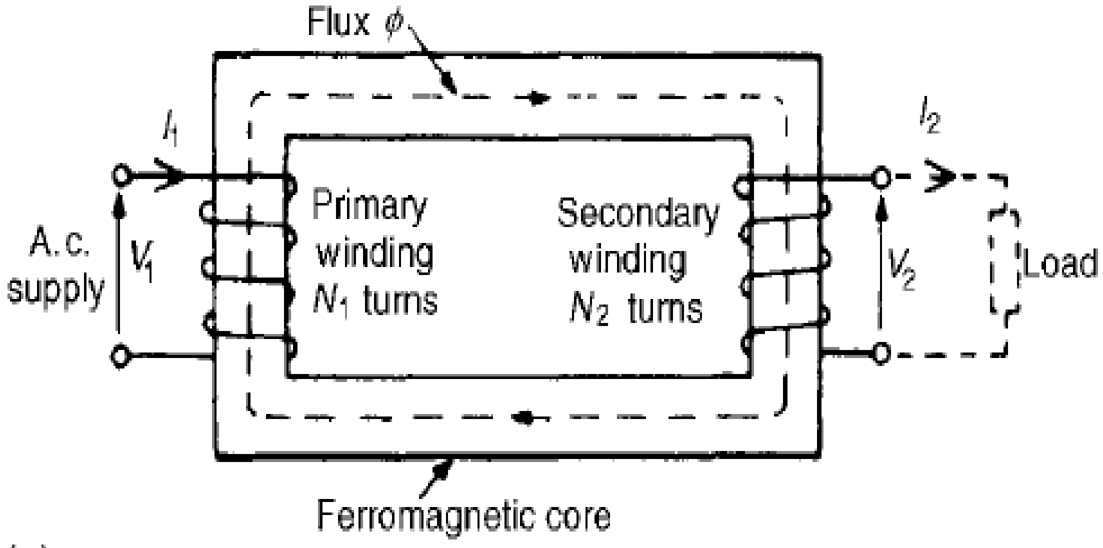
Lecture 4: Transformers

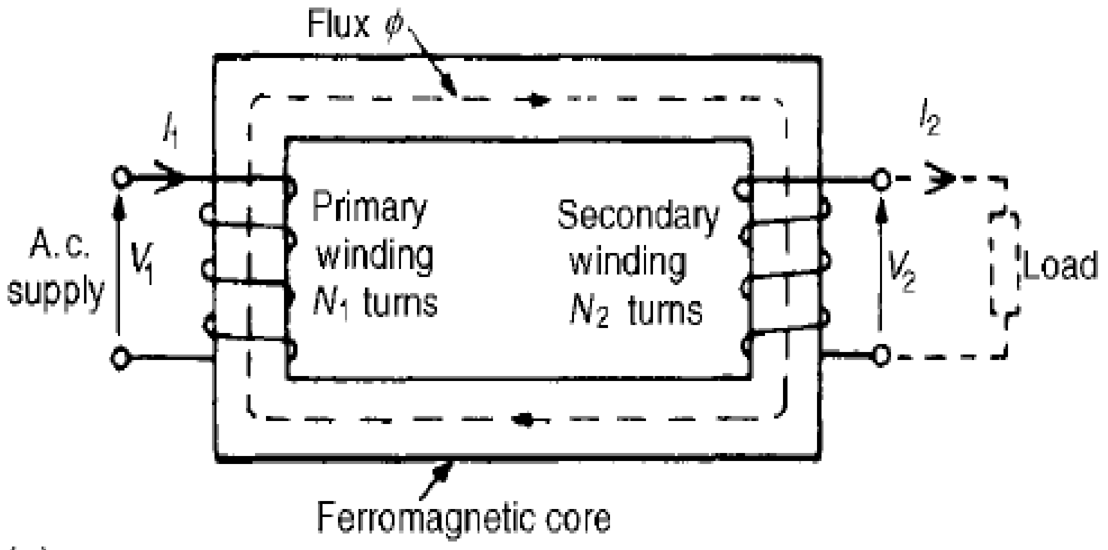
Introduction

A transformer is a device which uses the phenomenon of mutual induction to change the values of alternating voltages and currents. In fact, one of the main advantages of a.c. transmission and distribution is the ease with which an alternating voltage can be increased or decreased by transformers.

