your answers in a way that makes them easy to follow.
- Please submit your answers as PDF and use *PL-2025-A4-⟨your-last-name⟩* as the name of your file.
- For any questions or comments, please contact {m.degano, t.j.klochowicz}@uva.nl
- **Deadline: Tuesday 25 November 2025, 9 pm**

Note. Assignment 4 may be more challenging than the previous ones. Give it your best effort, and should you have any questions, do not hesitate to reach out. Good luck!

Exercise 1 [25 points]

We have seen that the notion of *exact* truthmaking does not coincide with *minimal* truthmaking. Can you find compelling examples of sentences/truths which have exact verifiers/truthmakers but do not have any minimal truthmakers? What about sentences/falsities which have exact falsifiers/falsemakers but do not have any minimal falsemaker? From a metaphysical perspective, would you accept the view there are truths/falsities which have exact truthmakers/falsemakers, but they do not have minimal truthmakers/falsemakers?

Use no more than 400 words

Exercise 2 [45 points]

We work in the propositional language with connectives $\neg$, $\wedge$, and $\vee$ only. A *literal* is either a propositional variable p or its negation $\neg p$.

A formula is in *disjunctive normal form (DNF)* if it is a disjunction of conjunctions of literals. A formula is in *conjunctive normal form (CNF)* if it is a conjunction of disjunctions of literals. You may assume that every formula is equivalent to some DNF and to some CNF.

For formulas φ, ψ we write $\varphi \vDash_T \psi$ for *tautological entailment.* This is defined via *explicitly* tautological entailment $\vDash_{ET}$.

$$\alpha \vDash_{ET} \beta \text{ iff:}$$

1. α is a conjunction of literals and β is a disjunction of literals, and
2. some literal occurring in α also occurs in β.

Let φ, ψ be formulas. Choose:

$$\varphi \equiv \varphi_1 \vee \cdots \vee \varphi_n \quad \text{(a DNF for } \varphi\text{, each } \varphi_i \text{ a conjunction of literals),}$$

$$\psi \equiv \psi_1 \wedge \cdots \wedge \psi_m \quad \text{(a CNF for } \psi\text{, each } \psi_j \text{ a disjunction of literals).}$$

Then:

$$\varphi \vDash_T \psi \quad \text{iff} \quad \text{for all } i = 1, \dots, n \text{ and } j = 1, \dots, m \text{ we have } \varphi_i \vDash_{ET} \psi_j.$$

(You may assume, without proof, that this definition does not depend on the particular choice of DNF for φ and CNF for ψ.)

We now consider the axiomatic system for tautological entailment. We want to show that this system is *sound*: everything provable in the system is a tautological entailment. In this exercise, we will check only one axiom and two rules. You are *not* allowed to assume soundness or completeness of $\vDash_T$ with respect to FDE or inexact truthmaking.

(a) Lemma on DNF-CNF conversion.

Let φ be a formula in DNF:

$$\varphi \equiv \varphi_1 \vee \cdots \vee \varphi_n$$

where each φ_i is a conjunction of literals, and let $\mathrm{Lit}(\varphi_i)$ denote the set of literals occurring in φ_i.

We define a set of formulas ("choice clauses") by

$$C(\varphi) \;:=\; \{\ell_1 \vee \cdots \vee \ell_n \mid \ell_i \in \mathrm{Lit}(\varphi_i) \text{ for each } i\}$$

Prove the following lemma:

> **Lemma 1.** The conjunction of all formulas in $C(\varphi)$ is a CNF equivalent to φ, i.e.
>
> $$\varphi \;\equiv\; \bigwedge C(\varphi)$$
>
> (Hint: prove this by induction on n.)

(b) Soundness of the disjunction axiom.

One axiom scheme of the system is the *disjunction axiom*

$$\varphi \vdash_T \varphi \vee \psi$$

We want to show that every instance of this axiom is a tautological entailment. Show that for arbitrary formulas φ, ψ,

$$\varphi \vDash_T \varphi \vee \psi$$

Hint. Put $\varphi \vee \psi$ in DNF. Apply Lemma 1 and use the definition of $\vDash_T$ via $\vDash_{ET}$.

(c) Soundness of rules: contraposition and transitivity.

We now check that two inference rules preserve tautological entailment.

(i) **Contraposition**

Rule:

$$\frac{\varphi \vdash_T \psi}{\neg\psi \vdash_T \neg\varphi}$$

Suppose $\varphi \vDash_T \psi$. Prove that $\neg\psi \vDash_T \neg\varphi$.

