

# Ph 20: Assignment #2

Maya Fuller

Due: April 16, 2018

## Problem 1

$$h_N = \frac{b-a}{N}$$

$$x_0 = a, x_1 = a + h_N, x_2 = a + 2h_N, \dots, x_N = b$$

$$\begin{aligned} \int_a^b f(x) dx &= \int_{x_0}^{x_1} f(x) dx + \int_{x_1}^{x_2} f(x) dx + \dots + \int_{x_{N-1}}^{x_N} f(x) dx \\ &\approx h_N \left( \frac{f(x_0)}{6} + \frac{4f(\frac{x_0+x_1}{2})}{6} + \frac{f(x_1)}{6} \right) + h_N \left( \frac{f(x_1)}{6} + \frac{4f(\frac{x_1+x_2}{2})}{6} + \frac{f(x_2)}{6} \right) + \dots + h_N \left( \frac{f(x_{N-1})}{6} + \frac{4f(\frac{x_{N-1}+x_N}{2})}{6} + \frac{f(x_N)}{6} \right) \\ &= h_N \left( \frac{f(x_0)}{6} + \frac{2f(\frac{x_0+x_1}{2})}{3} + \frac{f(x_1)}{3} + \frac{2f(\frac{x_1+x_2}{2})}{3} + \frac{f(x_2)}{3} + \dots + \frac{f(x_{N-1})}{3} + \frac{2f(\frac{x_{N-1}+x_N}{2})}{3} + \frac{f(x_N)}{6} \right) \end{aligned}$$

$$\begin{aligned} \text{Local error} : I &= f(a)H + f'(a)H^2 + f''(a)\frac{H^3}{2!} + f'''(a)\frac{H^4}{3!} + f^{(4)}(\eta)\frac{H^5}{4!} \\ I_{\text{simp}} &= \frac{1}{6}(f(a)H) + \frac{4}{6}(f(a)H + f'(a)\frac{H^2}{2}) + \frac{1}{6}(f(a)H + f'(a)H^2 + \frac{f''(a)}{2!}H^3 + \dots + \frac{f^{(4)}(\eta)}{4!}H^5) \\ &= f(a)H + \frac{f'(a)}{2}H^2 + \frac{f''(a)}{6}H^3 + \frac{f'''(a)}{24}H^4 + \frac{5}{576}f^{(4)}(\eta)H^5 \\ &\therefore O(H^5) \text{ is the local error} \end{aligned}$$

$$\begin{aligned} \text{Global error} : \text{global error} &= (I_{\text{simp}} - I)N \\ &= \frac{-19}{576}f^{(4)}(\eta)h_N^5N \\ &= \frac{-19}{576}f^{(4)}(\eta)h_N^5\frac{b-a}{h_N} \\ &= \frac{-19}{576}f^{(4)}(\eta)h_N^4(b-a) \\ &\therefore O(h_N^4) \text{ is the global error} \end{aligned}$$

$$\begin{aligned} I_{\text{simp}} - I &= \dots - \frac{5}{576}f^{(4)}(\eta)H^5 + \frac{f^{(4)}(\eta)}{24}H^5 \\ &= \dots - \frac{19}{576}f^{(4)}(\eta)H^5 \end{aligned}$$

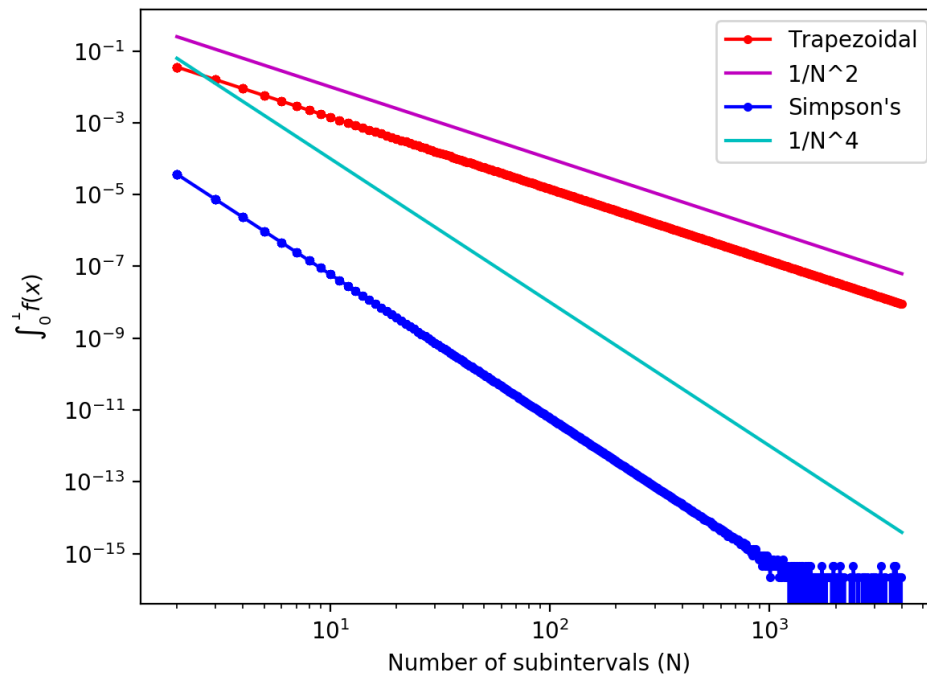
## Problem 2

See Python file

## Problem 3

See Python file

## Problem 4



See Python file

## Problem 6

See Python file

## Problem 7

See Python file