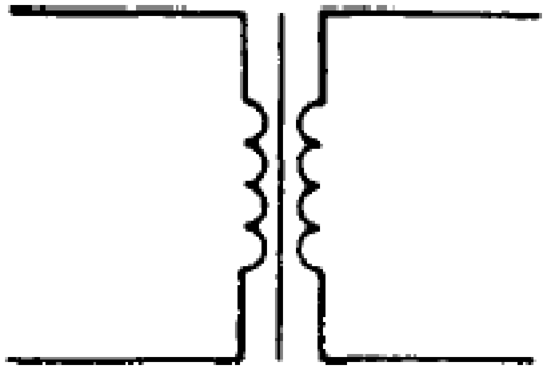


(a)

A transformer is represented in Figure (a) as consisting of two electrical circuits linked by a common ferromagnetic core. One coil is termed the **primary winding** which is connected to the supply of electricity, and the other the **secondary winding**, which may be connected to a load. A circuit diagram symbol for a transformer is shown in Figure (b).



(a)


(b)

Transformer principle of operation

When the secondary is an open-circuit and an alternating voltage V_1 is applied to the primary winding, a small current — called the no-load current I_0 — flows, which sets up a magnetic flux in the core. This alternating flux links with both primary and secondary coils and induces in them e.m.f.'s of E_1 and E_2 respectively by mutual induction. The induced e.m.f. E in a coil of N turns is given by:

$$E = -N\frac{d\Phi}{dt} \text{ volts,}$$

- where $d\Phi/dt$ is the rate of change of flux.
- In an ideal transformer, the rate of change of flux is the same for both primary and secondary and thus

$$E1/N1 = E2/N2,$$

i.e. **the induced e.m.f. per turn is constant.**

Transformer principle of operation (continued)

Assuming no losses, $E_1 = V_1$ and $E_2 = V_2$. Hence:

$$\frac{V_1}{N_1} = \frac{V_2}{N_2} \quad \text{or} \quad \frac{V_1}{V_2} = \frac{N_1}{N_2}$$

- *V*1/*V*2 is called the **voltage ratio.**
- *N*1/*N*2 the **turns ratio**, or the '**transformation ratio**' of the transformer.
- If N2 is greater then N1 then V2 is greater than V1 and the device is termed a step-up transformer.

Transformer principle of operation (continued)

When a load is connected across the secondary winding, a current I_2 flows. In an ideal transformer losses are neglected and a transformer is considered to be 100% efficient.

Hence:

input power = output power,

or $$V_1 I_1 = V_2 I_2$$

i.e. in an ideal transformer, the **primary and secondary volt-amperes are equal**.

THUS $$\boxed{\frac{V_1}{V_2} = \frac{I_2}{I_1}}$$ AND $$\boxed{\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}}$$

- The **rating** of a transformer is stated in terms of the volt-amperes that it can transform without overheating.

Problem 1. A transformer has 500 primary turns and 3000 secondary turns. If the primary voltage is 240V, determine the secondary voltage, assuming an ideal transformer.

Answer:

For an ideal transformer, voltage ratio = turns ratio, i.e.

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}, \text{ hence } \frac{240}{V_2} = \frac{500}{3000}$$

Thus secondary voltage $V_2 = \dfrac{(3000)(240)}{(500)}$

$$= \mathbf{1440\ V\ or\ 1.44\ kV}$$

Problem 2. A 5 kVA single-phase transformer has a turns ratio of 10:1 and is fed from a 2.5 kV supply. Neglecting losses, determine (a) the full load secondary current, (b) the minimum load resistance which can be connected across the secondary winding to give full load kVA, (c) the primary current at full load kVA.

(a) $\frac{N_1}{N_2} = \frac{10}{1}$ and $V_1 = 2.5\,\text{kV} = 2500\,\text{V}$

Since $\frac{N_1}{N_2} = \frac{V_1}{V_2}$, secondary voltage

$$V_2 = V_1\left(\frac{N_2}{N_1}\right) = 2500\left(\frac{1}{10}\right) = 250\,\text{V}$$

The transformer rating in volt-amperes $= V_2 I_2$ (at full load), i.e. $5000 = 250 I_2$

Hence full load secondary current $I_2 = \frac{5000}{250} = \mathbf{20\ A}$

(b) Minimum value of load resistance, $R_L = \frac{V_2}{I_2}$

$$= \frac{250}{20}$$

$$= \mathbf{12.5\ \Omega}$$

(c) $\frac{N_1}{N_2} = \frac{I_2}{I_1}$, from which primary current $I_1 = I_2\left(\frac{N_2}{N_1}\right)$

$$= 20\left(\frac{1}{10}\right)$$

$$= \mathbf{2\ A}$$

Transformer no-load phasor diagram

- The core flux is common to both primary and secondary windings in a transformer and is thus taken as the reference phasor in a phasor diagram.

