Math 3000. Spring 2018. Information about the final exam.

The final exam will be given on Thursday, May 10, 9am-12pm in our regular classroom (Kerchof 317). It will contain 6-8 regular problems and possibly a bonus problem. The exam will be cumulative, but with more emphasis on the material since the cutoff point for the second midterm: the second part of Lecture 20 (inverse images) and Lectures 21-26 (the corresponding sections of the book are 6.2, 6.3 and 6.4, and the corresponding homework assignments are 10 and 11). You should expect 1 or 2 long problems on the final (like the practice problem 1 for the second midterm). Also, problems on the final may combine different topics.
Here are some basic suggestions for preparing for the final:

(i) Go over your homework assignments; whenever you lost points on a problem, make sure to understand why. If you made a minor mistake, realizing what the mistake was is probably sufficient; if your solution was not close to being correct, redo the problem.

(ii) Work on the practice problems that were assigned throughout the semester. Many of those problems are solved in the solution manual

(iii) Go over the class notes and prepare questions about things you do not understand.

(iv) Go over the sections of the book that you did not fully understand on the first reading.

Additional practice problems

Note: This list only includes problems on the material covered since the second midterm. Therefore this list should NOT be considered a model for the final.

1. Let $f : A \to B$ be a function. Prove that the following two conditions are equivalent:

(a) f is injective

(b) $f^{-1}(f(C)) = C$ for every subset C of A.

Hint: Use contrapositive for the proof in one of the two directions.

2. Give a short proof of the furthermore part of Theorem 6.2.8(d) from the book using Problem 1 above, the first assertion of Theorem 6.2.8(d) and Theorem 6.2.9(d)

3. Let A and B be sets. Prove that the following are equivalent:

(i) there is an injection $f : A \to B$

(ii) there is a surjection $g : B \to A$

Hint: If you are not sure how to start, take a look at the proof of Theorem 23.4. This should help you at least with the proof of the implication "(ii)$\Rightarrow$(i)".

4. Construct an explicit bijection between $(-1, 1)$ and $[-1, 1]$. **Hint:** Imitate the construction of bijection between $[0, 1)$ and $[0, 1]$ from Lecture 24 or use the procedure from Problem 5 in HW#11.

5. Let $A = [-1, 1]$ and $B = (-1, 1)$. Define the functions $f : A \to B$ and $g : B \to A$ by $f(a) = \frac{a}{2}$ and $g(b) = b$ for all $a \in A$ and $b \in B$. It is straightforward to check that f and g are both injective, so one can apply the proof of the Schroeder-Bernstein theorem to those f and g to construct a bijection $\Phi : A \to B$. Find an explicit formula for such Φ (this is a good way to test if you understand the proof of the Schroeder-Bernstein theorem).

6. Let A be a set. Prove that $|A| \neq |\mathcal{P}(A)|$, that is, there is no bijection $\phi : A \to \mathcal{P}(A)$. **Hint:** Argue by contradiction: suppose that such ϕ exists. Consider the set $B = \{a \in A \mid a \notin \phi(a)\} \in \mathcal{P}(A)$, and show that B cannot lie in the image of ϕ. The contradiction you should get is very similar to the one in Russell's paradox.