Additional practice problems for the first midterm

1. Let A, B and C be independent events. Suppose that $P(A) = 50\%$, $P(B) = 60\%$ and probability that exactly two of the events A, B, C occur is 34%. Find $P(C)$.

2. Let X be a random variable and $C \in \mathbb{R}$ a constant. Prove that $Var(X + C) = Var(X)$.

3. Does there exist a random variable such that $EX = 4$ and $E(X^2) = 10$? If yes, give an example; if no, prove why.

4. Suppose that an n-sided die is rolled twice (where n is some natural number). Let X be the number obtained on the first roll, Y the number obtained on the second roll, and consider X and Y as random variables. Does there exist a constant C such that $X - Y$ and $X + Y - C$ are identically distributed? Justify your answer, and if the answer is 'yes', find such C.

5. Prove that if X and Y are independent random variables, then $E(XY) = EX \cdot EY$. **Note:** This is, of course, proved in the book (see § 2.7), but try to do this yourself, e.g. by imitating the proof of the equality $E(X + Y) = EX + EY$ given in class.

6-14. BT, Problems 2.6, 2.7, 2.13, 2.16, 2.23, 2.29, 2.38, 2.39, 2.40. Note: the solution to 2.29 can be phrased in terms of indicator functions (if you do this, you should see that it is a natural generalization of the solution to Problem 6 in HW#5.)