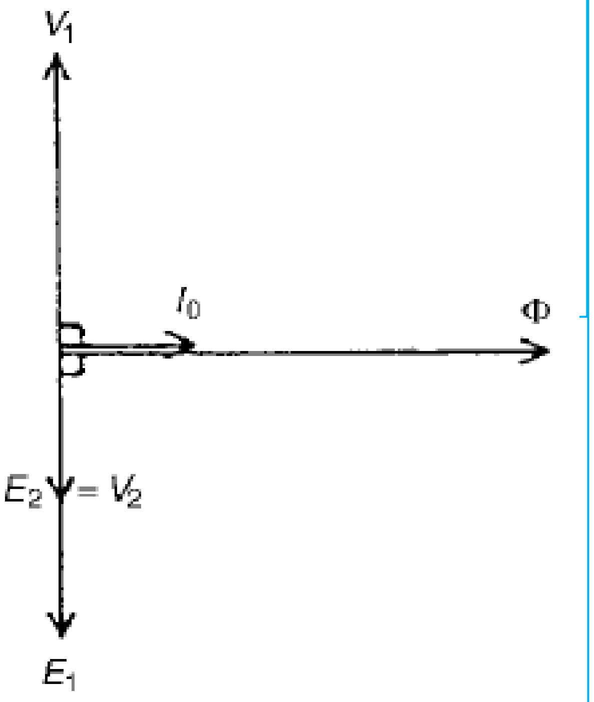
- On no-load the primary winding takes a small no-load current I_0 and since, with losses neglected, the primary winding is a pure inductor,

- Current lags the applied voltage V1 by 90◦.

- Current I_0 produces the flux and is drawn in phase with the flux.

- The primary induced e.m.f. E1 is in phase opposition to V1 (by Lenz's law) and is shown 180◦ out of phase with V1 and equal in magnitude.

- The secondary induced e.m.f. is shown for a 2:1 turns ratio transformer.



No-load phasor diagram for a practical transformer

When losses are considered then the no-load current I_0 is the phasor sum of two components:

- I_M, **the magnetizing component**, in phase with the flux,

- I_C, **the core loss component** (supplying the hysteresis and eddy current losses).

- From Figure

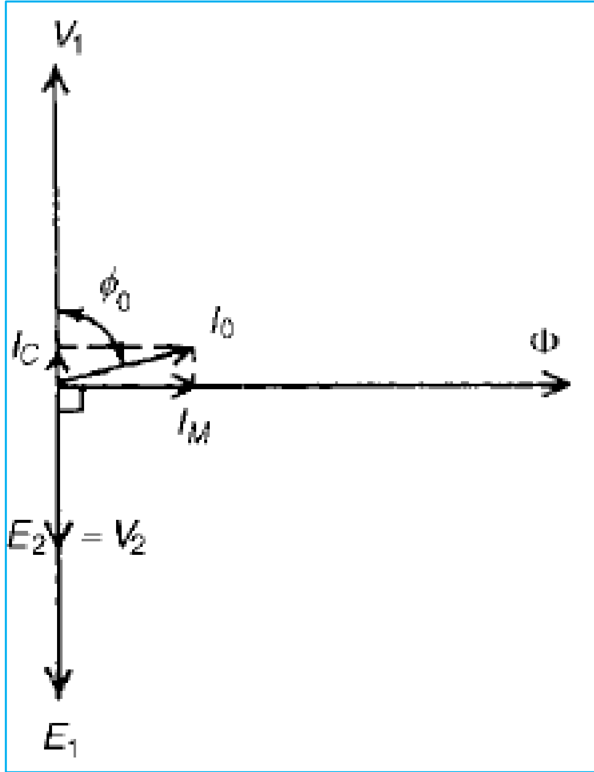
No-load current, $I_0 = \sqrt{(I_M^2 + I_C^2)}$, where

$$I_M = I_0 \sin \phi_0$$

and $$I_C = I_0 \cos \phi_0$$

$$\text{Power factor on no-load} = \cos \phi_0 = \frac{I_C}{I_0}$$

$$\text{The total core losses (i.e. iron losses)} = V_1 I_0 \cos \phi_0$$



Problem 3. A 2400V/400V single-phase transformer takes a no-load current of 0.5A and the core loss is 400W. Determine the values of the magnetizing and core loss components of the no-load current. Draw to scale the no-load phasor diagram for the transformer.