(ii) **Transitivity**

Rule:

$$\frac{\varphi \vdash_T \psi \qquad \psi \vdash_T \chi}{\varphi \vdash_T \chi}$$

Suppose $\varphi \vDash_T \psi$ and $\psi \vDash_T \chi$. Prove that $\varphi \vDash_T \chi$.

Exercise 3 [30 points]

We work with exact truthmaking models $\mathcal{M} = \langle S, \leq, I \rangle$, and we use the *inclusive* clause for disjunction.

Recall that logical consequence for exact truthmaking is defined as:

$$\Gamma \vDash_{\mathrm{TM}} \varphi \quad \text{iff} \quad \text{for all models } \mathcal{M} \text{ and states } s \in S,\ \big(\forall \psi \in \Gamma,\ s \vDash^{+} \psi\big) \Rightarrow s \vDash^{+} \varphi$$

Consider the following *distributivity* principle:

$$\text{(Dist)} \qquad (p \vee q) \wedge (p \vee r) \vDash_{\mathrm{TM}} p \vee (q \wedge r)$$

We have seen in class that (Dist) is *not* valid in general for exact truthmaking: there is a model $\mathcal{M}$ and a state s such that

$$s \vDash^{+} (p \vee q) \wedge (p \vee r) \quad \text{but} \quad s \nvDash^{+} p \vee (q \wedge r)$$

We now investigate whether (Dist) can be recovered under additional assumptions.

Upward closure. We say that a model $\mathcal{M} = \langle S, \leq, I \rangle$ is *upward closed* if, for every propositional letter p and all $s, t \in S$,

$$s \leq t \text{ and } s \vDash^{+} p \quad \Rightarrow \quad t \vDash^{+} p$$
$$s \leq t \text{ and } s \vDash^{-} p \quad \Rightarrow \quad t \vDash^{-} p$$

(Equivalently: if $s \leq t$ and $I^{+}(s, p) = 1$, then $I^{+}(t, p) = 1$; and if $s \leq t$ and $I^{-}(s, p) = 1$, then $I^{-}(t, p) = 1$)

(a) Does (Dist) hold on the class of upward-closed models?

More precisely: is it true that for every upward-closed model $\mathcal{M} = \langle S, \leq, I \rangle$ and every $s \in S$,

$$s \vDash^{+} (p \vee q) \wedge (p \vee r) \quad \Rightarrow \quad s \vDash^{+} p \vee (q \wedge r)$$

If yes, prove it. If not, construct an upward-closed countermodel and verify the relevant facts.

Non-vacuity. We say that a model $\mathcal{M} = \langle S, \leq, I \rangle$ is *non-vacuous* if, for every propositional letter p,

- there is some $s \in S$ with $I^{+}(s, p) = 1$ (i.e. some state that exactly truthmakes p), and
- there is some $s \in S$ with $I^{-}(s, p) = 1$ (i.e. some state that exactly falsemakes p).

(b) Does (Dist) hold on the class of non-vacuous models?

More precisely: is it true that for every non-vacuous model $\mathcal{M} = \langle S, \leq, I \rangle$ and every $s \in S$,

$$s \vDash^{+} (p \vee q) \wedge (p \vee r) \quad \Rightarrow \quad s \vDash^{+} p \vee (q \wedge r)$$

If yes, prove it. If not, construct a non-vacuous countermodel and verify the relevant facts.

Convex truthmaking. Fix a model $\mathcal{M} = \langle S, \leq, I \rangle$. We define a "convex" notion of positive truthmaking as follows:

$$s \vDash^{+,c} \varphi \quad \text{iff} \quad \exists s', s'' \in S\ (s' \vDash^{+} \varphi,\ s'' \vDash^{+} \varphi, \text{ and } s' \leq s \leq s'')$$

(c) Now consider the convex truthmaking relation $\vDash^{+,c}$ *on non-vacuous models.*

Does (Dist) hold for convex truthmaking on the class of non-vacuous models?

Is it true that for every non-vacuous model $\mathcal{M} = \langle S, \leq, I \rangle$ and every $s \in S$,

$$s \vDash^{+,c} (p \vee q) \wedge (p \vee r) \quad \Rightarrow \quad s \vDash^{+,c} p \vee (q \wedge r)$$

If yes, prove it. If not, construct a non-vacuous countermodel and verify the relevant facts.