Homework #6. Due Saturday, March 19th, by 11:59pm in filedrop

All reading assignments and references to exercises, definitions etc. are from our main book 'Coding Theory: A First Course' by Ling and Xing

Reading and plan for the next week:

1. For this homework assignment read 5.1-5.4 and 5.7.

2. Plan for next week: Tue (Mar 15) – Plotkin bound (5.6) and Gilbert-Varshamov bound (5.2). Thu (Mar 17) – Polynomial rings and basic theory of finite fields (3.2 and parts of 3.3; see also online Lecture 15 from Spring 2020).

Problems:

1. Recall that if C is a code of length n and I is a proper subset of $\{1, \ldots, n\}$, the punctured code C_I is a code of length $n - |I|$ obtained from C by puncturing the i^{th} coordinate for every $i \in I$ (from every $c \in C$). Prove part (b) of Lemma 13.1 from class:

$$d(C_I) \geq d(C) - |I|.$$

2. In parts (a) and (b) of this problem assume that $d \geq 2$.

(a) Prove that $A_q(n, d) \leq A_q(n, d-1)$. In other words, fix an alphabet A with $|A| = q$ and prove the following: if there exists an (n, M, d)-code C over A, there also exists an $(n, M, d-1)$-code C' over A. You should give a precise argument; do not try to say this is obvious or something like that.

(b) Now prove that $A_q(n, d) \leq A_q(n-1, d-1)$. **Hint:** use punctured codes.

(c) Now use (b) and induction to give another proof of the Singleton bound: $A_q(n, d) \leq q^{n-d+1}$.

3. Problem 5.7.

4.

(a) Show that the all-one vector $(1, 1, \ldots, 1)$ of length 24 lies in the extended binary Golay code G_{24}.

(b) Assume without proof that G_{24} contains (exactly) 759 words of weight 8. Use this fact and (a) to prove that the distribution of weights in G_{24} is given by Table 5.5 on page 109.

5. This problem deals with the Golay code G_{23}.

(a) Use Problem 3(b) to prove that possible weights of elements of G_{23} are $0, 7, 8, 11, 12, 15, 16$ and 23. Make sure to prove that each of those numbers actually arises as the weight of some element of G_{23}.

(b) Let $w \in \mathbb{F}_2^{23}$ with $wt(w) = 4$. Let $c_w \in G_{23}$ be the result of applying NND decoding (with respect to G_{23}) to w. Use (a) and the fact that G_{23} is perfect (as shown in class) to prove that $d(w, c_w) = 3$ and $wt(c_w) = 7$.

6. Problem 5.19. **Note:** Simplex codes $S(r, q)$ are defined at the end of 5.3.2, page 88.

7. Use the result of Problem 5.19 to show that the simplex codes $S(r, q)$ attain the Griesmer bound.