

Homework #11. Hint for Problem 4.

If you are not sure how to start, first try to prove

Claim 1: $\frac{2}{3} + \mathbb{Z}$ (considered as an element of \mathbb{Q}/\mathbb{Z}) has order 3.

If you successfully proved Claim 1, you should have no problems with part (a). If not, do the following:

1. Make sure you understand why elements of \mathbb{Q}/\mathbb{Z} are additive cosets $q + \mathbb{Z}$, with $q \in \mathbb{Q}$.
2. Write down the formula for the group operation in \mathbb{Q}/\mathbb{Z} :

$$(x + \mathbb{Z}) + (y + \mathbb{Z}) = \dots$$

3. Which coset is the identity element of \mathbb{Q}/\mathbb{Z} ?
4. If $x, y \in \mathbb{Q}$, under of what conditions on x and y do you have the equality $x + \mathbb{Z} = y + \mathbb{Z}$?
5. Now write down the definition of the order for elements of \mathbb{Q}/\mathbb{Z} : “An element $x + \mathbb{Z}$ has order $n \iff \dots$ ”

For part (b): Everything you need to know is contained in the above list of items 1-5. Perhaps an additional thing that would help is

6. Find a transversal of \mathbb{Z} in \mathbb{Q} .