Answer

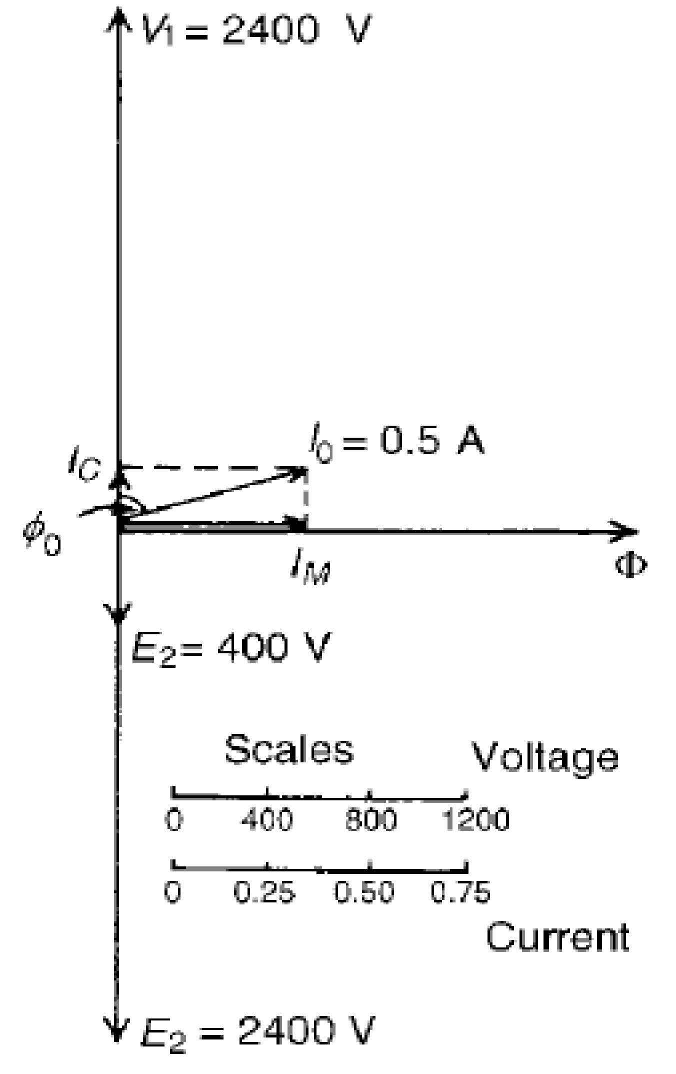
$V_1 = 2400\,\text{V},\ V_2 = 400\,\text{V},\ I_0 = 0.5\,\text{A}$

Core loss (i.e. iron loss) $= 400 = V_1 I_0 \cos\phi_0$

i.e. $400 = (2400)(0.5)\cos\phi_0$

$$\text{Hence}\ \cos\phi_0 = \frac{400}{(2400)(0.5)} = 0.3333$$

$$\phi_0 = \cos^{-1} 0.3333 = 70.53^\circ$$



e.m.f. equation of a transformer

- The magnetic flux (ϕ) set up in the core of a transformer when an alternating voltage is applied to its primary winding is also alternating and is sinusoidal.

- Let (ϕ_m) be the maximum value of the flux and (f) be the frequency of the supply.

- rms value of e.m.f. induced in primary winding:

$$\boxed{E_1 = 4.44 f \Phi_m N_1 \text{ volts}}$$

Where N1 the number of turns in primary winding

- and rms value of e.m.f. induced in secondary windings:

$$\boxed{E_2 = 4.44 f \Phi_m N_2 \text{ volts}}$$

Where N2 the number of turns in secondary winding

Problem 4. A 100 kVA, 4000V/200V, 50 Hz single phase transformer has 100 secondary turns. Determine: (a) the primary and secondary current, (b) the number of primary turns, and (c) the maximum value of the flux.

Answer

$V_1 = 4000\,\text{V},\ V_2 = 200\,\text{V},\ f = 50\,\text{Hz},\ N_2 = 100$ turns

(a) Transformer rating $= V_1 I_1 = V_2 I_2 = 100\,000$ VA

Hence primary current, $I_1 = \dfrac{100\,000}{V_1} = \dfrac{100\,000}{4000}$

$= \mathbf{25\ A}$

and secondary current, $I_2 = \dfrac{100\,000}{V_2} = \dfrac{100\,000}{200}$

$= \mathbf{500\ A}$

(b) From equation (20.3), $\dfrac{V_1}{V_2} = \dfrac{N_1}{N_2}$

from which, primary turns, $N_1 = \left(\dfrac{V_1}{V_2}\right)(N_2)$

$= \left(\dfrac{4000}{200}\right)(100)$

i.e. $N_1 =$ **2000 turns**

(c) From equation (20.5), $E_2 = 4.44 f \Phi_m N_2$

from which, maximum flux Φ_m

$$= \frac{E_2}{4.44 f N_2} = \frac{200}{4.44(50)(100)}$$

(assuming $E_2 = V_2$)

$= \mathbf{9.01 \times 10^{-3}\ Wb}$ or **9.01 mWb**

[Alternatively, equation (20.4) could have been used,

where $E_1 = 4.44 f \Phi_m N_1$

from which, $\Phi_m = \dfrac{E_1}{4.44 f N_1} = \dfrac{4000}{4.44(50)(2000)}$

(assuming $E_1 = V_1$)

$= 9.01$ mWb, as above]